How risky is my investment?

A paper presented to
the Society of Actuaries
in Ireland

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Contents

Link to Past Work - Kevin Murphy's Paper

The outcome from making an investment is not certain. When the outcome is not certain, the authors believe that investors might like to have access to the likely range and distribution of returns.

In a paper presented to the Society of Actuaries in Ireland on 24th May 2005 entitled, "It's the Outcome, Stupid", Kevin Murphy said that the actuarial profession needs investment tables to *"build a deep insight into the outcomes that are possible from the various products which we are involved in with our clients."*

Kevin's paper builds, in his own words, *"a simple investment table"* based on long-term investment assumptions. However, as we understand the point being made in Kevin's paper, there seems to be a requirement for actuaries to agree on the long-term assumptions as to return and volatility and as to correlation between asset classes. Kevin also notes in his paper that *"It will be more complex to include more modern assets such as hedge funds and private equity."*

Kevin's paper is the pioneering work in this area and we hope to build on that work by showing how the actuarial profession might make further progress without the need for agreement on long-term investment assumptions and how Kevin's concept could be extended further to some non-traditional asset classes.

In this paper, we focus exclusively on market risk and ignore other types of risk like credit risk, legal risk and operational risk. However, to the extent that interest rate risk and credit risk drives the prices of fixed income securities underlying the returns of the fixed income fund, these two risks are included in fixed income, consensus and managed funds.

Introduction

Most people in the investment business intuitively understand that, over the "long term", equities offer higher returns than bonds and that the potential for the higher returns from equities comes with the downside of greater risk than bonds.

However, it is difficult to quantify what that extra risk means for investors' portfolios. For example, how does the extra risk of equities over bonds translate into probabilities of how much an investor may gain or lose over a month, a year or a five-year period?

The aims of this paper are:

(i) to illustrate one possible way in which risk might be illustrated so that consumers might gain a better understanding of the market risk of different investment funds. The approach results in a relatively simple illustration of the likely risk of a fund generated using a simple statistical technique;

(ii) to remind investors of the need to adjust for risk when comparing the investment performance of different managers even where the managers have similar investment objectives; and

(iii) recognising that investors' choices are not restricted to single asset classes, to quantify the benefits of diversification for investors when risk is lowered without lowering potential returns by combining assets that do not have their periods of positive and negative returns at the same time.

Bootstrap Re-sampling Technique

Introduction

The approach to quantifying risk suggested in the paper is to show potential customers a table of the returns for certain types of proposed investment that is arrived at by repeated random sampling with replacement of the actual past daily returns of that investment.

The approach might typically be called the "bootstrap re-sampling technique". The sampling process builds up a distribution of the returns for the potential investment over a number of different time periods.

The potential investor can get a significantly better idea of the risk of the investment from such a table than is currently typically available from qualitative measures like low, medium and high risk or from colour coding investment funds from red, at the high risk end of the scale, through to purple at the low risk end of the scale. The distribution takes the form shown in Table 1 below.

Table 1
Irish Equity Fund
Distribution of Returns Net of Fund Management Fees

	1% of returns less than	5% of returns less than	25% of returns less than	Mean return	25% of returns greater than	5% of returns greater than	1% of returns greater than
Calendar Month	-10.5%	-7.1%	-2.3%	1.0%	4.3%	9.3%	13.0%
Calendar Year	-25.6%	-16.2%	-0.8%	12.9%	25.0%	47.4%	65.5%
Five Calendar Years *Annualised*	-6.9%	-1.9%	5.7%	12.9%	17.2%	26.2%	33.0%
Five Calendar Years ***Total***	-30.1%	-9.3%	31.8%	83.7%	121%	220%	316%

Nature of Approach

The figures in Table 1 were prepared by repeated random sampling with replacement of the actual daily returns of a unit-linked Irish Equity Fund. The set of daily returns used for the repeated random sampling consisted of all the daily returns of the Irish Equity Fund from the time it first started daily unit pricing, 1st October 1996, up to 21st November 2007, the date on which the analysis was performed.

For example, to build the return for a month, a computer program would randomly select an actual return for one trading day from among the more than 2,500 days of actual daily returns (net of fund management fees) in the sample of daily returns for the period from 1st October 1996 to 21st November 2007 and record that daily return. Having replaced that randomly selected daily return in the sample of over 2,500 actual daily returns, the computer would then randomly select a second daily return from the sample and record that daily return. This selection process would then be repeated 21 times to build up return figures for a single month. Then 250,000 such months would be created and the results summarised and tabulated in the manner shown in Table 1.

