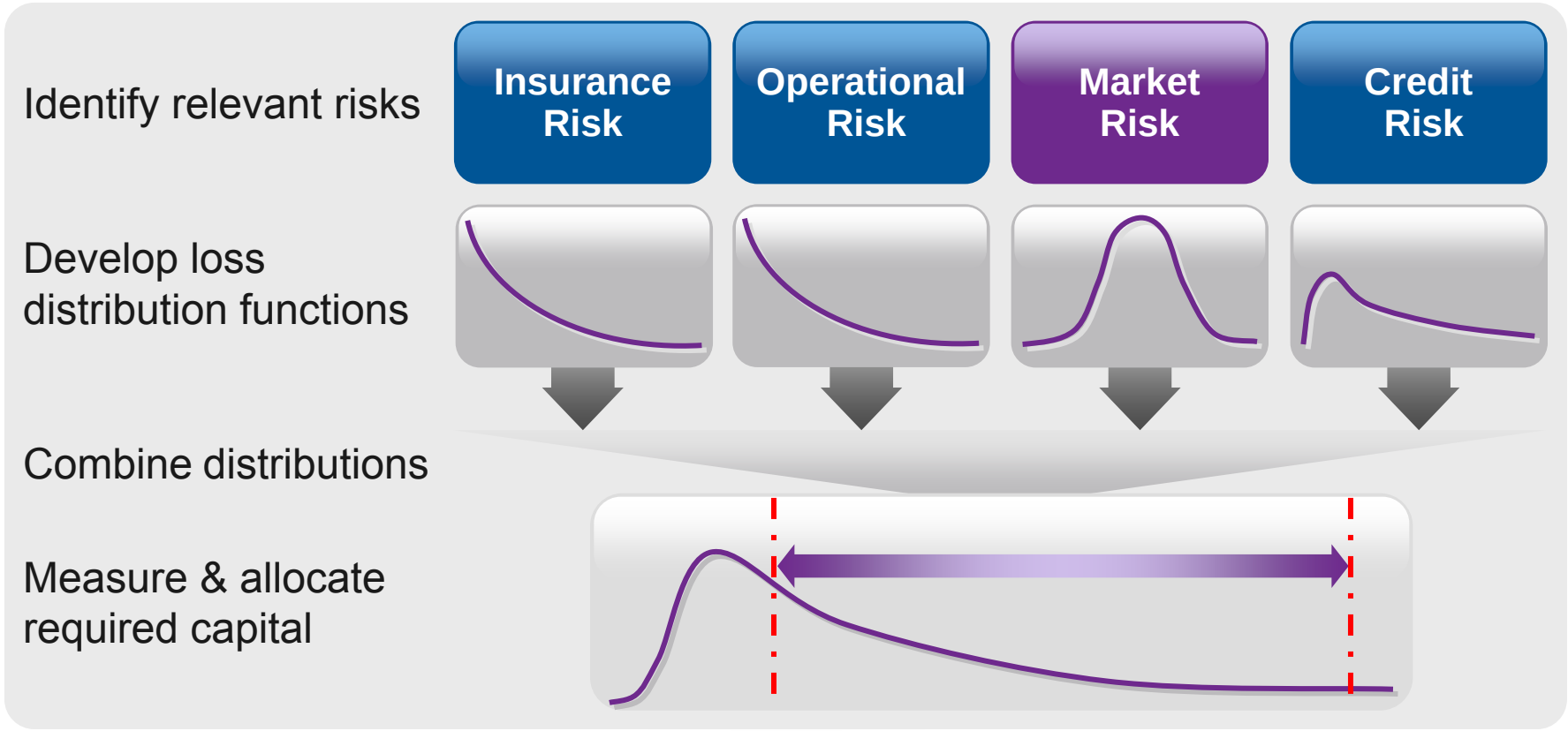
Capital Aggregation and Attribution

Curt Burmeister
Vice President, Risk Solutions

Algorithmics

Copula Based Risk Aggregation



Algorithmics

Copula Based Risk Aggregation

Copula Risk Aggregation Steps

1. Generate Correlated Market Risk Scenarios on IR Curves, EQ Indices, etc
2. Generate Correlated Non-Market Risk Scenarios on N(0,1) Risk Drivers
3. Value assets using market risk scenarios
4. Value non market risks using marginal risk distributions

