OTC Option Pricing for Insurers

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Abstract

The following note looks to investigate the sources of pricing mismatch that can arise between insurers and investment banks when looking at embedded derivatives as commonly arise in guaranteed products. The paper recognises the important role that insurers can play in delivering long term guarantees, and notes their key benefit of providing liquidity to the market in complex guarantees for the retail sector. In particular the note upgrades the risk neutral replication and no arbitrage pricing arguments through the inclusion of transaction costs and revisits the pricing of OTC options in the context of liquidity. In particular, the impact of having guarantees on underlying funds with differing liquidity profiles, and the variability of market liquidity over time all point to the need for margins in pricing. The note ends with some suggestions requiring further consideration and investigation in the context of delivering a sustainable guarantee offering from insurers.

Key Words : guarantee, implied volatility, insurance, liquidity, OTC option, no arbitrage, repo, risk neutral, transaction costs,

Disclaimer

This note is not, and does not purport to be, a piece of academic literature rather it aims to translate the academic and abstract literature into credible worked examples and illustrations to aid insurance company executives appreciate the factors and complexities that feed into seemingly irrational market prices for Over The Counter derivatives.

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Any and all mistakes, omissions or oversights within this note are my own

1. Introduction

The recent past has led to significant analysis and (re)investigation of market liquidity[1], whether in the determination of the appropriate discount rate for “risk free” valuations or in setting the market consistent valuation of assets and liabilities in banking and insurance company balance sheets.

This note seeks to explore the importance of market liquidity effects on (sold) option positions by insurance companies. In particular to consider the degree or otherwise to which market liquidity effects are incorporated in setting quasi/pseudo market consistent prices for embedded derivatives.

The note contrasts insurance companies and investment banks recognition of liquidity effects as regards to long dated and complex guarantees such as Guaranteed Annuity Options (GAO’s) and Guaranteed Minimum Benefits attaching to Unit Linked guarantees, so called GMxB’s.

This note specifically illustrates and analyses the nature and location of margins that are required to gross up theoretical option premiums into full “office” premiums to allow for transaction costs that are impacted by liquidity.

From this informed position we then seek to reconcile the apparent differences in prices between options on liquid and illiquid underlyings, even when the statistical risk profile of the positions are identical.

This difference in pricing, as found in financial markets, is inconsistent with an insurance pricing approach which would likely value them consistently.

Analogous to differing liquidity profiles for assets in a benign market environment, this analysis will illustrate that generalised market liquidity shocks (as appear in crises) have an amplified effect on the replication of OTC options and thus price.

As an overall finding, the note supports other the research findings[2] that sold option protection should be a premium business, with excess returns reflecting not just equilibrium market pricing parameters, but should allow for the uncertainty of future market states and liquidity.

Thus, notwithstanding the irrationality of markets, the OTC markets behave in a rational way in response to such uncertainties.

2. OTC options

This note is primarily concerned with the pricing and replication of European style put options on funds and other equity style indexes and stocks. In particular the note is concerned with the over the counter (OTC) market as opposed to the exchange traded market, however, the note will make considerable reference to the underlying exchange traded markets.

Thus, for the purpose of this note we are concerned with bilateral market arrangements that may adhere to greater or lesser degrees of market standards and documentation (ISDA). Areas of non standard structure as compared to exchange trades may include term/duration, payoff, strike, and collateral.

Furthermore, as our options become more exotic, (whether in terms of underlying, term, payoff or collateral) then the impacts of these features on liquidity become all the more amplified and thus impinge directly on price.

2.1. OTC options as Aggregators of Market Liquidity

From a quant or traders perspective, pricing in the OTC option market is a function of the replicability of the sold/bought position where the price is based on the availability and cost of underlying instruments, together with a residual margin for profit.

Furthermore, variation and uncertainty in either the supply or cost of these underlying components are fully taken into account. Thus the present value of lifetime transaction costs becomes a core part of the pricing of OTC options. A consequence of this is that due to the OTC market being a derivative of underlying market instruments; it is more prone to liquidity holes than the underlying market in a crisis[3].

In the context of the “short put”/”protection market” there is the further dimension of market disequilibrium where markets are in general net short thus there is a further argument and premise for a risk aversion premium for speculators to participate in the market.

Thus the OTC market by construction is expected to be a “premium” market.

2.2. Presentation in the OTC Market

The beauty of exchange traded instruments is the certainty of all elements bar price, thus there is an ability to recover information for prices and use that information for short hand or inference purposes. Thus, within a market, Implied Volatility (IVOL) being the unique descriptor of price of a particular option under the Black Scholes formula and the conditions of the market.

What happens however when these market conditions are no longer centrally set or the contracts pre-defined? In such a situation more information than either the option price or an option pricing parameter such as IVOL are required to capture the contract conditions and value.

In such situations we have to treat our market inferred parameters somewhat more tentatively and reflect upon the limitations of further inferences and extrapolations that can be made from then. Such situations may concern higher degrees of estimation risk (e.g. dividends, forward rates) and structure (collateral amounts, accretion) which are further amplified by duration.

As an illustration of this scope for ambiguity we will look to locate all (transaction and liquidity) pricing margins in our Implied Volatility metric and look to observe the consequent impact on shape and level of the resulting IVOL curve.

This then leads to the challenging question as to what happens when we calibrate market prices under the risk neutral space inferred by an augmented IVOL that includes Liquidity?

3. Liquidity Premiums - Positive and Negative

Market Liquidity can cut both ways and in particular where insurance companies may earn liquidity premiums on bond positions there is a cost or negative liquidity premium payable for owning OTC options.

Arising from this a number of questions worthy of consideration are:

- How or to what extent is this negative premium recognised by insurers?
- To what extent do they charge policyholders for it?
- To what extent are they rewarded for warehousing liquidity risks under dynamic replication hedge strategies?

In the following sections we will briefly revisit the positive illiquidity premium that is earned by bond holders before looking at the case of the holder of an OTC option who can expect to pay a liquidity premium for holding an illiquid asset.

3.1. Earning Liquidity Premiums

Over recent times we have become more than conversant with the liquidity premium discussion as it pertains to bond prices and in particular its significance for key classes of business such as annuities in the context of the emerging solvency II discussion.

The literature recognises the existence of illiquidity premia for traded assets such as stocks and bonds where for identical (or close matching) series of future payments an asset that is more liquid will deliver a lower return as compared to an illiquid asset.

The volatility of this liquidity premium, in particular during times of distress, is the principle argument for certain long term investors, such as insurers, seeking to ignore temporary liquidity driven changes in market values, to dampen or reduce the impact of cyclicality of markets.

The main discussion (in Insurance circles) is thus not so much on the existence of liquidity premia but the extent to which these premia should be anticipated in pricing and reserving. The key argument against such valuation being that once valued, the temptation is to either transfer the amount to policyholders through lower prices or transfer the amount to management and shareholders through higher profits. Thus where the liquidity premium has been capitalised the consequence of the amount not being realised may well fall to a different generation of policyholders or shareholders, or in the limit wider stakeholders in the case of systemic failure.

For the purpose of this discussion, however, we will accept the existence of a liquidity premium without seeking to either evaluate or comment on the impact of valuing it ex post or ex ante for pricing, profit or distribution purposes.

3.2. Paying Liquidity Premiums

In the case of sold guarantees, insurance companies are arguably selling OTC options to the retail market place and arguably should be able to assert pricing power, under an insurance or risk aversion context. Where these full prices are recovered from the retail investor, the insurance company is then in a position to purchase OTC options from the quasi efficient Investment Banking sector and earn a spread for the balance sheet intermediation.

However, there appears to be a persistent disconnect between the prices sought by the Investment Banking (IB) community and the insurance sector and this may in part be related to expectations or understanding of the role of liquidity and liquidity risk in OTC options. It is this very disconnect that we are seeking to explore with the aim of aiding in the process of reconciling the positions and perspectives of the participants.