1 *There is not a fixed number of samples that must be taken. The number of samples must be such that taking more samples does not radically alter the distribution of the returns.*

A table like Table 1 gives customers a reasonable understanding of the likely range of returns of the Irish Equity Fund. Samples are taken randomly from the Irish Equity Fund's historical data set. It's easy to see how the extreme values in the distribution of monthly returns in Table 1 arise. Sometimes the sampling process for a month will result in a set of daily returns all or most of which are large negative return numbers; this gives rise to the more extreme negative return observations for a month in Table 1. At other times, the sampling process may give rise to a set of daily returns all or most of which are large positive return numbers; this gives rise to the more extreme positive return observations for a month in Table 1.

Table 1 shows investors that while there is a 1 in 20 chance that the investor might gain 47.4% or more in a calendar year, there is also a 1 in 20 chance that she might lose nearly 16.2% of her capital over the same time horizon. The table also shows that there is about a 1 in 100 chance of losing more than 25.6% of her capital in a calendar year. If the investor could not bear a loss of capital of that magnitude with that level of probability, then the table has been particularly useful in identifying to her both the magnitude and associated probability of loss.

The table is useful in assessing the risk implications on risk of investing in a geared Irish equity fund. Consider a consumer who wishes to invest in a geared version of the Irish Equity Fund in table 1 with gearing of say, 2.0x †. Let's assume that the path of prices followed by the Irish Equity Fund during the twelve month period is not such as to wipe out all of the investor's original capital or that it does not cause the value of the investor's original capital to fall below some threshold figure set by the provider of the gearing, typically a bank, at which the loan becomes repayable or the gearing must be reduced. In such circumstances the 90% confidence interval for the returns in a calendar year widens from -16.2% on the downside and 47.4% on the upside to a loss of more than one third of the investor's original geared capital amount on the downside to a gain of somewhat less than 95% of the investor's geared capital amount on the upside. We say more than one third and less than 95% to allow for the rate of interest charged by the lender for providing the leverage.

To put a loss of one third of an investor's capital in context, after a fall of one third in the value of a customer's geared investment amount, the fund must rise in value by 50% to bring the value of the customer's geared investment amount back to its pre-fall value.

The table is also useful as a benchmark against which an investor might judge the performance of the investment particularly in assessing whether losses are outside the range of what she might expect and in assessing whether she has been extremely lucky in terms of return.

The bootstrap re-sampling technique incorporates the real market 'fat-tailed events' that have occurred in the past daily returns. Extreme events of this nature are never really allowed for when a manager's daily return distribution is modelled using a normal or lognormal distribution. As the data sample spans the bursting of the technology bubble in 2000, the bear market in equities from 2001 through to 2002 and the falls in Irish equities in 2007 (up to 21st November 2007), 'fat tailed events' are included in the data set from which we are re-sampling and work their way through to the distribution of returns.

Value-at-risk (VaR) is a measure of the size of loss over a specific time period with a specified probability. For example, a fund of €100,000 might have a VaR of €2,500 over a one-day time horizon with 95% confidence. So on average, we would expect the fund to suffer a loss of more than €2,500 once in every 20 days. When there is a large amount of daily return data from which to re-sample, the bootstrap re-sampling technique gives a much more accurate estimate of value-at-risk or VaR.

Our focus in this paper is on the order of magnitude and probability of outcomes that arise due to market risk. The unit prices used for the Irish Equity Fund in applying the bootstrap re-sampling technique are net of the fund management charge used in pricing the units. The returns are **not** net of all the charges on a policy linked to the fund. For any given time period or confidence level in table 1, the returns shown in table 1 will exceed the returns on a product or policy linked to the fund.

2 A gearing of 2.0x means that for a €10,000 investment, the investor has €20,000 of economic exposure to Irish equities.

Table 2 below summarises the key advantages and disadvantages of the bootstrap re-sampling technique which we understand first appeared in 1979 when it was pioneered by the statistician Bradley Efron.

Table 2

Issue	Bootstrap Re-Sampling Technique
Nature of Approach	Samples are taken randomly from a historical data set with each observation replaced after it is registered. In trying to model the performance of a month, this procedure is repeated 21 times and the procedure repeated a large number of times to build up the return distribution for large number of months. The number of months used is chosen as that number of months above which the distribution of returns remains relatively stable if a larger number of months is used.
Advantages	• Includes observed extreme events which are never really allowed for in a stochastic simulation based on a normal or log normal distribution of the daily returns. In the case of a daily return data set for the S&P 500 index, extreme events would include such daily moves as the four, 1 in 120-year moves experienced by the S&P 500 index in 2000 on the assumption that the daily returns of the S&P 500 index are normal. • If there is a large amount of daily data, the accuracy of the VaR estimate will be greater than if the amount of daily data is small. • Fewer assumptions are required for re-sampling techniques compared with traditional statistical methods. • As the drawings from the set of daily returns are replaced, we can take as many drawings as we wish. • There is no need to estimate any parameters to arrive at the distribution of returns.
Disadvantages	• The key assumption underlying the bootstrap technique is that the daily returns are distributed identically and independently over time. The assumption is not unreasonable in the case of the returns of certain managers but it would not be valid for equity return data where there is evidence of volatility clustering. • Results are dependant on the data set and therefore the approach does not deal with events that are not in the data set. The technique does not work very well for short volatility strategies such as writing options where there are few observations of the options having been exercised against the writer of the options or where past performance is otherwise a highly unreliable guide to future performance. In this regard, see section 4.1 of the paper *Hedge Funds* by John Caslin.

