Non-life Insurance Technical Provisions.
Prediction Errors
for Common Reserving Techniques
‘Ultimo’ and One-year perspectives

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Caveat

The views and opinions presented today are given in the personal capacity of the presenters and do not represent the views of the Central Bank or PwC

Introductory remarks

- Motivation
- Central Bank letters
  - Ultimo perspective
- Solvency 2
  - One year view
  - ORSA
- Document the wide variety of approaches to the one year problem

Introductory remarks: Agenda

- Overview of the basics
- Ultimo methods
- One-year methods
- Results of numerical examples

Methods covered: Analytic and Simulation

- Best Estimates:
  - Chain ladder
  - Bornhuetter Ferguson
  - GLM models

- Prediction error:
  - Mack
  - Alai, Merz & Wuthrich: BF
  - Bootstrap ODP
  - Bootstrap Mack
  - Merz & Wüthrich: One-year
  - Bootstrap: Claims Development Result (CDR)
  - Time scaling

Chain Ladder Assumptions

- A heuristic algorithm
- With some (apparently) straightforward assumptions
  - Assume that each origin year develops similarly
    - i.e. has the same development factors for each lag “j”
  - Assume each origin year “i” has a native level
    - Represented by the current cumulative claims
  - Assume that the claims $C_{i,j}$ from each origin years are (conditionally) independent
    - This is the problematic one!

Reformulating heuristic algorithms as statistical models

- Why?
  - recast a heuristic algorithm with no particular statistical foundation into a defined stochastic model with a sound statistical basis
- GLM framework
  - Generalizing to allow for calendar year features

Reformulating heuristic algorithms as statistical models

- Many of the stochastic models that correspond to the chain ladder are described as follows

$$\log(m_{i,j}) = c + \alpha_i + \beta_j$$

Constant

Origin year trend

Development year trend

- However if we extend the formula a little to also include calendar year effects then we can start to deal with issues like changes to inflation that are beyond the reach of the traditional chain ladder method

$$\log(m_{i,j}) = c + \alpha_i + \beta_j + \gamma_k$$

Calendar year trend

Mack method

- First two moments only
  – No assumption re distributional form
- Applies to pure chain ladder estimates only
  – Requires a complete triangle to function due to recursive nature of formula
- Nearly 20 years old
- Often cited as “simple” method

Mack formulae

$$m.s.e.((C_{i,n}))^2 = C_{i,n}{}^2 \sum_{j=n+1-i}^{n-1} \frac{1}{\hat{f}_j^2}\left(\left((s.e.(f_{i,j}))^2 + (s.e.(\hat{f}_j))^2\right)\right)$$

$$(s.e.(f_{i,j}))^2 = \frac{\sigma_j{}^2}{C_{i,j}} \quad \text{and} \quad (s.e.(\hat{f}_j))^2 = \frac{\hat{\sigma}_j{}^2}{\sum_{i=1}^{n_j} C_{i,j}}$$

$$\hat{\sigma}_j{}^2 = \frac{\sum_{i=1}^{n-j} C_{i,j-1}(f_{i,j} - \hat{f}_j)^2}{n-j-1}$$

$$m.s.e.((C_{i,j+1}))^2 = C_{i,j}{}^2\left((s.e.(f_{i,j}))^2 + (s.e.(\hat{f}_j))^2\right) + (s.e.(C_{i,j}))^2 \hat{f}_j{}^2$$

$$\left(m.s.e.\left(\sum_{i=n+1-i}^{n} C_{i,j+1}\right)\right)^2$$

$$= \left(s.e.\left(\sum_{i=n+2-j}^{n} C_{i,j}\right)\right)^2 \hat{f}_j{}^2 + \sum_{i=n+1-j}^{n} C_{i,j}{}^2.(s.e.(f_{i,j}))^2 + \left(\sum_{i=n+1-j}^{n} C_{i,j}\right)^2 (s.e.(\hat{f}_j))^2$$

Mack and tail risk work-arounds

$$\hat{\sigma}_j^2 = \frac{\sum_{i=1}^{n-j} C_{i,j-1}\left(f_{i,j} - \hat{f}_j\right)^2}{n-j-1}$$

- An adjustment is needed to Mack to allow for triangles that are not fully run-off
- Mack suggestion (1999) is to assume that if

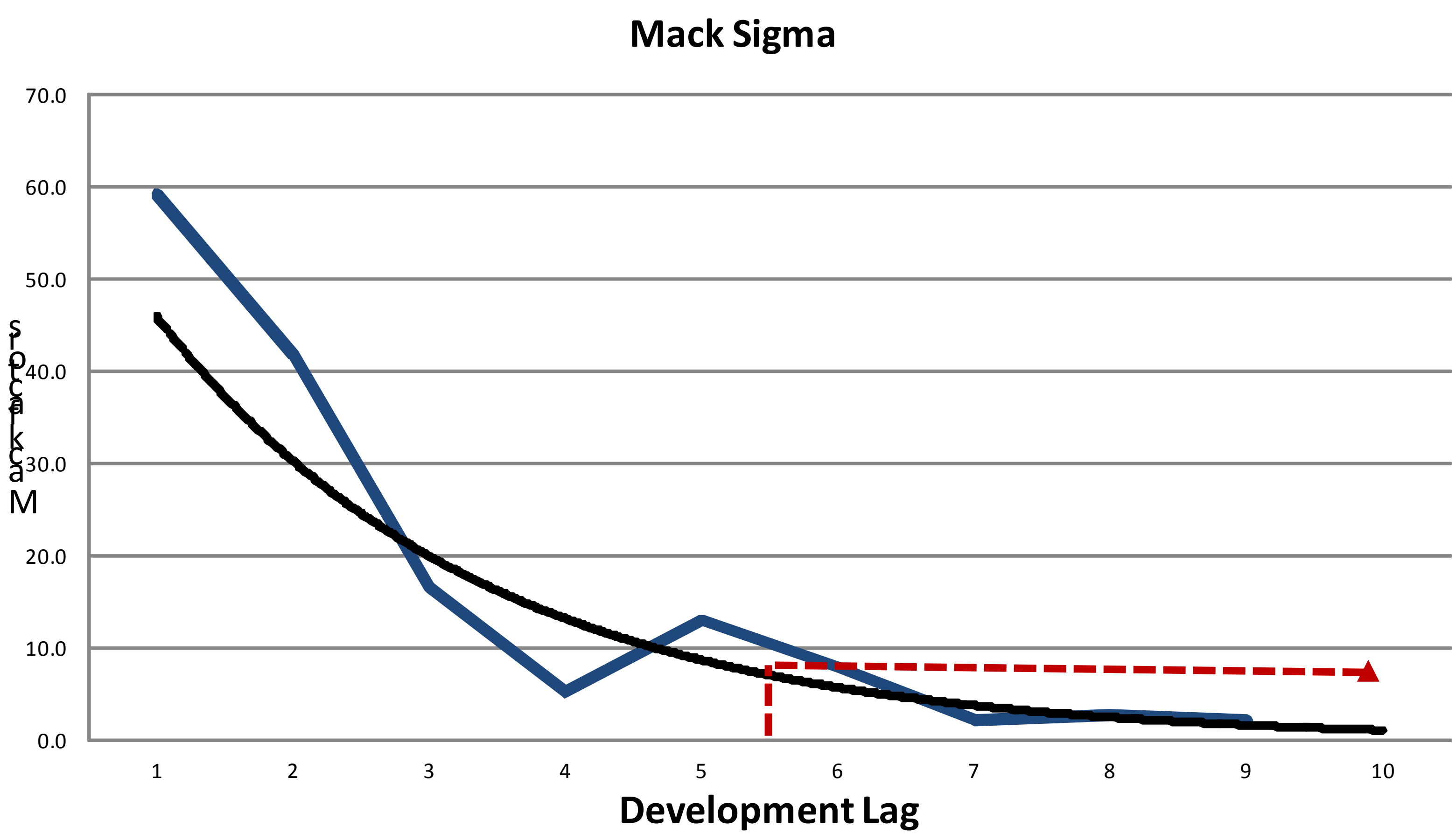
$$\hat{f}_{j-1} \leq \hat{f}_{\infty} \leq \hat{f}_j$$

  then

$$s.e.\left(f_{i,j-1}\right) \leq s.e.\left(f_{i,\infty}\right) \leq s.e.\left(f_{i,j}\right) \qquad s.e.\left(\hat{f}_{j-1}\right) \leq s.e.\left(\hat{f}_{\infty}\right) \leq s.e.\left(\hat{f}_j\right)$$

- Another approach used in ResQ is to apply the CoV from the main triangle reserve to the tail reserve effectively grossing up the prediction error for the tail reserve