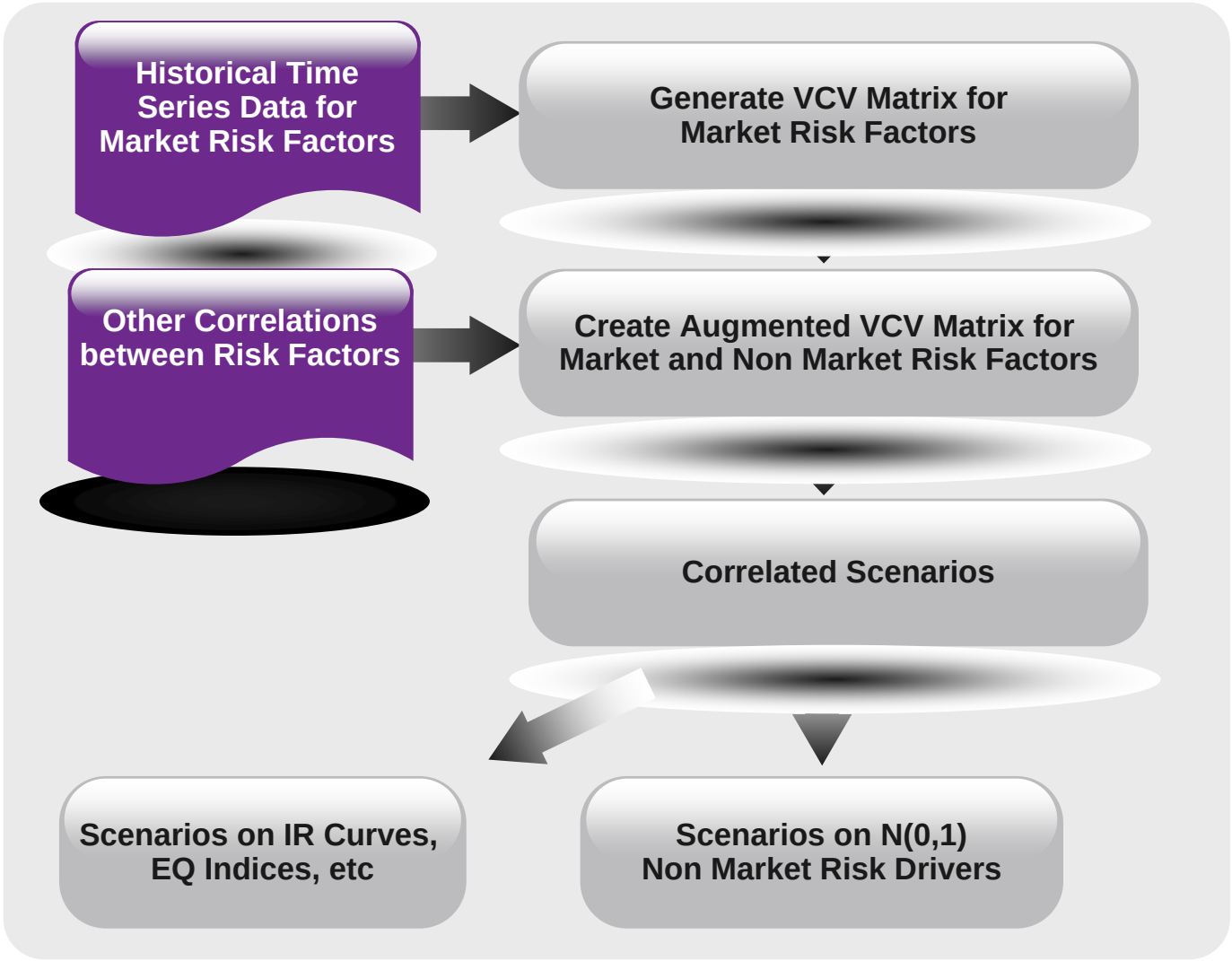
Scenarios are correlated $\Rightarrow$ values are additive

Key Benefits

- Full drill down into Market Risk
- Hedge analysis
- Stress tests

Algorithmics

Scenario Generation



Algorithmics

Capital Aggregation and Aggregation

Concepts

Aggregation Rules

- Determines how much capital each node can transfer to its parent node
- Examples include fungibility rules, caps on transfers, percentage ownership of subsidiary, tax considerations, etc.

Transferable Capital

- The amount of capital for each business hierarchy child node can share with its parent node
- The transferable capital is aggregated up the business hierarchy to calculate SCR

Algorithmics

Concepts

Capital Attribution

- Allocation of capital down the reporting hierarchy from a parent node its children
- Aggregation rules are typical non-linear (max, min, etc)

Methods for Aggregation & Attribution

- Quantile attribution - upper empirical cumulative distribution function value or UCEV (order statistic)
- Smoothed values – use an L-estimator such as Harrell-Davis weights
- Biting Scenario – use an L-estimator to smooth the risk factors to create a new scenario