Sample Distributions for a Number of Irish Unit-Linked Funds

Tables 3 to 6 show the operation of the bootstrap re-sampling technique in illustrating the risk of a number of different types of unit linked funds available in the Irish market.

Irish Equity Fund

Table 3
Distribution of Returns Net of Fund Management Fees

	1% of returns less than	5% of returns less than	25% of returns less than	Mean return	25% of returns greater than	5% of returns greater than	1% of returns greater than
Calendar Month	-10.5%	-7.1%	-2.3%	1.0%	4.3%	9.3%	13.0%
Calendar Year	-25.6%	-16.2%	-0.8%	12.9%	25.0%	47.4%	65.5%
Five Calendar Years *Annualised*	-6.9%	-1.9%	5.7%	12.9%	17.2%	26.2%	33.0%
Five Calendar Years ***Total***	-30.1%	-9.3%	31.8%	83.7%	121%	220%	316%

The estimated volatility of the return data is 17.1% per annum. The daily data set runs from 1st October 1996 to 21st November 2007.

Fixed Income Fund

Table 4
Distribution of Returns Net of Fund Management Fees

	1% of returns less than	5% of returns less than	25% of returns less than	Mean return	25% of returns greater than	5% of returns greater than	1% of returns greater than
Calendar Month	-2.8%	-1.7%	-0.4%	0.5%	1.4%	2.7%	3.7%
Calendar Year	-4.6%	-1.4%	3.2%	6.6%	10.0%	15.0%	18.8%
Five Calendar Years *Annualised*	1.4%	2.9%	5.0%	6.6%	8.1%	10.3%	11.9%
Five Calendar Years ***Total***	7.4%	15.4%	27.9%	38.0%	47.3%	63.0%	75.1%

The estimated volatility of the return data is 4.7% per annum. The daily data set runs from 1st October 1996 to 21st November 2007.

Consensus Fund

Table 5
Distribution of Returns Net of Fund Management Fees

	1% of returns less than	5% of returns less than	25% of returns less than	Mean return	25% of returns greater than	5% of returns greater than	1% of returns greater than
Calendar Month	-6.1%	-4.1%	-1.2%	0.8%	2.8%	5.8%	8.0%
Calendar Year	-15.0%	-8.5%	1.4%	9.4%	16.8%	29.3%	38.8%
Five Calendar Years *Annualised*	-2.5%	0.7%	5.4%	9.4%	12.3%	17.6%	21.4%
Five Calendar Years ***Total***	-11.8%	3.5%	30.1%	56.8%	78.8%	124%	163%

The estimated volatility of the return data is 10.5% per annum. The daily data set runs from 1st October 1996 to 21st November 2007.

Currency Fund

Table 6
Distribution of Returns before Deduction of Fees & before the Addition of Interest Income

	1% of returns less than	5% of returns less than	25% of returns less than	Mean return	25% of returns greater than	5% of returns greater than	1% of returns greater than
Calendar Month	-9.9%	-6.5%	-2.0%	1.0%	4.0%	8.6%	12.1%
Calendar Year	-23.5%	-14.3%	0.3%	13.2%	24.5%	45.5%	62.2%
Five Calendar Years *Annualised*	-5.5%	-0.8%	6.4%	13.2%	17.3%	25.7%	32.0%
Five Calendar Years ***Total***	-24.6%	-3.7%	36.6%	85.8%	122%	214%	300%

The estimated volatility of the return data is 15.2% per annum. The daily data set runs from 30th March 2005 to 21st November 2007.

The returns shown in Tables 3 to 6 are not net of all the charges on a product or policy linked to the funds. For any given time period or confidence level in Tables 3 to 6, the returns shown in these tables will exceed the returns on a product or policy linked to the fund.

Discussion of Distributions of Returns

The tables are not designed specifically to illustrate the relative return of the different funds, rather they are designed to illustrate the relative risk of the different funds. We could reduce the emphasis on the mean return by removing the mean return column from each table. However, most people would probably try to estimate it by finding the average of the upper and lower 25% confidence intervals. This is a reasonable approximation in the case of the four funds shown in tables 3 to 6. However, the mean may not always be equal to this average and we felt on balance that it should be disclosed. Besides, investment advisors can obtain the mean by calculating the average annualised return from a database containing the particular fund's returns.