Mack's sigma factors and tail risk

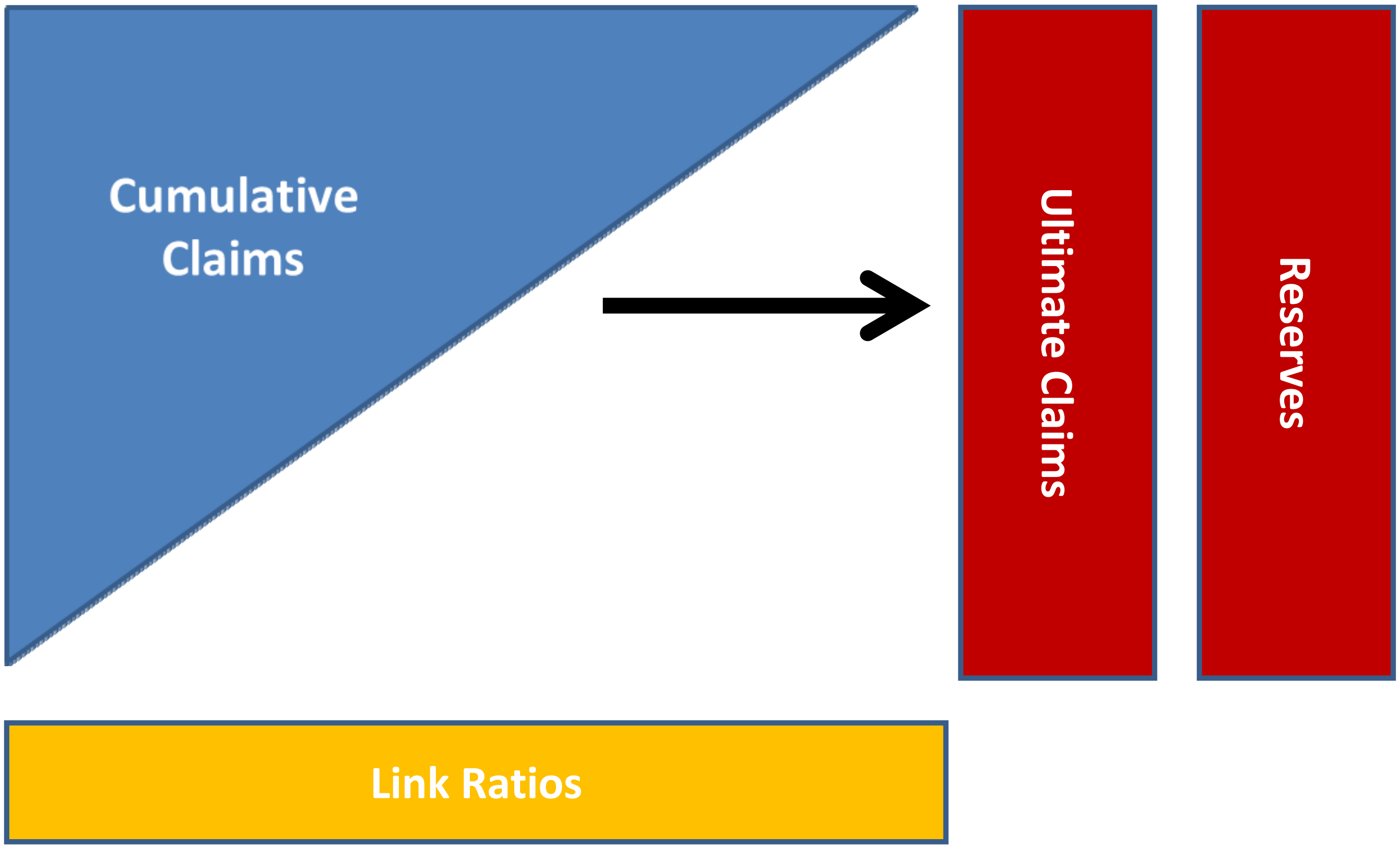


Bootstrap: Background

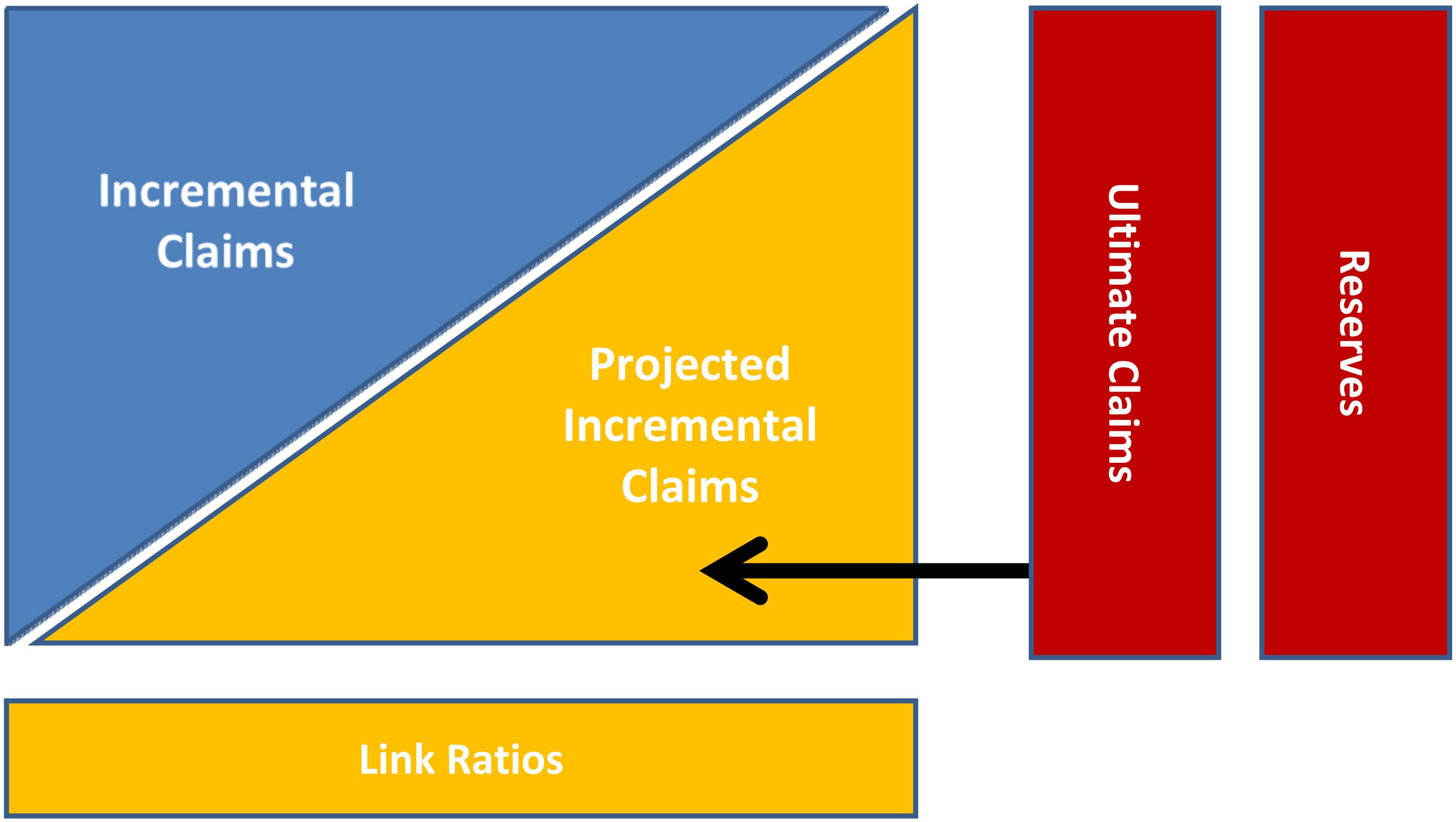
- England and Verrall
  - (see 2006 paper rather than original)
- ODP bootstrap
  - Negative increments
- Mack bootstrap
  - Link ratios
  - Negative increments handled o.k.

