Target-Risk Equity Funds

by

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Presented to the Society of Actuaries in Ireland
9 February 2016

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ACKNOWLEDGEMENTS

The authors thank Brian Flanagan, Pramit Ghose, George McCutcheon, Michael Madden, and Joe Mottley for their comments on the paper; Garrett Hickey for proof reading of the paper; and Rioghna Gannon for checking the cross references. All remaining errors are those of the authors.

1 EXECUTIVE SUMMARY

A target-risk equity fund aims to keep the volatility of an equity fund or equity index in a very tight range around the chosen level of risk. This is done by forecasting the risk of the fund and varying the exposure to the underlying risky assets inversely to the forecast risk so as to keep the risk of the fund in that tight range around the target-level of risk (Section 7.2). The choice of target-risk level is driven by the investor's appetite for losses over a given time horizon for a chosen level of probability.

Target-Risk equity funds have significant application in approved retirement funds, defined benefit pension plans, defined contribution pension plans, capital protection products, multi-asset portfolios, and general portfolio risk management.

For an investor in an approved retirement fund ("ARF"), the probability of the ARF not running out of money depends not just on the investment performance of the ARF portfolio but on the path of its investment performance and in particular the size and timing of large peak-to-trough falls in the value of the ARF portfolio. Where the risk of the equity portion of an ARF investment varies in line with market variations in risk, the chances of a large peak-to-trough fall in value in the early years of the ARF increase and it may be difficult to recover that loss because the value of the ARF upon which any recovery in investment performance is based is constantly being eroded by regular withdrawals.

In a defined benefit pension plan, a target-risk equity fund allows the trustees to choose the risk level at which the equity portion of the portfolio operates to meet the prudential requirements of the plan and control the size of the Funding Standard Reserve (Section 11.5). Where the risk of the equity portion of the plan's investment varies in line with market variations in risk, there is a greater chance of breaching the prudential requirements of the plan than if the risk of the equity portion is controlled using a target-risk approach.

Equities have historically delivered strong returns in the long-term and are an essential component of the investment portfolios of many insurance companies, defined contribution pension plans, and defined benefit pension plans. Changes in accounting standards and prudential regulation have meant that there is limited scope for such investors to absorb the impact of large peak-to-trough falls in the value of the equity component of such portfolios. Accounting standards and prudential regulation effectively require such institutional investors with equity exposure to control the risk of that component of their portfolio. Target-Risk equity portfolio management is likely to be a much better means of including equities in such portfolios than simply investing in equities and allowing the risk of the portfolio to vary as the market dictates.

In our research, we find that a target-risk equity fund based on the EURO STOXX 50® index with net dividends reinvested provides the same return as the underlying equity index every three to five years for one third of the risk and with just over one third of the maximum peak-to-trough fall in value. (Section 8.4, Table 3).

It is very difficult for investors to recoup losses in their portfolios which arise from large peak-to-trough falls in the value of those portfolios. In the decade ending 31 December 2010, major equity indices, such as the EURO STOXX 50® index, suffered losses of more than 50% of their value not once but twice. Losses in excess of 50% of value require returns of over 100% to recover to their pre-loss value.

Large losses like those cited in the previous paragraph are caused by an absence of risk control within such portfolios. Put simply, the risk or realised[1] volatility of equity funds and equity indices varies dramatically over time (Section 7.1, Chart 6) and can be more than five times the risk as measured by the annualised standard deviation of past returns. When risk rises, the probability of large losses increases (Section 7.1.1, Table 2). The variation in the risk of equity funds and equity indices leads to larger peak-to-trough falls in value than what investors might expect from a review of past risk.

Other approaches to managing the risk of an equity fund or equity index such as low-volatility funds suffer from a number of significant drawbacks relative to the target-risk approach (Section 9).

The ability to forecast equity market volatility is critical to the operation of a target risk equity fund. Poor volatility forecasting manifests itself in a distribution of daily returns for the target-risk fund with high kurtosis[2], significant variation in volatility, and large peak-to-trough falls in value.

[1] The realised volatility is computed by annualising the standard deviation of daily prices taken at the same time over five days.

[2] Probability mass is concentrated around the mean and in the tails of the distribution.

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3 The Distribution of Equity Returns

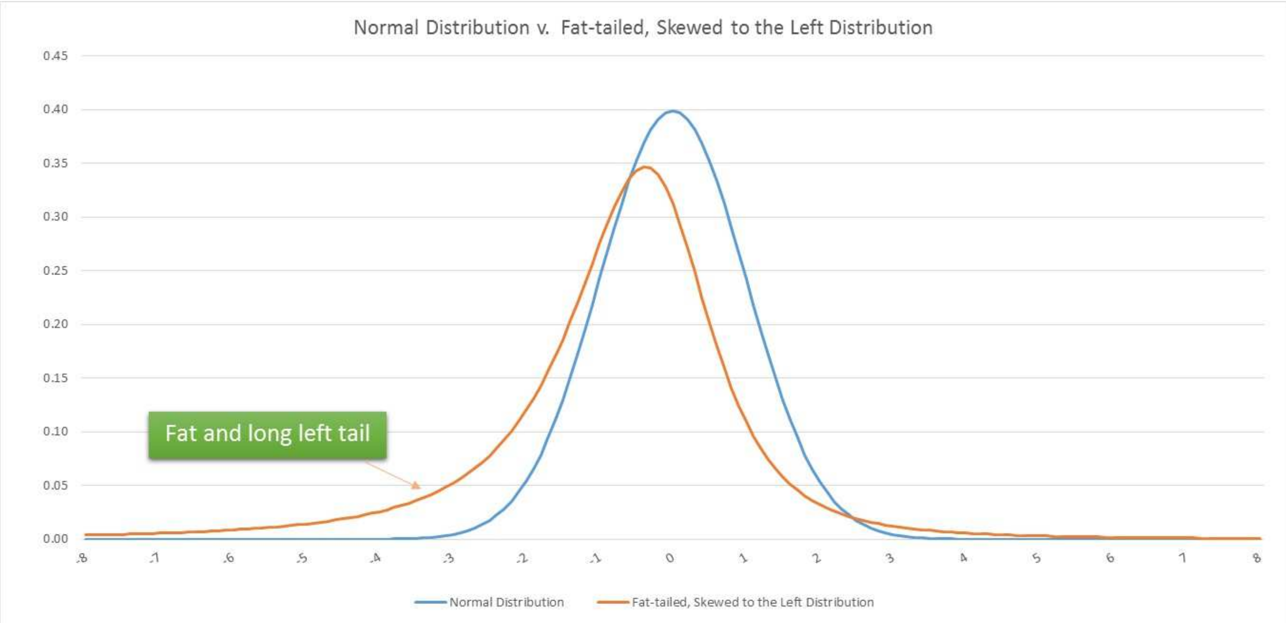
Empirical studies have shown[3] that the return distribution of equities is fat-tailed. The probability of extreme profits or losses is much larger than would be predicted by the normal distribution based on the average long-term volatility of a portfolio of equities.

Compared to the normal distribution, equity return distributions:

(i) Are more peaked around the centre of the distribution;
(ii) Show asymmetry between upside and downside potential with a fatter tail on the left hand side of the distribution, negative skew; and
(iii) Exhibit excess kurtosis.

Chart 1 illustrates the difference between the normal distribution of returns and a fat-tailed, skewed to the left distribution of returns. The two distributions have the same mode.

Chart 1



The empirical findings in relation to the return distribution of equities which show such large variations from the normal distribution are partly a result of the significant variation in equity market volatility over time.

Over the period 1 September 1998[4] to 31 December 2015 (the "Period"), the average annualised volatility of the EURO STOXX 50® index with net dividends reinvested was 24.4%. However, during the month of October 2008, the average annualised volatility of that index rose to nearly 80%. Looked at in the context of the average volatility, the 14.7% fall in value in October 2008 was a 2.09[5] standard deviation move whereas when the move is examined through the lens of the realised volatility that month the move was a mere 0.64[6] standard deviations. The probability of a 2.09 standard deviation move for a standard normal

[3] Poon & Granger; Hocquard, Ng, & Papageorgiou; and Ducoulombier.
[4] This is the earliest date from which reliable tick data is available for the EURO STOXX 50® index futures contract.
[5] 2.09 = 14.7/(24.4/12^0.5)
[6] 0.64 = 14.7/(80/12^0.5)

distribution of returns is of the order of 1.8%. The probability of a 0.64 standard deviation move for a standard normal distribution of returns is of the order of 26%. Seen in the context of the prevailing volatility at the time of the move, it is not a shock. However, seen in the context of the average volatility it is quite an unlikely move.

In the decade from 2000 to 2010, equity portfolios have exhibited two very large peak-to-trough falls in value. For example, the EURO STOXX 50® index with net dividends reinvested fell in value something of the order of 65% near the beginning of the period and 58% near the end of the period. Investors naturally ask, "Can we control the size of such peak-to-trough falls in value without giving up the upside potential of equities?" We shall briefly examine the drivers of the size of peak-to-trough falls in value experienced by a portfolio of equities before proceeding to examine the issue of controlling such peak-to-trough falls in value.

4 Key Drivers of the Size of Peak-to-Trough Falls in Value

The worst peak-to-trough fall in value experienced by a portfolio depends on a number of factors, the most critical of which might be as follows:

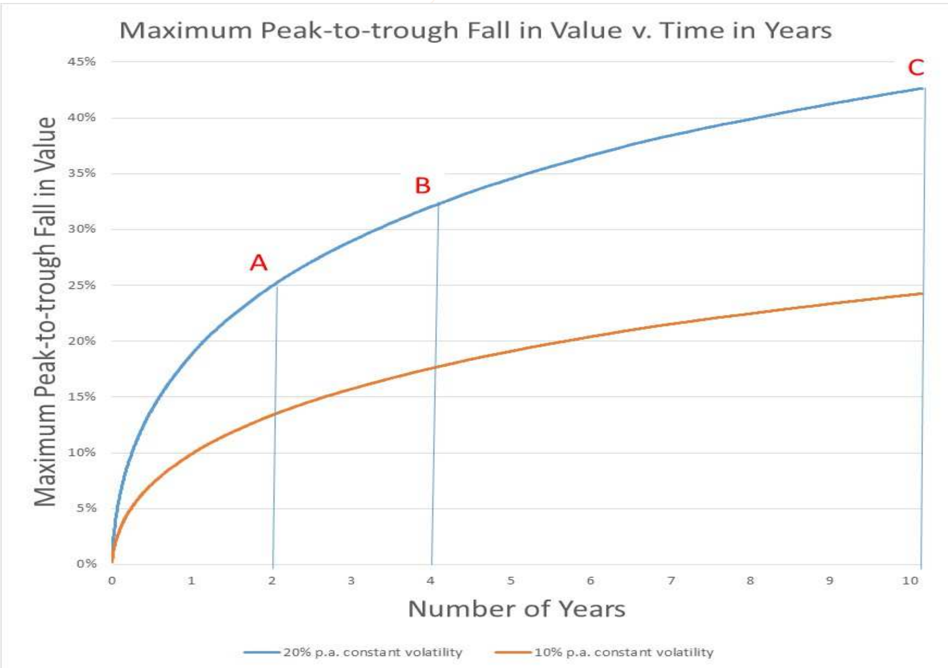
- Volatility level
- Time window
- The extent of variation in volatility
- Return

Let's look at each of these factors in turn.

4.1 Volatility Level

Chart 2[7] below illustrates the manner in which the level of volatility influences the size of maximum peak-to-trough falls in value over any given time period for two different levels of volatility, 10% per annum volatility and 20% per annum volatility.