Algorithmics

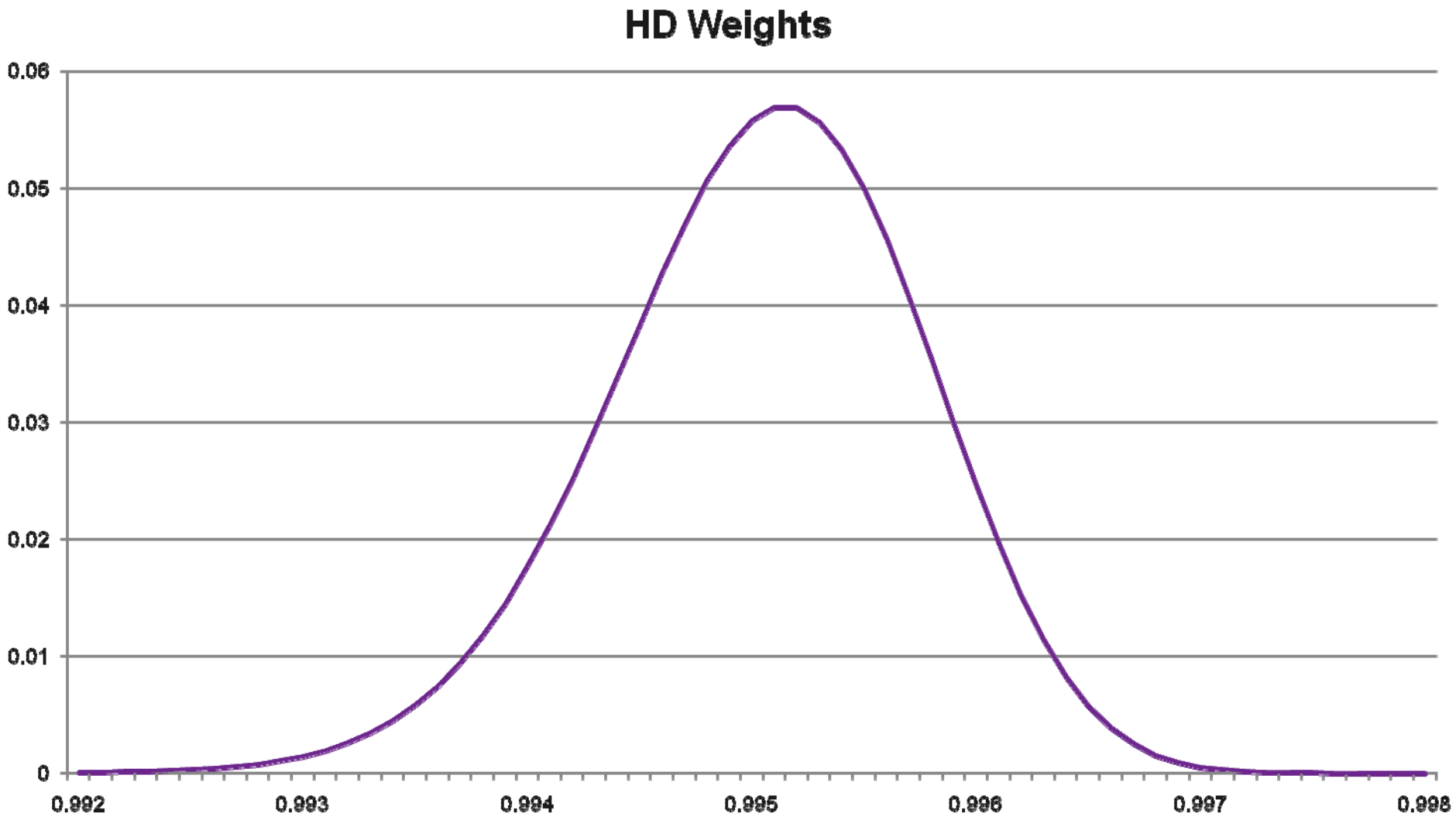
References for Harrell-Davis Weights

Harrell-Davis Weights

- Harrell, EE. And C.E. Davis, 1982, “A new distribution-free quantile estimator,” Biometrika, 69(3): 635-640.
- Mausser, H., 2001, “Calculating Quantile-based Risk Analytics with *L*-estimators,” Algo Research Quarterly, 4(4): 33-47

Algorithmics

Harrell-Davis (HD) Weights



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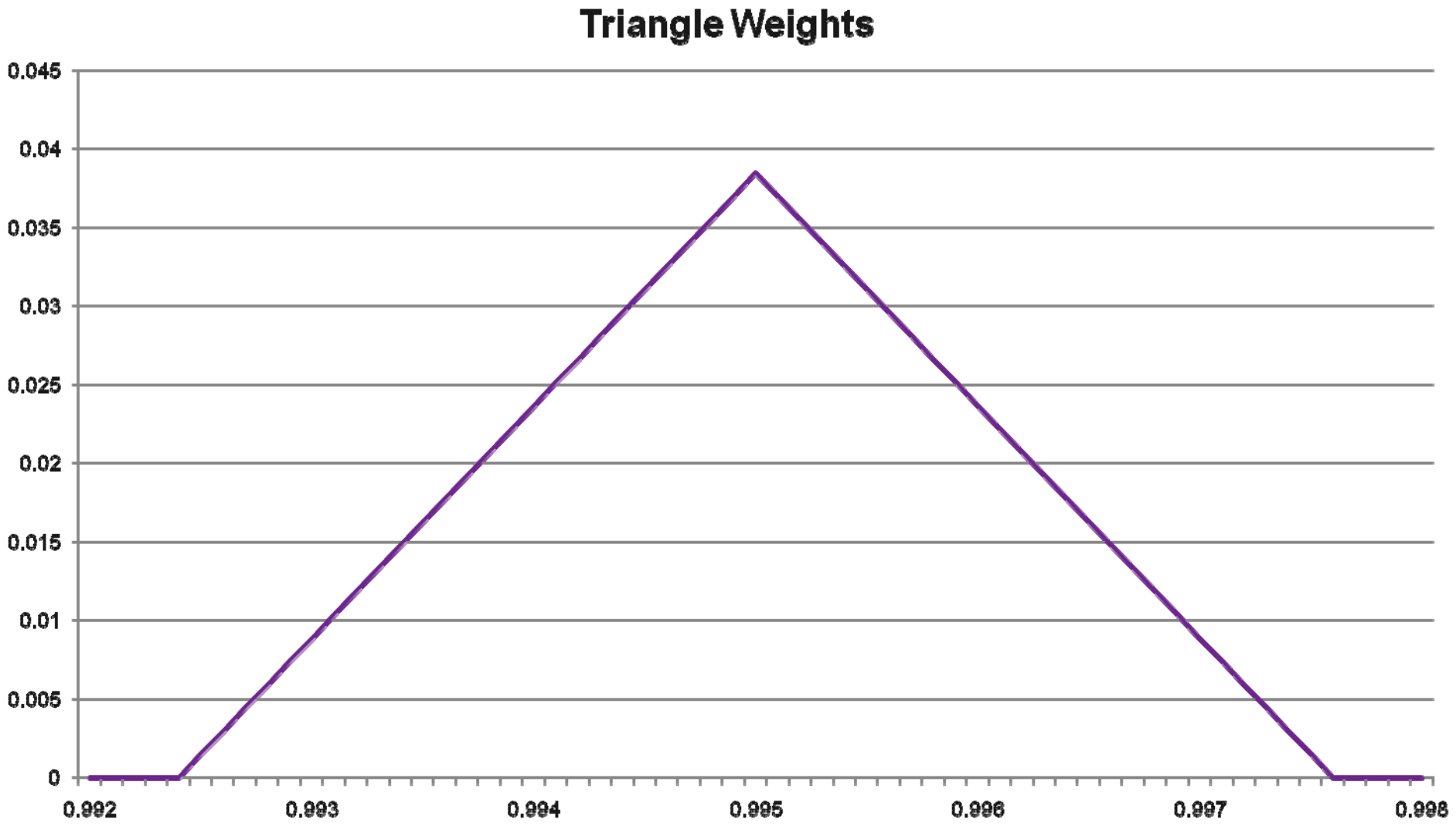
Triangle Distribution Weights