As a start, we can perhaps accept that there is an equilibrium level of "disequilibrium"[4] in markets such that the payment of frictional costs is accepted as a requirement to encourage and reward participants for creating a market. Furthermore it is perhaps uncontroversial to note that the degree of market disequilibrium and the speed at which it changes are not constant. Thus we move from states of high optimism to low optimism, passing through periods of dislocation and uncertainty as markets seek to find equilibrium. Thus in a (rapidly) declining environment we will have amplified uncertainty and risk aversion leading to a change in the excess of hedgers over speculators, creating a new and unstable equilibrium. In such a market the speculators will seek to increase the reward they receive for the service they provide to the market thus either withholding liquidity and/or increasing transaction costs.

Furthermore, these costs may also be amplified by regulations and regulators[5] whether in the imposition of capital requirements that increase demand (e.g. increased capital for position taking, counter party exposure) and/or reduce supply (increased capital for banks, restrictions on short sales, collateral requirements etc).

The impact of these changing levels of market sentiment and volume propagate through to market prices in the form of Bid/Ask Spreads, Borrowing Costs, and capital availability.

Thus the taking on of a position of long duration needs to countenance not just current market states but also future market states and conditions and the convexity of option prices to those changes.

We can thus think of the OTC Option market as being some hybrid pricing regime that incorporates both frictional costs and risk aversion and that these combined together lead to negative liquidity premiums for the buyers of options.

What then for the buyers of these instruments or the generality of OTC market dynamics? For an orderly market to exist there is a requirement that there is a ready supply of buyers of these instruments who are sufficiently risk averse to absorb these negative liquidity premiums, or in simple terms pay for insurance risk. Where these buyers are absent or are insufficiently risk averse then no market will operate as without the buyers there won't be sellers and the market won't clear. Thus we can talk about a market in complex derivatives noting that this is temporal as there can be theoretical markets or markets that only emerge in certain market states (of high optimism).

Thus the markets in complex instruments can be transient and temperamental.

What then can we identify for the insurance companies seeking to sell complex OTC options through GAO and GMxB? For one they straddle two markets, the retail market and the wholesale markets and it is reasonable to identify that these markets have differing levels of utility and risk aversion. Secondly, and a result of this pivot or market making position, it is clear that the ability to transfer risk en-masse through OTC options from retail to wholesale markets is unlikely to be stable where the utility for risk transfer on both sides is variable.

Thus there will be periods of increasing and decreasing opportunity to transfer risk between these markets and a need for insurance companies to play their role as a capital provider to manage out these imbalances.

In this way the insurance sector can become an important liquidity provider, intermediating between the wholesale and retail markets. However as a liquidity provider (a market maker of sorts) it is critical that insurance companies understand the dynamics of market liquidity so that their risk appetite can be established and that an appropriate risk reward profile is set in place.

4. OTC Market Liquidity and Relevance to Insurance Companies

An initial reaction or response by Insurance market participants and actuaries may well be to disregard the chaotic shenanigans of the financial market and focus on the liquidity argument of insurance companies. This would be consistent with the buy and hold policy for assets with the exception that in the case of these sold obligations it is a case of an unmatched “sell and hope strategy”.

The argument would likely focus on the surrender value of GMxB’s being nil in most cases, and the duration of the guarantees being sufficiently long (especially in the case of retirement guarantees). Such long durations facilitate the emergence of mean reversion and for the various risk premia in the underlying funds to assert themselves, thus dragging guarantees out of the money, with a high degree of confidence.

Thus with the expectation of low long term volatility and positive risk premia the economic costs of guarantees appear relatively inexpensive and the price is dominated by (regulatory) capital and expense charge considerations. Such a cultural ideology does not easily embrace the vagaries and apparent cost of market consistency, for either statutory or prudential reporting purposes. This perspective can lead to reluctant engagement with these valuation metrics and perhaps utilisation of reinsurance or regulatory arbitrage to facilitate a more benign treatment.

However, there are a number of concerns with this familiar (but thankfully decreasingly so) perspective that benefits from exploration. In the first instance there is the growing question as to whether stocks are really less volatile in the long run? Secondly, the question as to whether it is possible to hold positions over the long run, to realise the great actuarial “convergence trade”?

We will briefly address each of these before settling on the need to countenance market facing risk management, however reluctantly, and the far reaching consequences of such risk mitigation.

4.1. Are Stocks Less Risky in the Long Run?

This is somewhat off topic as it is not a discussion of market liquidity but the direction of prices over the long term. It is however pertinent to our discussion as to what is the fair insurance price of a sold guarantee in the absence of constraints and is thus still germane to the discussion.

The basic premise supporting the long run out performance, including the stability of returns, is that mean reversion exerts significant downward pressure on the variance of stock returns over time. This is clearly evident over short and medium terms and is recovered from historic time series data. This mean reversion effect is not to be challenged in this discussion, however, the concern is that the discussion is incomplete. In particular when we move from an assessment of historic deviations and seek to make predictions into the future we need to add back into the mix estimation error, to arrive at a comprehensive value for “predictive variance”. This estimation error increases as we move further into the future, thus we have competing pressures with a consequent impact of the predictive variance of returns being duration sensitive. Based on the analysis of Stamburgh and Pastor[6] they estimated that over longer horizons the impact of this uncertainty suggests that the annualised 30 year variance is of the order of 1.5 times the 1 year variance, which is certainly an alarming implication from a traditional actuarial perspective.

The point of this thought strand is not to analyse or posit the precise quantification of future uncertainty but to add back into the mix a need for humility in projecting both short term and long term outturns. This evidence itself is reflected in the recent expansion of real world economic scenario modelling to deliver ever increasing fat tails on distributions and underpins the need for humility in looking to price long term options on an insurance basis.

4.2. Convergence Trade?

Convergence trading is based on a no arbitrage concept where short term market anomaly’s can be taken advantage of due to the longer term adherence of markets to converge either structurally or through a resumption of market equilibrium. Thus market participants may enter into apparently well priced positions with the expectation of realising their profits, with the passage of time, when market factors converge to their appropriate state of equilibrium or maturity.

There are many examples of convergence trades[7] for example the Futures/Cash market trade, Currency Pairs, etc. or more commonly the Double or Quits trading rules for Roulette.

It is also commonly appreciated that these trades do not breakdown because of a failure of convergence but because of a failure of the market participant to stay in the market or with the position to its logical conclusion[8]. Thus we have LTCM betting the ranch on converging (currency related) trades and we have insurance companies looking to realise long term liquidity and risk premia through the ups and downs of market gyrations.

Specific examples that are rather close to home include UK/European mutual insurers seeking to maintain equity backing ratios through the dislocation at the start of the millennium and reinsurers in North America taking on GMDB exposures on an insurance basis at the same time. More recently we have seen a resumption of un-hedged and under-hedged GMxB in North America, at the time of the 2007/2008 crisis.

From a macro prudential policy perspective there are genuine arguments to support a regime that better facilitates long term position taking for insurers. And we see this in practice, to greater or lesser degrees globally and as adjusted from time to time.

For example the North American policy of historic cost accounting for assets better facilitates the buy and hold strategy for non derivative general account obligations and the recent amendments under IFRS for banks have, to a degree, improved the scope for hold to maturity pricing. All of these are supportive of a policy of avoiding pro-cyclicality and to better support the aspiration of long term investors.

In the (insurance) embedded derivative market we have had forbearance on elements of the basis for MCEV calculations, in particular in respect of IVOL calibrations, and the beneficial impact of own counter party rating adjustments for IFRS purposes. From a prudential perspective we can identify the policy aims of reducing pro cyclicality through initiatives for equity dampeners and liquidity premia. Thus there is recognition of the benefit of adapting requirements to the liquidity of obligations.

That said, having somewhat survived the recent financial crash it is clear that regulators are seeking to reduce the scope for future market risk position taking through capital requirements.

As a result, whether driven by increasing humility over the ability to predict future markets, the impact of statutory reporting or prudential requirements it is reasonable to identify that degree of risk associated with our so called “convergence trade” are high and likely unacceptable for all but incidental or modest volumes of exposure.

Thus, however reluctantly most insurers will be forced to engage with financial risk mitigation and thus recognise the impact of these on shortening their assets and liabilities.