Chart 2



[7] Chart 2 has been constructed from 10,000 simulations of the path of an investment with a normal distribution with mean return of 7% per annum and volatilities of 10% per annum and 20% per annum and plotting the average across the 10,000 simulations of the maximum drawdown over each period of time.

The higher the constant level of volatility, the greater the potential peak-to-trough fall in value over any time period. Chart 2 shows that peak-to-trough falls in value are not linearly proportional to volatility for any given time period. For example, looking at a ten-year period, doubling the constant volatility from 10% per annum to 20% per annum increases the average maximum peak-to-trough fall in value from 24% to 43% rather than from 24% to 48%. The former peak-to-trough fall in value requires a return of 31.6% to regain the previous high whereas the latter requires a return of 92.3%.

4.2 Time Window

The longer the time window over which one looks, the bigger the chances of observing a higher peak-to-trough fall in value compared with looking at a shorter period. Chart 2 shows how the average maximum peak-to-trough fall in value over 10,000 simulations using actual daily returns of a constant volatility portfolio varies with the length of the time window examined.

Based on 10,000 simulations, the average maximum peak-to-trough fall in value during a time period of 2 years is about 13.5% for the 10% constant volatility fund and just over 25% for the 20% constant volatility fund, vertical line A on Chart 2.

However, as the time window extends to four years, vertical line B on Chart 2, the average maximum peak-to-trough fall in value rises to 17.7% for a 10% constant volatility fund and 32.3% for a 20% constant volatility program.

If the time window is extended to 10 years, the average maximum peak-to-trough fall in value rises to 24.3% for the 10% constant volatility fund and 42.6% for the 20% constant volatility program, vertical line C on Chart 2.

The longer the time window, the greater the size of the maximum peak-to-trough fall in value likely to observed.

4.3 Extent of Variation in Volatility

If the volatility of the portfolio varies significantly so that the maximum volatility of the portfolio may become significantly different from the average volatility, then other things being equal, the probability of extreme peak-to-trough falls in value is related to the maximum volatility as well as the average volatility.

To investigate this point we need to run many simulations of the extent of variation in volatility similar to what we see in the equity markets. We measure the extent of variation in volatility using the kurtosis statistic and then create models of the market with similar kurtosis, so that we can simulate many thousands of iterations rather than just the historical path that we have seen.

We choose three typical methods for modelling volatility, GARCH, Regime Shifting GARCH and Exponential GARCH. For each method we choose parameters that mimic the kurtosis characteristics of the equity markets.

In this way we can then create a distribution of drawdown likelihood in the presence of varying volatility similar to real life markets.

By contrast we also simulate a normal distribution of returns where there is no variation in volatility over time to show how the probability of large drawdowns can be reduced dramatically by using the constant volatility approach.

Chart 3

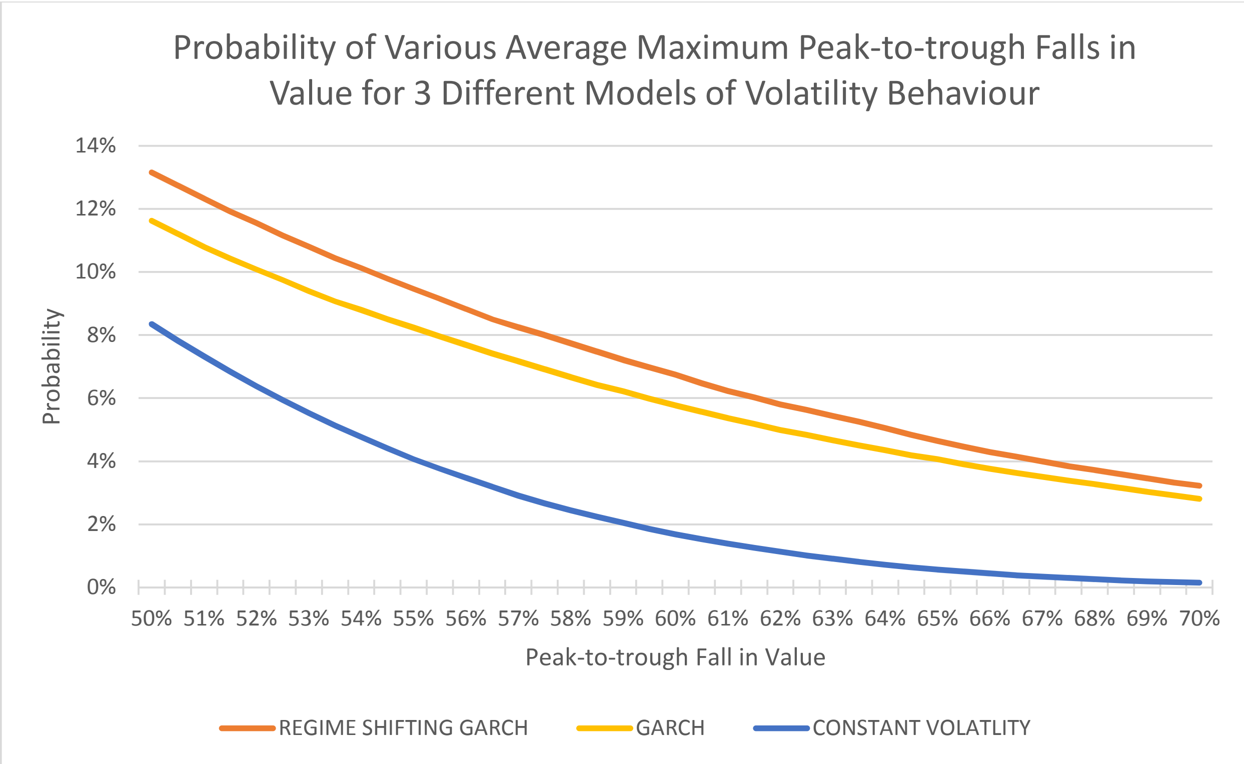


Chart 3 illustrates the point by reference to a number of different types of models of the behaviour of time-varying volatility over a three-year time period.

All of the models have been calibrated to the realised annual volatility of the EURO STOXX 50® Index with net dividends reinvested over the Period and, with the exception of the constant volatility model, to the realised fourth moment, kurtosis, of the EURO STOXX 50® Index. The most extreme model of time-varying volatility behaviour is the regime-shifting GARCH model where volatility can change significantly over a very short period of time.

Looking at the difference between the probabilities of a peak-to-trough fall in value of more than 50% over a three-year time period, the constant volatility model has a significantly lower probability than the regime-shifting GARCH model. For example, the probability of a peak-to-trough fall in value of more than 50% over a three-year time period is 8.4% for a constant volatility model but rises to 13.1% for a regime-shifting GARCH model of time-varying volatility behaviour.

Table 1 illustrates the effect that the different models of variations In volatility have on the probability of large drawdowns.

Table 1

Peak-to-trough Fall in Value	50%	60%	70%
Average of Three Models of Varying Volatility (1)	12.08%	5.83%	2.68%
Constant Volatility Model (2)	8.35%	1.68%	0.15%
Multiple of Probability (2)/(1)	45% higher	3.5 times higher	18 times higher

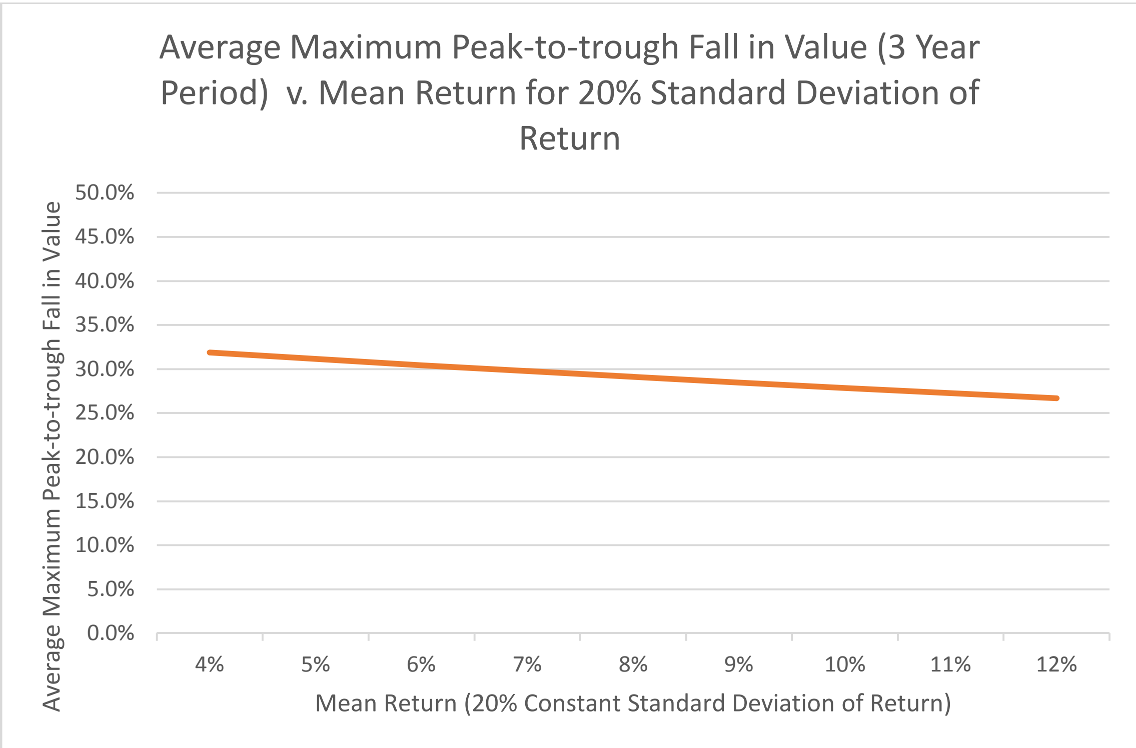
4.4 Return

Other things being equal, the higher the mean return level the lower the expected peak-to-trough fall in value over any given time period for any given level of probability. However, return has only a second order effect on peak-to-trough fall in values.

Chart 4 shows the average of maximum peak-to-trough falls in value over a three-year time period based on 10,000 simulations of different levels of mean return for a fund operating at a constant volatility of 20% per annum of the value of the fund.

Over a three-year time horizon, the average maximum peak-to-trough fall in value for a fund operating at 20% risk is reduced from 32% at 4% per annum mean return to 27% at 12% per annum mean return. Thus a fund with a significantly higher mean return will still suffer substantially similar peak-to-trough falls in value as a fund with a lower mean return.

Chart 4



4.5 Conclusions as to the Drivers of Peak-to-trough Falls in Value

Leaving aside the time window, the key drivers of the size of peak-to-trough falls in value is the average level of volatility and the degree of variation in volatility around that average level, particularly for large drawdowns. Funds with similar volatility characteristics but different mean returns do not exhibit marked differences in maximum peak-to-trough falls in value.

So to reduce the likelihood of large drawdowns one should keep the extent of variation in volatility low by targeting constant volatility.

5 Implications of the Distribution of Equity Returns

Even if large losses and large gains were equally probable and similar in size, the geometric compounding nature of returns shows that for a portfolio of unit value, a loss of r followed by a gain of r results in a portfolio of value $1 – r^2$, which is less than the original unit value of the portfolio[8]. The potential size of r varies with the volatility of the portfolio and the effect, in terms of the net loss in portfolio value, of a loss of r followed by a gain of r is magnified as the volatility of the portfolio rises.