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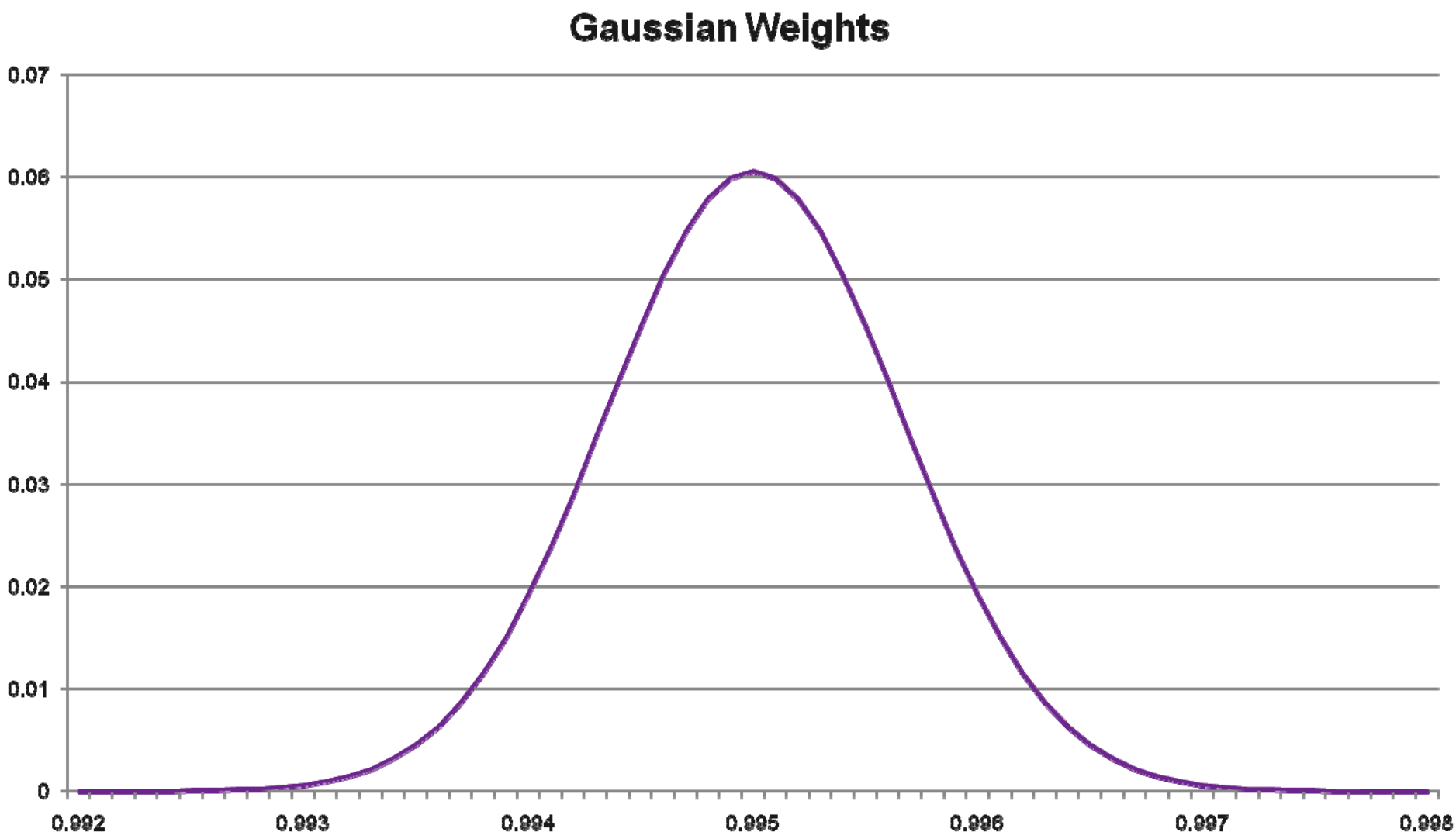
Gaussian (Truncated) Weights