4.3. Regulations and Risk Management Shorten Liabilities

The recent crisis has accelerated and deepened the interest of regulators in market risk positions. Furthermore, the recognition of the role of credit and liquidity risk in the crisis has further broadened the range of risks that regulators and risk managers are taking into consideration.

For example, the imposition of a VaR or stress test regime as countenanced by Solvency II mean it is almost impossible to write any material aggregate exposure to embedded derivatives without introducing at least a first order hedging program.

It is the introduction of a new set of assets and liabilities, through the hedge program, that shortens the duration of the insurance companies’ balance sheets, irrespective of the liquidity of the policyholder exposure.

In particular, there are two related but distinct elements that will shorten the insurance companies’ liability profile.

In the first instance, the hedging positions will be brought into account on a mark to market basis. The amount or direction of this change can be positive or negative thus creating assets or liabilities for the insurer. For effective offset then this will require the liability will move in an opposite but contemporaneous fashion.

Secondly, and more unambiguously, to the extent that the value of hedge instruments are collateralised or settled continuously, these paper marks to market variations will become physical movements of cash and collateral thus copper fastening the liquidity requirement.

This second effect is perhaps somewhat underappreciated owing to past experience of transfers from the IB sector to the insurance sector in terms of hedges during and through the crisis. Recent time will perhaps have led to some return of cash from insurers to banks as markets have to some degree recovered, which may cause timing issues, however in many cases the insurers are still cash positive. However what happens when and if guarantees move fully out of the money? Future market movements would then lead to net cash transfers from insurers to their counterparties, backed only by fair value assets in the hands of the insurer thus creating a net cash strain. This situation introduces a borrowing requirement for insurers, or in a general sense, introduces an additional consideration for funding.

Thus by virtue of risk management alone the illiquid policyholder liabilities become liquid obligations in the hands of the insurer.

Thus, it is critical that insurers appreciate why, how and to what degree market microstructure in liquidity effects option positions to enable them assess their risk appetite and pricing expectations of the same

5. Analysis of Transaction and other Costs

Using the example of a European, at the money forward, put, we will look at a range of replication microstructure considerations and walk through where and how margins can and are added. We will look at maturities of 5 years out to 20 years so that we can see the duration dependent sensitivities and scale of these elements.

5.1. A Black Scholes Simplified Example

We will start with the basic no arbitrage, risk neutral pricing proposition as follows:

The Black Scholes formula for a European put is as follows:

- $\text{Exp}(-T*R_f)*\text{Strike}*N(-d2) - \text{Forward}*N(-d1)\}$
- For $d1 = \{\text{Ln}(\text{Spot}/\text{Strike}) + (R_f + .5*\sigma^2)*T\} / \{\sigma*\sqrt{T}\}$, $d2 = d1 - \sigma*\sqrt{T}$
    - R_f = Risk Free Rate (based on N month Interbank Swap Rate)
    - σ = Volatility
    - Strike = Forward = $\text{Spot}*\text{Exp}(R_f*T)$ = “At The Money” Forward
    - T = 5 to 20 years (Long Expirations)

What we want to do is to introduce adjustments to reflect real world transaction costs and to elaborate on these elements through discussion:

From a hedging perspective we can represent this as

$Exp(-T*R_c)*Strike*N(-d2) – Forward*N(-d1)$

For $d1 = \{ Ln(Spot/Strike)+ T*(R_r +.5*IVOL^2)\} /\{ IVOL*\sqrt{T}\}$, $d2 = d1-IVOL*\sqrt{T}$

$Forward = Spot*Exp(R_r*T)$

In this we exchange

1. IVOL for σ to reflect the adjusted statistical volatility

2. R_r for R_f to reflect the "repo rate" as the appropriate no arbitrage drift

3. R_c for R_f to reflect the earning rate for positive cash.

Additionally we will add to this amount an idiosyncratic adjustment "RP" being a risk premium to reflect the market participants' level of risk aversion or expected excess margin. In the absence of two way prices (Bid/Ask) this will include the ask margin over the mid price.

5.2. Analysing the Components

The Elements of the basis to be explored are thus:-

1. Volatility Adjustments
2. Borrow Cost/Repo of The Short
3. Investing Rate of the Cash
4. Risk Premium

5.2.1. Volatility Adjustments

The first adjustment to our model is to adjust the measurement of volatility to reflect a) the nature of stock prices processes as different to Geometric Brownian Motion (implicit in Black Scholes) and b) to introduce the transaction costs of dynamic replication.

At this early stage we will not adjust our IVOL for parameters that are not volatility related relying on a rich enough model to reflect these parameters in their appropriate places. As a final step, in section 6 below, we will locate all our margins in the IVOL parameter and identify the implications of the same.

5.2.1.1. Statistical Adjustments

We know that the Implied Volatility measurement is a recovered parameter from observed option prices using the Black Scholes formula and that it includes a combination of pricing adjustments and statistical adjustments. The statistical adjustments include amounts for a) heteroskedasticity, being the non constant volatility parameter, b) Allowances for specific jumps (whose impact is short lived) c) the structural relationships between interest rate levels and volatility levels, d) adjustments for trading days and other conventions.

These changes are either approximated through the parameter or more formally through the model.

For the purpose of this exercise we will ignore the statistical adjustment or assume that it has already been made to our parameter as we wish to focus on properties other than the direction and volatility of the underlying index.

5.2.1.2. Bid/Ask of Replication

We now start to look at market micro structure and in particular introduce the first element whereby we see an option price as a synthesis of underlying market liquidity.

In the case of a sold put option, the theoretical no arbitrage pricing is predicated on the absence of market jumps and the absence of transaction costs. Thus, allowing our pricing to follow an idealised policy of continuous hedge rebalancing can replicate the position with no exposure.

However, in the presence of transaction costs, continuous (infinite) rebalancing leads to the transaction costs become infinite and thus risk free replication through continuous rebalancing breaks down.

As a result, in the presence of transaction costs hedging occurs in discrete time and there is an optimisation of residual hedging error and hedging costs to determine the fair cost of hedging. Thus we can identify our optimal position is related to hedge costs, volatility and rebalance frequency.

An example of such an optimisation or formula is that developed by Leland[9] (and expanded further by Wilmott[10] into a generalised formula depending on whether we are short or long gamma). This approach seeks to adjust the volatility parameter, on the basis that the need for trading is proportionate to the volatility of the underlying and in the case of our short put position leads to an increase in the statistical volatility as follows:

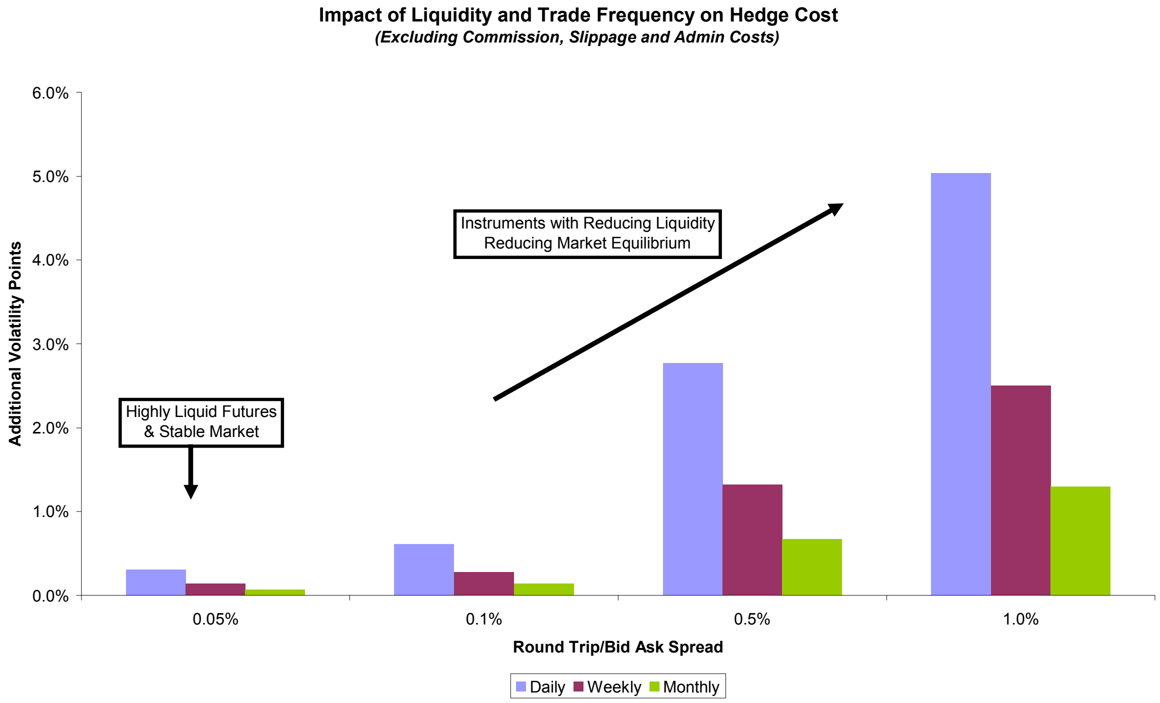
$$\sigma_{tc} = \sigma * \{ 1 + (2/\pi)^{1/2} * k/(\sigma * \sqrt{dt})\}^{1/2}$$