We can see that the Irish Equity Fund, the Consensus Fund and the Currency Fund are significantly more risky than the Fixed Income Fund. There is a 1 in 20 or 5% chance that an investor could lose more than 7.1% in a single month in the Irish Equity Fund whereas, in the Fixed Income Fund there is a 1 in 20 or 5% percent chance that an investor could lose more than 1.7% in a month. So on average, over the long term, at least once in every two years, investors in the Irish Equity Fund will lose at least 7.1% in a month whereas the corresponding figure for the Fixed Income Fund is 1.7%.

The lower risk of the Consensus Fund versus the Irish Equity Fund or the Currency Fund illustrates the benefits of diversification across a number of different asset classes. Despite the heavy equity weighting in the Consensus Fund, the addition of fixed income, property and cash to the mix reduces the risk of the Consensus fund relative to that of the Irish Equity Fund or the Currency Fund. While this diversification comes at the cost of lower returns, many may believe that it is worth accepting the lower returns as there is a substantial reduction in risk.

The authors suggest that consumers can make better judgements about their return appetite in the light of the risk profiles in tables 3 to 6 which are based on historic daily data which includes extreme events. In addition, they suggest that the benefits of diversification, as evidenced by the lower risk of the Consensus Fund without too much return sacrifice, are clearly illustrated to consumers.

Assumptions Underlying the Bootstrap Re-sampling Technique

As a general rule, the daily returns on equity indices and funds tend to be serially correlated; see for example, *Introductory Econometrics* with *Applications* by *Ramu Ramanathan* (5th Edition) published by South-Western.

The bootstrap re-sampling technique has many technical advantages, in particular the use of past daily data which include extreme events, the ability to extract more accurate estimates of VaR and the relatively easy translation of the results to a format that can help potential investment product consumers to better understand the risk profile of investments. However, it is important to bring one disadvantage to the attention of readers.

That disadvantage is that the bootstrap re-sampling technique relies on an assumption that daily returns are identically distributed and independent of each other over time or, put another way, do not exhibit significant serial correlation.

Positive serial correlation will tend to underestimate risk so that downside returns are underestimated while negative serial correlation will tend to overstate the risk of downside returns. Adjustments can be made for serial correlation but there is no method of doing so that is likely to meet with uniform approval. No test for serial correlation has been performed on any of the data sets used in this paper, nor has any attempt been made to correct for serial correlation.

Despite this disadvantage, the authors suggest that this illustration of risk for consumers is a very significant improvement on what is currently available in the Irish and UK markets.

The advantages to consumers of the method of presentation proposed in this paper might be summarised as follows:

1. Other methods impose a potentially unsuitable statistical distribution on returns, which can often lead to problems such as the underweighting of extreme values in the data.

2. The method can be applied to any investment where daily data is available and the past is not an unreasonable guide to the future.

3. The method is very efficient - i.e. extracts a lot of information from the available data set.

4. The method gives good information on the range of returns and likelihood of a return falling within a given range.

Opportunity for the Actuarial Profession

In relation to point number 2 above, we emphasise the need for an actuary to exercise judgement in relation to the data set before applying the bootstrap re-sampling technique. A fund that writes out-of-the-money options on, say an equity index, may go for several years before an option is exercised against it; the past performance of such a fund is therefore not a guide to its future performance and it would in the opinion of the authors be inappropriate to apply the bootstrap re-sampling technique to the daily data of such a fund and represent it as a good estimate of the likely risk of the fund. Property funds may be subject to long periods of suspended dealings and to substantial and sudden reductions in value. The authors do not feel it is appropriate to apply the bootstrap re-sampling technique to the daily returns of property funds to arrive at a reasonable estimate of the risk of such funds. In view of the inherent lack of liquidity of the underlying assets of a private equity fund, many of the same arguments might apply to private equity funds.

To produce the distribution of returns for a guaranteed fund, the authors believe that it would be inappropriate to apply the bootstrap re-sampling technique to the past daily returns of a similar guaranteed fund. Instead, the authors suggest that the actual method of trading proposed for the guaranteed fund should be applied to the actual daily returns of the underlying assets of the return generating component of the guaranteed fund.

It is in this area of judgement that we believe that there is a significant opportunity for the actuarial profession to add value.

Assessing the Technique

In assessing the bootstrap re-sampling technique as applied to the daily return data for investment funds where the method might be suitable, it must be borne in mind that in the actuarial world, while the past is not a perfect guide to the future, it often is the only guide available to use in making projections.

