KPMG

FINANCIAL SERVICES

Aggregation

Presentation to Society of Actuaries, Dublin

5th April 2011

AUDIT ▪ TAX ▪ ADVISORY

Agenda

- Key Stakeholders
- Key Risks
- Dependency and Modelling
- Copulas
- Practical Aggregation
- Algorithmics Case Study

Key Aggregation (Diversification) Stakeholders

Customer

Aggregation of **independent risks** (e.g. term life insurance) **creates value** for the insurance consumer.

Aggregation of risks **within a product line or between entities** can **creates value** for the consumer if they manifest as **lower premiums.**

Shareholder

Aggregation can deliver shareholder value by allowing companies to operate from a **lower capital base**.

Regulators

Regulators will be concerned over whether aggregation of risks leaves **enough capital** within businesses to **protect consumers**.

They may have concerns over whether the aggregation is **conservative enough**.

Governments

Governments may be concerned over aggregation between firms could mean multiple failures and **political pressure** to **compensate consumers.**

But they also want a competitive insurance market delivering **affordable insurance** for their consumers and (possibly) to **generate exports**

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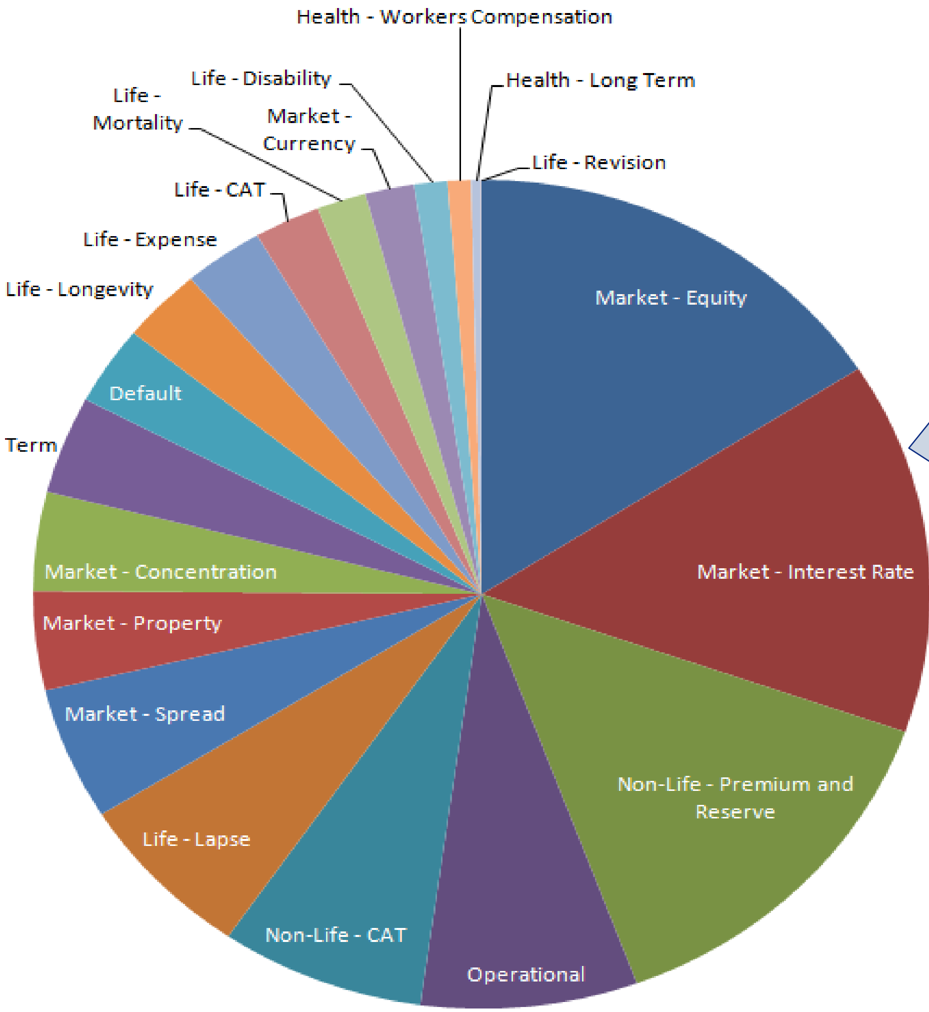
Risk Types – EU Insurance Risk Profile - QIS-4 (2008)



- Under QIS-4 (31 Dec 2007) the EU Insurance Sector was dominated by market risks.
- The largest non-market risks at non-life premium and reserve
- OpRisk, CAT Risk and Lapse Risk and also significant

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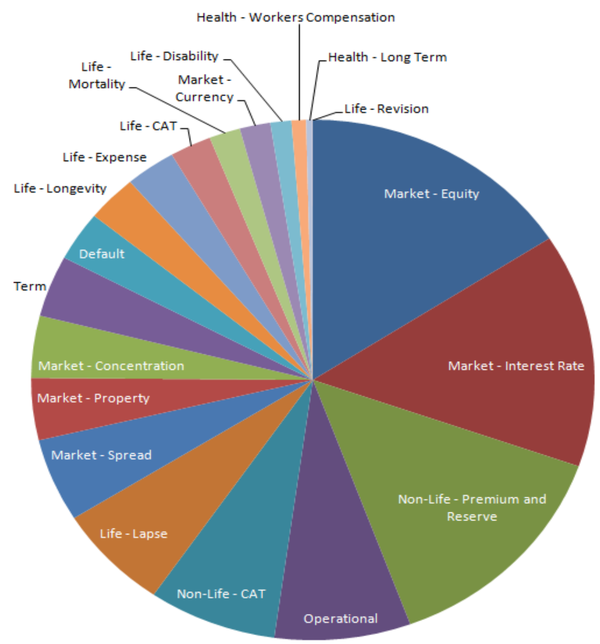
Risk Types – EU Insurance Risk Profile – Typical Internal Model