Algorithmics

Aggregation & Attribution Case Study

Algorithmics

Case Study

Reporting hierarchy with four product lines

10,000 Monte Carlo scenarios

Aggregation Rules applied to NAV and Attribution applied to ΔNAV

Limits applied to transferable capital up the hierarchy

Calculate SCR

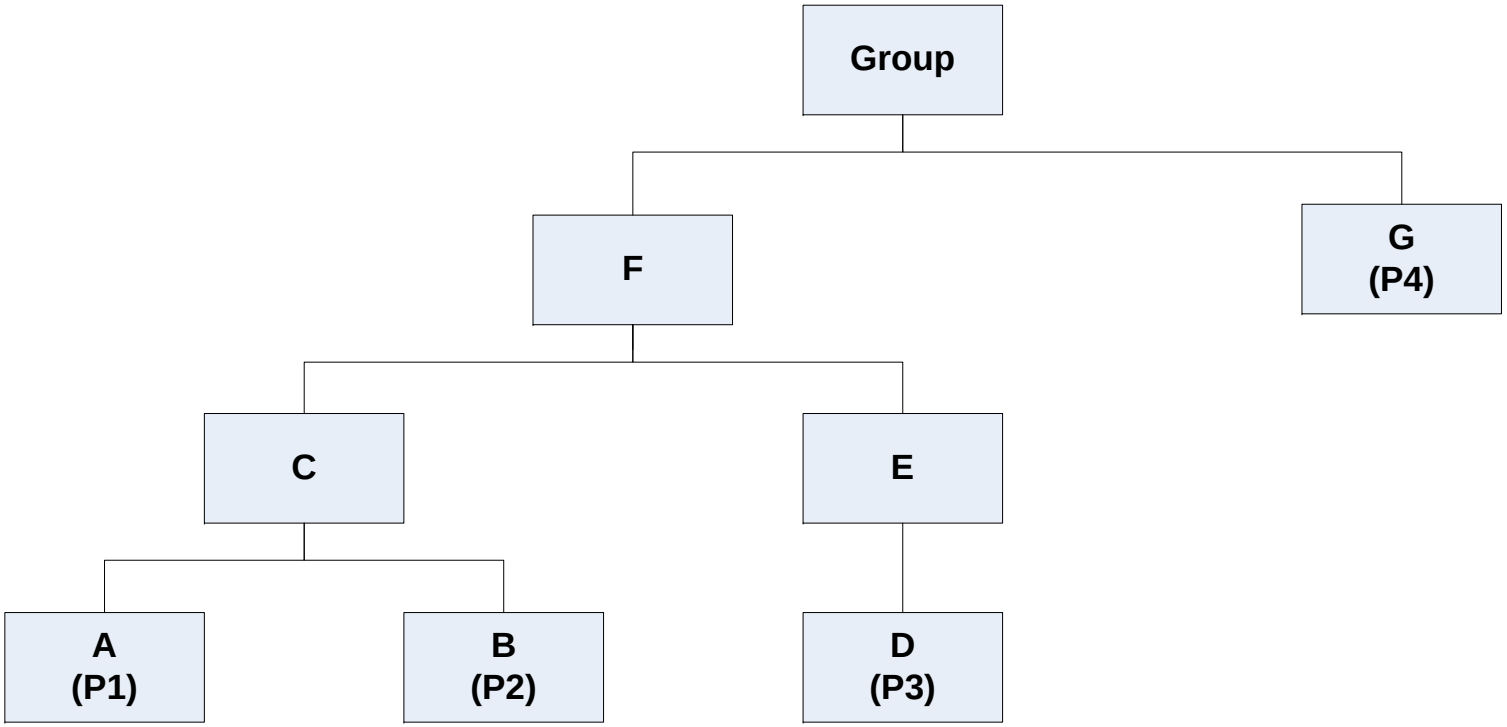
1. Quantile
2. Use HD Weights as to smooth SCR values
3. Use HD Weights to smooth risk factors for biting scenario

Two alternative attribution methods using HD Weights:

1. Use HD Weights to smooth SCR values
2. Use HD Weights to smooth risk factors for biting scenario

Algorithmics

A Simple Reporting Hierarchy



Algorithmics

NAV Sorted by Group

Order	Scenario	A	B	D	C	E	F	G	Group
45	SSMC_5474	5.7471	28.3337	25.8269	8.5202	6.4567	3.7442	31.0991	8.7108
46	SSMC_1594	6.6216	19.3178	28.9337	6.4848	7.2334	3.4296	31.4823	8.7280
47	SSMC_824	7.4212	16.2818	30.5021	5.9258	7.6255	3.3878	31.5251	8.7282
48	SSMC_5908	6.6182	22.3473	27.9640	7.2414	6.9910	3.5581	31.3725	8.7326
49	SSMC_7811	5.3369	30.5722	25.1125	8.9773	6.2781	3.8138	31.1211	8.7337
50	SSMC_5762	7.6980	11.2231	32.4361	4.7303	8.1090	3.2098	31.7287	8.7346
51	SSMC_3041	6.6725	21.3790	28.2554	7.0129	7.0638	3.5192	31.4387	8.7395
52	SSMC_1908	6.6234	22.7543	28.3017	7.3444	7.0754	3.6050	31.3536	8.7396
53	SSMC_98	6.1500	25.1527	26.6300	7.8257	6.6575	3.6208	31.3915	8.7531
54	SSMC_3487	5.4300	29.7629	25.5939	8.7982	6.3985	3.7992	31.2822	8.7703
55	SSMC_5497	6.3882	24.4721	27.1761	7.7151	6.7940	3.6273	31.4636	8.7727

Algorithmics

NAVs for

	A	B	D	C	E	F	G	Group
SSMC_7811	14	9981	21	9701	21	7281	38	49
SSMC_5762	554	93	9791	42	9791	21	62	50
SSMC_3041	120	8086	414	1711	414	509	48	51

Algorithmics

NAV for Group scenarios 49, 50, 51

- Good for diversification
- Hard to use for

	A	B	D	C	E	F	G	Group
SSMC_7811	14	9981	21	9701	21	7281	38	49
SSMC_5762	554	93	9791	42	9791	21	62	50
SSMC_3041	120	8086	414	1711	414	509	48	51

Algorithmics

Aggregation Rules

Node	*A*	*B*	*D*	*C*	*E*	*F*	*G*	*Group*
Max Transferable Profit	10	20	20	25	17	35	*20*	None
%Profit	25%	25%	25%	25%	25%	25%	25%	None
%Loss	80%	80%	80%	80%	80%	80%	80%	None

Algorithmics

Quantile Aggregations Calculations

For leaf nodes NAV for each scenario

value from asset and/or liability model

For aggregation nodes, NAV for each scenario

$$NAV_{Parent\ Node} = \sum_{i=Child\ Nodes} TNAV_i$$

For all nodes, TNAV for each scenario

aggregations rules applied to NAV

Algorithmics

Quantile Aggregations Calculations

.995% NAV (SCR)

1. Calculate the NAV for each of the Products for all 10,000 scenarios using valuation models

2. Sort the NAV from smallest to largest

3. Select scenario 50

.