In calculating the funding rate for a pension scheme, actuaries make assumptions including those about the future rates of return on different investment portfolios or asset classes, about future interest rates, about the relationship between interest rates and inflation rates and the variation around those assumptions for the purpose of showing clients a range of likely outcomes. All of these assumptions are informed by past experience, amongst other considerations. Similarly, past mortality data is an important consideration in pricing long-term life assurance contracts and annuity contracts.

If experience deviates adversely from the assumptions, adjustments are made to the assumptions for future use. Similarly, life assurance and annuity contracts for new entrants are re-priced to reflect deviations from past assumptions.

The bootstrap re-sampling technique is not different in this regard; when more daily data becomes available, the sample size from which the re-sampling takes place is increased and the distribution of returns is recalculated to reflect the richer past history.

According to Pramit Ghose, in the last 226 calendar years, the worst five calendar years for the performance of the Irish market were:

- -47% (1974 - Oil crisis & global recession),
- -37% (1862 - Catholic Emancipation),
- -29% (1990 - Gulf War)
- -28% (2002 - Technology Bubble Collapse) and
- -25% (1920 - War of Independence).

Source: Bloxham Wealth Management Snippets, 30th November 2007.

If we examine the results in Table 3, the distribution of returns for an Irish Equity Fund, we note that the table estimates that 1% of returns should be less than -25.6%. The performance of the Irish equity market is not the same as that of the Irish Equity Fund and the results in Table 3 ignore the impact of serial correlation on the downside performance of the fund.

Let's assume that the downside risk in Table 3 is understated by something of the order of 2.0% - 2.5% for serial correlation effects and ignore the difference between the performance of an actively managed fund and the Irish market. Under these assumptions, table 3 is not a bad guide to downside risk across the entire 226-year period as somewhere between two and three observations are beyond the adjusted lower limit of the table (-25.6% less 2.0% or 2.5%) which should only occur once per 100 calendar years. In 226 calendar years, we would expect about 2 or 3 years to be beyond the 1 in 100 year return level. Therefore the table seems

broadly consistent with the frequency of extreme losses in value experienced over a period of more than 200 years.

In the next section, we propose a sample of how the risk table for one of the funds might be presented to investors. We use the Currency Fund as an example and draw the attention of investors to a number of points including the fact that the distribution of returns is not a guarantee as to actual returns or the frequency of observation of returns and that the downside limit shown in the table is not a 'stop-loss' limit beyond which losses cannot go.

Sample Presentation of Results & Associated Caveats for a Hypothetical Firm Called FX Manager

Introduction

FX Manager believes that its customers should have a reasonable understanding of the likely range of returns of its Currency Fund as a stand alone investment.

Table A gives figures for the expected returns of Currency Fund. They have been prepared by FX Manager by repeated random sampling with replacement of the actual daily returns (before interest & fees) of FX Manager's oldest traded account for the period from 30th March 2005 to 21st November 2007. For example, to build the return for a month, a computer program would randomly select an actual return for one trading day from among the more than 600 days of actual daily returns (before interest & fees) produced by FX Manager for the period in question. This selection process would then be repeated 21 times to build up return figures for one month. Then 250,000 such months would be created and the results tabulated in the manner shown in table A below.

No Guarantee that Parameters will be within the Ranges Shown

The information in the tables represents FX Manager's best estimate of the various parameters that can be provided using current information but it cannot guarantee that actual returns will be within the ranges shown in the tables or that the frequency of actual observations of the various statistics will be as shown.

Currency Fund - Estimated Performance Characteristics

FX Manager's estimates of the expected returns of the Currency Fund and the variation around those expected values are set out in table A below.

Table A

Distribution of Returns before Deduction of Fees & Before the Addition of Interest Income

	1% of returns less than*	5% of returns less than	25% of returns less than	Mean return	25% of returns greater than	5% of returns greater than	1% of returns greater than
Calendar Month	-9.9%	-6.5%	-2.0%	1.0%	4.0%	8.6%	12.1%
Calendar Year	-23.5%	-14.3%	0.3%	13.2%	24.5%	45.5%	62.2%
Five Calendar Years *Annualised*	-5.5%	-0.8%	6.4%	13.2%	17.3%	25.7%	32.0%
Five Calendar Years ***Total***	-24.6%	-3.7%	36.6%	85.8%	122%	214%	300%

**This is not a stop loss limit. FX Manager estimates that there is at least a 1/100 chance of exceeding these values. Put another way, in the case of monthly returns, one would expect that over a long period of time, on average, at least one month in every 100 months of trading would show a loss of more than 9.9%.*

Table A shows that the average monthly return is +1.0% but in 90% of cases the monthly return may range from a loss of 6.5% to a gain of 8.6%. The returns are not net of all the charges likely to apply on a product or policy linked to the Currency Fund. For any given time period or confidence level in Table A, the returns on a product or policy linked to the fund will be less than those shown.