Market

- Equity
  - Indices / Implied Volatilities / Dividend Yield
- Nominal Yields
  - Level / Slope / Twist
- Property / Real Estate
  - Indices / Rental Yield
- Inflation
- FX

Credit

- Spread
- Default
- Downgrade

Underwriting / LOB

- Claims & Premium
- Lapse
- Longevity / Mortality
- Morbidity
- Expense
- NAT-CAT
- LIFE-CAT

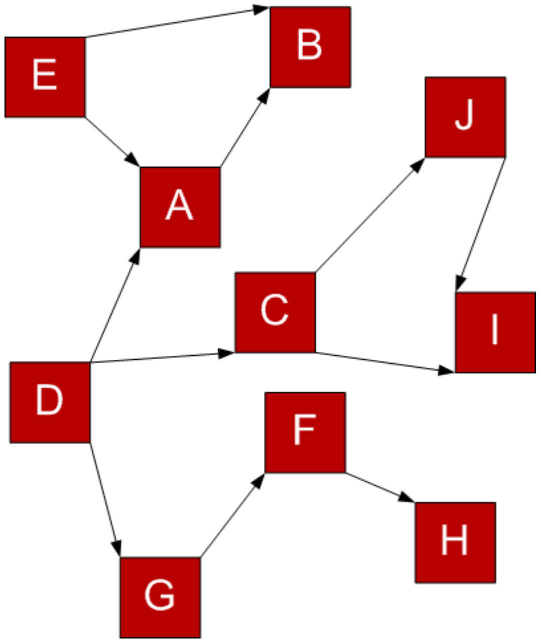
Business Unit Specific

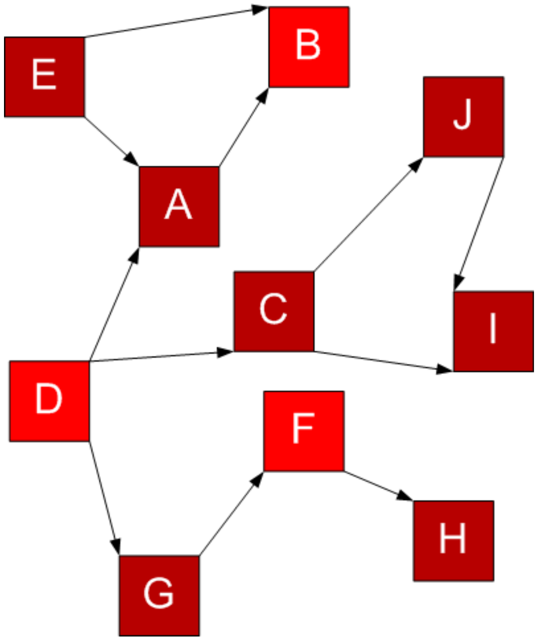
Operational Risk

Causation : Event Dependency

•Each block below represents an **event**. Some events lead naturally to **increased / decreased probability** of other events.
•Events dependency has a **time dimension** which further complicates the inter-event relationships.
•Building a model of the world can be **challenging** if instructive.

•Some events can **observed more easily** while others cannot.
•Some events are more clearly **drivers of risk**.





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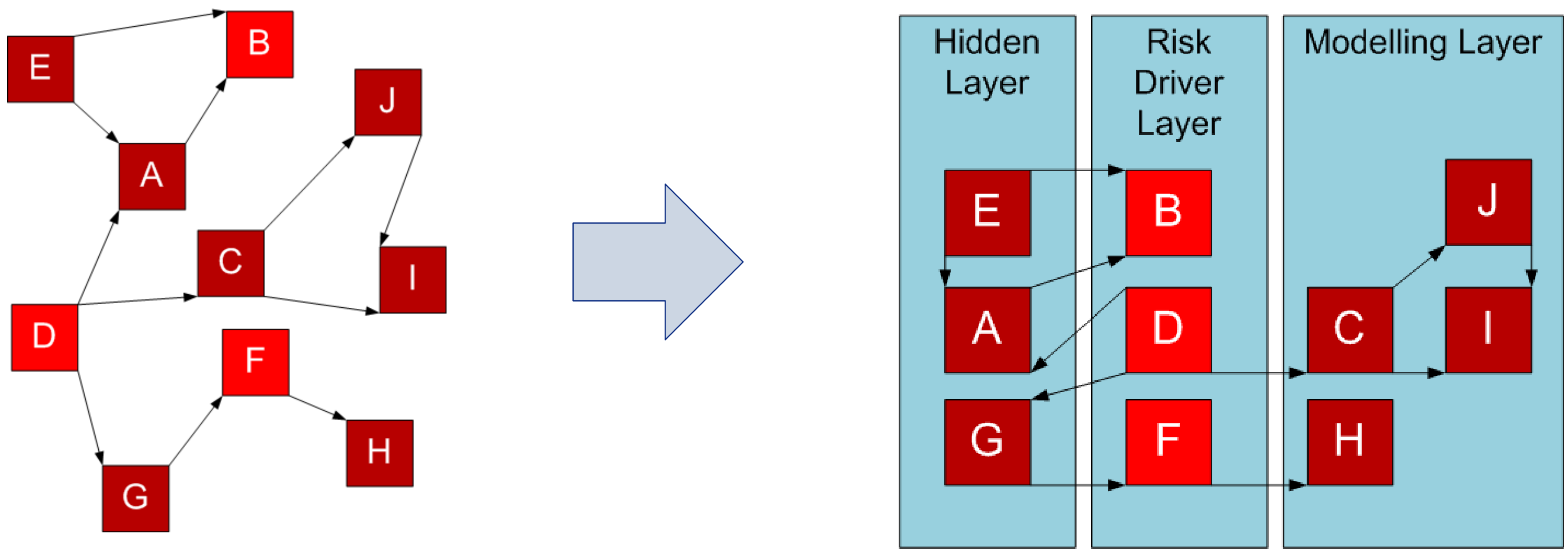
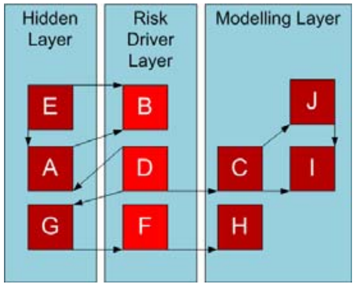
From Causation to Association Modelling