Bootstrap: Process #1

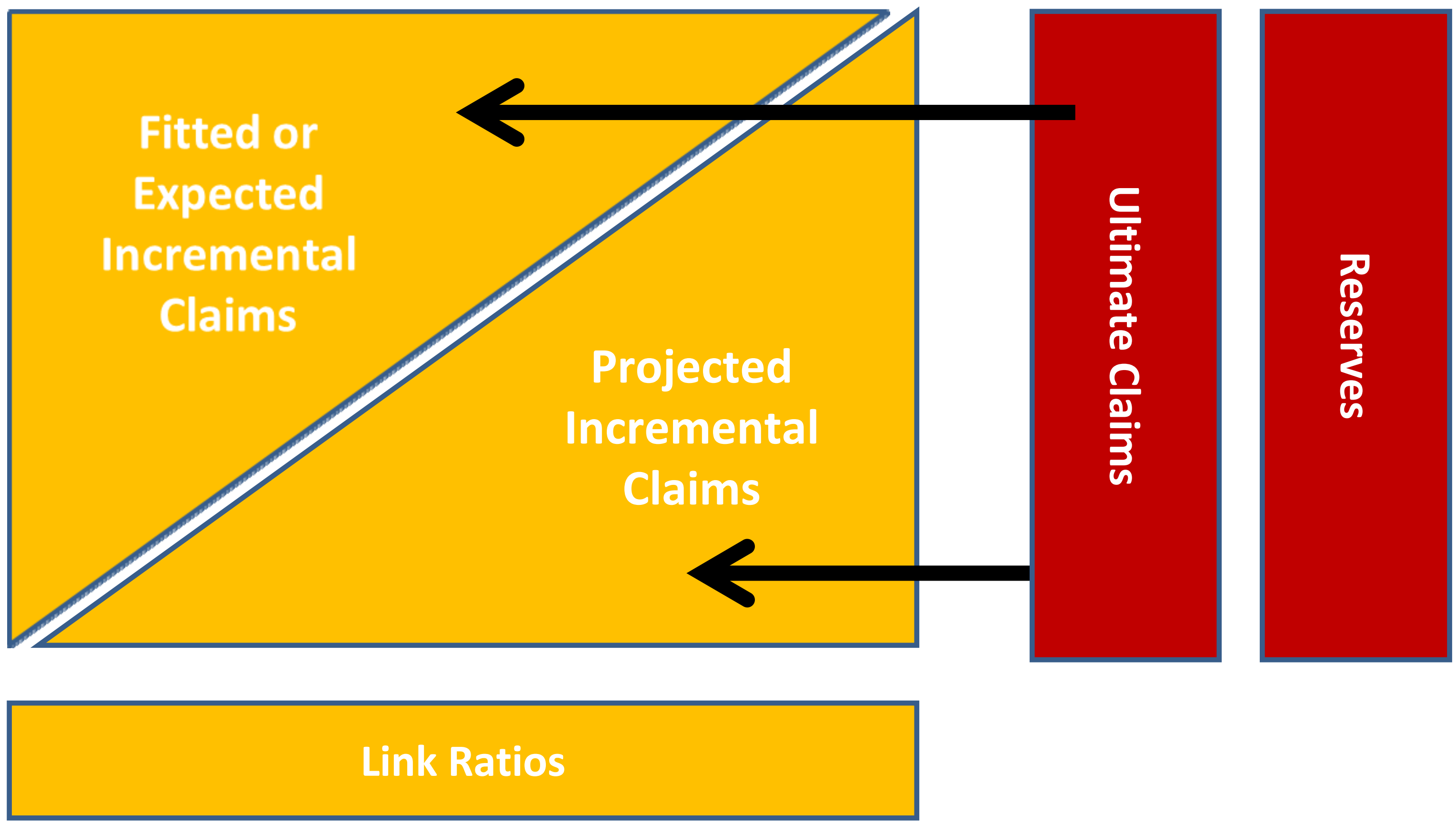
Bootstrap: Process #2



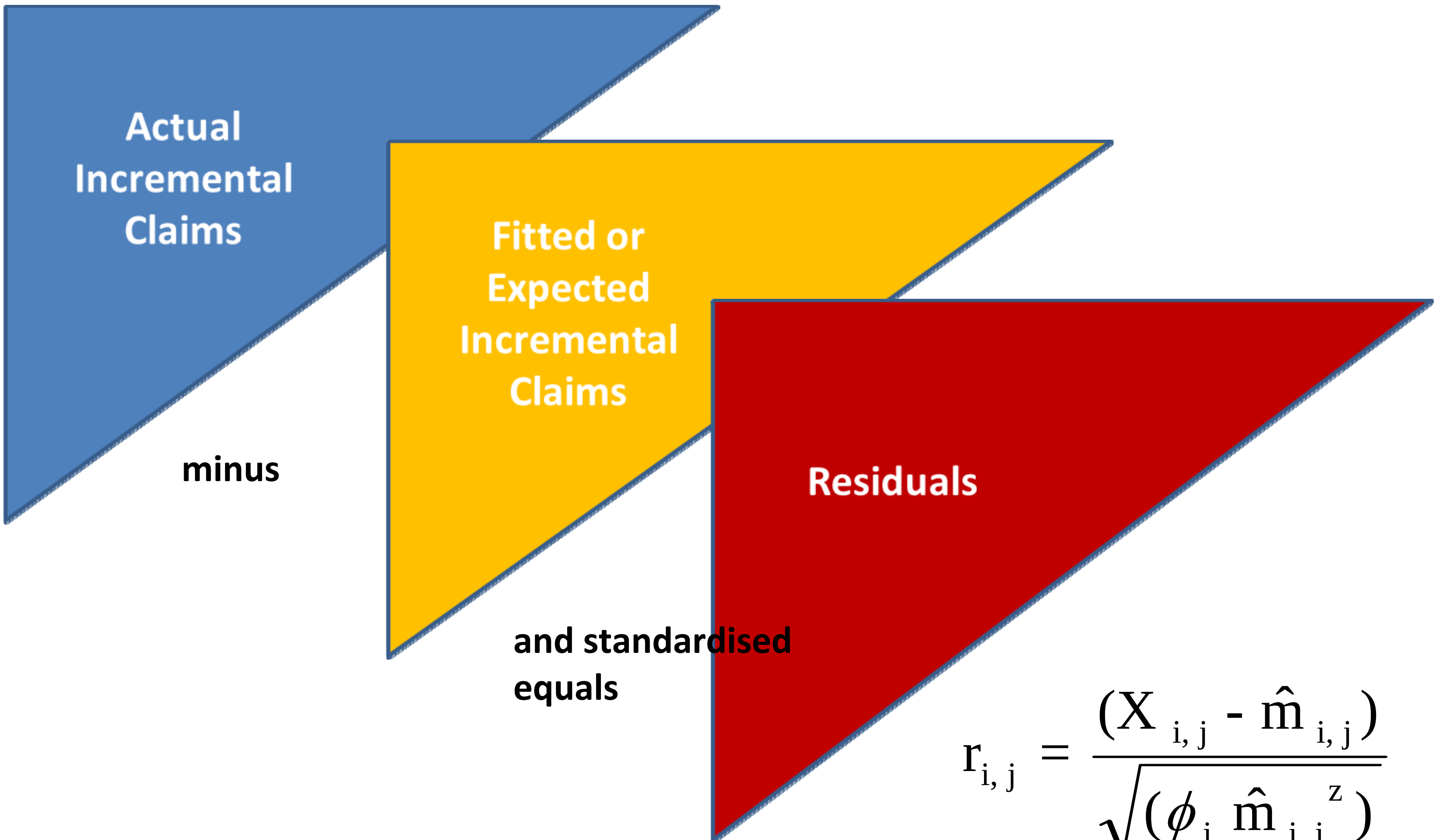
Bootstrap: Process #3



Bootstrap: Process #4

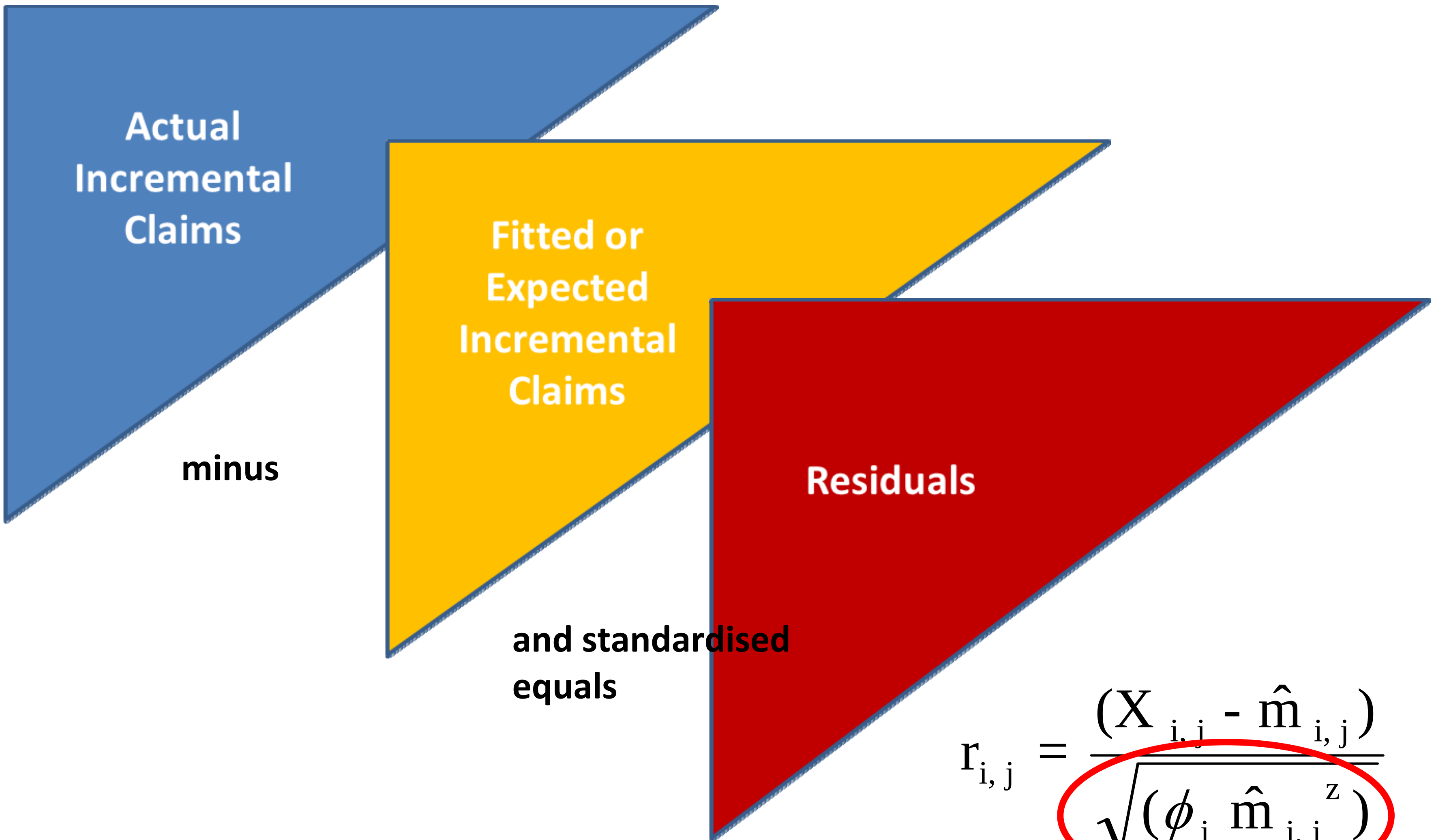


Bootstrap: Process #5