where

σ = historic/expected volatility of the underlying unit
σ_{tc} = is our volatility parameter adjusted for hedge costs
k = the round trip transaction cost and
dt the discrete time step, and $1/\sqrt{dt}$, a function of the frequency of trading:

From this formula we can see that (frequency of trading) * (round trip transaction costs) drive the adjustment and we seek to illustrate this in the following chart:

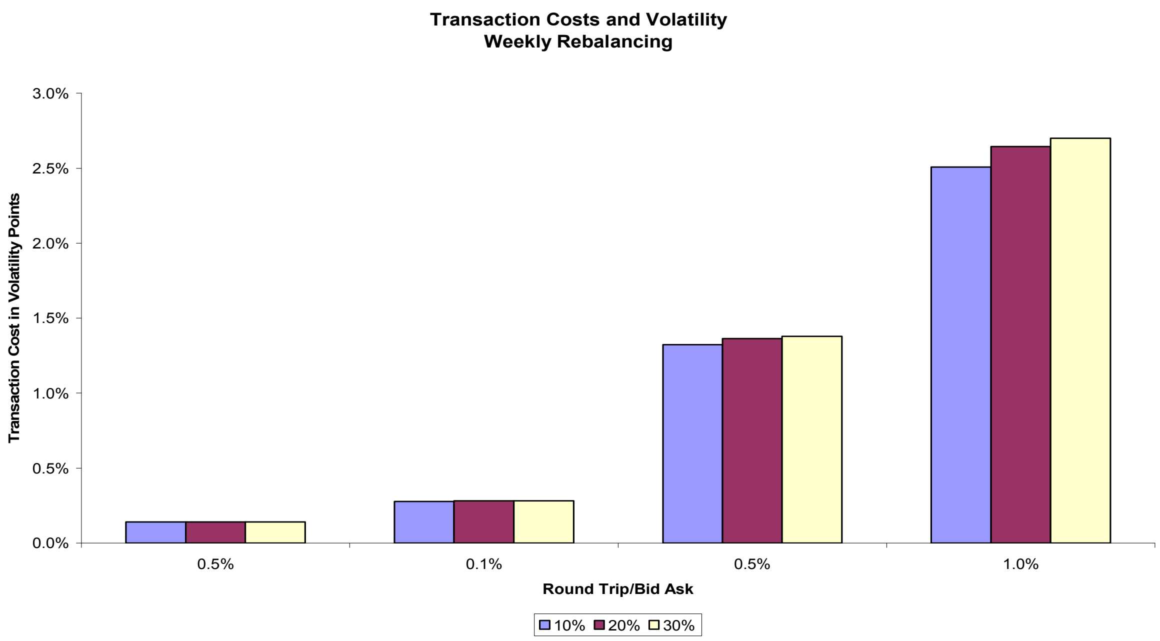
Chart 1



Thus we can readily identify that while markets are stable and liquid the cost of bid/ask can be relatively benign, as a proportion or amount of volatility, however as markets dislocate or we seek to use expensive underlying instruments (in terms of Bid/Ask) that the costs escalate rapidly and materially.

For completeness, when we look to see how sensitive the adjustment is to the underlying level of volatility we can identify that it is very much a second order effect as illustrated below:

Chart 2



Thus it becomes readily apparent that the liquidity of the underlying instrument, as manifested in Bid/Ask prices are direct inputs to the option replication price, through the volatility parameter. This leads to the simple, but important result that options on liquid instruments are cheaper than options on illiquid instruments, all other elements equal.

Indirect consequences of this are a) In the limit, options on illiquid underlying units become unhedgeable/not replicable due to price considerations and b) Variation in market implied volatility (vega risk) countenances changes not just in expected volatility but also to changes in the liquidity of instruments used to replicate them.

5.2.1.3. Are insurers charging the right Bid/Ask?

Where an insurance company sells an option on an illiquid index and seeks to hedge with a liquid index what then is the correct price for the option?

Ignoring all other elements (in particular repo which we will cover below) then the sold position should reflect the illiquidity of the underlying fund, on the basis that the insurance company will incur either expensive replication costs, or increased hedging errors in providing the protection.

In this regard the insurer is transforming the illiquid obligation and requires recompense and reward for this.

Thus the IVOL parameter should have regard to the sold obligation in addition to the replication strategy.

5.2.2. Borrow Cost/Repo Cost

To create a short position (to create our delta neutral position), we either need to “borrow” the fund or stock directly or use some other proxy or derivative instrument. The decision over which instruments to select for hedging to optimise the net cost of replication will depend on the cost and availability of the instruments, assuming we can ignore basis risk for now.

Depending on the scarcity of supply, or the degree of demand, the stock lender will be able to assert pricing power for the loan of stock. Furthermore, to the extent that there are credit and operational risks in either recovering the stock (or the value of the stock) these will become factors of the fees charged for the loan.

It is interesting to reflect on the crisis of 2008 wherein the perfect storm for stock lending arose with increases in demand by hedgers, market intervention to limit (naked) short selling by regulators and increased concerns over operational and credit risks of stock loaned (as experienced in the case of Lehman brothers). Such shocks to the system led to significant increases in the level of repo rates during the crisis and in certain markets to a sustained elevation in cost and availability of repo rates subsequent to the crisis.

The way in which the charge for stock lending arises is pertinent to our discussion not least as it impacts directly on our no arbitrage assumption for the price at which our spot price can grow.

Again starting with our theoretical no arbitrage argument, the theoretical proposition is that spot prices for assets grow at the rate at which we can borrow cash to by shares, or sell shares and invest cash and buy futures. However our real world differentiates not just between borrowing and lending but discriminates for credit risks and collateral structures[11]. Thus we now need to introduce the cost of borrowing or investing and in particular look at the repo (repurchase) rate for our stock lending position or equivalently our implied repo rate for futures position. In the following we illustrate the position of a stock lending arrangement and expand it to countenance the treatment for futures and total return swap positions:

Stock Lending[12]

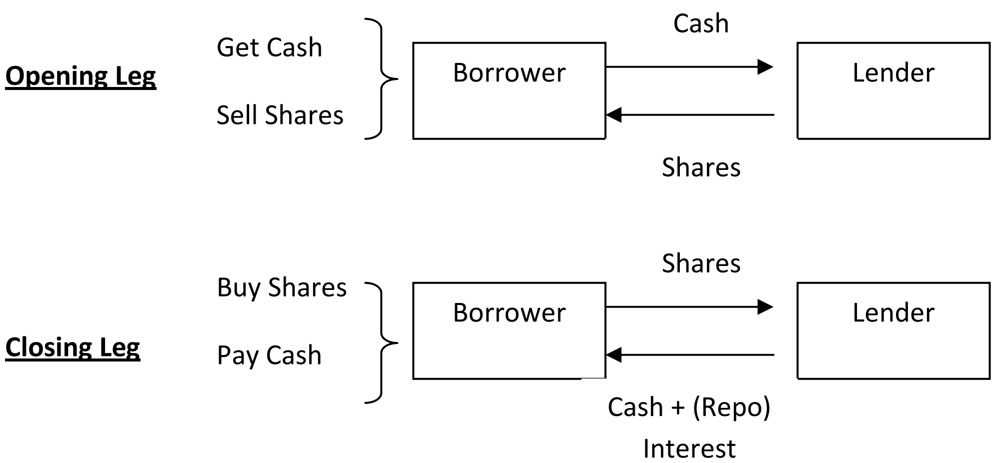
As an illustration, in order to protect the lender of shares the borrower usually deposits cash equal to the proceeds of the sale (and perhaps a margin on top) with the lender against the future redemption.