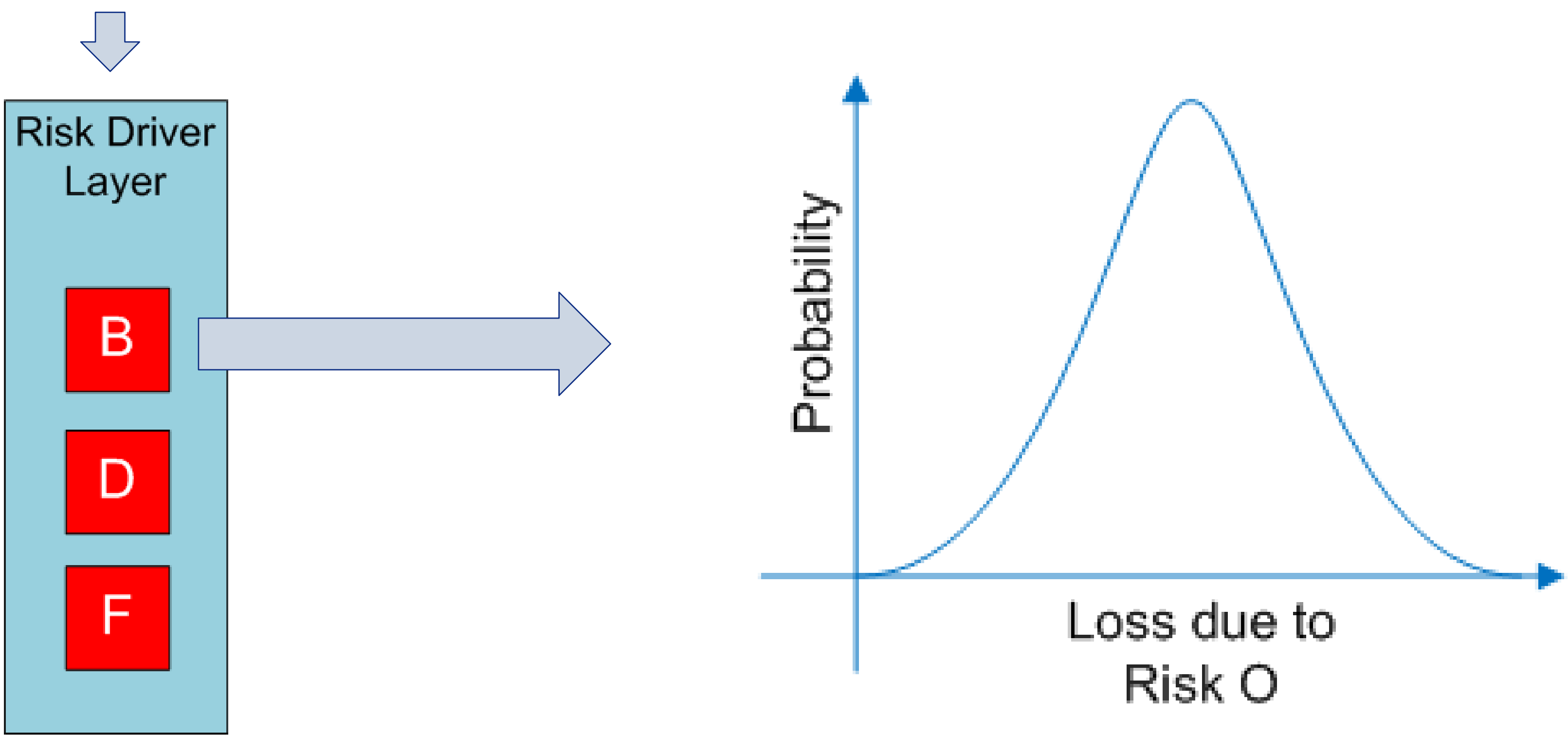
•Typically firms simplify their models by using risk factors considered to be **observable / quantifiable**. This could be termed the **risk driver layer**.
•Subsequent (dependent) events that are caused by the risk drivers can be captured in a **modelling layer**.
•But in doing so we lose some information to a **hidden (fundamental) layer** of the modelling. Changes in this layer will impact our models (in particular aggregation) through the need for re-calibration.