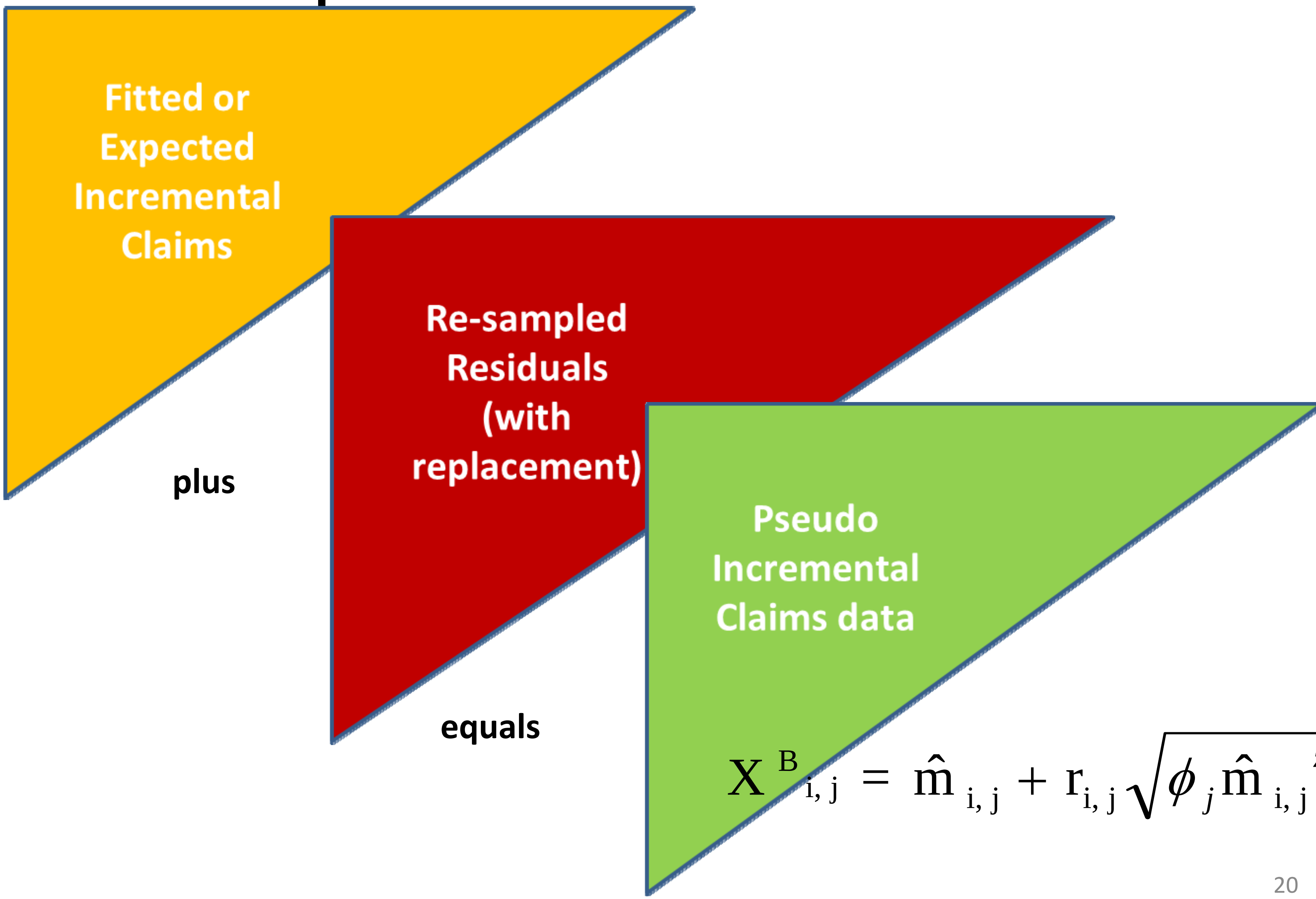
$$r_{i,j} = \frac{(X_{i,j} - \hat{m}_{i,j})}{\sqrt{(\phi_j \, \hat{m}_{i,j}^{\,z})}}$$

Bootstrap: Process #6



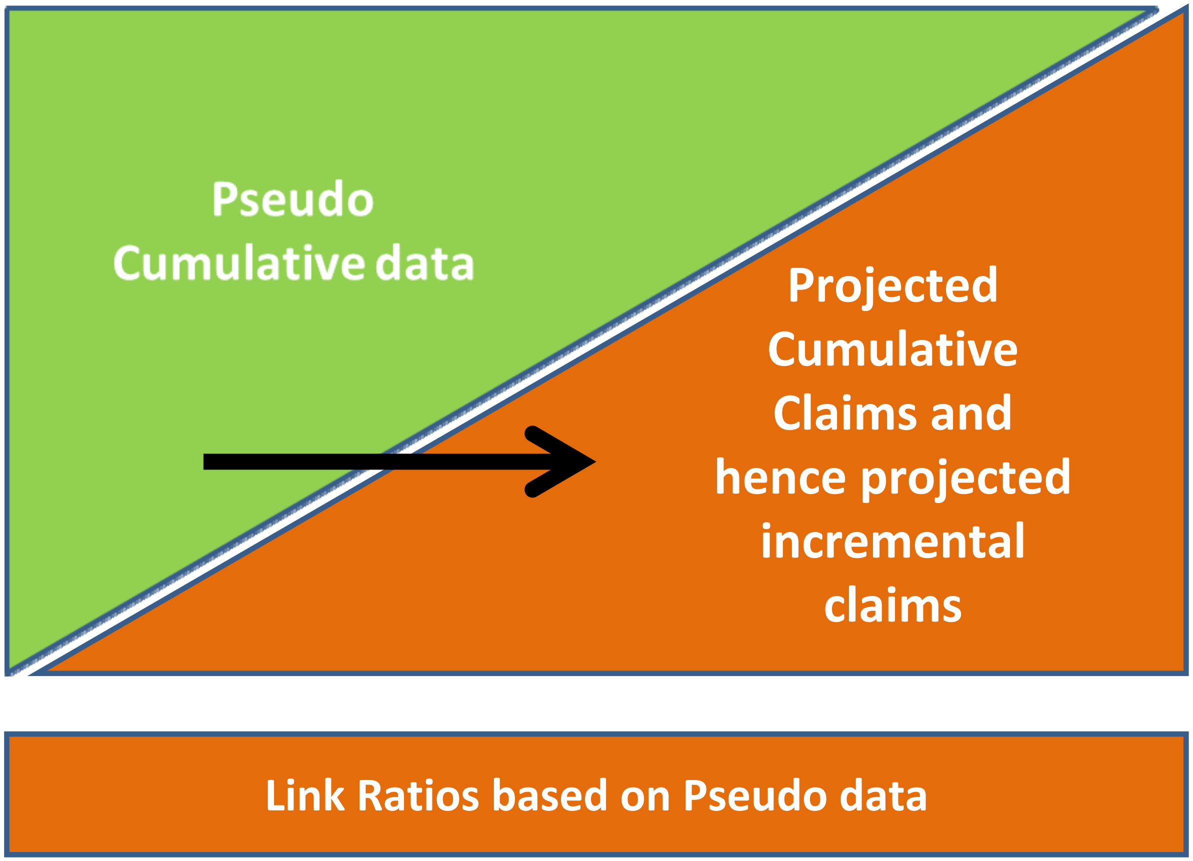
$$r_{i,j} = \frac{(X_{i,j} - \hat{m}_{i,j})}{\sqrt{(\phi_j \hat{m}_{i,j}^{z})}}$$