The asymmetry between upside and downside potential and the fatter tail on the left-hand side of the distribution of equity returns have implications for investors. Large losses are not just more probable than large gains but they are bigger in magnitude than large gains[9]. A target-risk equity fund aims to keep the risk of the fund in a tight range around the target-risk level and thereby reduce the asymmetry between upside and downside returns and to eliminate the fat tail on the left-hand side of the distribution of returns.

Prudential regulation and accounting standards now mean that: (i) large losses can pose significant short-term problems for investors holding equity portfolios where risk is not actively controlled by forecasting volatility and varying exposure inversely to the forecast of volatility; put simply, the portfolio is at the mercy of the prevailing volatility in the market; and (ii) large losses can cause the portfolio to require an injection of capital if it is to continue to meet its long-term objectives.

Chart 5

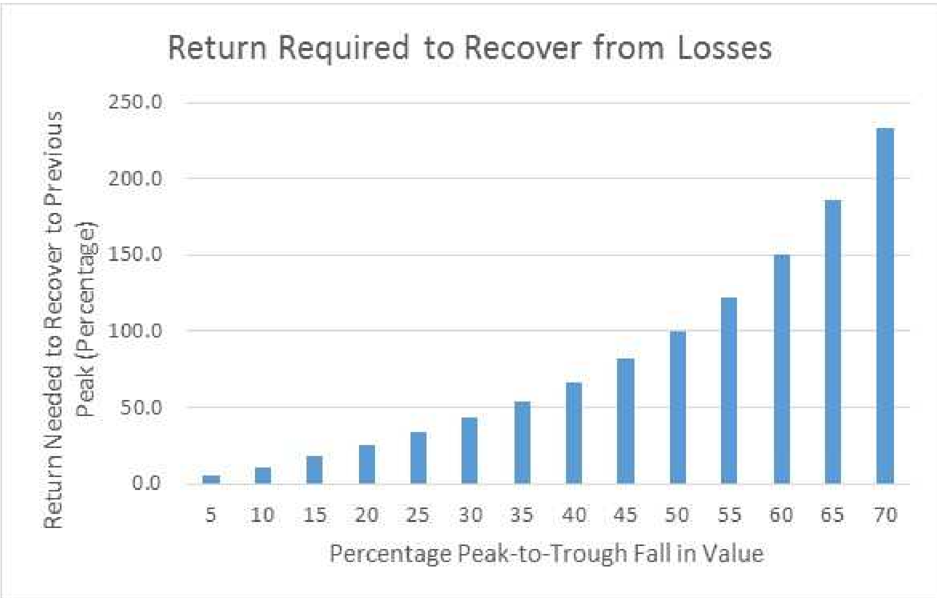


Chart 5 shows the exponentially increasing rate of return required to recover from linearly increasing rates of loss in steps of 5%. During the Dot-Com Crisis and Market Downturn in 2001 and 2002, the EURO STOXX 50® index with net dividends reinvested fell 64.5%. Chart 5 shows that a loss of that magnitude requires a return of 182% to get back to the previous high point of the index just before the fall. The index took twelve (12) years to reach its previous high point. The losses in the index occurred during a period which was characterised by high volatility in the index while the gains needed for the recovery of the index took place in a relatively lower volatility environment.

[8] The result arises from the following equation: $(1 – r)(1 + r) = 1 - r^2$

[9] Poon & Granger.

One way to avoid such large losses is to control the volatility of the portfolio around a target level to suit the investor's risk appetite so that losses are proportional to the target-risk level and not driven by the prevailing volatility in the market.

6 The Changed Nature of Equities in Institutional Portfolios

In the 1980s, there was a widely held belief that, unlike short-term investors in equities, defined benefit pension plans could endure large peak-to-trough falls in the value of their substantial holdings in equities because they were long-term investors unaffected by short-term market movements.

At the time, the view was perhaps reinforced by the consistency of method used to value assets and liabilities for solvency and funding purposes; a method that was not particularly sensitive to the market valuation of either assets or liabilities.

6.1 Accounting Standards for Irish, Defined Benefit, Pension Plans

Today, pension accounting standards like IAS19: (i) use a market discount rate, the yield on high-quality, corporate bonds, to value the liabilities of pension schemes; (ii) value the scheme assets at fair value which is essentially market value; and (iii) put the 'pension deficit' or 'net defined benefit liability' on the balance sheet of the sponsoring employer. A sponsoring employer with a large pension deficit may suffer increased borrowing costs through a lower credit rating compared with a sponsoring employer with no pension deficit or one with a small pension surplus on its balance sheet.

6.2 Prudential Regulation for Irish, Defined Benefit, Pension Plans

The march of prudential regulation in relation to defined benefit plans introduced a 'funding standard' in 1991 in order to: (i) set out the minimum assets that a defined benefit scheme must hold in order to satisfy the funding standard under the Pensions Act 1990 as amended (the "Pensions Act"); and (ii) specify the steps required if the assets of the scheme fell below this minimum.

Section 42 of the Pensions Act generally requires that trustees of funded, defined benefit, pension schemes submit an actuarial funding certificate ("AFC") at regular intervals to what is now the Pensions Authority. In the AFC, the scheme's actuary certifies whether the scheme does or does not satisfy the funding standard at the effective date of the AFC. If the AFC shows a shortfall, the trustees must prepare a funding proposal which is designed to eliminate the shortfall over an agreed period.

Although the trustees can choose the effective date of the AFC, the period between successive AFCs prepared and submitted to the Pensions Authority must be no longer than three years. AFCs must be submitted to the Pensions Authority within nine months of their effective date.

In the intervals between AFCs, the trustee annual report must state whether the actuary could certify that, at the scheme year end, the scheme would have satisfied the funding standard. If the actuary cannot make such a statement, the trustees must notify the Pensions Authority, and a revised AFC must be submitted to the Pensions Authority within 12 months of the last day of the reporting period, with an effective date that falls during that 12 month period.

In effect, prudential regulation means that long-term investors like defined benefit pension funds are subject to short-term constraints which may cause trustees to closely examine the risk characteristics of the investment portfolio and the size of likely peak-to-trough falls in the value of the portfolio.

From 1 January 2016, funded, defined benefits pension schemes are required to hold a Funding Standard Reserve commonly known as the 'risk reserve' which is equal to the sum of two calculations, (a) and (b) as defined below:

(a) 0.1 x (Minimum Fund Standard ("MFS") liabilities *less* the amount of the fund held in prescribed assets[10]); and
(b) The increase in MFS liabilities if long-term interest rates were reduced by 0.5% *less* any corresponding increase in the assets as a result of the interest rate reduction.

If a defined benefit pension plan satisfies the Funding Standard but not the Funding Standard Reserve, it must prepare a funding proposal for the Pensions Authority unless a previously-submitted, funding proposal is on track to ultimately meet both requirements.

In an article in the winter 2015 edition of the Irish Pensions Magazine, Shane Wall, Consulting Actuary, notes that according to figures provided by the Pensions Authority, the most recent certification on the 700 defined benefit pension plans not in wind-up showed total funding standard liabilities of EUR53.5 billion. The corresponding disclosed risk reserve figure is EUR5.4 billion or about 10% of the Funding Standard Liabilities.

6.3 Impact of Accounting Standards & Prudential Regulation

The volatility of a pension plan's asset portfolio, the volatility of its liability portfolio, and the extent to which it is not fully funded now have implications for the volatility of the sponsoring employer's financial statements and in some cases the sponsoring employer's borrowing costs.

If the trustees' annual report states that the actuary could not certify that, on a specified date, the scheme would have satisfied the funding standard, the trustees must notify the Pensions Authority. Ultimately, this may require the trustees to put a funding plan in place to eliminate the shortfall.

A brief examination of the risk reserve shows that part (a) of the risk reserve would be zero only if the pension plan were fully funded in accordance with the MFS and all the assets of the defined benefit pension plan were held in a portfolio consisting of euro-denominated bonds and cash deposits of similar duration to the liabilities. Any departure from the fully-matched and the fully-funded MFS positions will cause the risk reserve to increase. In effect, the risk reserve encourages pension plans to invest in assets perceived to be low risk and to be highly correlated with the liabilities on the one hand, and to avoid investment in more volatile assets like equities notwithstanding their potential for higher returns.

Accounting standards and the prudential regulation of defined benefit pension plans have increased the sensitivity of pension fund returns to equity market volatility for the plan sponsor, the shareholders of the plan sponsor, the creditors of the plan sponsor, and the beneficiaries of the plan.

It is no longer appropriate to consider only the question of how to increase the returns on a pension plan's portfolio of assets. Significant attention must now be paid to potential variations in these returns over relatively short time horizons.

In the current low interest rate environment, investors are being pushed towards more volatile assets with greater variation in volatility and greater maximum peak-to-trough falls in value in pursuit of returns that may, if realised, lower the cost of pension provision. But how can the risk of such volatile assets be controlled so as not to imperil the solvency of the scheme for MFS purposes, the borrowing costs of the

[10] Euro-denominated bonds including corporate bonds provided their yield is within 3% of the yield on a German government bond if the term is less than 10 years, or within 4% if the term is more than 10 years, and deposits with credit institutions.

employer, and the size of the risk reserve? Ideally, pension plan trustees would like to capture the upside potential of equities to achieve their funding goals but at the same time avoid the worst of the downside risk which can lead to very significant losses. The larger the extent of variation in volatility, the greater the likely maximum peak-to-trough fall in value; following a more conservative asset allocation to manage the size of the maximum peak-to-trough fall in value will have negative implications for return.

6.4 Impact of Prudential Regulation on EU Regulated Insurance Companies

Under Solvency II, the investment assets of insurance companies must meet the '*prudent person principle*' rather than meet defined restrictions or limits in relation to such assets. The '*prudent person principle*' requires that the assets held to cover the technical provisions are invested in a manner appropriate to the duration and nature of the liabilities. Supervisors in the various EU Member States are likely to challenge the way the *prudent person principle* is reflected in the investment policy of insurance companies as part of their supervisory work.

Under Solvency II, the sum of:

- *Own funds*

must exceed the greater of:

- Solvency Capital Requirement; and
- Minimum Capital Requirement

The portfolio of assets of an insurance company will have to be examined for its impact on the market risk component of the Solvency Capital Requirement ("SCR"). Under Solvency II, equities contribute significantly to the market risk component of the SCR. Target-Risk equity portfolio management is likely to be a better means of including equities in *own funds* than simply investing in equities and allowing the risk of the portfolio to vary as the market dictates.

Solvency II also requires insurance companies to:

- IDENTIFY,
- MEASURE,
- MONITOR,
- MANAGE,
- CONTROL, and
- REPORT

the risks involved in investment and to ensure the security, quality, liquidity, and profitability of the portfolio as a whole. We have identified significant variations in the risk of equities. At least from a regulatory point of view, measuring, monitoring, managing, and controlling the risk of equities is now more important than ever for insurance companies. Investing in equities via a target-risk equity fund provides a more robust framework for demonstrating the identification, measurement, monitoring, management, and control of equity risk than investing in equities where the risk of the portfolio is simply dictated by the market.

While investing in equities might not meet the nature and duration asset-liability, matching concept is embodied in the '*prudent person principle*' for certain types of liabilities, one might conclude that the free assets of insurance companies may be invested in equities in order to capture the potential higher returns of that asset class.

However, such an investment policy for the free assets will contribute to the market risk component of the Solvency Capital Requirement ("SCR"). Further, as equities exhibit substantial variations in their volatility, such an investment policy for the free assets may contribute to negative ratings for quoted insurance companies that exhibit significant swings in economic capital ratios due to equity market movements.