The lender will pay an interest rate on this collateral deposit, which will in most circumstances be a penal rate reflecting the fee for borrowing the stock.

Thus the cash deposit with the lender = Δ Shares, or Δ Shares *(1+ Margin) where an additional margin is required.

This deposit earns a rate of LIBOR-Lending Fee = Repo Rate and thus we get the adjustment to our Black Scholes pricing model where we replace the risk free rate R_f with our repo rate R_r that incorporates the real world no arbitrage price for the short position.

The following illustrates the elements of the opening and closing leg of our stock lending operation.



Note we addressed the cost of trading the shares, the Bid/Ask transaction cost above, however here we are concerned with the rent to be paid for borrowing the shares or the proxy.

Futures[13]

In the case of hedging with future positions the following equivalent scenario arises:-

Δ Shares equivalent of Futures are "sold" and accrete at the Implied Repo Rate of the futures market. Noting the general condition of backwardation in Futures markets where the future trades below the cash market this short position will grow faster than the cash market to maturity. The unwind of this implied repo rate is equivalent to the payment of a lending fee except that it operates at the level of the futures markets.

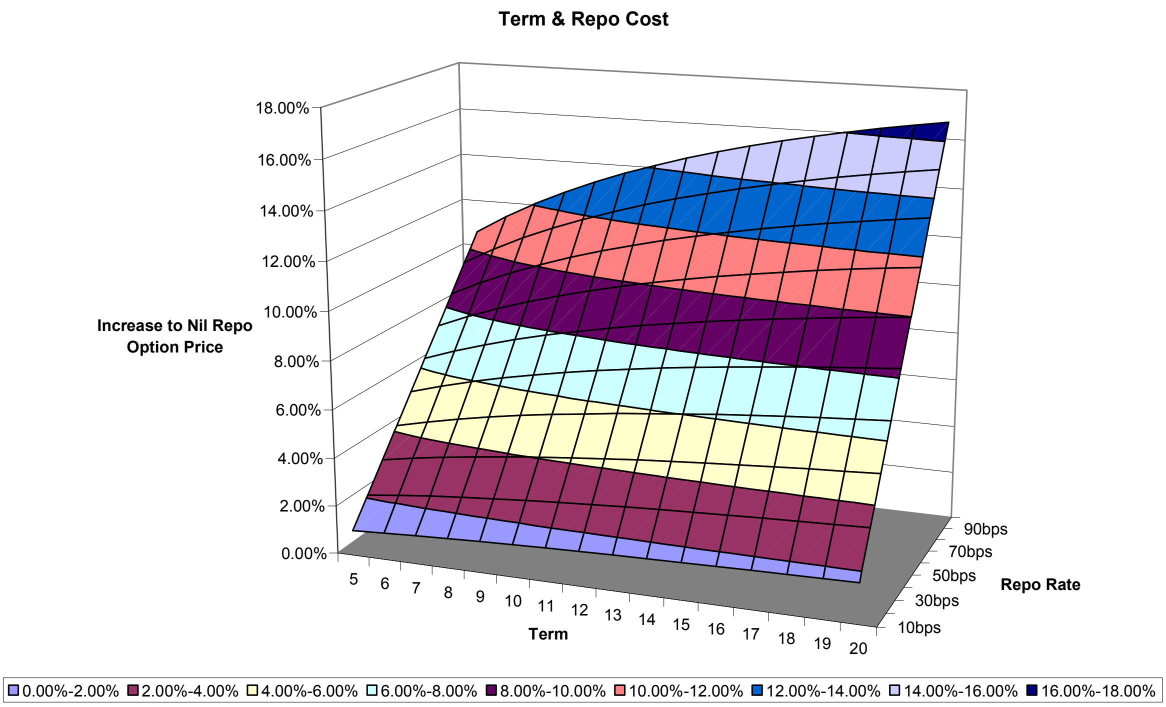
Total Return Swap

A more comprehensive short position can be achieved using over the counter derivatives themselves such as Total Return Swaps wherein you effectively sell the performance of the fund to a buyer being either a bank or asset manager. The charge for this facility is explicit and similar to the arrangement in a stock loan wherein the insurer will deliver the performance of the index to the counterparty and receive an adjusted interbank rate as recompense.

The benefit of these instruments is that these can incorporate the uncertainty of the specific fund or a closer matching fund thus for example the effects of total return funds (allowing for dividends, correlation etc) can be better tracked or where the underlying indices are not traded futures the basis risk can be eliminated. These better matching instruments come at higher repo rates and higher bid/ask rates.

In the following chart the impact of differing levels of fixed lifetime repo rates for our long dated options are illustrated as a percent of the nil repo price of the option. The price illustrates the impact for our “at the money” options and thus our positions only have time value. As options go more in the money (as is the case with many in-force programs post the crisis) the impact of repo is all the higher as the delta of an “in the money” put options is higher than those for “at” and “out of the money” options

Chart 3



5.2.2.1. Repo Rate Uncertainty

As a further consideration, repo rates are by no means stable over time and are prone to dislocation. At times of crisis with a consequent alteration to the supply/demand equilibrium repo rates, even in liquid markets such as futures, increase vigorously leading to mismatches and roll over risks which will have consequent impacts for the replicators' bottom line.

Thus, a seller of an option of duration longer than the repo markets used to hedge the position is taking on a position in repo liquidity. In such a case it is then reasonable for the option seller to assess their repo risk and retain a margin against future liquidity uncertainty. Such a margin would recognises some combination of spot market repo, long term repo, interaction between repo market costs and the levels of markets and the cost of convexity effects.

5.2.2.2. Optimising Basis Risk, Repo and Transaction Costs

Noting the sensitivity to the repo rate over a long duration and the sensitivity of repo to market dislocation we can appreciate a considerable incentive to utilise a liquid instrument such as a future to minimise the aggregate cost of repo over the life of the deal. Furthermore we identified in the previous section the impact of Bid/Ask spreads and general liquidity of hedge instruments on our volatility parameter.

Where for example we have purchased an option on a future we are in pretty good shape in so far as our option and our hedging instruments are aligned and thus we have a very efficient replication proposition. Extensions of these are OTC solutions for insurers that look to utilise an index measured from future markets rather than cash markets.

Where however we move away from these exchange traded positions and move to total return indices and onwards into active and managed funds we start to confront a choice between increasing our basis risk and increasing our total hedge replication costs above those payable in the liquid futures market.

Thus we need to arrive at an optimised hedge program that countenances the trade off between the residual hedge error and the cost of replication, noting that now we have a wider set of choices (i.e. not just hedge, not hedge or frequency of rebalancing).

Thus we need to develop a model or framework that considers a minimisation of (marginal excess repo + transaction cost – marginal excess basis risk), where the excess is the excess over the beta replication cost and residual exposure.

This also gives some information or guidance as to the proper location of margins for basis risk, being some combination of a volatility adjustment and repo adjustment under a no arbitrage argument.

Thus, indifference between basis risk and repo for example would indicate a clearing price for basis risk in an efficient market.

5.2.3. Cash Settlement & Collateral

The third element on our list is to work out how much we earn on our cash deposits, after adjusting for credit risk.

Under our Black Scholes model we roll forward our current reserve at the risk free rate such that our cumulative position after allowing for movements in our delta hedge and growth with interest we will have the appropriate amount of cash to settle our obligation.