Bootstrap: Process #7

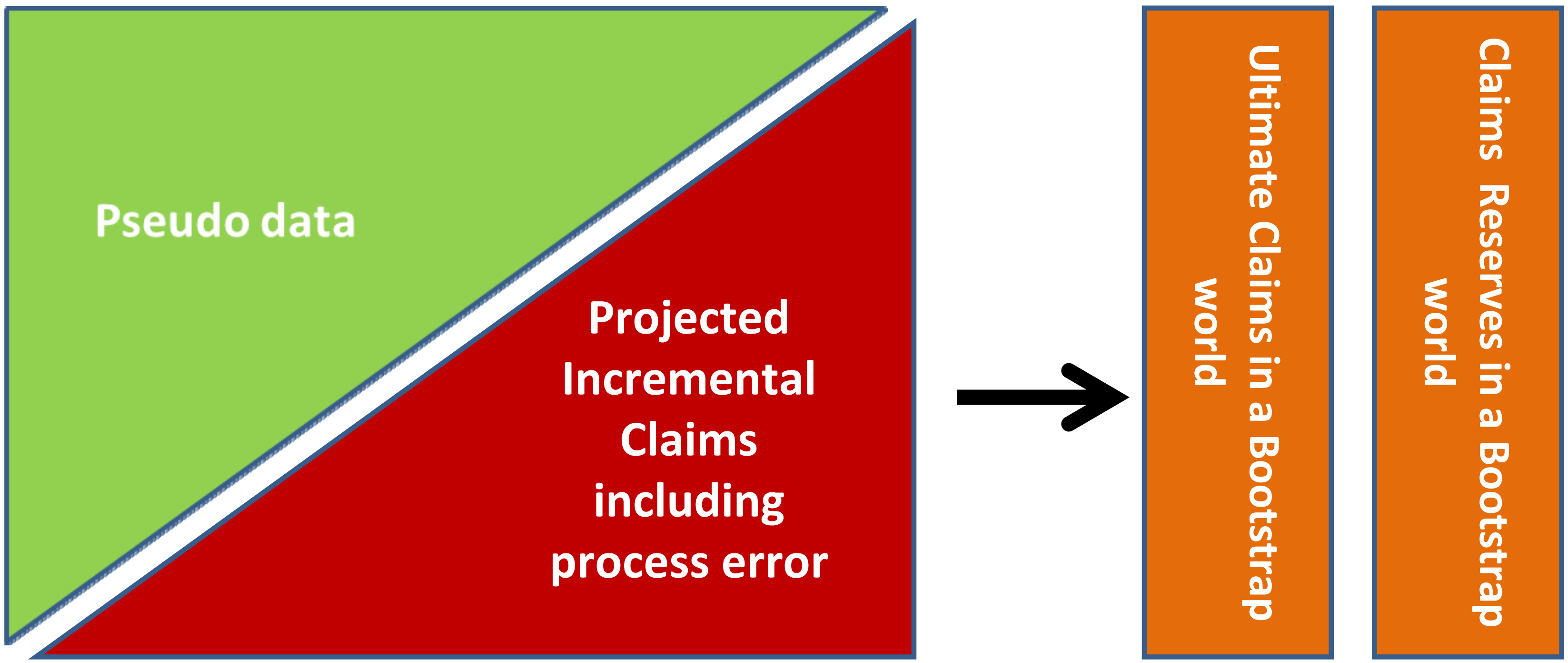


$$X^{B}_{i,j} = \hat{m}_{i,j} + r_{i,j}\sqrt{\phi_{j}\hat{m}_{i,j}^{z}}$$

Bootstrap: Process #8



Bootstrap: Process #9



$$\Gamma\left(\hat{m}^{B}{}_{i,j}, \phi_j \hat{m}^{B}{}_{i,j}{}^{z}\right)$$

Bootstrap: Mack variation

- Link ratios not incremental claims

$$r_{i,j} = \frac{\sqrt{C_{i,j}}\,(f_{i,j} - \hat{f}_j)}{\hat{\sigma}_j}$$

- Recursive
- Calculate ultimate and back out incremental pseudo claims

$$f^{B}_{i,j} = \hat{f}_j + r^{B}_{i,j}\,\frac{\hat{\sigma}_j}{\sqrt{C_{i,j}}}$$

- Include process error in the usual way

$$\Gamma\left(\hat{m}^{B}_{i,j}, \phi_j \hat{m}^{B}_{i,j}{}^{z}\right)$$

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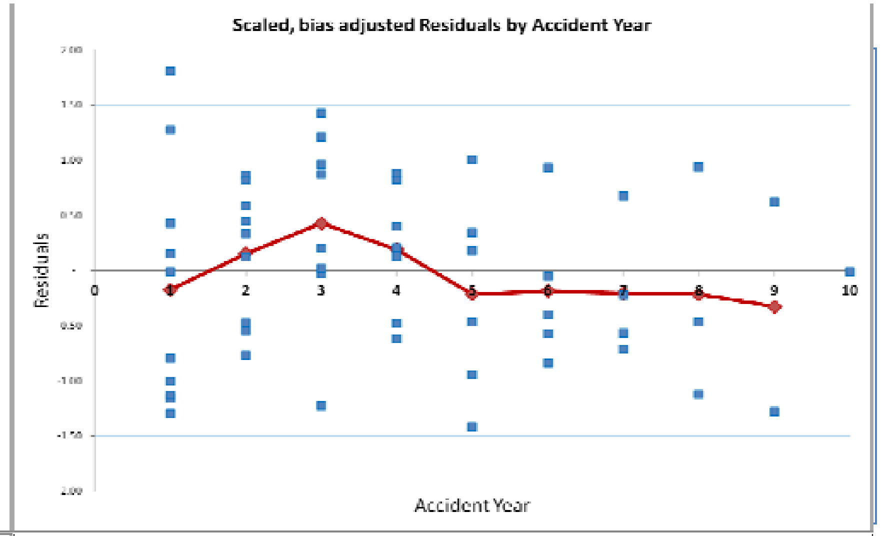
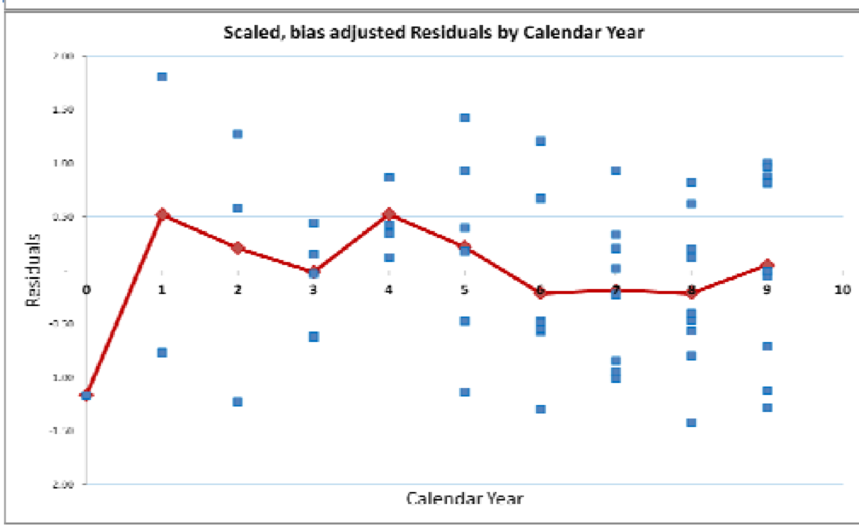
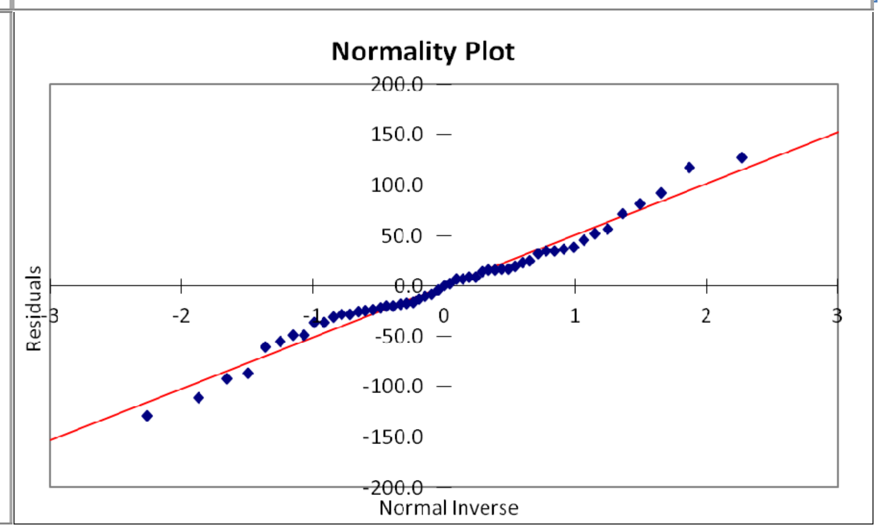
Bootstrap: Diagnostics





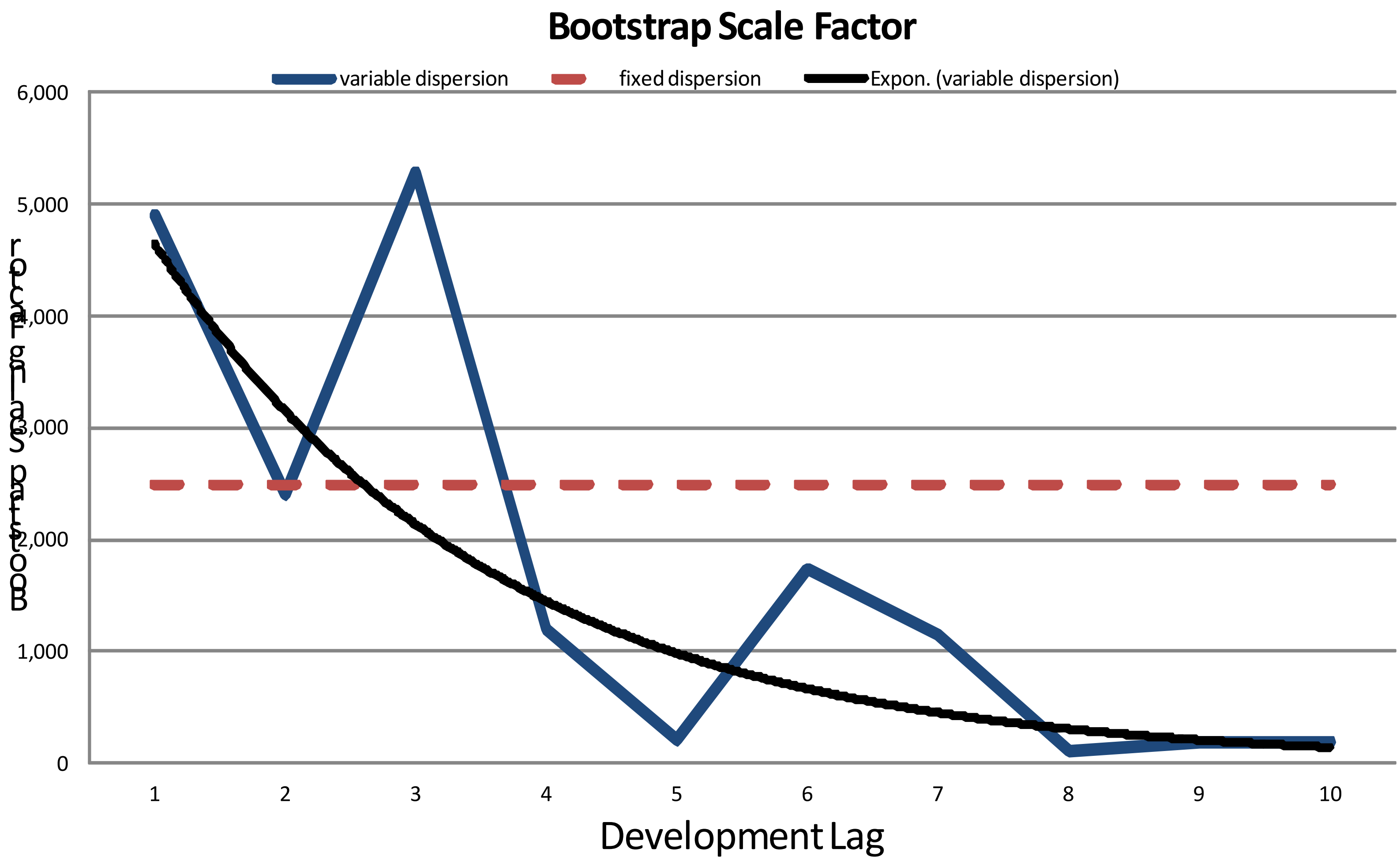


Bootstrap Choices:

Scaling, Bias adjustment and non-zero mean residuals

- Dispersion parameter
  - Variable by development lag or fixed
- Bias adjustment factor
  - Hat matrix versus degrees of freedom factor
  - Removal of zero residuals
- Setting mean of the residuals to equal zero