6.5 Equity Analysts' Views

Aside from regulatory issues, equity analysts examine the sensitivity of shareholders' equity, economic capital, earnings, and embedded values to moves in equity markets. The equity analysts look at a range in equity market moves from -30% of current market value to +30% of current market value. Generally speaking, investors in the shares of insurance companies don't like big swings in the capital base of their insurance companies.

Big swings in the capital base of an insurance company create uncertainty in the minds of investors and typically come at a cost in terms of the market requiring the insurer to hold an extra buffer of capital which adds to the cost of equity capital of the insurer.

7 Capturing Upside Potential of Equities; Avoiding the Worst of Downside Risk

There are at least two distinct ways to control the risk of a portfolio with a substantial holding in equities. One approach has been to diversify the portfolio using asset classes that historically have shown little or no correlation to equities and which have provided the same or a similar long-term return as equities. This method of controlling the risk of a portfolio will fail unless the risks of the constituents of the portfolio are fully controlled. Another approach is to control the risk of the equity component of the portfolio. It is also possible to use a combination of the two methods of risk control.

7.1 Diversification

The risk of an asset class, like equities, is not stable and varies considerably[11]. Properly implemented diversification calls for the inclusion of assets in the portfolio which can offer similar long-term returns, which do not have their periods of positive and negative performance at the same time as the other assets in the portfolio, and which have stable or controlled levels of risk.

Chart 6 below illustrates the huge variation in the volatility of equities as represented by the EURO STOXX 50® index (net dividends reinvested).

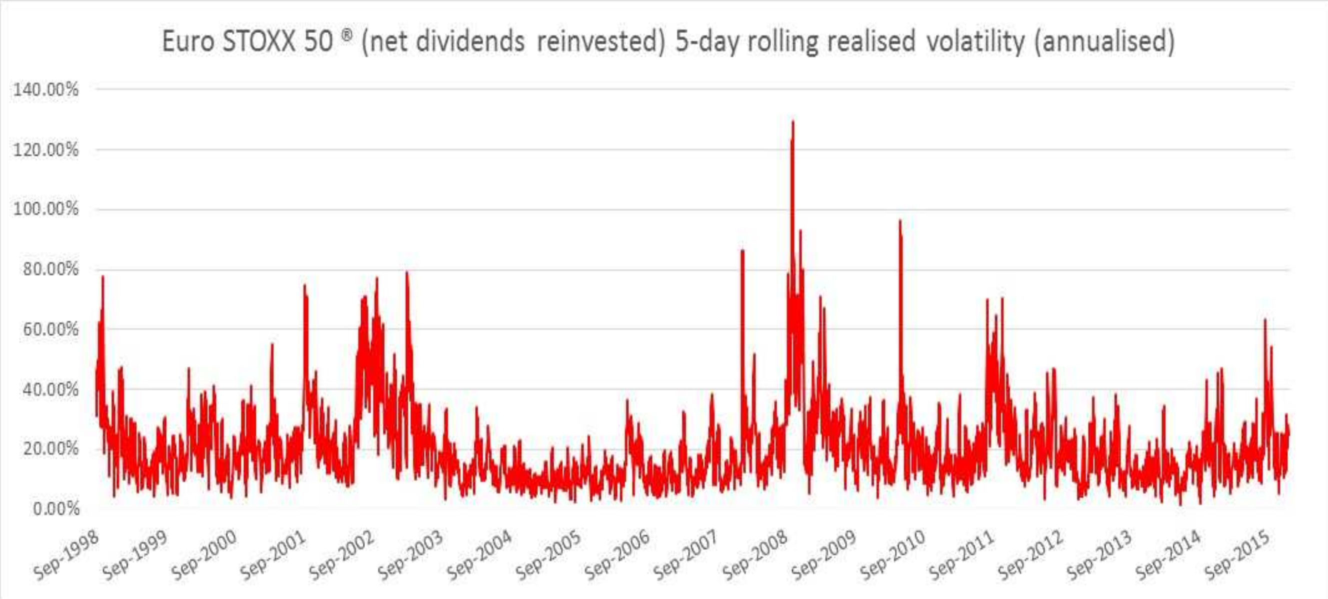
The annualised, five-day-rolling, realised volatility[12] ranges from a low of 1.5% to a high of 129.2% over the period covered. The average annualised risk across the 4,489 trading days of the data set[13] is 24.4%. In the case of the data set, the maximum risk is many times the average risk and therefore the standard deviation of past returns is not a good guide to the size of peak-to-trough falls in the value of the index.

[11] Hocquard, A., Ng, S., and Papageorgiou, N.

[12] The annualised, five-day-rolling, realised volatility is calculated by taking the standard deviation of the daily return on each day for five consecutive trading days and multiplying this result by the square root of 259.

[13] The dataset covers the period from 1 September 1998 to 31 December 2015.

Chart 6



7.1.1 Implications of Varying Risk for Diversification

Exposure to a range of asset classes may give the impression of diversification but if there is an asset class in the portfolio that has huge swings in its volatility and is significantly riskier than each of the other asset classes, equities for example, then the portfolio's risk behaviour may be dominated by the equity allocation despite the apparently low initial percentage allocation to equities.

Underlying efficient frontier analysis in the determination of strategic asset allocation, is the idea that the volatility of the various assets in the portfolio remains unchanged. Chart 6 shows how the volatility of equities varies over time; the assumption of volatility remaining constant is clearly flawed. Therefore efficient frontier analysis is unlikely to be a successful means of asset allocation to achieve a desired expected return for a given level of risk.

To illustrate the point, suppose that we have a portfolio consisting of 60% bonds and 40% equities. Let's assume: (i) that the risk, annualised standard deviation of return, of these two asset classes are 8% and 18% respectively; and (ii) the mean annual return on the portfolio is 4%.

On the face of it, the portfolio is dominated by bonds but perhaps surprisingly, even if we assume that there is no correlation between bonds and equities, 69.2% of the total variance of the portfolio comes from the 40% allocation to equities.

We examine the impact of the risk of the equity component of the portfolio increasing due to variation in the volatility of equities. We look at what happens if the risk of the equity component of the portfolio: (i) doubles to 36%; and (ii) rises to the highest level of realised, annualised volatility sustained for a 12-month period, namely 43%.

Table 2 has the details and Charts 7 and 8 illustrate the results graphically.

Table 2

	Portfolio of 60% Bonds and 40% Equities (Assumed Bond-Equity Correlation: Zero)						
	RISK		Percentage of Portfolio Variance		Portfolio Risk	Loss in the event of a 2 Standard Deviation Negative Move[14]	Probability of a Loss of 20% in a Calendar Year
	Bonds	Equities	Bonds	Equites			
Constant Volatility Risk Scenario	8%	18%	30.8%	69.2%	8.6%	-13.3%	0.3%
Equity Risk Doubles Scenario	8%	36%	10.0%	90.0%	15.1%	-26.4%	6.0%
Equity Risk Rises to 43% Scenario	8%	43%	7.3%	92.7%	17.6%	-31.7%	9.0%

Chart 7 shows the source of portfolio variance for different levels of equity risk. Despite the portfolio's 60% allocation to bonds, the equity allocation accounts for nearly 70% of the portfolio's total variance on the basis of the equity component of the portfolio maintaining a constant volatility of 18% per annum. If the volatility of the equity component doubles to 36% per annum while that of the bond component remains fixed at 8% per annum, then 90% of the portfolio's total variance is accounted for by the equity component of the portfolio. Should the volatility of the equity component rise to the highest level of realised, annualised volatility sustained for a 12-month period, namely 43%, while that of the bond component remains fixed at 8% per annum, then nearly 93% of the portfolio's total variance is accounted for by the equity component of the portfolio.

Chart 7

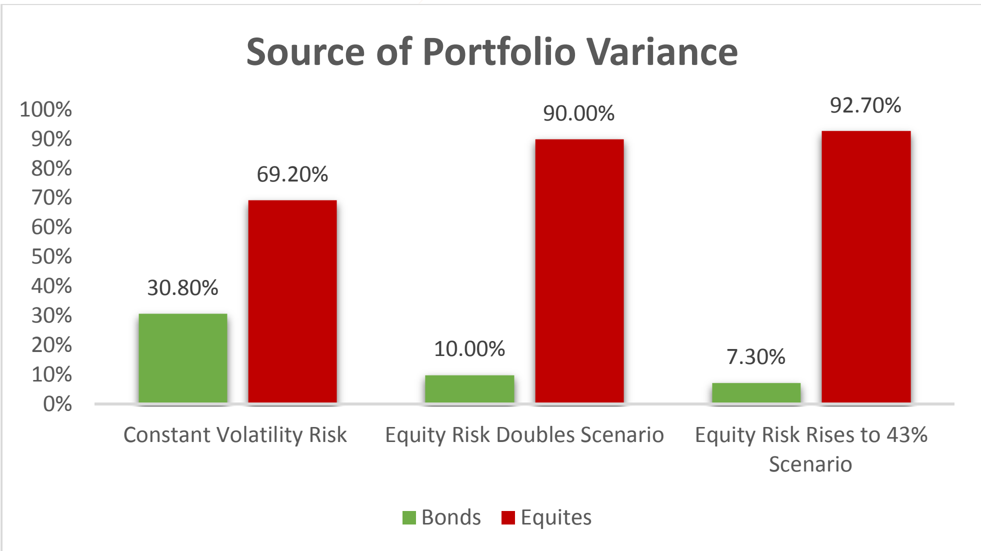
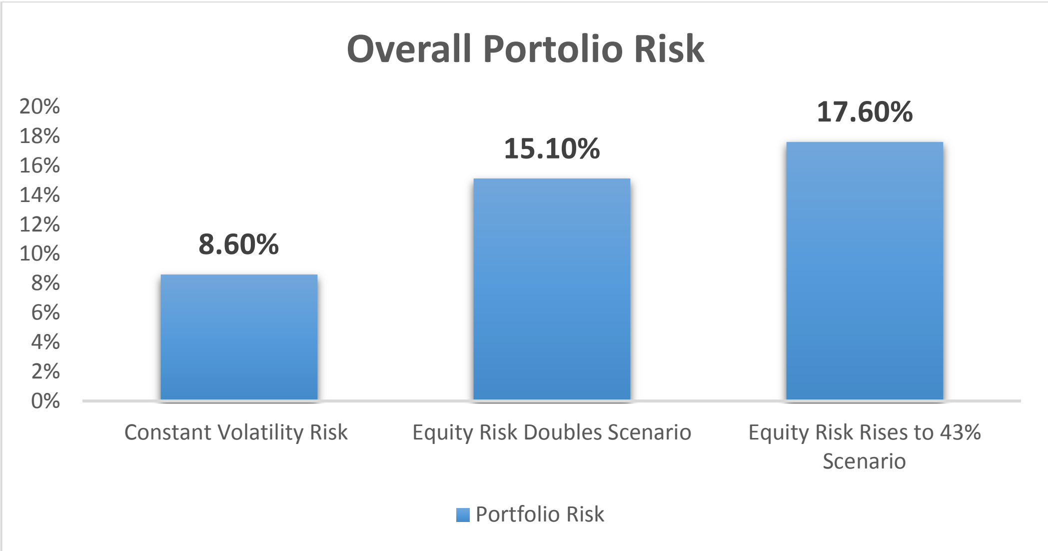


Chart 8 shows the impact that increases in the volatility of the equity component have on the risk of the overall portfolio.

14 Assuming a Mean Annual Return of 4% per annum.

Chart 8



As the risk of the equity component of the portfolio rises due to variation in the volatility of equities, the risk of a 20% loss in a calendar year rises exponentially from 0.3% to 9% as the risk of equities increases by a factor of 2.4 from 18% to 43%[15].