Making the Tables Available to Consumers

The authors have considered a number of possible options for making the information available to consumers.

One possible way to distribute this information would be for the Financial Regulator to provide generic tables which are reasonably representative of the different types of unit-linked or other funds in the Irish market, and publish those tables on its www.itsyourmoney.ie website. It would be important for the Financial Regulator to point out the benefits of diversification across a number of asset classes in terms of lower risk without significant sacrifice in return and to provide illustrative examples of such diversification.

Yet another possibility would be for a data vendor to provide the tables as part of its product offering.

If the product producers were to produce the tables we suggest, the tables might significantly increase competition in the investment product market. Publication of these tables might also shift some of the competitive focus away from comparison of the mean return towards management of the volatility of returns around the mean.

In all cases, guidance notes for the preparation of the tables would probably need to be developed in order to prevent competitive distortions. We set out some possible guidance notes in this regard in the next section of the paper.

Guidance Notes for Preparing the Disclosures

The authors believe that guidance notes may be necessary for the preparation of the tables to ensure a level playing field from a competition point of view. The overriding guidance might be that the table is fair, clear and not misleading within the context shown and that the assumptions made are clearly stated.

One possible approach would be to require product providers to make their daily return data available, for example, on their websites or on request, to persons interested in re-sampling the data to check on the accuracy of the tables disclosed by investment managers. This we believe would provide a reasonableness check on the figures disclosed by product providers.

The authors also believe that the data sets that product producers use in the re-sampling process for calculating the risk disclosures should be the full track record for the particular investment strategy. If the strategy changes at some point in time, then provided that the investment manager made a public announcement of the change by for example changing its marketing material and, where relevant, getting the investors to agree to the change in investment strategy, then the investment manager may use the shorter time period relevant to the new investment strategy.

Product providers might be required to update their tables at least every nine months.

Life Assurance Disclosure Regulations

How do the tables sit alongside the maximum illustration rates for life assurance and pension products? Clearly the expected annual return from bootstrap re-sampling will differ from the maximum growth rate prescribed for life assurance illustrations. We do not believe that the two tables are inconsistent. The current disclosure statements focus on expense and mortality charges rather than on market risk. In fact, the current tables give a highly misleading picture of risk as they assume a constant rate of return and ignore, in the case of, for example, equities, the significant volatility around any assumed illustration rate.

We believe that the current disclosure table is useful in its own right, in particular in that it allows simple comparisons of charging structures across providers. Illustration of the range and distribution of returns, in the manner we have proposed, would complement the disclosure table by illustrating the risk of the investment. Remember, the outcome from making an investment is not certain, therefore investors might like to see the likely range and distribution of returns before making an investment.

Comparing the Returns of Investment Managers

Introduction

For investors with an understanding of risk, the authors feel that the next step is to ensure that comparisons of investment performance by different managers are informed by an adjustment for the risk of the different managers being compared.

Benchmarking - Limitations

Care must be taken when comparing the investment performance of different managers or of managers against an index of the performance of managers. For a valid comparison of investment performance, at the very least, the following conditions should be met:

1. The volatility of the managers being compared should be very similar
2. The investment strategy should be similar
3. The opportunity set of the two managers being compared should be similar

Many might consider comparing the performance of managed funds to be relatively straightforward - however differences in the volatility at which managers operate can make even this seemingly simple comparison invalid. The problem is compounded for more specialist funds and asset classes.

Despite the appearance of homogeneity among managed funds, managers tend to follow different strategies. For example, at least one manager in the pooled pension fund survey in Ireland does not invest in property as an asset class. In the 'search for alpha' in looking at returns relative to a benchmark or comparing the returns of managers, it is important to bear in mind the different opportunity sets of the different managers. A manager who does not invest directly in property may appear to have alpha when direct property investment does poorly; 'alpha' may be illusory if the opportunity sets of the investment managers being compared are not similar.

Comparing the Performance of Managed Funds

As a first step in comparing the performance of managed funds, one might wish to attempt to control for volatility differences between the managed funds.

Table B below ranks the annualised returns of all of the managers listed in the Hewitt Managed Fund Performance Report for the 10-year period ending 30th September 2007 and also shows the annualised standard deviation of monthly returns ('volatility') for those managers for the same 10-year period.

The average annualised 10-year return of the managers in Table B is 7.0% per annum and the difference in annualised return between the best and the worst manager in Table B is 3.6% per annum.