Bootstrap Choices: Scaling, graph of ϕ



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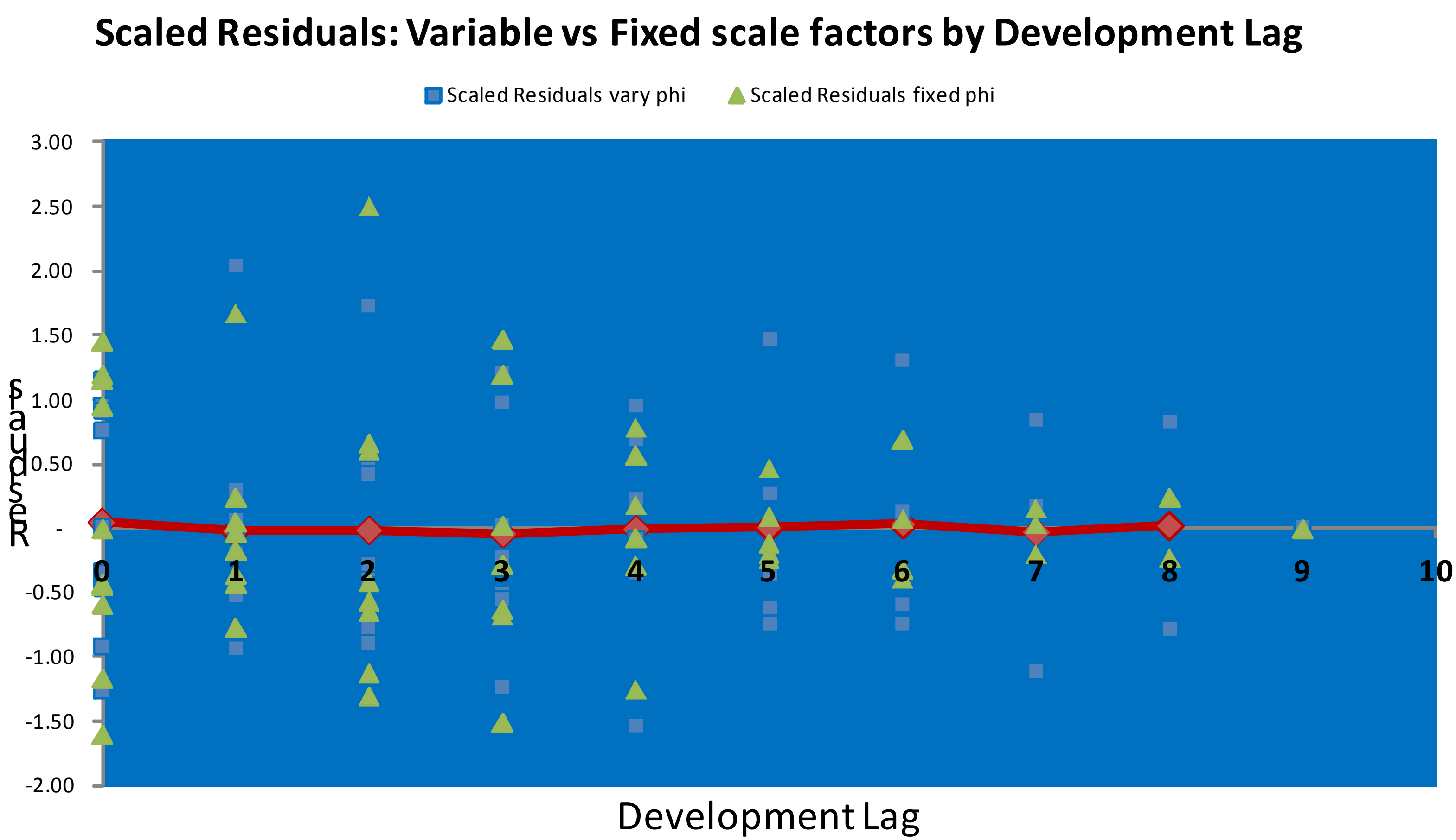

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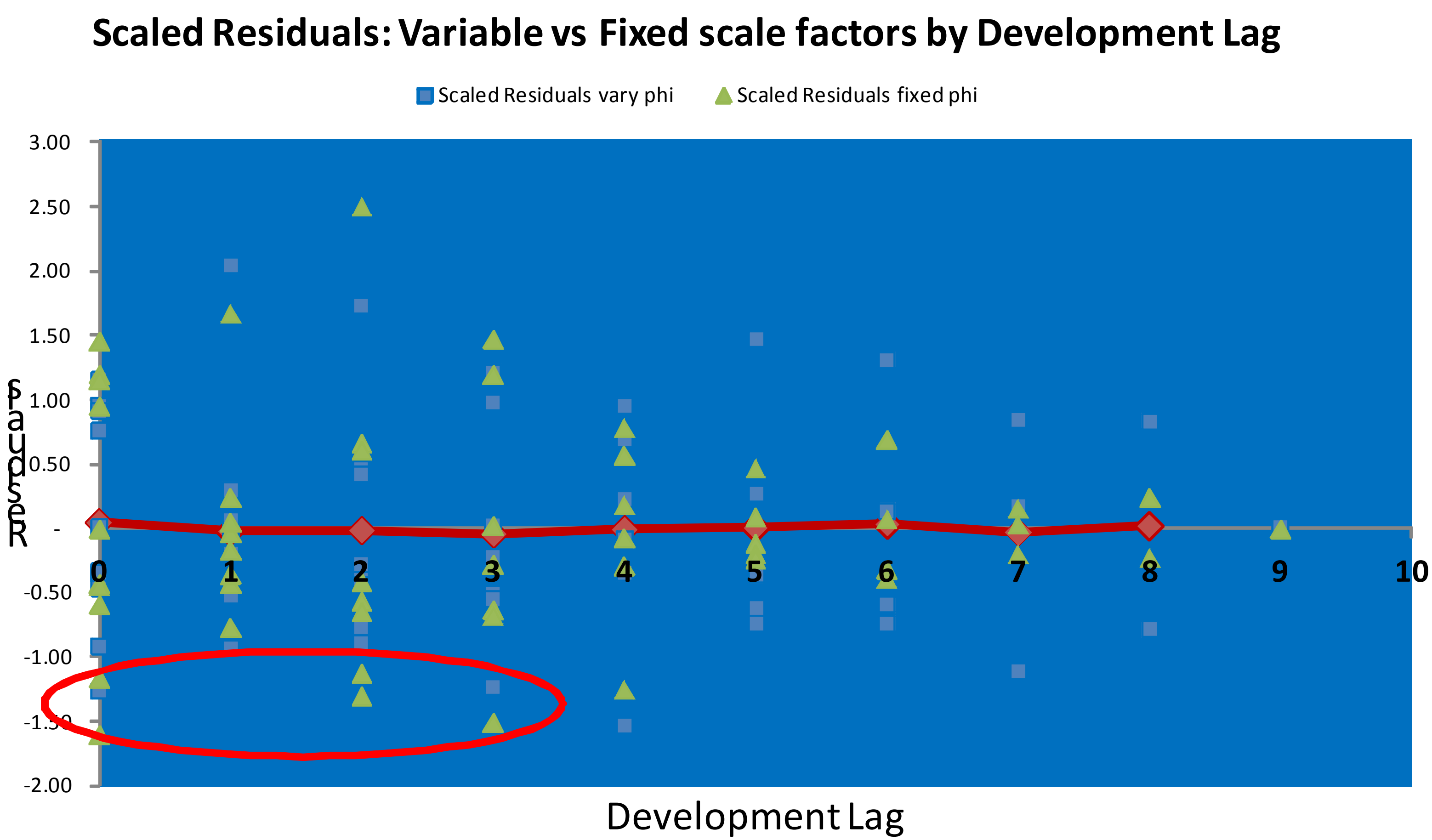

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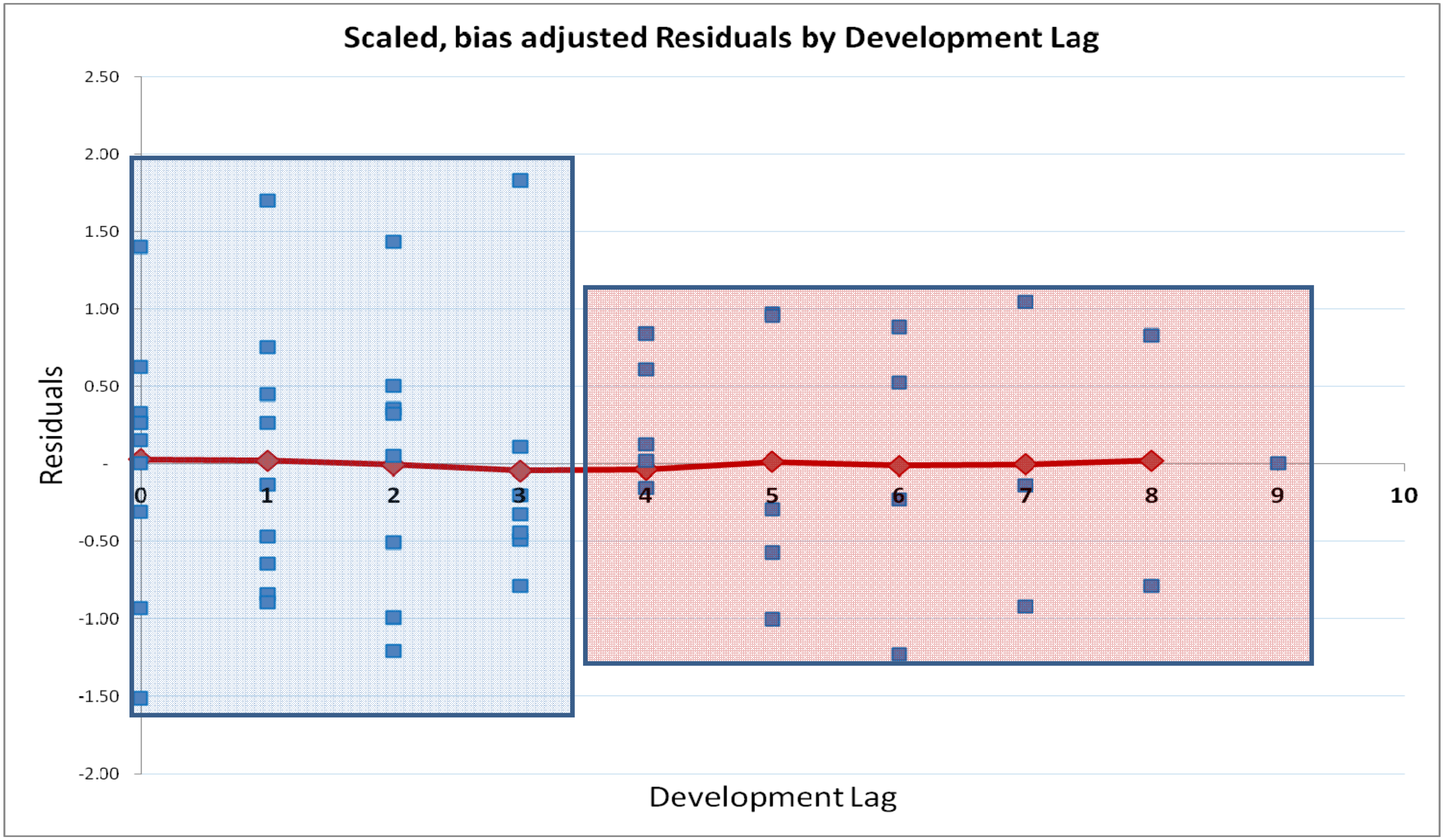
Bootstrap Problems: Non “iid” residuals