7.1.2 Failure in the Implementation of Diversification

Some might argue that given the portfolio's 60% exposure to bonds and only 40% exposure to equities the risk of the portfolio taking into account "diversification" ought to be low. However, as Table 2 shows, this approach to 'diversification' fails at least when viewed from a risk-control point of view. However, it is not diversification that has failed but rather the implementation of diversification that has failed, as no account was taken of the significant additional risk of equities versus bonds and the significant potential for the risk of the former to vary widely especially in a crisis. Poorly implemented diversification is not a tool for the management of tail risk. For effective diversification, the allocation to the different assets in the portfolio has to be on the basis of the relative risk of the different asset classes and needs to take into account the stability of the risk of the different assets in the portfolio.

7.1.3 Varying Bet Size in a Series of Wagers with Uncertain and Unpredictable Outcomes

If you were to vary your bet size in a series of wagers where you might win or lose an amount of uncertain size, while you could be very lucky and win big you could also be very unlucky and lose a significant portion of your wealth were a wager to go against you in circumstances where you took a big bet. One risk management strategy for such a game would be to make the same size bet each day.

As the risk of the equities in a portfolio consisting of 60% bonds and 40% equities varies from day to day, so too does the risk of the portfolio and by implication we are varying our exposure to tomorrow's return despite the fact that we have no idea of the sign of that return. In an environment where the sign of tomorrow's return cannot be predicted with any accuracy, investors ought to seek to maintain the same risk exposure each day.

[15] In the period from 1/9/1998 to 31/12/2015, the highest level of realised, annualised volatility sustained for a 12-month period was 43%.

Investment portfolios that do not manage the monetary value of the exposure to risky asset classes like equities are effectively not managing their risk as when equity volatility rises, the risk of the portfolio rises with consequent implications for loss.

7.2 Controlling the Risk Directly

The other means of controlling the risk of an equity portfolio is to choose a target-risk level for the portfolio, forecast the risk of the portfolio, and vary the exposure to the underlying basket of equities inversely to the forecast risk. For example, if we wish to target a risk level, annualised standard deviation of return, of 8% per annum of the value of the fund and we forecast volatility to be 32% per annum, then the exposure of the portfolio to the underlying equities would be 25% (8/32) with the balance of the portfolio invested in a combination of cash and short-dated government bonds.

Chart 9 illustrates an idealised variation in exposure to the underlying equity portfolio with forecast volatility for a target-risk level of 8% per annum in which no leverage is permitted. In Chart 9, exposure to the underlying equity portfolio is limited to 100% of the net asset value of the portfolio. This limitation in leverage is used throughout the paper as institutional investors like pension schemes and insurance companies rarely seek geared exposure to equities.

Chart 9

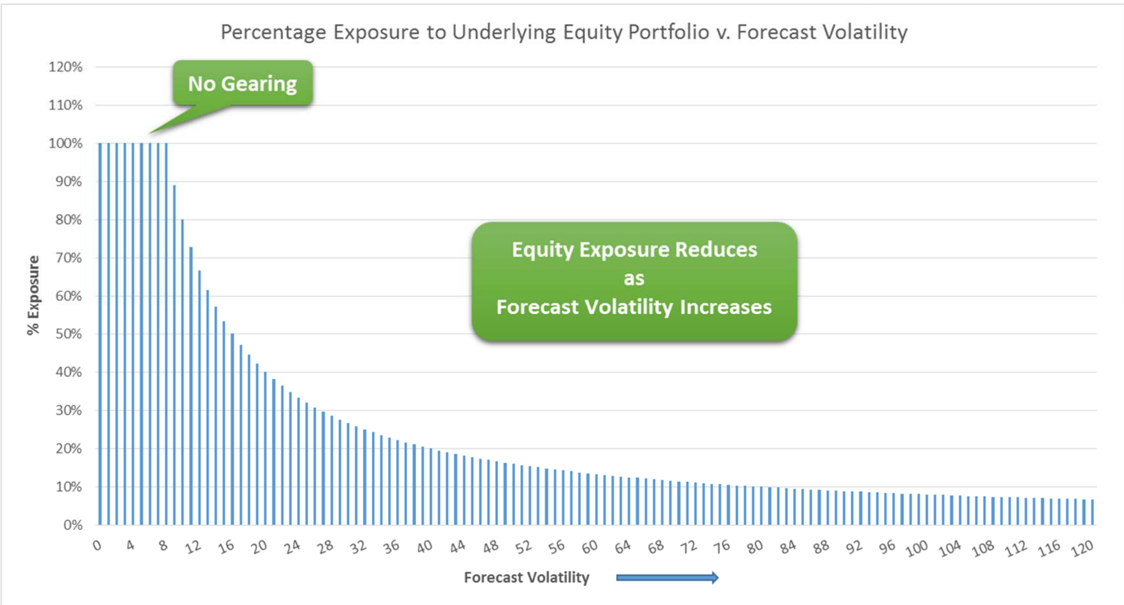


Chart 9 shows the idealised exposure. In practice, there are trading costs involved in varying exposure. The trade-off between the size of those costs for the frequency of trading and the impact of not reacting quickly enough to variations in forecast volatility will determine the size of the range around the target-risk level in which forecast volatility is allowed to vary before taking any action to change exposure.

8 Design of a Target-Risk Equity Fund

Portfolio design might be approached by asking: How much can the investor afford to lose in the context of the investor's long-term objectives and short-term reporting requirements? What are the implications of that loss for the investor's objectives over the given period of time? Only by controlling the risk of an investor's portfolio can the size of the loss suffered by the investor for any given time period and level of probability be controlled.

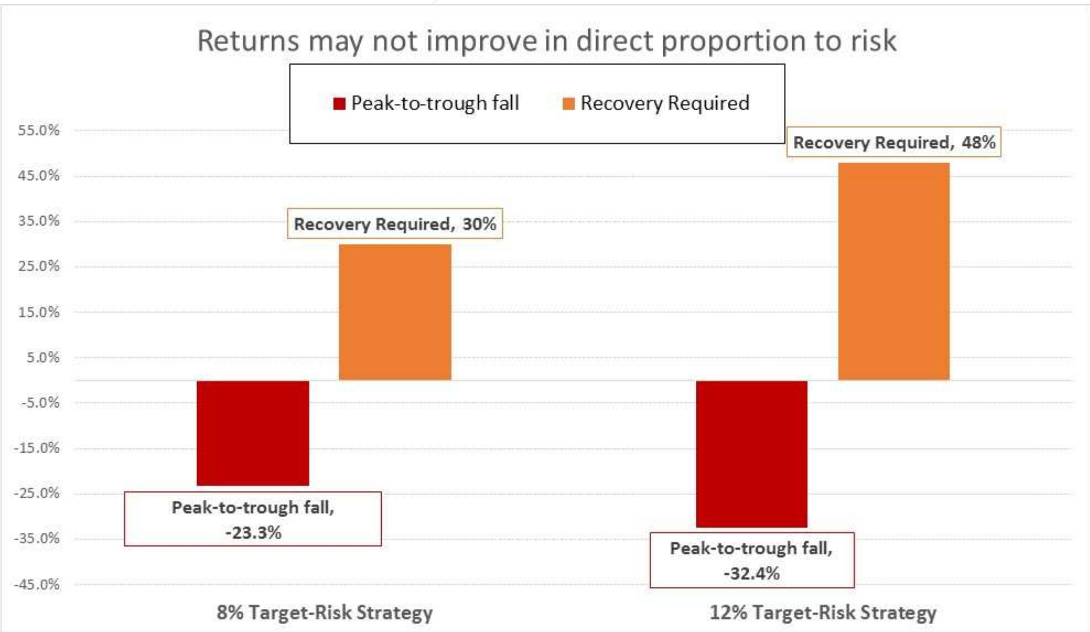
8.1 Choice of Risk Level

Perhaps the first step in the design of a target-risk equity fund is to choose a risk level to target. Given the relationship between risk and maximum peak-to-trough fall in value, this decision may be informed by the size of the maximum loss over a given period of time that the investor is willing to bear and the expected return on the target-risk equity portfolio relative to that on the underlying index. Thus the decision is driven by the investor's risk aversion level perhaps measured by the maximum peak-to-trough fall in value that the investor would be willing to accept with a certain probability over a given time horizon.

In our research, we simulated the returns of a target-risk equity fund based on the EURO STOXX 50® index with net dividends reinvested from a target risk level of 8% per annum of the value of the fund over the period 1 September 1998 to 31 December 2015 (the "Period") by forecasting volatility five times each trading day and adjusting the exposure of the fund to the underlying index to achieve the 8% target-risk level.

If we increase the risk of a target-risk equity fund based on the EURO STOXX 50® index with net dividends reinvested from a target risk level of 8% to say 12%, the returns will not improve by 50% but the risk will rise by 50%. As shown in Chart 10, at 8% target-risk, our simulated returns show a maximum peak-to-trough fall in value of the order of 23.3%. When a fund falls by 23% it has to grow by just under 30% to get back to its pre peak level. If we raise the risk level to 12%, the estimated peak-to-trough fall in value will be about 32.4%[16]. However, when a fund falls 32.4%, it has to grow by 48% to get back to the peak level.

Chart 10



[16] 1- $(1-0.23)^{(12/8)}$ = 0.324328. This is very close to the value obtained from our simulations of the Eurozone Equity Fund at 12% risk, namely a peak-to-trough fall in value of 33%.

Higher risk levels give rise to higher peak-to-trough falls in value which require ever higher returns to recover to peak level and this affects overall returns. Geometric compounding acts against the investor as the risk level rises. The simulations show that raising the risk from 8% to 12% raises the return by only 43% for precisely this reason.

In this paper, we choose the 8% risk level partly to maximise the return to peak-to-trough fall in value ratio, partly to limit the size of peak-to-trough falls in value to around 20%, and partly to avoid the need to leverage the fund.

8.2 Leverage

Leverage increases the risk of losing all of the money invested. A leverage factor of h will cause the portfolio to fall to zero for a 1/h drop in the value of the underlying. As an example, if a fund based on the EURO STOXX 50® index with net dividends reinvested were leveraged two (2) times and failed to cut its leverage, as a percentage of the initial leverage, with increasing losses, it would have lost all of the money invested during the Dot-Com Crash and Market Downturn in 2001 and 2002 and again during the Global Financial Crisis as the peak-to-trough falls in value on both of these occasions exceeded 50% (½).

In the discussion of the choice of target-risk level above, we looked at raising the target-risk to 12%. At this level of risk, the fund would have at times become a geared equity fund in order to reach the target risk level as there have been periods where the risk of the underlying index fall below 12% so gearing would be necessary to achieve the target risk level.

Generally speaking, insurance companies and trustees of pension funds are somewhat reluctant to allow geared exposure to equities notwithstanding the fact that in the case of a target-risk equity fund the risk would be controlled in a tight range around 12%.