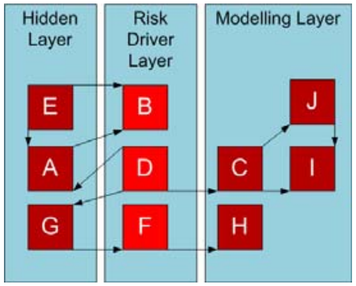
Considerations in the design of a multi-variate risk distribution



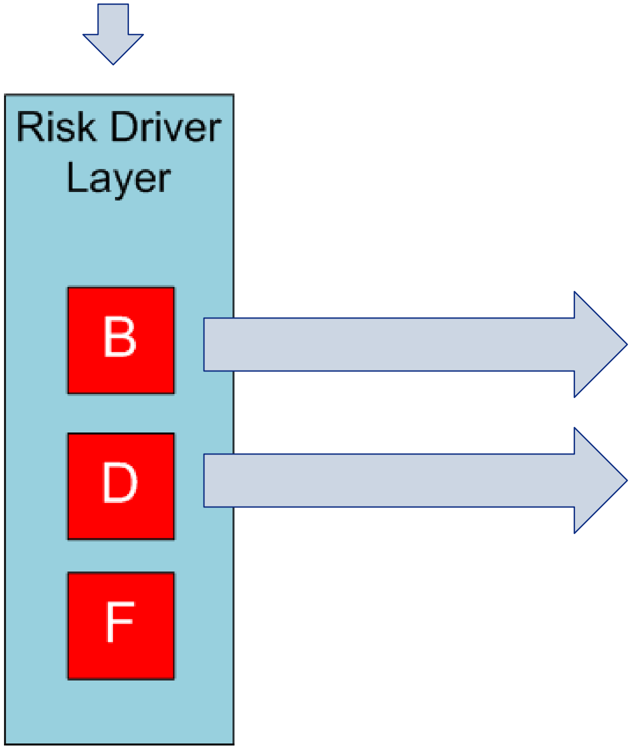
•Each risk driver will have a **marginal distribution** – typically estimated using **historical data** – although many risk factors have limited data available so **expert judgement** is in evidence in calibration.

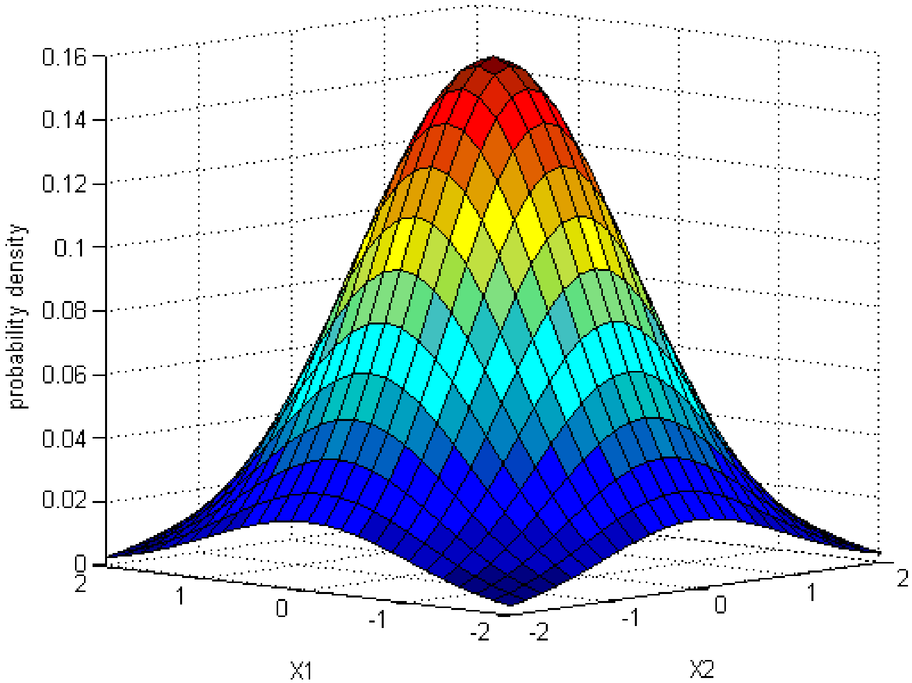


Considerations in the design of a multi-variate risk distribution



•Pairs of risk drivers will have a **joint distribution** – again typically estimated using **historical data** –and again many risk factors have limited data available so **expert judgement** is in evidence in calibration.

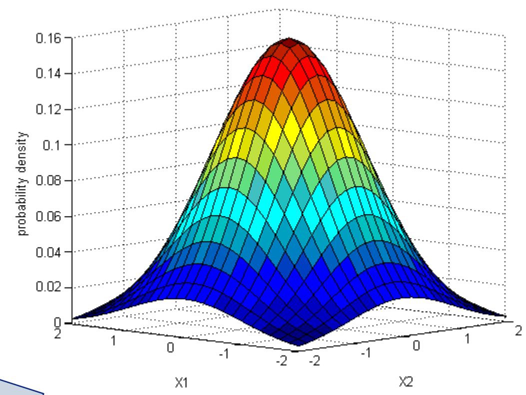




Considerations in the design of a multi-variate risk distribution

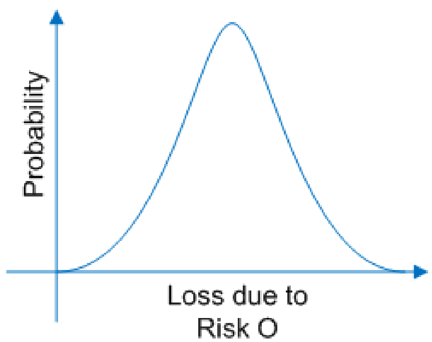
•A multi-variate distribution can be broken into component parts.