Quantile Aggregations Calculations

. 995% TNAV (Transferable NAV)

1. Apply the aggregation rules to NAV for each of the 10,000 scenarios to calculate TNAV

2. Sort the TNAV from smallest to largest

3. Select scenario 50

.

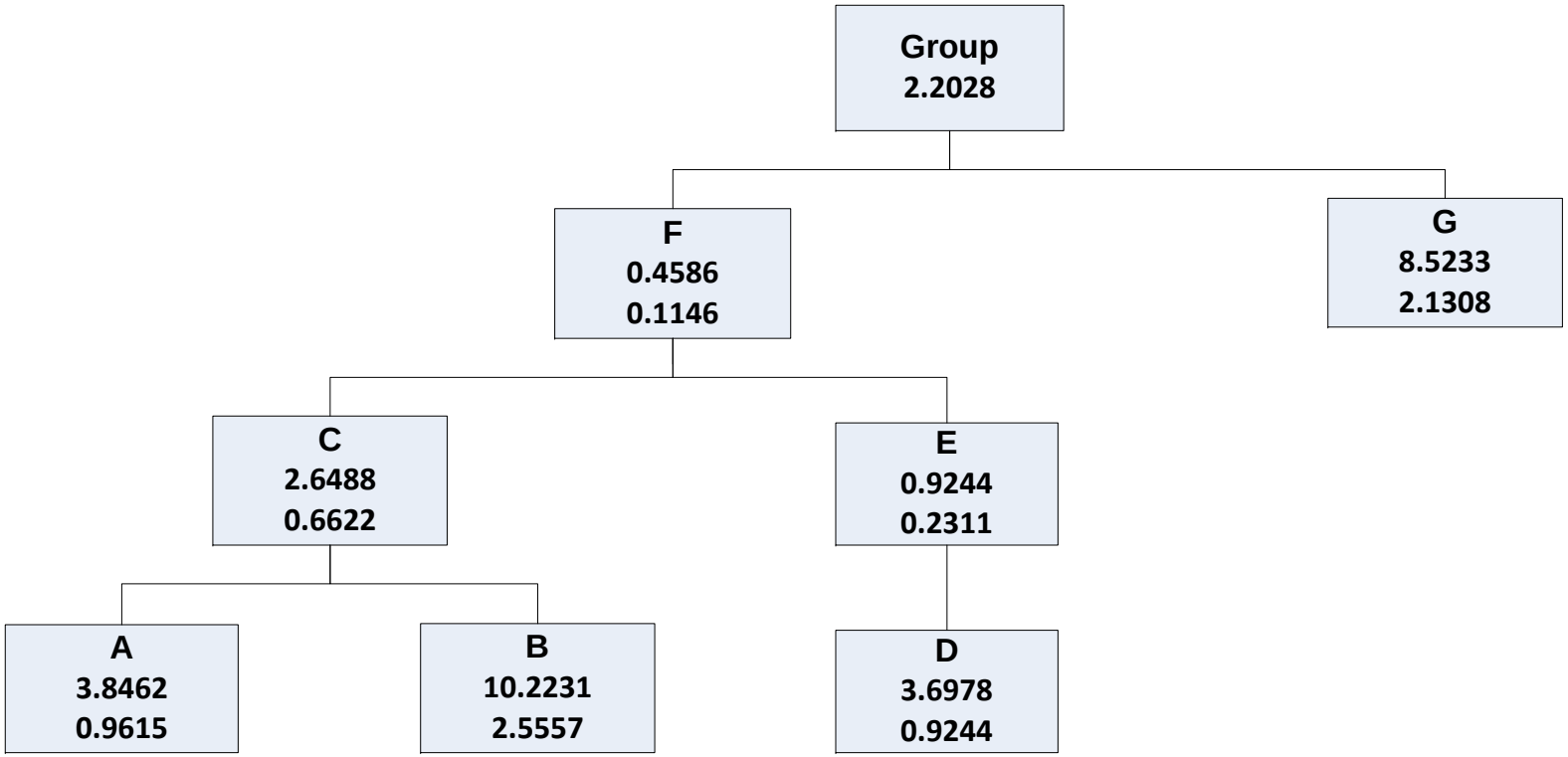
Quantile Aggregations Calculations

Compute NAV and TNAV for each of the aggregation nodes

1. $$NAV^{Scenario=S} = \sum_{i=Child\ Nodes} TNAV_i^S$$
2. Sort the NAV from smallest to largest
3. Select scenario 50 for .995% NAV (SCR)
4. For each scenario apply aggregation rules to NAV to determine TNAV
5. Sort the TNAV from smallest to largest
6. Select scenario 50 for .995% TNAV