This requires we explore in greater depth the correct rate for our invested cash as this to a degree needs adjustment for risk and to reflect any constraints we may have on the amount or availability of cash to invest[14].

Prior to the crisis it was common for long term option prices to reference long term forwards priced to the interbank SWAP market, in most case some measure of "n" month LIBOR such as 3months or 6 months. This reference rate reflected that the treasury functions of investment banks commonly set the reference rate for their trading desks and acted as an internal depositor/lender and thus absorbed the costs and profits of deviations of actual funding from the internal financing arrangements.

As exposed by the crisis it is becoming evident that interbank loans are not risk free and in particular the negligible or at least stable and benign spread between overnight money (compounded) and longer dated (uncollateralised) money is no longer a trivial consideration.

Thus there is a need to contemplate whether or not the investing rate is the same as the borrowing rate and what element of the structure impact on the choice of rate. For example, where there are requirements for collateral to be posted, then the rate at which this collateral credits or accretes interest (over night for exchange trades, LIBOR for OTC?) becomes pertinent to the discussion.

In particular, where the cash to be rolled forward is tied up in a collateral agreement earning over night interest then this indicates a limit to what we can earn. However, what of the case where the option seller has discretion as to the allocation of the premium and seeks to make some risk or liquid spread above risk free, for example by seeking to earn Libor through longer dated loans. Is there an argument here for a liquidity premium of some sort to be taken into account in setting premiums?

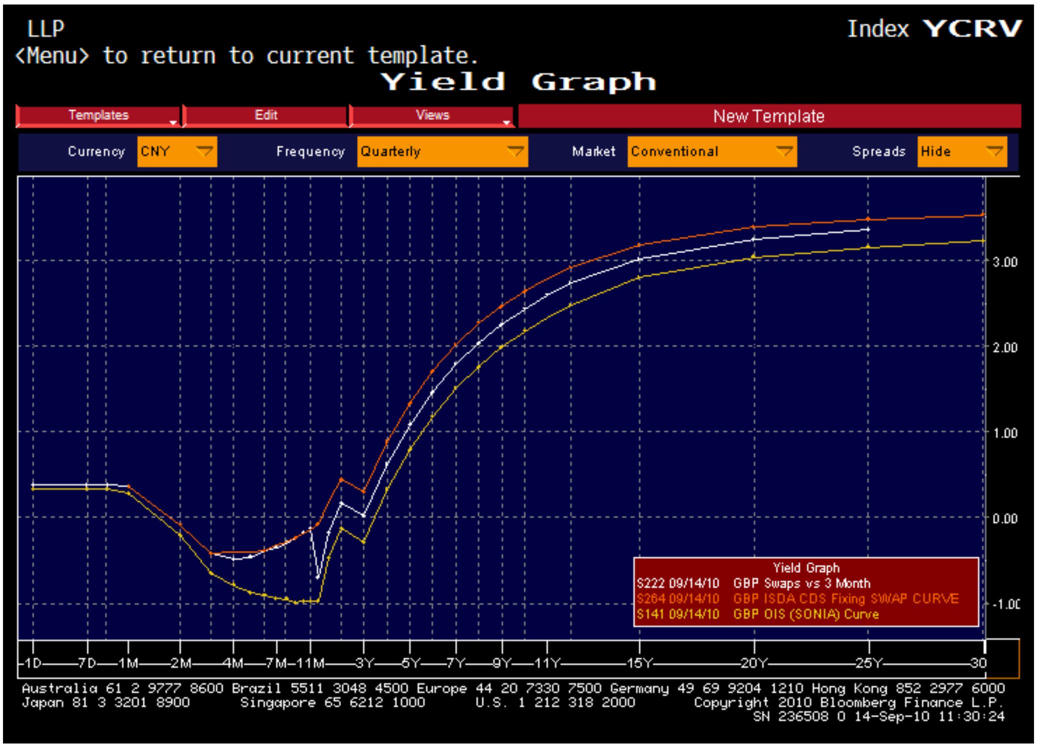
Whatever the strategy taken then there needs to be a consideration as to the freedom and utility that the deposit/premium taker has for cash and this will find its way into the pricing of the OTC option.

As a summary, we now have departed from our single rate for risk free and identified at least three possible interest rates being a) the repo rate for our risk neutral drift, b) the discount rate for rolling forward our cash deposits and c) an unspecified borrowing rate that may be required to fund either a haircut/margin on a repo agreement or to fund settlement of amounts for out of the money options where insurers get premiums in the future.

5.2.3.1. Scale of Swap Spreads

In order to assess the importance of these differing rates it is important to at least benchmark current levels and to then assess how they have diverged over different periods of time.

Chart 4



From the attached chart we can see the OI swap curve trades at a pretty consistent discount (circa 30bps+) across all durations to our 3 month and 6 month Libor rates. Furthermore, the relationship between the differing swap curves is unstable over time, having moved from spreads close to the range 0bps to 10bps pre crisis to levels in excess of 100bps during the crisis and settling back to muted but still elevated levels. Thus decisions on discount curves and bases are not trivial and the impacts can be material with particular impact on longer dated positions.

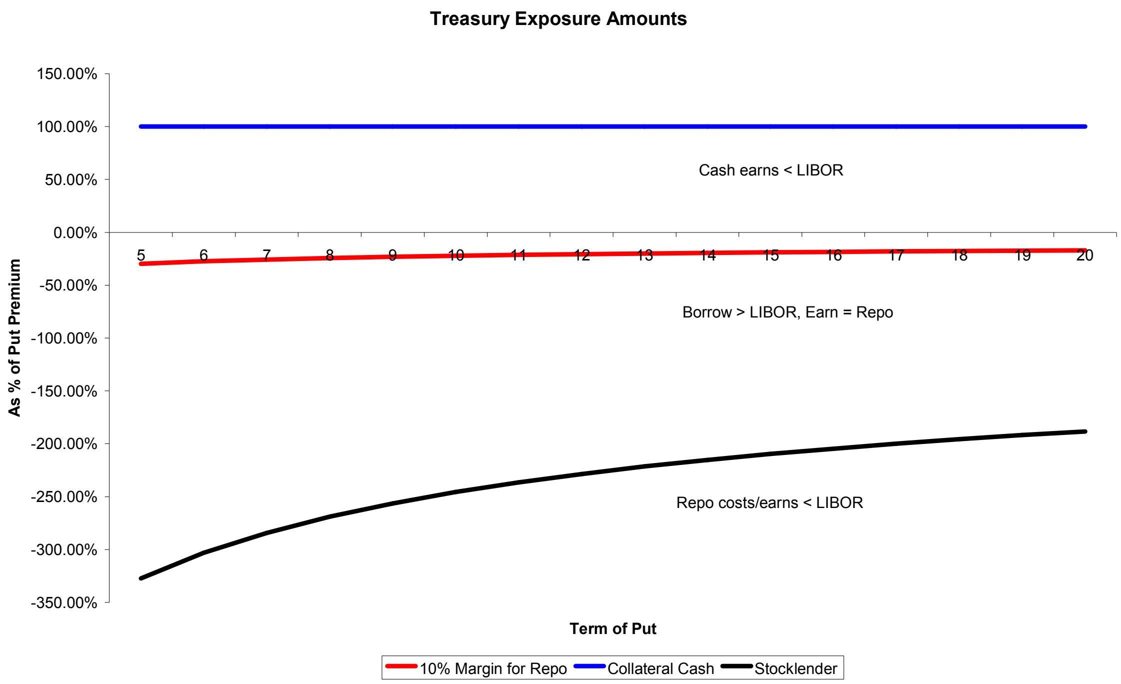
For a full exposition on the relative risks and merits of different rates it is worthwhile reviewing the May 2010 submission to CEIOPS[15] by the CRO Forum and CFO Forum regarding the risk free rate for QIS 5 and furthermore to keep abreast of current work being carried out by the Actuarial Profession in the UK.

5.2.3.2. Relative Size of Treasury Exposures

To appreciate the full impact of these treasury requirements (investing collateral, repo for stock and net borrowing and investing for excess collateral) it is important to understand not just the rate differentials but also the quantum that is exposed. The following chart illustrates these amounts as a % of the option premium and it becomes clear that the amounts are material.