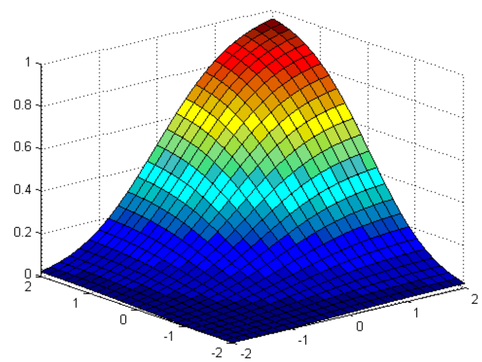
•This bi-variate distribution can be separated into **2 marginal distributions**…

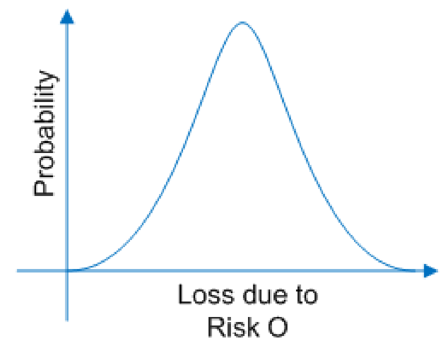


… and a **copula function** which represents the **dependency** between the two risk factors.

"Copula functions are to aggregation what distributions functions are to random variables."

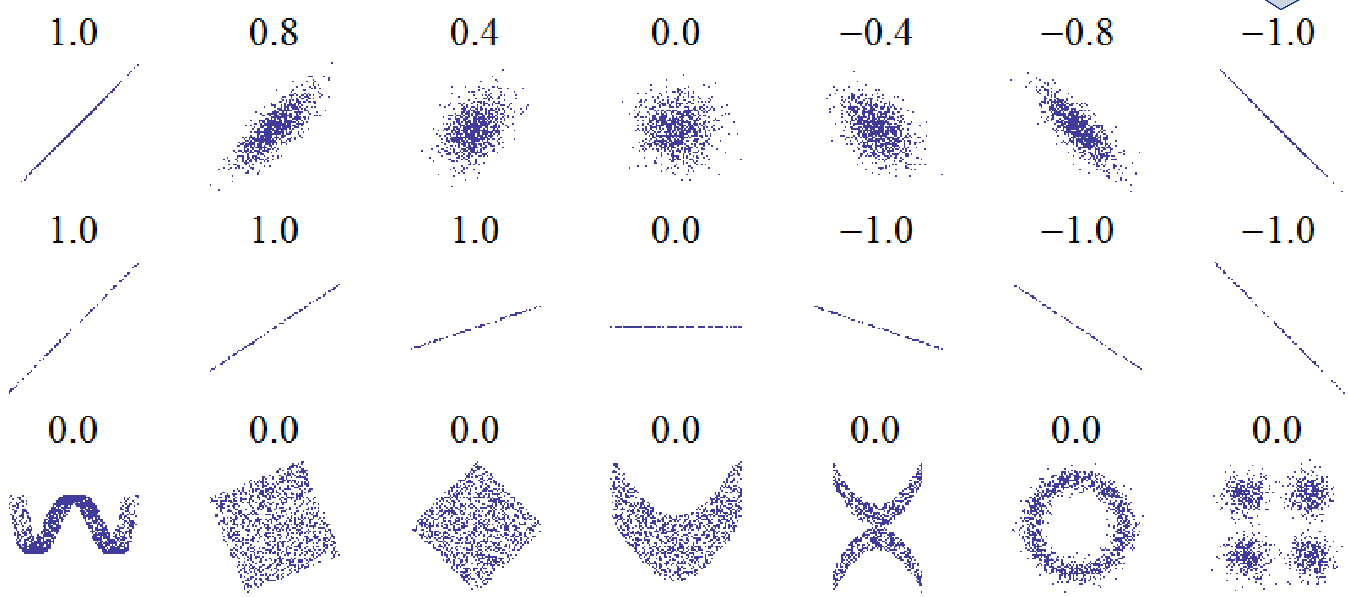




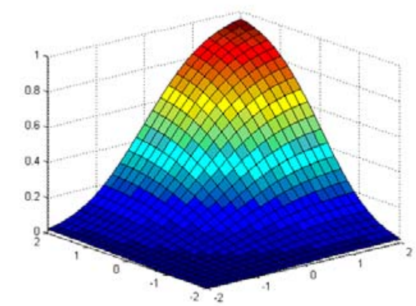


Why (Pearson) Correlation and Dependency are Different

•All the **numbers** below are (Pearson) **correlation coefficients** between 2 risk drivers..
•But the **dependency** can be **very different** as these extreme example illustrate.
•For **complex dependency** a **copula function** would be required.
•For **simple dependency** a **correlation matrix** can suffice.



$$\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$



Types of Copula

Fundamental Copulas

Monotonic
Counter-Monotonic

Empirical Copulas

Flexible

Challenging in Many Dimensions

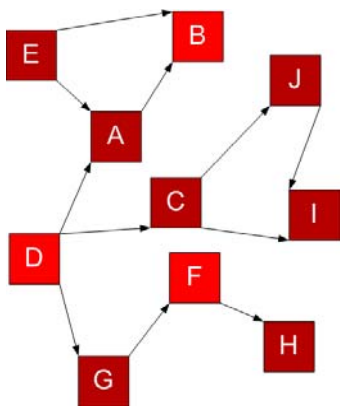
Explicit Copulas

Gumbel
Clayton

Implicit Copulas

Gaussian
t-Copula

Tractable



Practical Aggregation – Stress & Correlation Matrix

•**Stresses** applied to risk factors. Typically these stress factors represent the probability of an event happening over **1 year** with **1-in-200 probability**.