Algorithmics

Quantile Aggregated NAV and TNAV



Algorithmics

HD Smoothed Aggregation Calculations

1. Order scenarios by Group

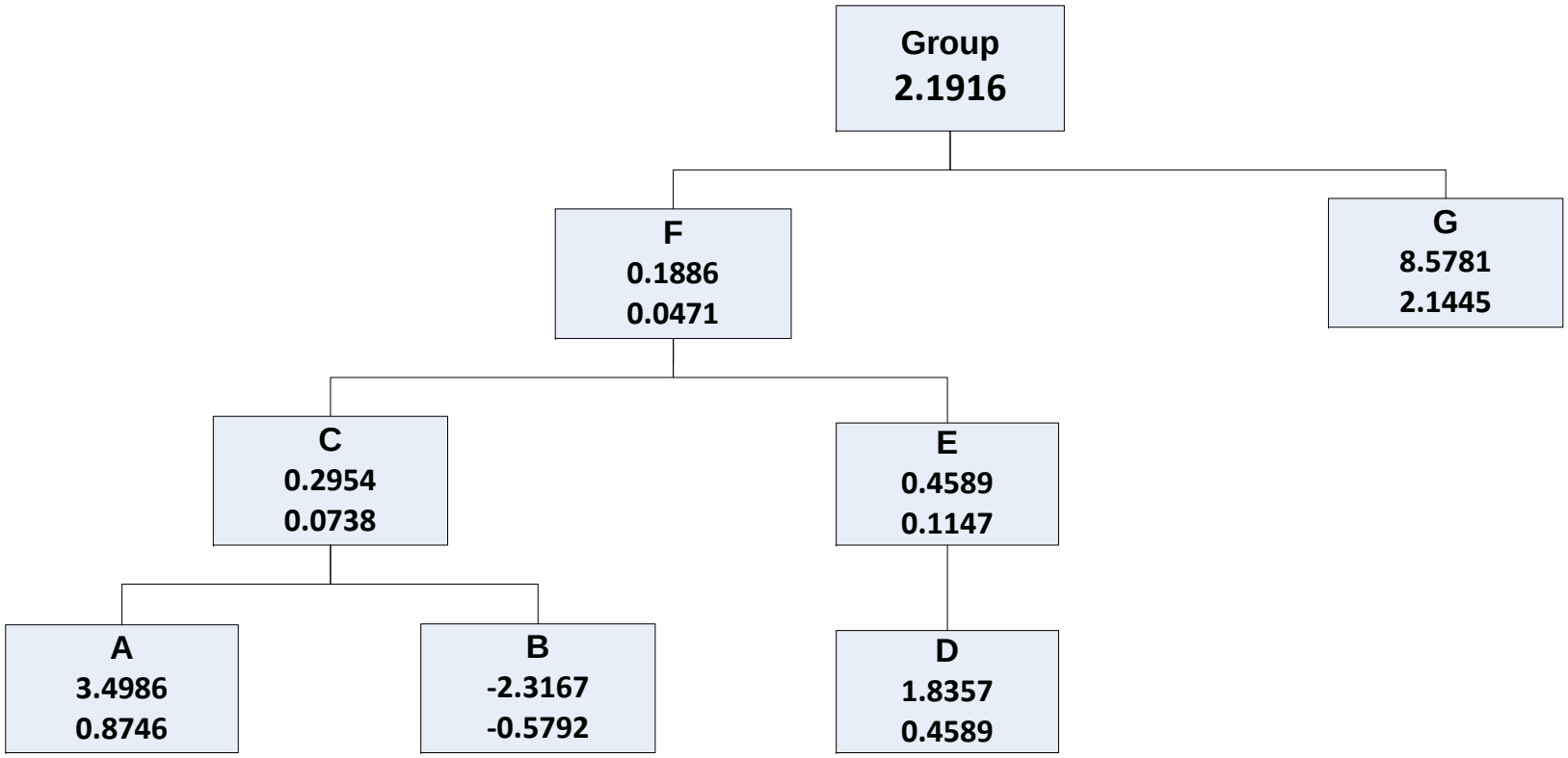
1. Calulate the HD Weights

2. .SCR = 995% HD NAV $= \sum_{10,000}^{s=1} HDWeight_s * NAV_s$

1. .995% HD TNAV $= \sum_{10,000}^{s=1} HDWeight_s * TNAV_s$

Algorithmics

HD Smoothed Aggregated NAV and TNAV



Algorithmics

Comparing Quantile and HD SCR

Node	A	B	D	C	E	F	G	Group
Aggregated Quantile SCR	3.8462	10.2231	3.6978	2.6488	0.9244	0.4586	8.5233	2.2028
Aggregated HD SCR	3.4986	-2.3167	1.8357	0.2954	0.4589	0.1886	8.5781	2.1916

