Unconstrained Multi-Asset Investing
Seeking diversification through efficient portfolio construction using cash-based and derivative instruments

22.01.13
“Uncertainty is the only certainty there is, and knowing how to live with insecurity is the only security”

John Allen Paulos
Professor of Mathematics at Temple University, Philadelphia, USA
Background

- Working party paper prepared in 2012
  - Presented at Risk and Investment Management conference, June 2012

- This paper formed core part of sessional paper to be presented to Institute and Faculty audiences in London and Edinburgh in 2013

- Themes
  - Measurement of portfolio diversification
  - Diversification benefits of unconstrained multi-asset investment
  - Use of derivatives to implement investment strategies
  - Risk measurement of an unconstrained multi-asset portfolio

- Question
  - Where does the actuarial profession believe it has most to add in the realm of portfolio construction?
Traditional approach fails to deliver consistent returns

Annualised rolling 3 year returns – Standard Life Balanced Fund

Uncertainty is the only certainty there is

Source: Standard Life Investments

Uncertainty is the only certainty there is.
Finding diversity is increasingly difficult

Correlation of Global Equities with other market sectors

Source: IPD UK Monthly Property Index, All Property; Federal Reserve Trade-Weighted Exchange Value of US Dollar vs 6 Countries; Dow Jones UBS – Commodity Index; Barclays Capital Global Corporate Index, Excess Returns; Barclays Capital Emerging Markets Index, Excess Returns; Barclays Capital US High Yield Index, Excess Returns; Standard Life Investments, 31 December 2011
Portfolio diversification measures

- No single measure of diversification
- Growing academic literature on measures of systemic risk
- Need for some definite measures of diversification:
  1. Assess current state of the world
  2. Observe how it is changing through time
- We want to consider measures for:
  1. Diversification in a given portfolio
  2. Diversification potential in an investment universe
Introduction to principal components analysis

• Analysis is based on the **weighted covariance matrix**:

1. Covariance matrix of asset returns:

   \[ \Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1N} & \sigma_{2N} & \cdots & \sigma_N^2 \end{pmatrix} \]

2. Portfolio weights

   \[ \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{pmatrix} \]

   \[ \Sigma^w = \begin{pmatrix} w_1^2 \sigma_1^2 & w_1 w_2 \sigma_{12} & \cdots & w_1 w_N \sigma_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ w_1 w_N \sigma_{1N} & w_2 w_N \sigma_{2N} & \cdots & w_N^2 \sigma_N^2 \end{pmatrix} \]
Introduction: Eigenvalues

• For every linear combination of assets, represented by a vector \( v \), we can compute the size of the projected variance

\[
\lambda = v^T \sum v
\]

• The first \textbf{eigenvalue} is the projected variance associated to the vector for which this projection is maximised. This is typically interpreted as the market

• Later eigenvalues are associated with vectors that maximise this expression under the constraint that they are orthogonal to vectors computed earlier

• This process creates an ensemble of uncorrelated strategies called \textit{principal portfolios}. This process is called \textbf{principal component analysis}
Effective Number of Assets

• We define the **effective number of assets** based on the entropy of the eigenvalues of the weighted covariance matrix.

• **Entropy** $H(p)$ is a measure of the information in the probability distribution $p$. It has been used widely in physics, information theory, signal processing, etc.

• Hence the effective number of assets is a measure of the concentration of eigenvalues.

• It takes values between 1 and $N$.

**Effective number of assets:**

$$N_w = \exp \left( H \left( p^w \right) \right)$$

Diversification within a given portfolio.
Diversification Potential

- The **diversification potential** is the maximum over the effective number of assets as a function of the portfolio weights.

**Diversification potential:**

\[ P = \max_w N_w \]

- Well defined optimisation problem since the measure varies smoothly with the portfolio weights \( w \).
- Because of symmetries, we are looking for the local minimum near the weights that define an equal weighting in terms of risk.
- Can be solved with a local optimiser (Newton–Raphson type).