- Assumption is of “iid”
  - Invalidates re-sampling procedure if not true
  - And points to potential problem with fitted model
- Solution is stratified sampling
  - Constrain the re-sampling procedure so that only residuals from selected strata can be sampled when generating pseudo claims for that corresponding strata

Bootstrap Problems: Non “iid” residuals



Bootstrap Problems: Non “iid” residuals

	0	1	2	3	4	5	6	7	8	9
0	-0.32	1.695	-0.515	-0.498	0.016	0.967	-1.233	1.041	-0.795	0.000
1	1.395	-0.845	0.351	-0.793	-1.678	-0.298	-0.234	-0.927	0.823	
2	-0.939	0.751	0.322	0.105	0.607	-1.007	0.880	-0.144		
3	0.328	0.445	-0.996	-0.449	0.124	0.950	0.518			
4	-1.520	-0.898	1.426	1.828	0.838	-0.577				
5	0.262	-0.139	0.044	-0.331	-0.156					
6	0.147	-0.650	0.496	-0.206						
7	0.623	0.263	-1.212							
8	0.260	-0.474								
9	0.000									

Bootstrap Problems:

Missing data, Outliers, Zero Residuals

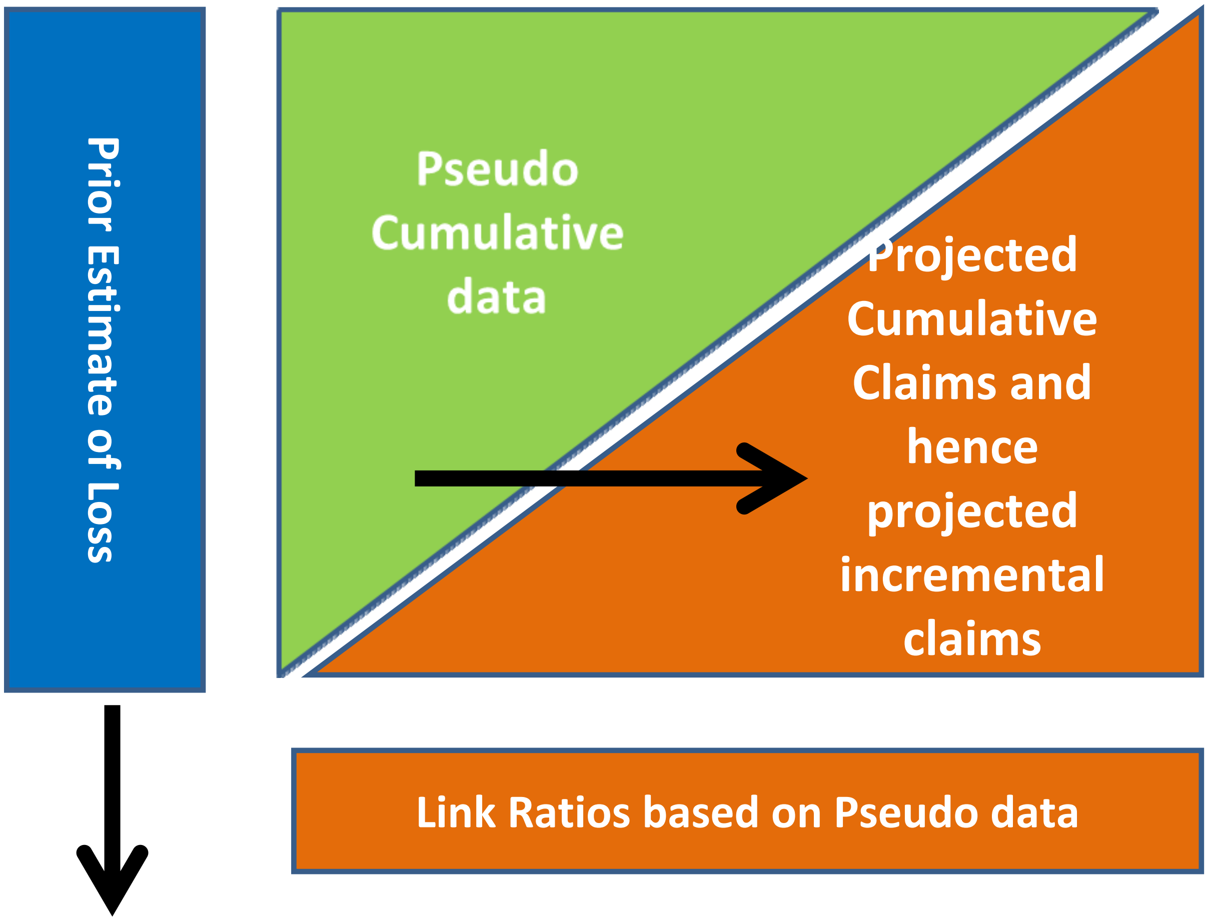
- Missing Values
  - Ignore, and delete from re-sampling procedure
  - Or impute
- Outliers
  - Tricky deciding what is an outlier, whether it is a feature of the data or something that can be deleted
- Zero residuals
  - Will occur in the corner of the triangles so remove
- Can also remove zero residuals from the re-sampling process as argue the model is over-parameterised at that point
- Will inevitably need some rescaling process to target actual booked reserves
  - Additive: shifts distribution
  - Multiplicative: stretches distribution

Bootstrap Problems:

Immature years and credibility weighted priors

- Bootstrap as described here is susceptible to same problems as chain ladder method
- Can be overcome by incorporating a credibility type step into the procedure e.g. BF or Cape Cod

Bootstrap: Process #8 with BF



This must include some element of stochastic variability e.g. AR(2) process

Bootstrap Problems:

Immature years and incorporating credibility weighted priors

- Will most probably reduce the variability of the estimate in the most recent years
  - Unless the variability associated with the prior estimate is large
- Prior variability can include
  - Correlated effects
  - Or model as a time series to reflect underwriting cycle a.g. ARMA process

Bootstrap Useful output for Solvency 2

- Payment patterns
  - General consistency with reserves in each simulation
- Can “industrialise” the process
  - Speed and facility of delivery of estimates to stakeholders
- Precursor calculation into the “claims development result”
  - which gives us a one-year view of risk

One Year View

Modelling the volatility of the Claims Development Result (CDR)

$$CDR_{t+1} = (R_t - X_{t+1}) - R_{t+1}$$

In English:

How much are the reserves liable to move in one year (after allowing for claims paid in the period)?

- E(CDR) = 0
- Want variability of the CDR

Ultimo versus One-year

How much are the reserves are liable to move in one year (after allowing for claims paid in the period)?

Ultimo

– estimating the volatility in the **claims**

One-year

– estimating the volatility in the **reserves**

Reserve changes

Sources of change (Wacek):

- Year-end claims different from expected
- Extra claims experience may result in different selection of development factors

But also (White and Margetts):

Actuaries will take into account information not contained in the triangle

Example – inflation

- Period of high inflation
- Reserving – take partial credit
- Underlying uncertainty has increased
  - how to capture this increased uncertainty

Long-tail classes

- Most susceptible to high inflation
- Very little information emerges over one-year

But

- When inflation is recognised hits all years at once
- Double (triple?) whammy – new business is under-priced (and BF *a priori* loss ratios are revised upwards)

Question

- Can the one year volatility be greater than the ultimo volatility?