8.3 Targeting a Risk Level and the Realised Risk in the Data Set

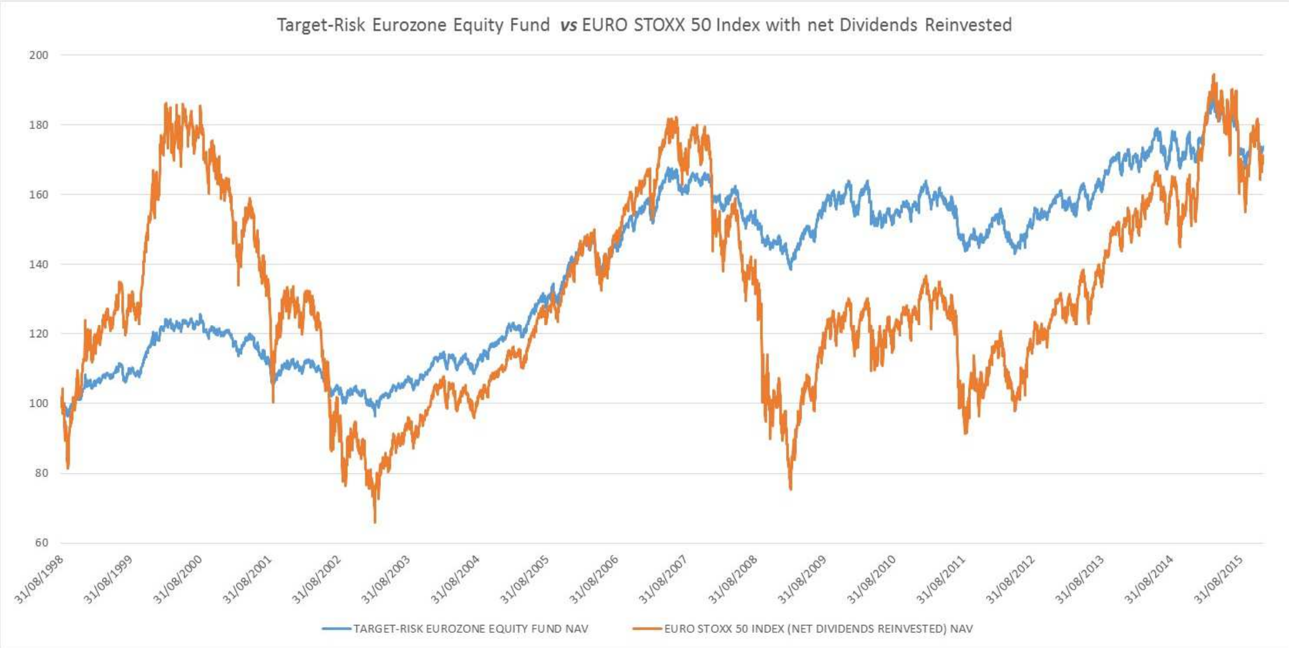
It is interesting to note that the lowest estimate of forecast volatility for the EURO STOXX 50® index never fell below the target-risk level of 8% per annum during the Period. Thus there was never a need to consider leveraging the target-risk equity fund to reach its target volatility.

Where the target-risk level is above the lowest estimate of forecast volatility in a data set and it is likely that such a feature will persist in the future, the issue of leveraging the target-risk fund will arise from both a governance point of view and a risk-return point of view. Governance may demand no leverage while permitting leverage may improve the return of the target risk fund.

8.4 Simulation of Results

Chart 11 shows the simulated results of operating a target-risk approach on the EURO STOXX 50® index with net dividends reinvested. The target-risk level is 8% and no leverage is permitted.

Chart 11



In our research, we found that every three to five years, the target-risk strategy produced the same return as the underlying equity index and did so with considerably less volatility. This can be seen by the high number of points at which the two portfolios cross in Chart 11.

Table 3 shows some key performance statistics for the target-risk strategy and the EURO STOXX 50 Index with net dividends reinvested over the period from 1 September 1998 to 31 December 2015 (the "Period").

The target-risk equity fund provides a very similar return to the underlying equity index, has a significantly lower maximum peak-to-trough fall in value than the underlying index, and maintains the volatility of the target-risk equity fund in a tight range around the 8% target-risk level.

Table 3

Fund / Parameter for the Period	8% Target-Risk Fund	EURO STOXX 50® Index with Net Dividends Reinvested
Annualised daily volatility (%)	**8.1**	24.4
Maximum peak-to-trough fall in value (%)	**23.3**	64.6
Annualised Return (%)	**3.2**	3.1
Simple Sharpe Ratio	**0.40**	0.13

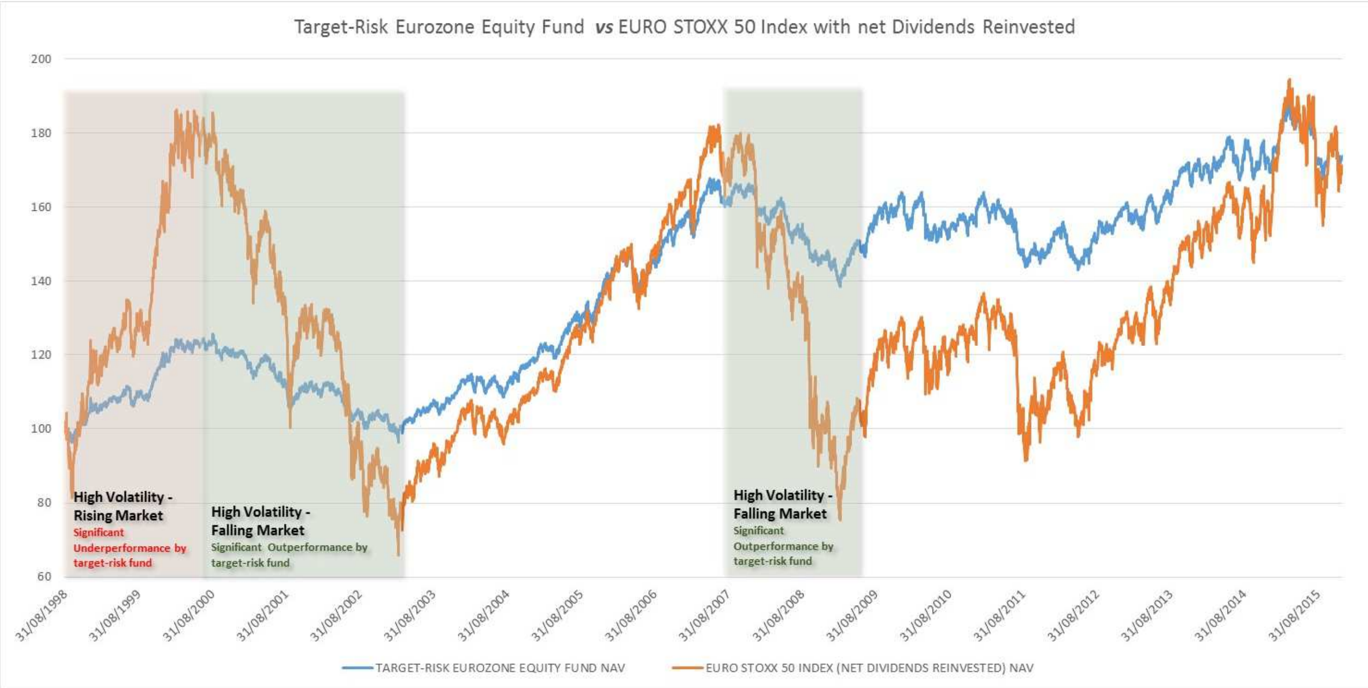
8.4.1 Managing Investors' Expectations

High-Volatility, Rising Market

Looking at the simulated performance of the target-risk equity fund we can see that it will underperform the underlying equity index in a high-volatility rising market. For example, during the period from 1

September 1998 to 6 March 2000, the annualised, daily, realised volatility was 25% or more than three (3) times the target-risk level. Not surprisingly, the target-risk equity fund delivered a return of 24.4% while the underlying index delivered a return of 86.3%.

Chart 12



High-Volatility, Falling Market

By contrast, the target-risk equity fund will outperform the underlying equity index in a high-volatility falling market. Two such periods are marked on Chart 12: (i) the Dot-Com Crash and Market Downturn in 2001-2002; and (ii) the Global Financial Crisis which began in 2007. For example, during the period from 16 July 2007 to 9 March 2009, the average realised, annualised daily volatility of the underlying equity index was 34.6% or more than 4.3 times the target-risk level and not surprisingly the target-risk fund suffered a loss of just over 17% while the underlying index suffered a loss of just under 59%. A loss of 59% requires a return of over 143% to recover to the previous peak whereas a loss of 17% requires a return of just 21% to recover to the previous peak.

One might argue that given the difference in average volatility between the target-risk fund (8%) and the underlying index (24.4%), it ought to be easy for the index to make up the 143% compared with the 21% required by the target-risk fund. However, the amount to be made up to recover to the previous peak is 2.6[17] times the risk level in the case of the target-risk equity fund compared with 5.9[18] times the risk level in the case of the underlying index.

Low-Volatility, Rising Market

In a low-volatility, rising market, where the volatility of the underlying index is close to that of the target-risk level, the target-risk fund and the underlying index ought to perform roughly in line. For example, during the period from 21 June 2004 to 16 July 2007, the annualised, daily, realised volatility was 12.9%, just over 1.5 times the target-risk level and the target-risk fund produced a return of 47% while the

[17] 2.6 = 21/8

[18] 5.9 = 143/24.4

underlying index produced a return of 75%. While the higher return of the underlying index is recognised, one must note the more controlled risk of the target-risk equity fund throughout all market conditions.

Low-Volatility, Falling Market
We were unable to identify any persistent, low-volatility, falling markets in the data set.

8.4.2 Frequency of Trading

To prepare the simulated data for the paper, we forecasted the volatility of the underlying equity index five times on each trading day. Each forecast was for a period of one day from time say, T, on one trading date to the same time, T, on the next trading day. Following each forecast of volatility, the exposure of the target-risk fund to the equity index was adjusted within a short time frame following the production of the forecast of volatility.

8.4.3 Transaction Costs

In order to minimise transaction costs, all adjustments of exposure to the index were made using index futures contracts. Futures contracts are often more liquid than the underlying index. Transaction costs will vary with the extent of variation in volatility. In our research we assumed transaction costs of EUR2.0 per contract per round turn and a spread of two pips on each trade.

8.4.4 Band around the Target Risk Level

It would be prohibitively expensive in terms of transaction costs to maintain a target-risk equity fund at exactly the target-risk level. In practice, there will be a band around the target risk level in which no change in exposure will take place and hence no transaction costs will be encountered. The width of this band is determined by a trade-off between the size of transaction costs and impact on the risk-return trade-off arising from failure to adjust exposure in relation to changes in forecast volatility.

Some target-risk funds forecast volatility once per day and take between one and three trading days to make the adjustment to the underlying risky asset. While this approach saves on transaction costs, it does not control the risk of the fund as well as a fund which makes intra-day forecasts of volatility and corresponding portfolio exposure adjustments.

The frequency of portfolio adjustment in response to volatility changes is also important for another reason. When we forecast risk, the actual level of risk experienced may be different to that forecast. Even if the best possible use of the information available beforehand is made, the actual experience may be an extreme observation. Certain events may effectively be unpredictable given the limitations of the data and the statistical techniques used by any forecasting system. A volatility forecasting system may be unable to predict certain events or the events are out of character with past observations, this could, in certain circumstances, lead to losses occurring between trading times. However, at the next trading session, the change in volatility will be incorporated into the volatility forecast and where there has been a rise in forecast volatility, the exposure to the index will be reduced. In this manner, subsequent losses are likely to be proportional to the risk level targeted by the Fund. The frequency of the review of volatility and adjustment for changes in volatility are particularly important in such circumstances.

8.4.5 Advantages of the Target-Risk Equity Fund

Properly implemented, a target-risk equity fund ensures that volatility of the risky asset is managed within a tight range around the target-risk level. Risk is controlled and therefore the size of peak-to-trough falls in value for any given time horizon and probability level is carefully managed to meet the investor’s risk appetite.