The maximum amount of effective assets that any given portfolio universe could have...
Effective number of stocks is low compared to universe
Effective number of stocks in FTSE 100
Equally weighted and maximum potential

Maximum potential is three times that of a market cap weighted approach
Effective number of assets in UK pension portfolio

Low number of effective assets reflects dominance of equity and interest rate risk
Defining the investment challenge

• “How can I build a better diversified investment portfolio but without impacting long term return expectations?”

• **Allow a broader global investment universe**
  - currencies
  - asset classes and sectors
  - yield curves across different geographies
  - volatility

• A number of these strategies (such as specific currency pairs) will necessitate the use of derivative contracts

• **Allow unconstrained dynamic asset allocation**
  - offers access to the full diversification potential of the chosen investment universe
Money making strategies and diversification

Strategy correlation with global equity

- UK Equity
- High yield credit
- Russian Equity
- Mexican 10 year bonds
- Global inflation-linked bonds
- UK corporate bonds
- Europe bond yield steepener
- US Dollar v Euro
- US equity large v small cap

Derivative-based strategies can offer the most diversification potential

Source: Standard Life Investments, July 2012
## Portfolio Themes – choice of implementations

<table>
<thead>
<tr>
<th>Profit opportunities in areas of growth</th>
<th>Implementation choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>• US equity</td>
<td>Physical or equity futures/options</td>
</tr>
<tr>
<td>• US technology equity vs US small cap equity</td>
<td>Physical and / or equity futures/options</td>
</tr>
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<td>• Korean equity</td>
<td>Physical or equity futures/options</td>
</tr>
</tbody>
</table>

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<tr>
<th>Corporate balance sheet strength</th>
<th>Implementation choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>• US large cap equity vs US small cap equity</td>
<td>Physical and / or equity futures/options</td>
</tr>
<tr>
<td>• European high dividend-yield equity</td>
<td>Physical or equity futures/options</td>
</tr>
<tr>
<td>• Investment grade and high yield credit</td>
<td>Physical or index CDS contracts</td>
</tr>
</tbody>
</table>

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<thead>
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<th>Fiscal advantages of resource-based economies</th>
<th>Implementation choices</th>
</tr>
</thead>
<tbody>
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<td>• Mexican government bonds</td>
<td>Physical bonds</td>
</tr>
<tr>
<td>• Brazilian Real vs Australian Dollar</td>
<td>Currency forward</td>
</tr>
<tr>
<td>• Russian equity</td>
<td>Physical</td>
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<td>• US Dollar vs Canadian Dollar</td>
<td>Currency forward</td>
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<th>Challenging Eurozone resolution</th>
<th>Implementation choices</th>
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<td>• US Dollar vs Euro</td>
<td>Currency forward</td>
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Wide range of strategies that can make money
Effective factors for unconstrained multi-strategy portfolio

Portfolio’s greater number of ‘moving parts’ allows for more return consistency
Portfolio risk management
Combining physical and derivative holdings

- Risk-based portfolio construction
  - Size of each position
  - Volatility of each position

- Ensures consistency
  - Some assets physically invested others by derivatives
  - Different asset classes have different volatility

- Stand-alone risk measures the risk any of any one investment strategy
  - US Equity has volatility: 23.2%
  - Portfolio allocates 10% investment to US equities
  - Stand-alone risk of US equities to portfolio is 2.3%

Source: Standard Life Investments, 31 March 2012
Absolute Return investing
Risk-based portfolio construction

Stand-alone risk aligns size of position with its volatility

Source: Standard Life Investments, sample absolute return portfolio, March 2012
Risk-based portfolio construction
Seek a number of different risks

Total aggregated risk 22.4% (total nominal exposure 220%)

Source: Standard Life Investments, APT system, March 2012
# Leverage and portfolio risk

<table>
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<tr>
<th>Portfolio</th>
<th>Leverage factor</th>
<th>Strategy volatility</th>
<th>Portfolio risk</th>
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<tr>
<td>US equity</td>
<td>1.0</td>
<td>23.2%</td>
<td>23.2%</td>
</tr>
<tr>
<td>UK corporate bonds</td>
<td>3.0</td>
<td>7.7%</td>
<td>23.1%</td>
</tr>
<tr>
<td>Australian short-term interest rates</td>
<td>10.0</td>
<td>0.9%</td>
<td>9.0%</td>
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- In theory lowest risk portfolio is the most leveraged
- However, actual outcome is driven by the behaviour of only **ONE** factor
- Leveraged credit is a strategy that failed in the GFC
Historical scenario analysis