Mathematically

– no

Intuitively

– no, otherwise reserving process is adding uncertainty

(One exception, when duration of liabilities < 1; in this case the "additional" risk is more likely to be captured as premium risk)

One-year approaches

- Merz-Wüthrich
- “Actuary-in-a-Box”
- Time-scaling
- Ultimate unadjusted

Merz Wüthrich

- One-year version of Mack method
- Gives an unbiased estimator of the standard deviation of the CDR
- User then chooses distribution to apply (recall E(CDR) = 0)
- Assumes triangle is run-off – workarounds available
- Dependency between AYs – user can overlay own dependency structure

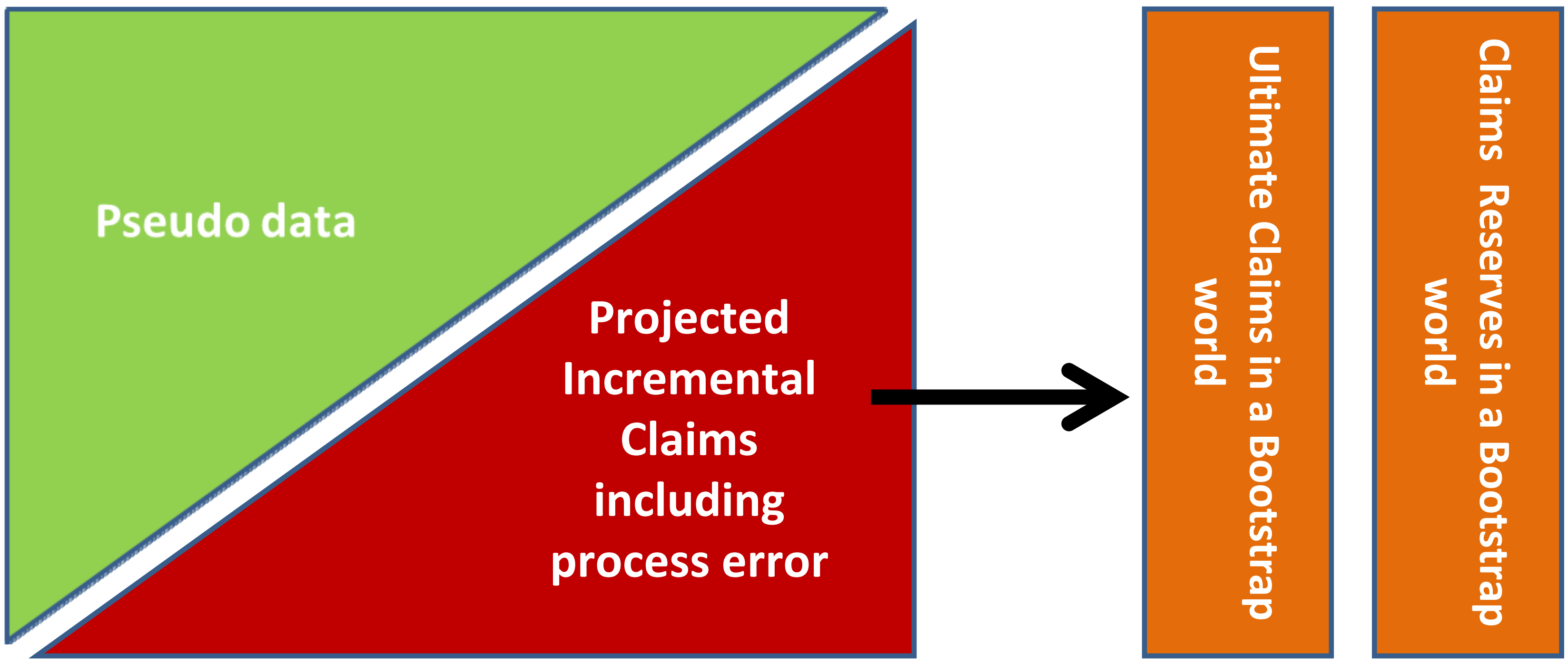
“Actuary-in-a-Box”

- Many variations

Bootstrap approach:

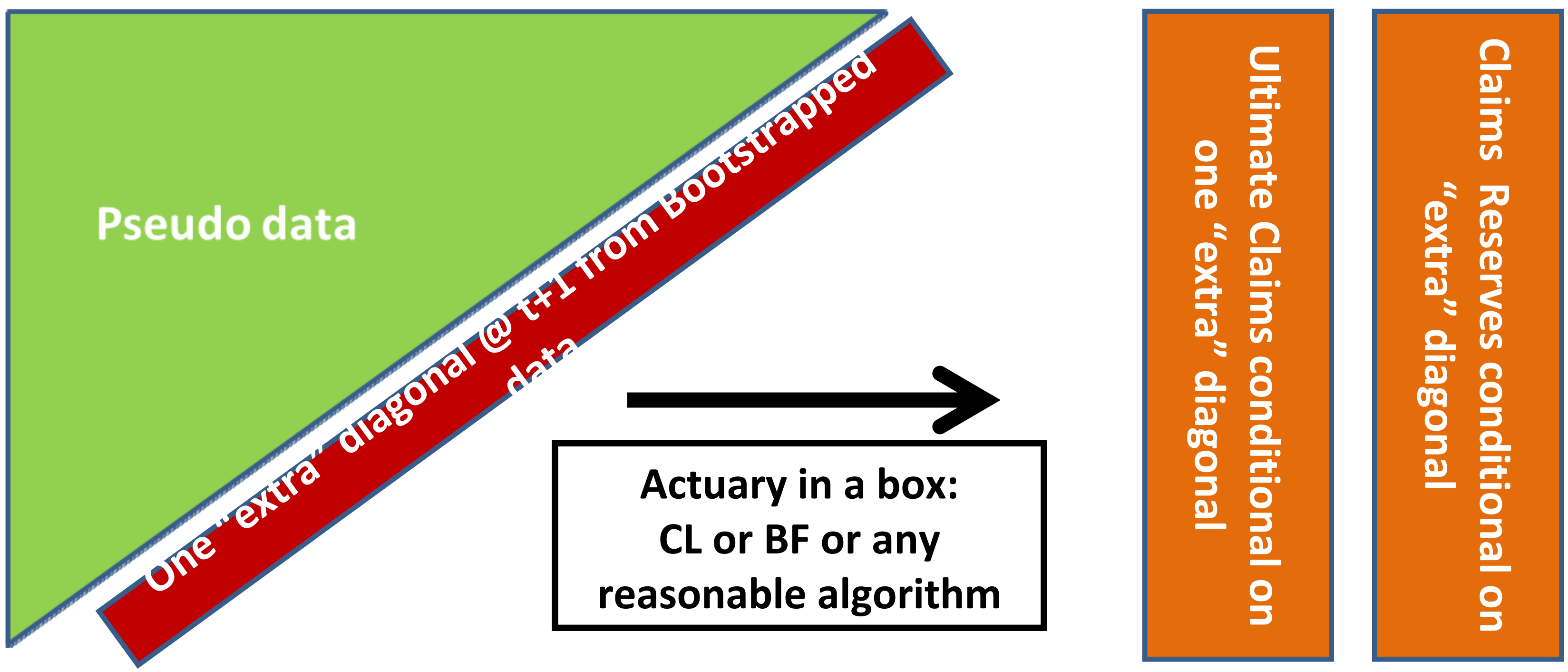
- Take the next diagonal from the simulated lower triangle
- Re-reserve at the end of the next period (using mechanical method)
- Gives a simulated CDR for each iteration of the Bootstrap

Bootstrap: Process #9 a quick recap



$$\Gamma\left(\hat{m}^{B}_{i,j},\ \phi_j \hat{m}^{B}_{i,j}{}^{z}\right)$$

Bootstrap CDR: Process #10 ("Actuary-in-a-Box")



$$CDR^{B}_{t+1} = \left(R_t - X^{B}_{t+1} - R^{B}_{t+1}\right) = \left(U_t - U^{B}_{t+1}\right)$$

Time-scaling

- Derive a Capital Signature for the reserve run-off
- From this, derive a duration λ for the capital run-off
- $p = 0.995^{\lambda}$
- Take the corresponding percentile p from the ultimate distribution

Ultimo unadjusted

- Tried and tested methods
- No additional work, systems already in place
- May be appropriate for some undertakings

In practice

- Variety of methods in use
- Expert Judgement and other factors:
  - Choice of method
  - Segmentation
  - Treatment of outliers, exclusions
  - Aggregation across lines of business
  - Aggregation across territories

One-year versus Ultimo

Wacek:

"…a large proportion of the potential variation in ultimate estimates can be present in the first year of future development"

"Actuaries have done a good job in getting clients to understand that ultimate loss estimates are subject to large potential variation, but many clients seem to expect that variation to emerge only in the distant future, if at all"

Results & Comparisons

		Ultimo			One-year		One-year / Ultimo	
		Mack (Includes tail load)	Bootstrap - ODP	Bootstrap - Mack	Merz-Wuthrich	Bootstrap - CDR	Merz-Wuthrich as a % of Mack	Bootstrap - CDR as a % of Bootstrap ODP
Property Reinsurance	Variability €'000	64,906	60,947	64,848	57,756	53,749		
	CoV	22.2%	20.9%	22.2%	19.8%	18.4%	89%	83%
Motor	Variability €'000	124,417	211,119	176,453	89,556	114,856		
	CoV	6.9%	11.7%	9.8%	5.0%	6.4%	72%	65%
Liability	Variability €'000	276,365	501,309	518,564	122,200	331,070		
	CoV	7.4%	13.4%	13.9%	3.3%	8.8%	44%	64%

Sources and References

- Useful papers
  - http://www.casact.org/pubs/forum/08fforum/21Merz_Wuetrich.pdf
  - http://www.cassknowledge.com/sites/default/files/article-attachments/371~~richardverrall_-_predictive_distributions_of_general_insurance_outstanding_liabilities.pdf
  - http://www.casact.org/pubs/forum/10fforum/ShaplandLeong.pdf
  - http://www.casact.org/pubs/forum/07wforum/07w345.pdf

- Data (publicly available)
  - Insurance Blue book available at www.centralbank.ie
  - ACE http://investors.acegroup.com/phoenix.zhtml?c=100907&p=irol-reportsannual#STS=g76ftr6z.19zv
  - AXIS http://www.snl.com/irweblinkx/ShowFile.aspx?Output=XLSX&KeyFile=1001159354&Format=XLSX
  - Partner Re http://www.partnerre.com/inc/docs/content/downloads/PartnerRe_Loss_Development_Triangles_2010.pdf
  - XL http://phx.corporate-ir.net/External.File?item=UGFyZW50SUQ9NDI1ODc1fENoaWxkSUQ9NDQxMzA4fFR5cGU9MQ==&t=1

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Questions?