Note in this example we introduce a margin requirement of 10% of the repo amount to illustrate its significance, we will not include this margin requirement in our final pricing exercise.

Chart 5



From above it is clear that the amounts in question are material (relative to the option premium) and thus are proportionately relevant to the pricing of the option.

5.2.3.3. Future Premiums and Cash Funding

In our example we have assumed that the OTC option position was paid for by an upfront premium. In the case of insurance obligations these guarantees are often funded through future premiums.

Having our deposit invested in future premiums is a material consideration when it comes to treasury aspects and in particular the value that is placed on this asset and how the position is treated for collateral purposes.

For example, where the premium leg is discounted at the same rate as the guarantee we have at least a consistency at outset, however as we advance through time our matching position is a combination of discounted premiums and accrued cash balances.

Thus the key message here is that over long durations the treasury effects of different interest rate spreads are critical to the successful pricing and security of positions. As such, the cost of these treasury effects needs to be properly contemplated as does the scope for divergences in the levels of these rates at times of crisis.

5.2.3.4. Solvency II and Cash positions

A final consideration that will have a consequent impact on risk management strategies is the current proposals under solvency II to include cash as a Type 1 asset with a consequent full credit default premium requirement. Such a position appears to be inappropriately conservative as it will penalise an overnight deposit with the same capital charge as a 12 month deposit, (and still worse when compared to an unsettled reinsurance balance due).

This treatment seems to be counterintuitive from a risk management perspective as it does not incentivise the holding of cash resources. This may potentially result in either greater risk taking in investment of collateral or a reduced appetite for cash as collateral to back exposures.

5.2.4. Risk Premiums

As a final part of the process a residual minimum expense and risk margin needs to be determined to reward un-hedged risks and return on capital requirements as augmented by the scope to assert any market dominance through an ask price margin. These are the parts of the pricing that depart from models and move more in house and reflect the liquidity of the particular OTC market in question.

To note the existence of a margin we will assume a relative margin to the net replication price and note that this relative margin will to a degree incorporate market dynamics and liquidity.

Thus RP = Replication Cost * $RP_{j,\ t}$ where j reflects the particular market and t reflects the fact that the state of risk aversion of the market time may differ from time to time. Thus $RP_{j,\ t+1} > RP_{j,\ t}$ as a market gets more risk averse or illiquid and $RP_{j,\ t+1} < RP_{j,\ t}$ as confidence and liquidity return.

6. An OTC Option "Office Premium"

In order to draw together the discussion we will look to present some illustrative pricing to exhibit the nature and scale of these margins. These are not purporting to represent current pricing of any particular asset or country but to exemplify the elements. Furthermore we will present two sets of parameters representing our stable and dislocated market scenarios. In particular it is important to note that our dislocated market scenario solely seeks to expand on the liquidity price components with no change to either interest rate or volatility parameters.

6.1. Basis and Assumptions

Theoretical Parameters

Element of the Basis	Stable Market	Dislocated Market
Statistical Volatility	20%	20%
Risk Free Rate*	5%	5%

**"n" month interbank swap rate*

Illustrative Costs

Element of the Basis	Stable Market	Dislocated Market
Transaction Costs (Volatility)	+ 2% (20%+2% = 22%)	+ 3% (2%*150%)
Uncollateralised Borrowing	5%	5%
Repo Rate	30bps	45bps (30bps*150%)
Overnight Investment	20bps	30bps (20bps*150%)
Risk Margin/Profit Loading	+5% (Price*105%)	+10% (5%*200%)

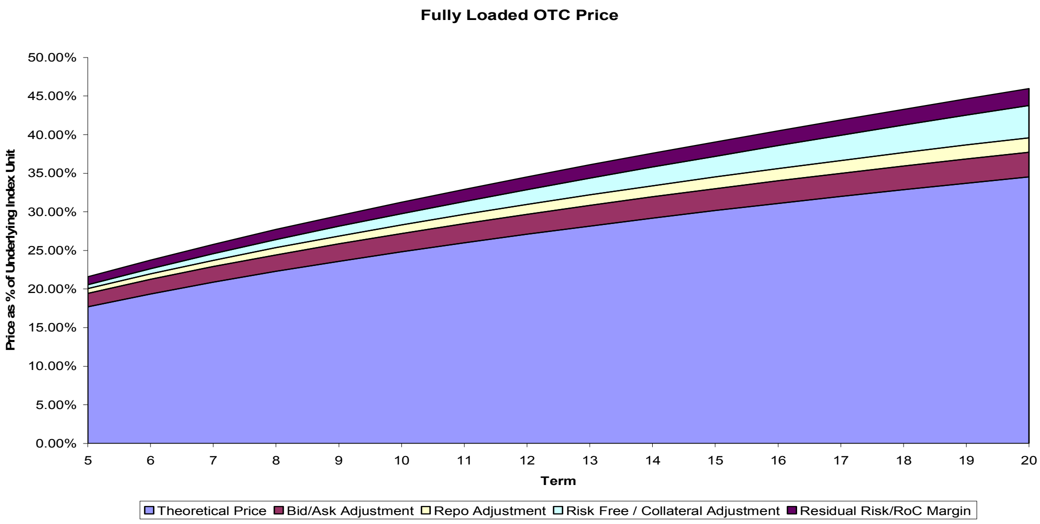
Collateral & Funding

- Premium Paid Upfront and Deposited Back/Settled
- Collateral Earns Overnight
- No net borrowing requirement (i.e. no haircut on repo)

6.2. Illustrative Office Premiums

Putting these parameter into our Black Scholes pricing for a European Put, with spot price of 1 and a strike of Exp(r_f*T) we get the following range of prices and components :

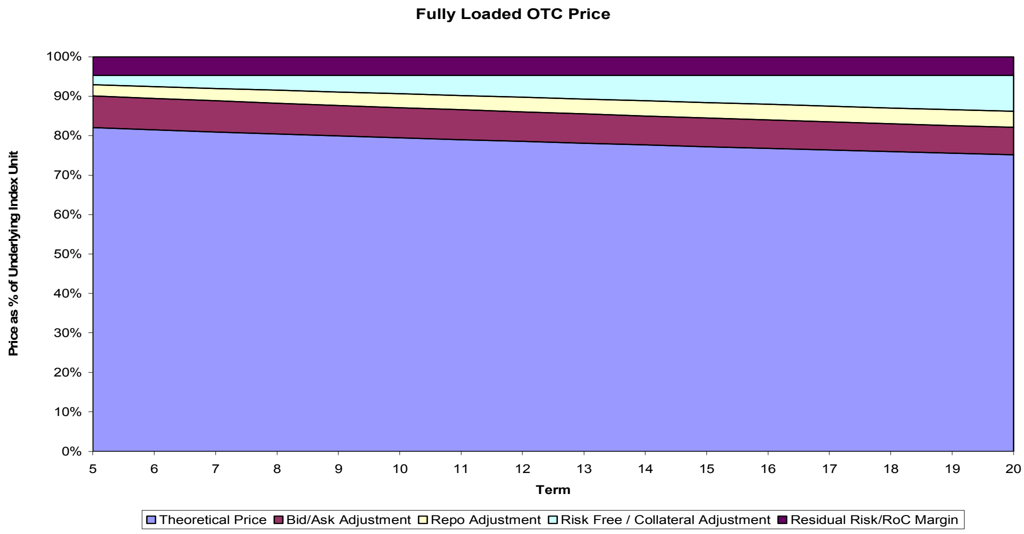
Chart 6



Notwithstanding our exercise is illustrative it is perhaps worth noting that the scale of the premiums are still material, as a % of the protected fund, and are duration dependent. From an insurance perspective, owing to the benefit of reserves being set net of future premiums, and thus nil or negative at outset, the relative size of the claim provisions for long term guarantees can often be underappreciated.