•Stresses could be -40% **shock to equity**, parallel **shock to the yield curve** or a **longevity improvement**.

•Each stress is applied and the **change in net assets** due to the stress is calculated.

•This are arranged in a **vector** as shown.

$$\begin{pmatrix} 58 \\ 47 \\ 21 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0.5 & 0 \\ 0.5 & 1 & 0.25 \\ 0 & 0.25 & 1 \end{pmatrix}$$

•A **correlation matrix** is defined to describe the dependency between the risk factors.

•A **Gaussian copula** is assumed when using this correlation matrix approach.

•**Practical issues** involve ensuring that the matrix has the right mathematical properties (i.e. that it is **Positive Semi-Definite**).

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Practical Aggregation – Stress & Correlation Matrix

•The aggregation is done using some **simple matrix algebra** as shown below.

•The resulting capital **96.1** is less than the sum of the capital requirements **126** giving a **diversification benefit** of:

126 - 96.1 = **20.9**

•**Key assumptions** underlying this type of aggregation are:

- •A) the risks are linked by a Gaussian copula
- •B) the capital requirement varies linearly with each risk type.

$$\begin{pmatrix} 58 \\ 47 \\ 21 \end{pmatrix}$$

$$\sqrt{\begin{pmatrix} 58 \\ 47 \\ 21 \end{pmatrix}^T \begin{pmatrix} 1 & 0.5 & 0 \\ 0.5 & 1 & 0.25 \\ 0 & 0.25 & 1 \end{pmatrix} \begin{pmatrix} 58 \\ 47 \\ 21 \end{pmatrix}} = 96.1$$

$$\begin{pmatrix} 1 & 0.5 & 0 \\ 0.5 & 1 & 0.25 \\ 0 & 0.25 & 1 \end{pmatrix}$$

Practical Aggregation : What do firms do internal model?

•This matrix using **Red** for correlations nearer to **1** and **Green** for correlations nearer to **-1**.

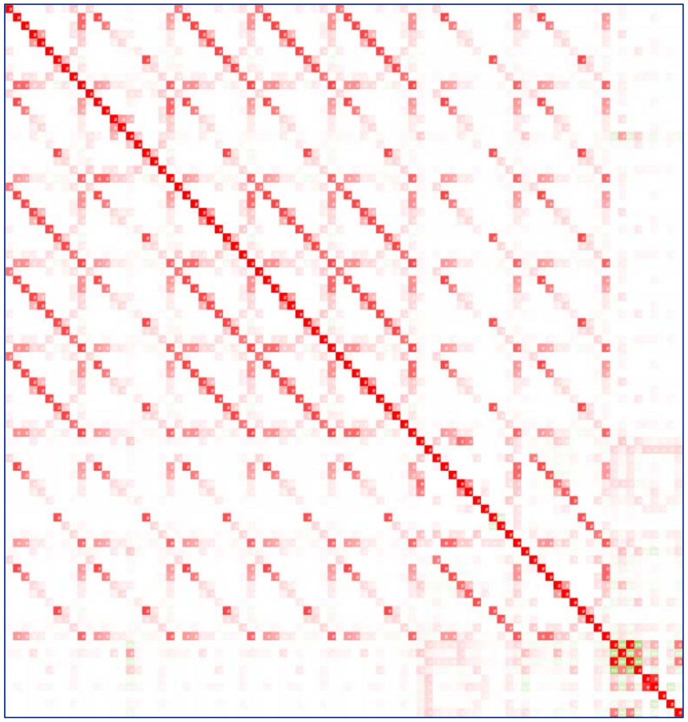
•**This is 1/25 of full matrix!**

•Groups typically have **large numbers of risk drivers** to
•consider. 500+ is not untypical.

•The volume of data leads insurers to **tractable methods** to capture dependency.

•Several **Practical Challenges** even with Gaussian Copula:

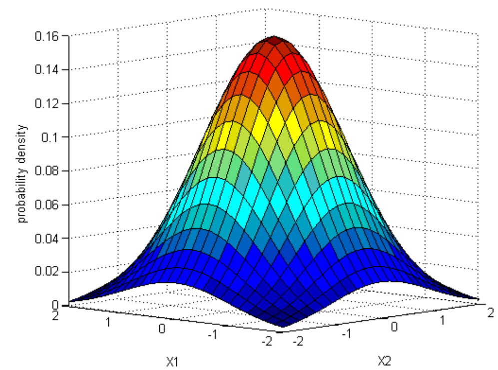
- •Populating the matrix with **robust correlations** even where little data exists.
- •Ensuring the matrix has the right mathematical properties – ie is **positive semi-definite**.
- •**Governance** of the matrix at Group and BU levels.