- Bank Meltdown 2008 (September 12 - October 15, 2008)
- Subprime Debacle 2007 (July 15 - August 15, 2007)
- Emerging Market Sell-Off 2006 (May 1 - June 8, 2006)
- Bond Sell-Off (June 14 - July 31, 2003)
- Bond Rally (May 1 - June 13, 2003)
- Gulf War 2 (March 1-23, 2003)
- Equity Rally (October 10 - November 27, 2002)
- Equity Sell-Off (August 23 - October 9, 2002)
- Sept 11th
- Tech Wreck (April 7-14, 2000)
- Russian/LTCM
- Asian Crisis 1997
- Mexican Crisis 1995
- Rate Rise 94
- Gulf War 1990
- Black Monday 1987

Source: UK £ GARS Regulated Unit Trust, RiskMetrics, 30/09/2012
Analyse actual return experience

Distribution of weekly returns of unconstrained multi-asset portfolio vs MSCI Global Equities

<table>
<thead>
<tr>
<th></th>
<th>Equity</th>
<th>Multi-asset</th>
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<tbody>
<tr>
<td>WORST three equity weeks</td>
<td>-16.6%</td>
<td>-5.5%</td>
</tr>
<tr>
<td></td>
<td>-9.3%</td>
<td>-3.1%</td>
</tr>
<tr>
<td></td>
<td>-8.3%</td>
<td>-3.0%</td>
</tr>
<tr>
<td>BEST three equity weeks</td>
<td>+9.9%</td>
<td>+4.3%</td>
</tr>
<tr>
<td></td>
<td>+8.3%</td>
<td>+3.1%</td>
</tr>
<tr>
<td></td>
<td>+7.6%</td>
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Source: Standard Life Investments, net performance from 12/06/2006 to 30/11/2012
Portfolio performance is based on the £, institutional pooled pension portfolio
* Source: Thomson Datastream, MSCI World (£) (net of tracker fund fee)
Summary

• Traditional multi-asset funds offer insufficient diversification opportunities

• Derivatives significantly broaden the range of investment strategies that a portfolio can utilise

• Combining traditional assets with derivative-implemented strategies produces more risk efficient portfolios
  ▪ increase the likelihood of ‘good’ outcomes for your portfolio

• Actuaries well placed to add value in understanding and communicating investment diversification
Appendices
Absorption Ratio: related measures

- Several related indicators based on correlations and principal components exist:
  - Billio et. al. (2010): Measuring Systemic Risk in the Finance and Insurance Sectors
  - Golts et. al. (2010): Diversification, Volatilities, and the Supercurrency
  - Fenn et. al. (2010): Temporal Evolution of Financial Market Correlations
  - Meucci (2009): Managing diversification

- There are several ways of measuring the concentration of eigenvalues:
  - Absorption Ratio
  - Herfindahl-Hirschman index (HHI)
  - Gini coefficient
  - Entropy

- But which one is mathematically justified? What properties do they satisfy?
Properties of ‘effective factors’

- **Invariant** measure
- Depends **continuously** on the portfolio weights
- High level of **stability**
- Varies **intuitively** under **addition of extra assets**
- **Interpretable** as the number of independent risk factors required to capture the risks of the underlying asset positions without losing information
- Rigorous **mathematical** interpretation:
  - Known in the signal processing literature as the effective rank
  - Satisfies a property that is analogous to Shannon’s noisy coding theorem
Risk-based portfolio guidelines

- Limit the amount of risk allocated to any one broad asset group
  - Eg equities, interest rates, credit etc.

- Limit the total aggregate risk that can be taken
  - Risk models underestimate volatility and overestimate diversification benefits in stressed markets

- Continuous monitoring of portfolio’s actual risk behaviour versus what is expected
The information shown relates to the past. Past performance is not a guide to the future. The value of investment can go down as well as up.

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