Focusing however on our liquidity consideration, we can see from the chart that the aggregate premium over the theoretical price is material, being of the order of 25% of the theoretical premium. Furthermore it appears that contribution to the total margin contributed by the interest rate related components appears to increase with duration. In particular we can better observe these relative components of price when we standardise the chart as follows:

Chart 7

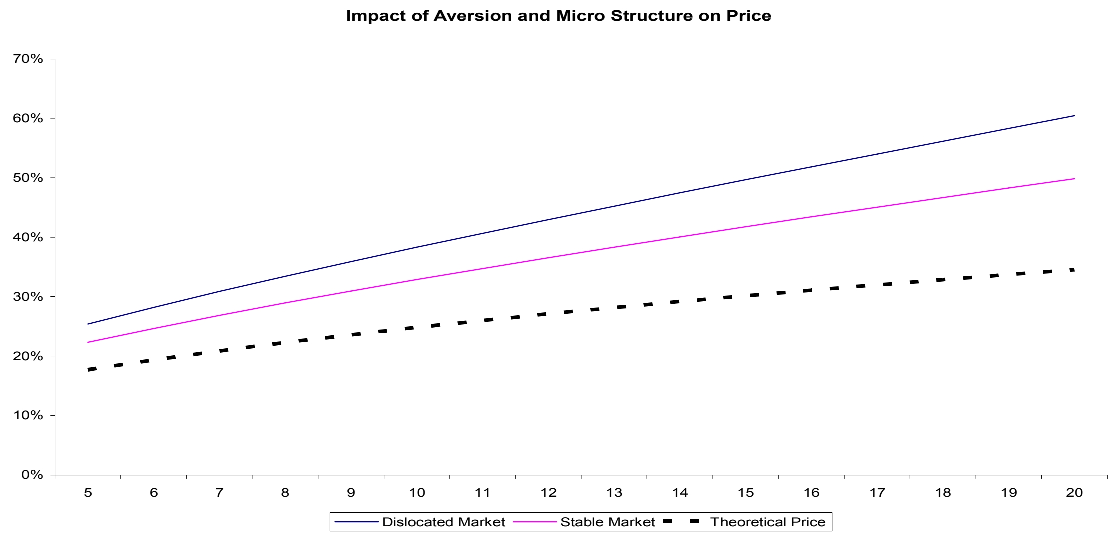


From above we can see the loadings increase with duration and furthermore the interest rate sensitivities (collateral and repo) asserting a greater impact on price.

6.3. Impact of Liquidity Shock

When we look at the sensitivity of our premium to changes in these parameters we can further identify the potential scale of variability of these parameters. In the following chart we look to stress each of these parameters simultaneously, thus we look to increase the level of liquidity charges by 50% in the underlying market (thus Bid/Ask, Repo and LIBOR/OIS spreads all blow out by 50%) and we look to double the risk aversion charge of the OTC market, thus $RP_{j,\ t+1} = 2^{*}\ RP_{j,\ t}$.

Chart 8



Note we are not changing either the level of interest rates or the expectation of volatility so as to solely isolate the liquidity effects on the option price.

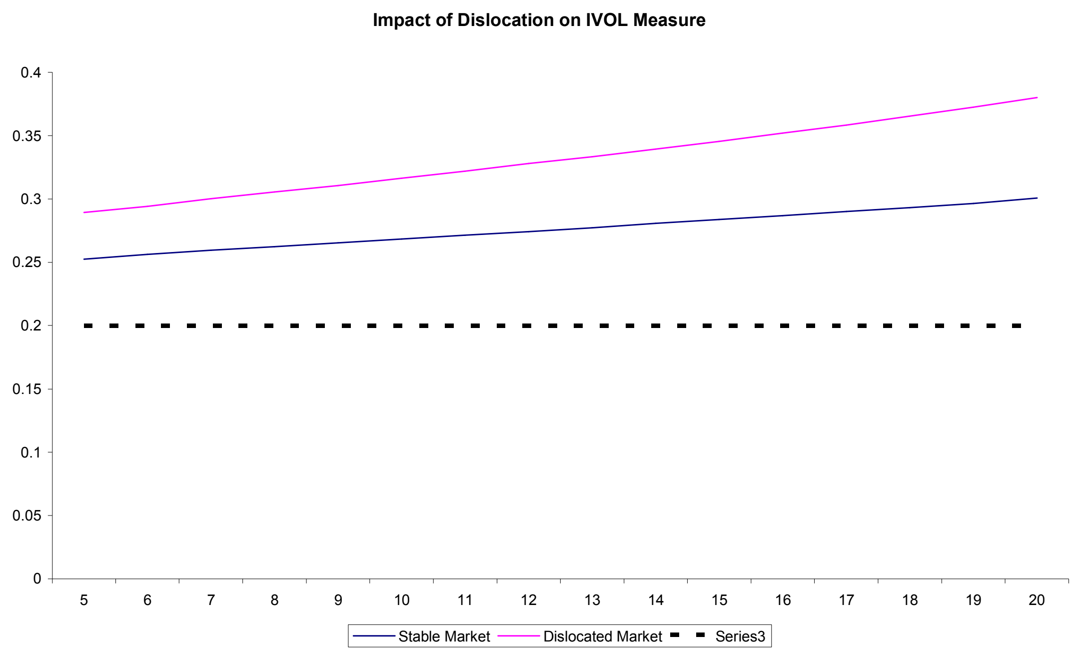
Through the above illustration it becomes clear that even seemingly modest levels of underlying replication costs can manifest themselves in material margins on option prices. Furthermore, that the impact of widening these underlying margins propagates directly into the option market even before allowing for changes in expected risk.

Thus it is imperative that reserving countenances these quantities, either directly through explicit allowance for market micro structure cost allowances or implicitly through a suitable modified IVOL parameter as per the next section.

7. Presentation and Implied Volatility

The final part of this presentation is to take a look at the usefulness of Implied Volatility as a presentation metric in the OTC market. In the final chart we take our office premiums under our initial theoretical pricing framework, our stable market scenario and our dislocated scenario and look to identify the impact on the IVOL parameter:

Chart 9



Thus, when we locate our transaction and other hedge costs in the IVOL parameter we can observe that a) the level of add on is significant (between 5% and 10%) and b) the implication is an upward sloping curve according to duration.

Furthermore, from above it is clear enough that even without changing our underlying market parameters the cost impact of changing the liquidity margins in the underlying option premium have a material effect on the IVOL curve.

So bringing this all together it points to the need for increased clarity and precision in describing metrics, models and measurements to ensure that all information is on the table and captured within the metric under discussion.

8. Implications for Insurance Company Guarantees

The purpose of the preceding note was to broaden insurance company executives understanding of the granular considerations that need to be incorporated into product design, pricing and risk management of guarantees.

In particular it points to the challenge of inference on pricing relationships between instruments of similar risk profile but differing liquidity and seeks to relate the pricing of liquidity and basis risk through an equilibrium or optimisation argument.

Finally there are considerable treasury considerations as to borrowing and investing that need careful consideration.

All of these point to the need for great care where insurance companies perceive options as being expensive or cheap when viewed solely through an insurance or actuarial lens.

So what are the direct consequences and considerations requiring further thought?

- Insurers can not dismiss the liquidity pricing information included in OTC prices.
- Where insurers believe that Investment Banks are over pricing for liquidity risk and as a result look to hold that risk and uncertainty it is important that insurers understand the cost and risks associated with this decision.
- In the limit insurers should perhaps sell protection to the Investment Banking community where they believe such price anomalies exist.
- This is of particular interest where Insurers interpose their balance sheet between the retail and wholesale markets as this introduces a further set of pricing utility to contend with.
- That basis risk and liquidity risk are related and that for cost optimisation, product design and hedge innovation for newer products are critical to a sustainable future.
- That many legacy positions may be underpriced when adjusted for liquidity
- That due to the cost of negative gamma and repo for sold put positions that the traditional investment bank structured product of Zero Coupon + Call may be an enduringly cheaper way to deliver guaranteed products. (Perhaps US style equity index annuities will play a greater roll in our future?)
- That where companies do seek to deliver alpha/illiquid fund based guarantees that security of hedge cost (and supply) is essential. Thus, permanent access to long term repo with fixed bid/ask through a direct relationship with an asset manager, could be a source of sustainable competitive advantage for an insurer.

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