The median volatility for the group of managers in Table B for the 10-year period is 11.23% per annum. If we standardise the 10-year returns of the managers in Table B so that they all operate off the median volatility, 11.23% per annum, then we arrive at the figures in Table C. The returns are standardised by dividing each manager's return in table B by that manager's own annualised standard deviation of monthly returns and multiplying the result by the median volatility (11.23%). Table C shows a slightly different ranking of the managers to that of table B. Between Table B and Table C, while there is little change in the ranking of the worst five managers, there is a new entrant into the group making up the top three managers in Table C.

When the returns of managers are compared using a common volatility, manager G jumps to 3rd place compared with 7th place when the rank of managers is compared without any adjustment for the level of volatility at which managers operate their portfolios.

Table B

Manager	10-Yr. Annualised Return to 30th Sept. 2007	Rank	0-Yr. Annualised Standard Deviation 1 of Monthly Returns to 30th Sept. 2007
A	9.0	1	12.09
B	7.8	2	10.32
C	7.7	3	10.99
D	7.5	4	11.03
E	7.5	5	11.19
F	7.4	6	11.12
G	7.2	7	9.86
H	6.9	8	11.73
I	6.9	9	11.02
J	6.6	10	11.39
K	6.6	11	11.39
L	6.3	12	11.38
M	6.1	13	11.90
N	5.2	14	11.26

Source: Hewitt with authors' restatement of joint rankings.

Table C

Manager	10-Yr. Annualised Adjusted Return to 30th Sept. 2007	Annualised Adjusted Returns to 30th Sept. 2007
B	8.5	1
A	8.4	2
G	8.2	3
C	7.9	4
D	7.6	5
E	7.5	6
F	7.5	7
I	7.0	8
H	6.6	9
J	6.5	10
K	6.5	11
L	6.2	12
M	5.8	13
N	5.2	14

Source: Hewitt with authors' calculations.

The difference between the highest and the lowest risk-adjusted return in Table C is just 3.3% per annum compared with 3.8% per annum in table B.

Diversification

As we noted earlier, the Consensus Fund exhibits less risk than either the Irish Equity Fund or the Currency Fund and this illustrates the benefits of diversification across a number of different asset classes that have their periods of positive and negative returns at different times.

To achieve a diversified portfolio, an investor must put together assets that have their periods of positive and negative returns at different times. Diversification is not simply a case of the number of funds that you invest in; it's the number of funds you invest in that have their periods of positive and negative returns at different times. Investing in lots of funds that have their periods of positive and negative return at the same time is a case of 'di-worsification' - lots of monitoring of the performance of different funds for no significant reduction in risk.

Looked at as stand-alone investments, the downside risks of the Irish Equity Fund and the Currency Fund look unappealing. In the case of the Irish Equity Fund, in at least one month in 100 months the fund is likely to show a loss of at least 10.5% and in the case of the Currency Fund, in at least one month in 100 months, the fund is likely to show a loss of at least 9.9%.

Table 7
Distribution of Calendar Year Returns for the Funds in Tables 3 to 6

	1% of returns less than	5% of returns less than	25% of returns less than	Mean return	25% of returns greater than	5% of returns greater than	1% of returns greater than
Fixed Income Fund	-4.6%	-1.4%	3.2%	6.6%	10.0%	15.0%	18.8%
Consensus Fund	-15.0%	-8.5%	1.4%	9.4%	16.8%	29.3%	38.8%
Currency Fund	-23.5%	-14.3%	0.3%	13.2%	24.5%	45.5%	62.2%
Irish Equity Fund	-25.6%	-16.2%	-0.8%	12.9%	25.0%	47.4%	65.5%

The relevant notes to tables 3 to 6 concerning time period and whether returns are net or gross of fees also apply to the corresponding rows in table 7.

If we were to invest in a combination of funds that have their periods of negative and positive performance at different times, then the risk of an investment in a combination of the two funds would be lower than that of either of the funds taken as a stand-alone investment.

An obvious combination is to diversify the Consensus Fund further by the addition of the Currency Fund. Before doing this, we have re-run the Consensus Fund over the shorter time period for which Currency Fund daily return data is available, in the interests of consistency. This yielded the table for the Consensus Fund, pictured overleaf.

Consensus Fund - Shorter Time Period

Table 8
Distribution of Returns Net of Fund Management Fees

	1% of returns less than	5% of returns less than	25% of returns less than	Mean return	25% of returns greater than	5% of returns greater than	1% of returns greater than
Calendar Month	-5.2%	-3.4%	-0.9%	0.8%	2.5%	4.9%	6.6%
Calendar Year	-10.4%	-4.7%	3.9%	10.7%	17.1%	27.4%	35.0%
Five Calendar Years *Annualised*	0.6%	3.4%	7.4%	10.7%	13.2%	17.6%	20.8%
Five Calendar Years ***Total***	3.0%	18.0%	42.9%	66.2%	86.2%	125.1%	157%

The estimated volatility of the return data is 8.9% per annum. The daily data set runs from 30th March 2005 to 21st November 2007.