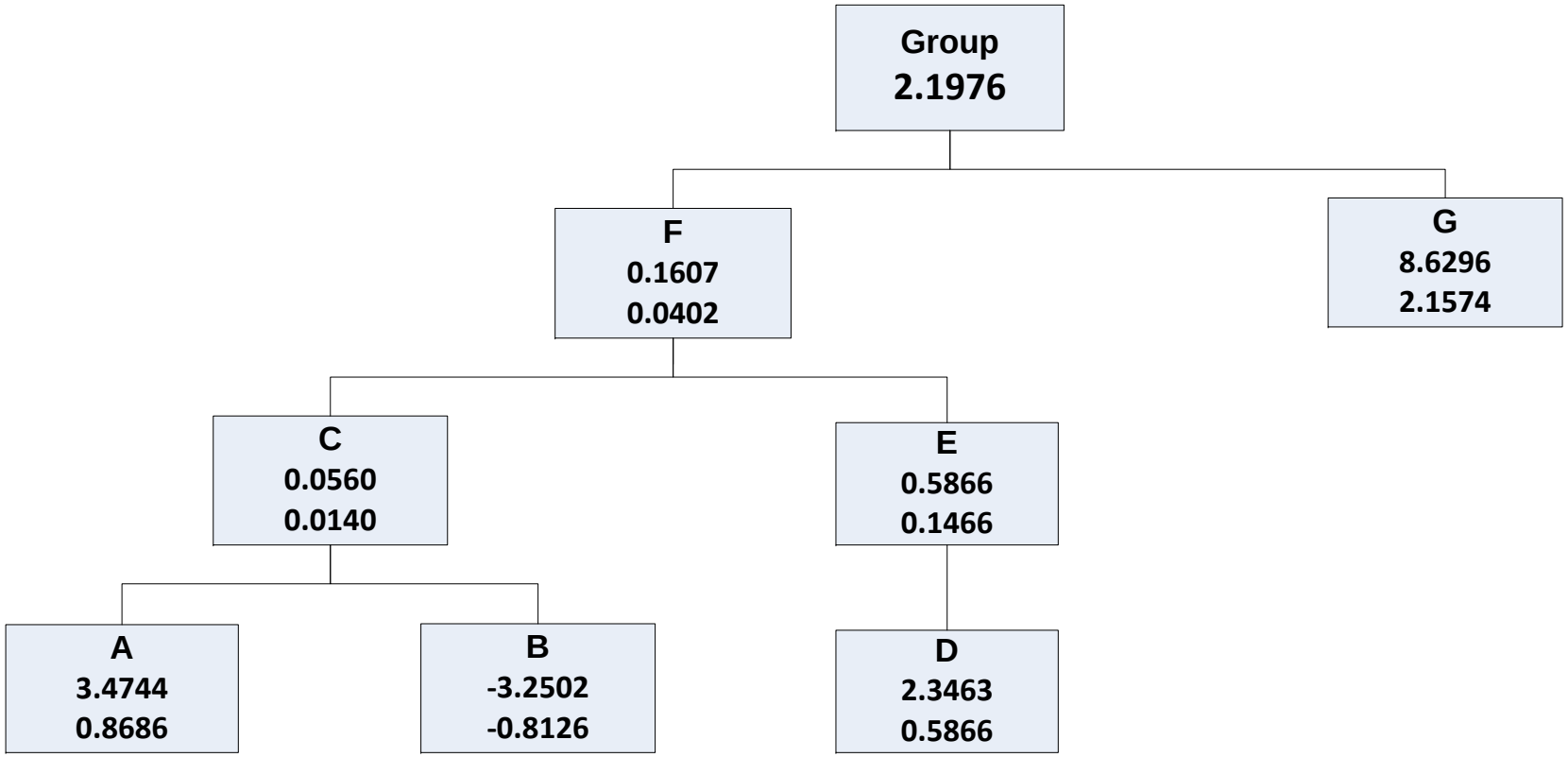
Algorithmics

Biting Scenario Calculations

1. Sort the Group NAV from smallest to largest
2. Apply HD Weights to the risk factor values in each scenario to compute a new biting scenario
3. Recalculate the NAV for the biting scenario
4. Calculate the TNAV for the biting scenario by applying the aggregation rules to the NAV

Algorithmics

Biting Scenario NAV and TNAV



Algorithmics

Comparing Quantile, HD, & Biting Scenario SCR

Node	A	B	D	C	E	F	G	Group
Aggregated Quantile SCR	3.8462	10.2231	3.6978	2.6488	0.9244	0.4586	8.5233	2.2028
Aggregated HD SCR	3.4986	-2.3167	1.8357	0.2954	0.4589	0.1886	8.5781	2.1916
Aggregated Biting SCR	3.4744	-3.2502	2.3463	0.0560	0.5866	0.1607	8.6296	2.1976

Algorithmics

HD Attribution

Take advantage of the fact that $NAV_{Parent\ Node} = \sum_{i=Child\ Nodes} TNAV_i$

Construct attribution weights $w_{i=Child\ Node} = \frac{TNAV^{.995}_i}{NAV^{.995}_{Parent\ Node}}$

Attributed SCR = $w_i * SCR^{HD}_{Parent\ Node}$

Algorithmics

Comparing Quantile, HD, & Biting Scenario SCR

Node	A	B	D	C	E	F	G	Group
Aggregated Quantile SCR	3.8462	10.2231	3.6978	2.6488	0.9244	0.4586	8.5233	2.2028
Aggregated HD SCR	3.4986	-2.3167	1.8357	0.2954	0.4589	0.1886	8.5781	2.1917
Aggregated Biting SCR	3.4744	-3.2502	2.3463	0.0560	0.5866	0.1607	8.6296	2.1976
Attribute HD SCR (Group)	0.0547	-0.0362	0.0287	0.0185	0.0287	0.0472	2.1445	2.1917

Algorithmics

HD Attribution

Take advantage of the fact that $NAV_{Parent\ Node} = \sum_{i=Child\ Nodes} TNAV_i$

Construct attribution weights $w_{i=Child\ Node} = \frac{TNAV^{.995}_{\ i}}{NAV^{.995}_{\ Parent\ Node}}$

Attributed SCR = $w_i * SCR^{Biting\ Scenario}_{Parent\ Node}$

Algorithmics

Comparing Quantile, HD, & Biting Scenario SCR

Node	A	B	D	C	E	F	G	Group
Aggregated Quantile SCR	3.8462	10.2231	3.6978	2.6488	0.9244	0.4586	8.5233	2.2028
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Aggregated Biting SCR	3.4744	-3.2502	2.3463	0.0560	0.5866	0.1607	8.6296	2.1976
Attribute HD SCR (Group)	0.0547	-0.0362	0.0287	0.0185	0.0287	0.0472	2.1445	2.1917
Attributed Biting SCR (Group)	0.0543	-0.0508	0.0367	0.0035	0.0367	0.0402	2.1574	2.1976

Algorithmics

Questions?

Algorithmics