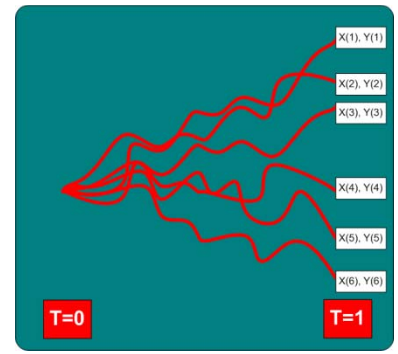


End to End Process – Stage 1 – Risk Driver Generation

	Mean	Volatility
Risk A	μ(A)	σ(A)
Risk B	μ(B)	σ(B)

	Risk A	Risk B
Risk A	1	ρ(A,B)
Risk B	ρ(A,B)	1





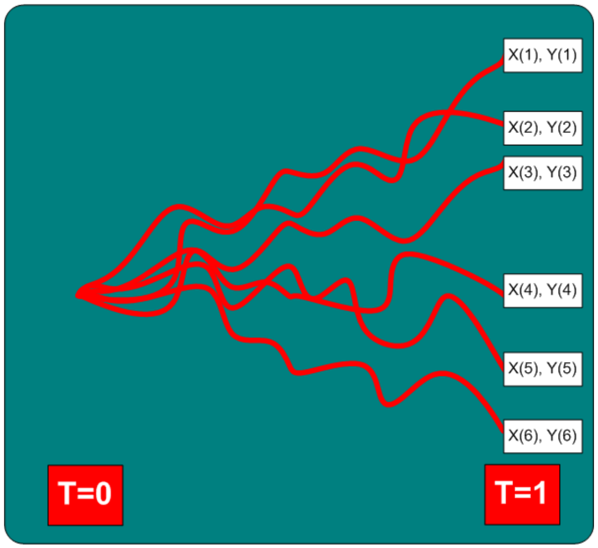
The **statistical properties** of the risk drivers are calculated or otherwise set. Higher order moments / empirical distributions may be calculated too.

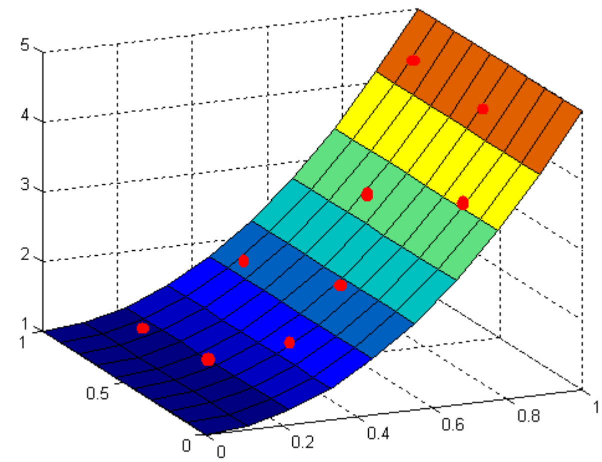
The statistical properties form a **multi-variate (**typically **Gaussian**)

This becomes distribution for the **risk drivers** that the firm is **exposed** to.

The multi-variate distribution is used to create **many scenarios** for the risk drivers respecting the measured statistical properties **as far as is possible using** the chosen distributions.

End to End Process – Stage 2 – Net Asset Calculation

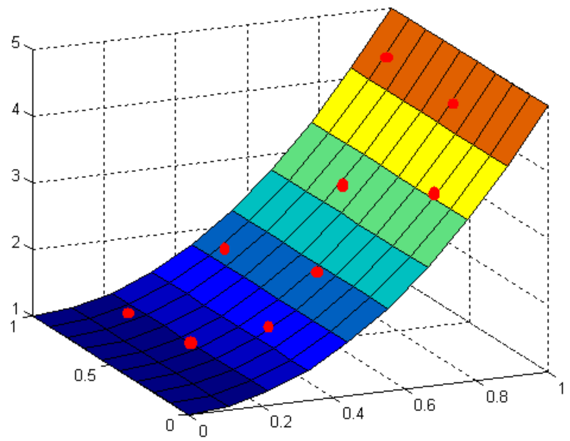


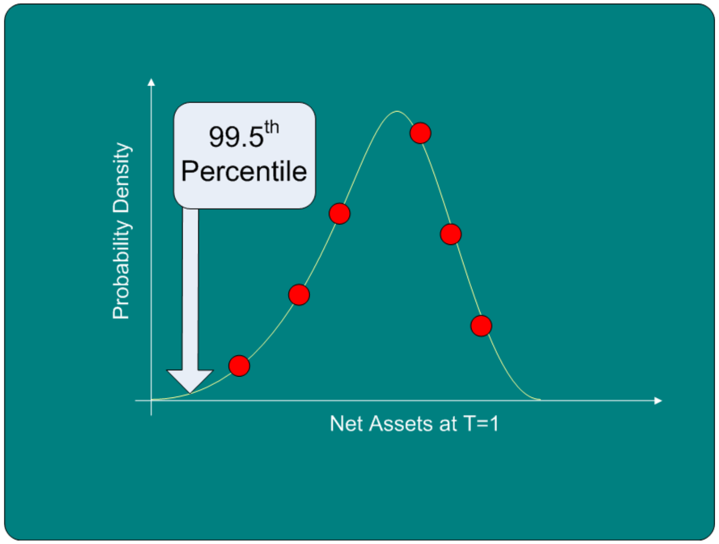


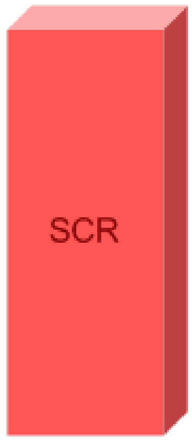
Each stochastic scenario needs to be converted into a net asset position. This is done by taking the risk driver values in each scenario and using a **non-linear function** to convert the position of all the risk drivers into valuation. Capturing non-linearity is a **key feature** of the aggregation.

Ideally the *function* would be the ALM (MCEV) model used to calculate the time zero net assets but this would in many cases lead to a **nested stochastic solution.** Instead the two approaches most widely used in the market are **replicating portfolios** or **curve fitting**.