Every investor has a threshold at which he or she will be forced to reduce risk. Investors tend to hold portfolios which exhibit increasing losses far too long. The decision to cut risk often comes at or close to the 'bottom' thereby missing the upswing or at the very least only slowly increasing exposure to the recovering market. Investors thus suffer the worst effects of a large peak-to-trough fall in value without fully participating in the upswing.

A target-risk equity fund ensures that the likely depth of peak-to-trough falls in value is controlled to the investor's risk appetite in the first place obviating the need for this kind of sub-optimal investor behaviour.

9 Low-Volatility Equity Funds v. Target-Risk, Equity Funds

There has been considerable growth in the number of low-volatility equity funds available in the market. Typically, these funds are made of up of equities with low beta or equities which have lower volatility than the benchmark index in which they are included.

9.1 Summary Table

Table 4 below summarises the key differences between low-volatility equity funds and target-risk equity funds.

Table 4

Comparison Heading	Low-Volatility Equity Fund	Target-Risk Equity Fund
Peak-to-trough fall in value	Likely to be significantly in excess of that implied by average past volatility due to the range of variation in volatility	Likely to be in line with target-risk level
Extent to which portfolio represents the underlying universe of stocks	Limited Only certain stocks or sectors are likely to be included in the portfolio	Fully representative of all sectors
Extent to which the risk of the portfolio varies	Substantial Risk of portfolio varies with that of the underlying universe of stocks	Minimal Risk kept in a tight range around the target level
Most likely conditions for underperformance when the underlying universe of stocks is rising	Sectors making up the portfolio underperform the universe of stocks	High-volatility, rising market
Most likely conditions for outperformance when the underlying universe of stocks is falling	Sectors making up the portfolio fall less than the universe of stocks	High-volatility, falling market
Active Management of Portfolio Volatility	Limited May be reviewed monthly or quarterly	Active, intra-day volatility management to meet the target-risk level

9.2 Assumptions Underlying the Comparison in Table 4

9.2.1 Low-Volatility Equity Funds

No two low-volatility funds are identical so for the purpose of this comparison we take the term 'low-volatility equities' to mean an investment strategy which selects a portfolio of equities from a particular universe of equities on the basis of their historical volatility being below say the 25th percentile of historical volatility of the stocks in the particular universe. Such a portfolio might include stocks from sectors which have exhibited lower volatility than other sectors and often includes consumer staples, telecoms, and healthcare stocks.

9.2.2 Target-Risk Equity Funds

We take the term 'target risk equity fund' to mean an investment management strategy that varies the exposure to a basket of equities several times a day to keep the forecast volatility of the portfolio of equities in a tight range around the target level.

9.3 Low-Volatility Equity Funds

The low-volatility equity fund is chosen on the basis of low historical levels of volatility relative to other equities in the universe from which the equities are selected. Thus the low-volatility equity portfolio is really a low *relative volatility* equity portfolio.

The low-volatility equity fund is likely to be less representative of the universe of equities as only stocks and sectors with low relative volatility are likely to be included. The low relative volatility fund is therefore likely to be less diversified than the universe of equities from which it is drawn. Even in a rising market, sectors go through periods of underperformance relative to the overall market while others tend to go through periods of outperformance. There may be long periods during which the low relative volatility fund underperforms or outperforms the overall market.

9.4 Low Relative Volatility does not Imply Low Absolute Volatility

Low relative volatility does not necessarily imply low absolute volatility. As the volatility of the universe of equities rises, the low-volatility stocks may indeed maintain their low-volatility badge relative to other stocks in the universe but their absolute volatility will rise.

The size of peak-to-trough falls in value in a portfolio is driven by the level of volatility and closely related to the extent of variation in the volatility of the portfolio. As the absolute volatility of the low relative volatility portfolio rises, investors are highly likely to experience peak-to-trough falls in value that are in excess of what the average risk might suggest.

Typically, there is no intra-day, active risk management of the volatility of the portfolio in terms of managing its absolute volatility once the portfolio is constructed. The low relative volatility portfolio is typically only recalibrated on a quarterly basis.

9.5 Target-Risk Equity Funds

In the target-risk investment strategy, the exposure to the underlying universe of equities is varied several times each day to maintain the forecast risk of the portfolio in a tight range around the target-risk level. Thus there is a highly active risk management strategy to control the absolute volatility of the investment portfolio.

Although exposure to the underlying universe of equities varies with forecast volatility, all sectors of the universe of equities are represented in the exposure that is taken by the portfolio. Thus the exposure is to the entire universe of equities rather than to a limited number of sectors or stocks.

The target-risk equity portfolio will underperform the market when the market is rising in a high volatility environment and will outperform the market when the market is falling in a high volatility environment. The extent of underperformance or outperformance will be determined by how high market volatility is above the target-risk level.

The size of peak-to-trough falls in value in a portfolio is related to the highest level of risk of the portfolio rather than the average level of risk. As the absolute volatility of the target risk portfolio is maintained in a tight range around the target level, investors are unlikely to experience peak-to-trough falls in value that are in excess of what might be implied by the target risk level.

10 Difficulties in Forecasting Volatility

Volatility is difficult to measure. To estimate volatility we must measure price changes over time. However, during the measurement time interval volatility may be changing. It would appear that instantaneous volatility cannot be measured.

Errors in the forecasting of volatility arise not only from the varying nature of volatility but also from discontinuities in the underlying process. Many equity markets close overnight or at the very least during public holidays. Gaps between the closing price on one trading day and the opening price on the next trading day would suggest that forecasts of volatility based on the closing price for each day in the past are likely to give inflated estimates of volatility for two main reasons: (i) the discontinuities in the process; and (ii) the inability to capture anything but close to close movements in price. Consideration has to be given to the effect of opening-price gaps on estimates of volatility.

The bouncing of transaction prices between the bid and ask sides of the market introduces a systematic bias to the data which can cause serious problems in forecasting volatility. We can partially reduce the systematic bias arising from the bouncing between bid and ask prices by using the mid-price rather than the transaction price but measurement error remains.

Stale prices also affect the quality of volatility forecasts. Futures prices are more active than those of the corresponding cash markets and hence diminish the risk of stale prices being used in the volatility forecasting process.

There is a significant body of empirical evidence which shows the persistence of volatility levels over time[19]; a kind of volatility clustering. If we plot any measure of volatility against time, the graph will show that volatility clusters. Chart 6 is one such graph illustrating not only volatility clustering but reversion to a long-run level of volatility. Volatility clustering suggests that volatility can be forecastable in the sense that recent realised volatility seems to be a useful guide to short-term risk.

There is empirical support for the idea that volatility adjusts relatively more slowly and is relatively more persistent in low volatility environments than in higher volatility environments where it tends to adjust relatively more quickly and with less volatility persistence.

The volatility implied by equity option markets is a possible source of volatility forecasts. However, a liquid equity option market is required to harvest reliable implied volatility estimates. In this regard, the absence of liquid, equity option markets for Asian equities makes the use of implied volatility a less reliable source for forecasting volatility for such markets. Further, the bid-offer spread on short-term options is wider than on longer term options and generally speaking the shortest time to maturity offered on option contracts is one month. Thus if one is trying to forecast volatility over a short time horizon such as 24 hours, the bid-offer spread on even the shortest term options is likely to be too wide to be of any use.

Two studies[20] find high frequency intra-day data can produce more accurate time series forecasts than implied volatility.

In judging the success of any volatility forecasting system, its performance in out-of-sample testing will be one of the critical assessment criteria.

[19] Ducoulombier

[20] Fung and Hsich (1991) and Li (2002).

11 Uses of Target-Risk Equity Funds

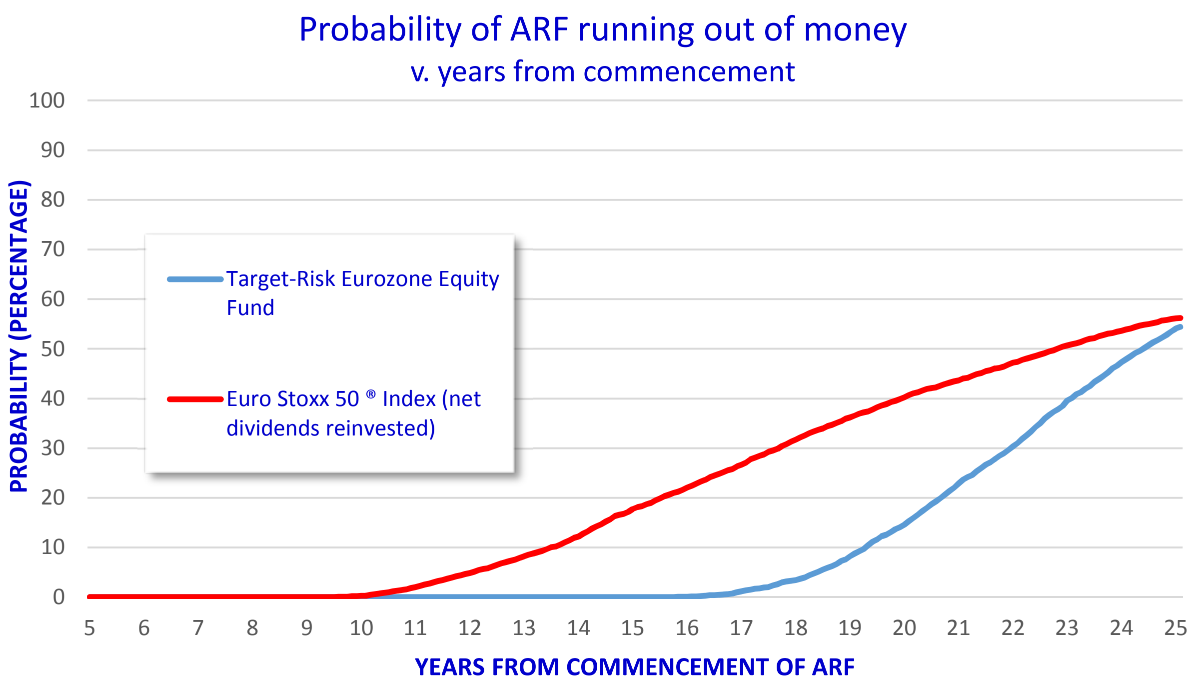
We have identified a number of portfolios where the use of target-risk equity funds is particularly appropriate namely:

- Approved retirement funds
- Capital protected products
- Multi-asset portfolios
- Defined contribution pension plans
- Defined benefit pension plans
- Own-funds of insurance companies

11.1 Approved Retirement Funds

Unlike an annuity, a personal retirement investment fund or approved retirement fund ("ARF") does not offer a guarantee from an institution that the income stream will continue until the death of the person drawing the income. Assuming a person survives for twenty five years after commencing to draw income from his or her ARF, the probability of the ARF running out of money depends not just on the investment performance of the ARF portfolio but on the path followed by the investment performance and in particular the size and timing of large peak-to-trough falls in value of the ARF portfolio.

Chart 13[21]



[21] **Notes to Chart 13**
Chart 13 has been constructed from simulations of the performance of a target-risk Eurozone equity fund targeting a risk level of 8% per annum of the value of the fund and the actual performance of the EURO STOXX 50® Index (net dividends reinvested) over the period from 1 September 1998 to 31 December 2015. The simulations use repeated, random selection of six continuous blocks of five-years of simulated or historic as appropriate daily returns of the investments to generate a 30-year investment return path. The daily returns used in the simulations of: (i) the target-risk Eurozone equity fund are before fund management fees but net of transaction costs; and (ii) the EURO STOXX 50® Index (net dividends reinvested) are before fund management fees. The target-risk Eurozone equity fund figures are derived from simulations of trading the EURO STOXX 50® index futures contract to achieve the annualised forecast volatility target of 8% per annum of the value of the fund.

Chart 13 shows the probability of an ARF running out of money versus the number of years from the commencement of the ARF for two different investment strategies: (i) a target-risk Eurozone equity fund; and (ii) the EURO STOXX 50® INDEX (net dividends reinvested).

Chart 13 assumes that 5% of the initial value of the ARF is drawn each year by monthly instalments with the payment stream beginning at the end of the first month. For each of the investment strategies, a fee of 0.5% of the value of the ARF is deducted from the value of the ARF at the beginning of each year.

The simulations involve the use of continuous blocks[22] of the simulated or historical, as applicable, daily returns of the two investments to construct thousands of possible paths for the progress of the value of the two investments. The probabilities are arrived at by counting the proportion of paths that give rise to an absence of funds in the ARF after different periods of time as a result of the combination of investment gains or losses and the fixed monthly withdrawals from the ARF portfolio.

Table 5 summarises the key features of Chart 13. The higher risk investment, the EURO STOXX 50® INDEX with net dividends reinvested, is almost certain to last just over nine (9) years. After that time period, the probability of an ARF invested in the EURO STOXX 50® INDEX lasting decreases almost linearly so that there is a 40.2% chance that the ARF will have run out of money within 20 years and almost a 56.1% chance that the ARF will have run out of money within 25 years.

Table 5

Investment Strategy	Chances of lasting at Least 15 Years	Chances of Lasting at Least 20 Years	Chances of Lasting at Least 25 Years
EURO STOXX 50 ® INDEX	82.2%	59.8%	43.9%
Target-Risk Eurozone Equity Fund	Almost Certain	85.4%	45.9%

The Target-Risk Eurozone Equity Fund, is almost certain to last 15 years which is a substantial improvement relative to the ARF invested in the EURO STOXX 50® INDEX. After that time period, the probability of the ARF lasting decreases almost linearly so that there is a 54.1% chance that the ARF will have run out of money within 25 years.

The higher the risk of an investment strategy, the greater the probability of a large peak-to-trough fall in value of the investment over any given time period, all other things being equal. Thus with a high-risk investment, if there is a large peak-to-trough fall in the value of the ARF in the early years, it is difficult for the ARF to recover that loss because the value of the ARF upon which any recovery in investment performance is based is constantly being eroded by the fixed, monthly withdrawals.

11.1.1 Key Risk Considerations in Creating an ARF Investment Strategy

An ARF is very different from say a normal investment portfolio which seeks capital growth because of the frequent withdrawals of capital which are generally fixed in euro terms. For this reason great care must be taken in the portfolio construction and management of an ARF so as to avoid as far as possible large peak-to-trough falls in the value of the ARF.

[22] The block size is five years.

An ARF investment policy constructed using historical risk measures like the annualised standard deviation of return of the assets to be included in the ARF portfolio would suffer from the flaw that the risk of the underlying investments may vary significantly over time and often rise substantially during a market downturn within an asset class.

For example, over the period from 1 September 1998 to 31 December 2015, the average risk of the EURO STOXX 50® INDEX is 24.4% per annum of the value of the index. However, ten percent of the time, the rolling, annualised, five-day, historical volatility is at least 50% higher than this average level. This is why investors get surprises. They think that they are in a portfolio with a risk of say, 24.4%, but don't realise that the risk varies. Suddenly they find themselves invested in a portfolio with say, 36% risk and exposed to the peak-to-trough falls in value associated with that higher risk level.

The size of peak-to-trough falls in value is proportional to the risk of the portfolio. Higher risk implies larger peak-to-trough falls in value all other things being equal. Rising risk can significantly damage the prospects of an ARF running out of money while the beneficiary of the ARF is still alive.

Bearing these points in mind, the portfolio would need to be reviewed regularly at least to ensure that there were no significant changes in the assumed risk levels of the assets. The difficulty is that the risk of the constituent components of the portfolio can change rapidly and it is unlikely that any reasonable frequency of review, such as monthly, can adapt quickly enough for an ARF portfolio where the continuous monthly withdrawals deplete the capital base on which any recovery can be built following a large peak-to-trough fall in value.

11.1.2 ARF Risk Management

In view of the key risk considerations in creating an ARF, it is important to control the risk of the ARF in a tight range around the chosen target-risk level. The target-risk Eurozone equity fund aims to keep the risk of the fund in a tight range around 8% per annum of the value of the fund. This target-risk management of the fund reduces significantly the size of peak-to-trough falls in the value of the fund compared with an investment in the EURO STOXX 50® INDEX where the average volatlity is much higher and varies significantly over time. For example, on 16 October 2008, the rolling, annualised, five-day, historical volatility of the EURO STOXX 50® INDEX spiked to just over 118% whereas that of the Target-Risk Eurozone Equity Fund was just under 9%.

11.2 Capital Protected Products

It is significantly cheaper to develop a capital protected product on an underlying asset that has a volatility that is kept in a tight range around a target level. The control of the volatility of the underlying makes the capital protected product more affordable for investors.

11.3 Stabilising the Risk of a Multi-Asset Portfolio

As illustrated in Chart 6, the volatility of an equity portfolio varies significantly over time. The impact of that variation in volatility on a simple 40% equity, 60% bond portfolio is shown in Table 2. Despite the higher allocation to bonds, we showed that the portfolio’s risk is heavily dominated by the equity component because of its higher average risk and the variations in that risk over time. Controlling the risk of the equity component of the portfolio by using a target-risk equity fund will dampen the large peak-to-trough falls in value of the portfolio.

11.4 Defined Contribution Pension Plans

For defined contribution pension plan investors, a target-risk equity fund allows investors to choose the level of loss with which they are comfortable over a given time and for a given probability rather than have equity risk simply washing through their portfolio. Further, a target risk equity fund allows investors to better control the absolute risk of a multi-asset portfolio by ensuring that the risk of the equity component of the portfolio does not vary wildly over time.

As we pointed out earlier, every investor has a threshold at which he or she will be forced to reduce risk and this seems to be particularly true for investors in defined contribution pension plans. Defined contribution investors tend to hold portfolios which exhibit increasing losses far too long. The decision to sell or cut risk often comes at or close to the 'bottom' thereby locking in significant loss. Defined contribution investors also often miss the upswing following a sale near the 'bottom' of the market or at the very least increase exposure to the recovering market only very slowly. By cutting their losses late in the cycle and delaying getting back into a recovering market, they suffer the worst effects of a large peak-to-trough fall in value.

A number of trustees of defined contribution plans report that after a large peak-to-trough fall in value, some active members become disillusioned by the loss and discontinue their contributions leaving themselves with hopelessly underfunded pension provisions.

11.5 Defined Benefit Pension Plans

For defined benefit pension plan investors, a target-risk equity fund allows the trustees to choose the level of loss with which they are comfortable over a given time and for a given probability rather than have equity risk simply washing through their portfolio. It allows the trustees to demonstrate their control of the risk of one of the most volatile, liquid assets and of the portfolio as a whole, by ensuring that the risk of the equity component of the portfolio does not vary wildly over time.

Further, in the context of the Funding Standard Reserve, to the extent that target-risk equity funds have less than 100% of a defined benefit pension plan's equity allocation exposed to the underlying basket of equities and hold the balance of the equity allocation in cash or Euro-denominated bonds meeting certain criteria, more of the fund will be invested in prescribed assets compared with a fund that invests its entire equity allocation in equities.

12 Comparing Studies

When comparing studies of target-risk equity funds, it is important to bear in mind that almost every study uses a different data set, a different time period, a different index futures contract, a different volatility forecast time horizon, a different volatility forecasting model, different rules in relation to the use of leverage, and critically, a different interval of time between the price readings used to forecast volatility,

13 Concluding Remarks

Target-Risk equity funds have significant application in approved retirement funds, defined benefit pension plans, defined contribution pension plans, capital protection products, multi-asset portfolios, and general portfolio risk management.

In our research, we found that over the Period, a target-risk equity fund based on the EURO STOXX 50® index with net dividends reinvested and operating at a risk level of 8% per annum of the value of the fund provided the same return as the underlying equity index every three to five years for one third of the risk and with just over one third of the maximum peak-to-trough fall in value. (Section 8.4, Table 3).

It is very difficult for investors in approved retirement funds, defined benefit pension plans, and defined contribution pension plans to recoup losses in their portfolios which arise from large peak-to-trough falls in the value of those portfolios. In the decade ending 31 December 2010, major equity indices, such as the EURO STOXX 50® index suffered losses of more than 50% of their value not once but twice. Losses in excess of 50% of value require returns of over 100% to recover to their pre-loss value.

Large losses like those cited in the previous paragraph are caused by an absence of risk control within such portfolios. Put simply, the risk or realised volatility of equity funds and equity indices varies dramatically over time (Section 7.1, Chart 6). Risk as measured by the annualised standard deviation of past returns can rise to more than five times that level. When risk rises, the probability of large losses increases (Section 7.1.1, Table 2). The variation in the risk of equity funds and equity indices leads to larger peak-to-trough falls in value than investors might expect from a review of past risk.

Target-Risk equity funds aim to keep the volatility of an equity fund or equity index in a very tight range around the chosen target level of risk. They do this by forecasting the risk of the fund and varying the exposure to the underlying risky asset inversely to the forecast risk so as to keep the risk of the fund in that tight range around the target-level (Section 7.2). The choice of target-risk level is driven by the investor's appetite for losses over a given time horizon for a chosen level of probability.

Equities have historically delivered strong returns in the long-term and are an essential component of the portfolios of many insurance companies and defined benefit pension plans. The march of accounting standards and prudential regulation has meant that there is limited scope for large peak-to-trough falls in the value of the equity component of such portfolios. Accounting standards and prudential regulation effectively force institutional investors such as defined benefit pension plans and insurance companies with equity exposure to control the risk of that component of their portfolio. Target-Risk equity portfolio management is likely to be a better means of including equities in such portfolios than simply investing in equities and allowing the risk of the portfolio to vary as the market dictates.

Other approaches to managing the risk of an equity fund or equity index such as low-volatility funds suffer from a number of significant drawbacks relative to the target-risk approach (Section 9).

The ability to forecast equity market volatility is critical to the operation of a target-risk equity fund. Poor volatility forecasting potentially manifests itself in a distribution of daily returns for the target-risk fund with high kurtosis[23], significant volatility of volatility, and large peak-to-trough falls in value.

For an investor in an approved retirement fund ("ARF"), the probability of the ARF not running out of money depends not just on the investment performance of the ARF portfolio but on the path that that investment performance follows and in particular the size and timing of large peak-to-trough falls in value of the ARF portfolio. Where the risk of the equity portion of an ARF investment can vary in line with market variations in risk, the chances of a large peak-to-trough fall in value in the early years of the ARF increase and it may be difficult to recover that loss because the value of the ARF upon which any recovery in investment performance is based is constantly being eroded by regular withdrawals.

In a defined benefit pension plan, a target-risk equity fund allows the trustees to choose the risk level at which the equity portion of the portfolio operates to meet the prudential requirements of the plan and control the size of the Funding Standard Reserve (Section 11.5). Where the risk of the equity portion of the plan's investment can vary in line with market variations in risk, there is a greater risk of breaching the prudential requirements of the plan than if the risk of the equity portion is controlled using a target-risk approach.

END

[23] Probability mass is concentrated around the mean and in the tails of the distribution.

14 List of References

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