End to End Process – Stage 3 – Ranking Risk







Several thousand (say 5,000 - 100,000) scenarios are run and the net assets calculated.

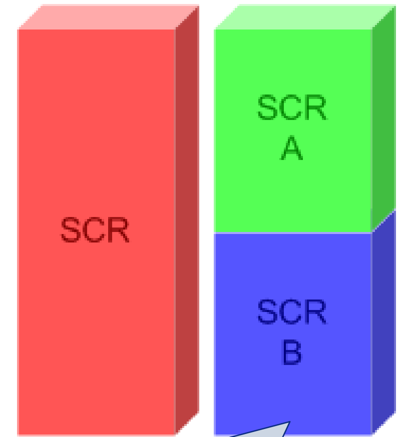
The net assets are **ranked** in order of size.

The ordering leads to a **distribution function** (and **density function**) as shown in the right hand diagram).

Reading off the **99.5% percentile** of the distribution yield the SCR.

De-Aggregation

$$r(\boldsymbol{\lambda}) = \sum_i \lambda_i \frac{\partial r}{\partial \lambda_i}(\boldsymbol{\lambda})$$



The SCR only calculated at solo or group level is of **limited use** as a management tool for **decision making.**

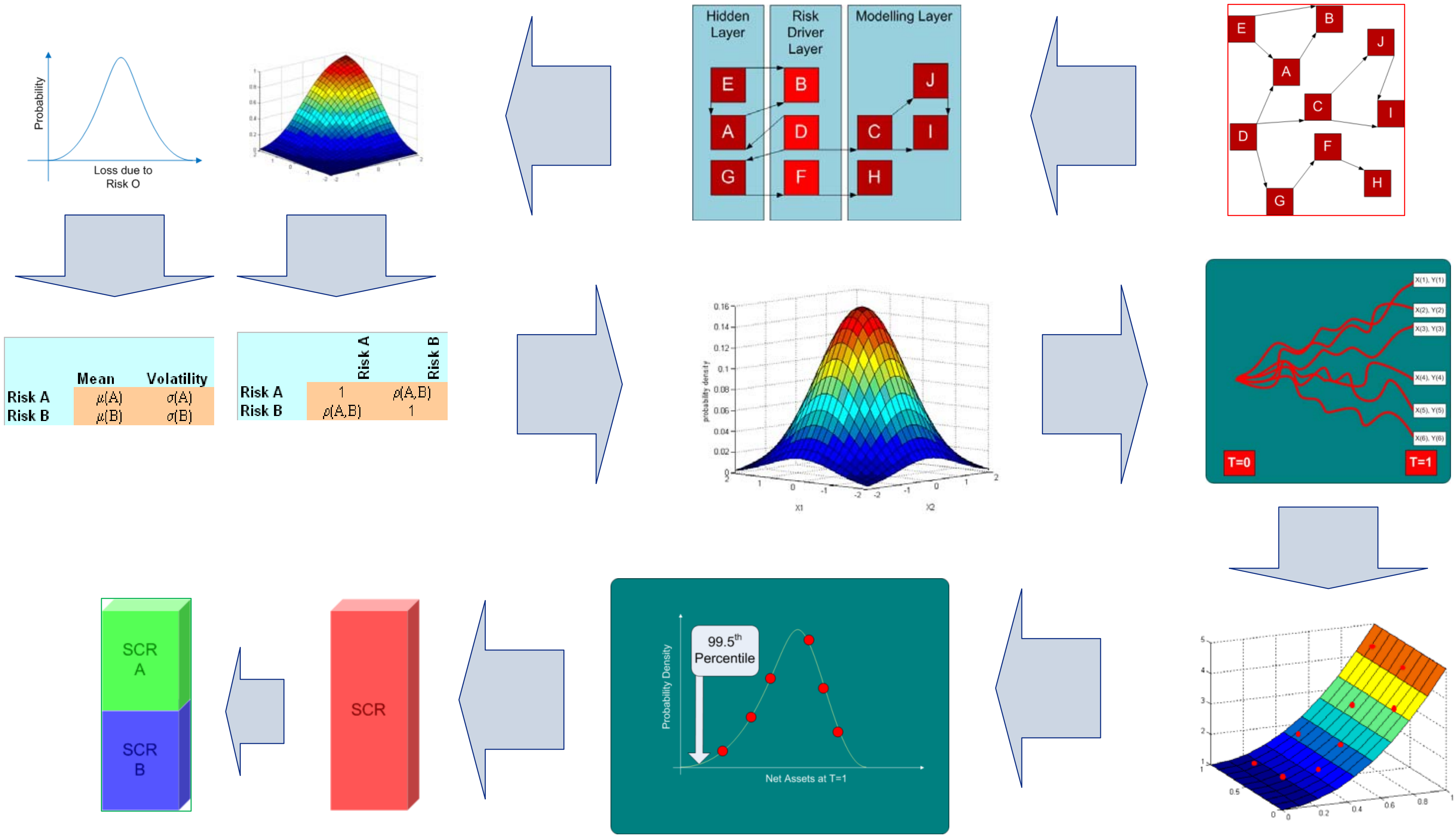
Allocation of capital to Business Units or Product Lines is desirable for **managing a business**.

Allocating capital to ensure **all sub-components add to the diversified capital** requires care.

Euler allocation is typically used to allocate capital so it sums to diversified capital.

However **practical issues** can arise.

Putting it all together



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Thank-You

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