Compared with Table 5, which samples daily returns of the Consensus Fund over a period of about 11 years, the distribution in table 8 is narrower on both the upside and the downside. The 98% confidence interval for monthly returns runs from -6.1% to 8.0% in Table 5 and from -5.2% to +6.6% in Table 8.

The calendar returns in Table 8 are also narrower that those in Table 5 reflecting the stronger investment performance over this shorter time interval and fewer extreme downside daily returns.

This illustrates the simple point that one should use all of the relevant daily data available to get as accurate a picture as possible of the likely distribution of returns.

Table 9 overleaf shows the distribution of returns arising from applying the bootstrap re-sampling technique to a data set consisting of daily returns based on 75% of the daily return of the Consensus Fund used in generating table 8 and 25% of the return for the corresponding day of the Currency Fund used in generating Table 6.

Table 9
Distribution of Returns for a Combination of 75% Consensus Fund & 25% Currency Fund

	1% of returns less than	5% of returns less than	25% of returns less than	Mean return	25% of returns greater than	5% of returns greater than	1% of returns greater than
Calendar Month	-4.3%	-2.7%	-0.6%	0.9%	2.4%	4.5%	6.0%
Calendar Year	-7.2%	-2.1%	+ 5.5%	11.4%	17.1%	26.0%	32.6%
Five Calendar Years *Annualised*	2.6%	5.0%	8.6%	11.4%	13.7%	17.6%	20.3%
Five Calendar Years ***Total***	13.7%	27.7%	50.8%	71.9%	90.3%	124%	152%

The daily data set runs from 30th March 2005 to 21st November 2007.

Table 9 suggests that the 75%/25% Consensus/Currency has a higher mean return relative to the Consensus Fund but more importantly it has lower risk than the Consensus Fund in that the risk of a negative return or a low positive return over a five-year period for the 75%/25% combination is quite small.

What Does it Mean to Reduce Risk?

A relatively small reduction in risk can cut the probability of a large fall (graphically, a large peak-to-trough fall in the value of an investor's portfolio quite significantly. However, risk is a difficult concept to come to grips with in advance of making a large gain or suffering a large loss or, looked at graphically, a peak-to-trough fall in the value of an investment.

To illustrate this point, take a typical managed fund and a 'diversified managed fund' which we will say consists say of 75% invested in the managed fund and 25% in a fund that does not have its positive and negative returns at the same time as the managed fund. The Currency Fund in table 6 would be such a fund. If we assume that both the managed fund and the diversified managed fund produce net-of-fees returns of 6% per annum and that their returns are normally distributed we can simulate the difference that the reduced risk of the diversified managed fund means for investors in terms of reduced chances of large peak-to-trough falls in value.

Table D
Illustration of the Meaning of Risk Reduction

Event Peak-to-Trough Fall in Value Over the Next 10 Years of More Than:	Managed Fund Chances of Event Assuming a Risk Level of 10.1% p. a.	Diversified Managed Fund Chances of Event Assuming a Risk Level of 8.8% p. a.
20%	50%	27%
25%	24%	9%

Source: MoneyMate with authors' computations. The standard deviation figures shown for the Managed Fund and the Diversified Managed Fund were derived for the period 1st March 2001 to 31st January 2007 for the average managed fund in the MoneyMate database and for the simulated returns of the Diversified Managed Fund. The standard deviation of the Diversified Managed Fund is calculated from an index of the prices of a 75%/25% combination of Friends First Mixed Pension Fund and the Insight Currency Fund. The risk calculations are based on bid to bid returns, include the reinvestment of any income, profits and losses generated on the assets of the specified funds, are net of trading costs, fund management charges and performance fees above the relevant 'high water' mark and are on an investment in the specified funds and not premiums paid under policies linked to the specified funds.

Table D above shows the reduction in risk achieved when a currency fund is added to a managed fund. The risk of the managed fund, in terms of annualised standard deviation of monthly returns, is 10.1% per annum whereas that of the 'diversified managed fund', consisting of 75% invested in the managed fund and 25% invested in the currency fund, is 8.8% per annum.

Cutting the risk by 13%, reduces by about half the chances of a 20% peak-to-trough fall in the value of the investment and cuts the chances of a 25% peak-to-trough fall in value from 1 in 4 to 1 in 11.

Summary & Conclusions

The approach to quantifying risk suggested in the paper is to show potential customers a distribution of the returns for any proposed investment that is arrived at by repeated random sampling with replacement of the actual past daily returns of that investment. The approach might typically be called the bootstrap re-sampling technique. The sampling process builds up a distribution of the returns for the potential investment over a number of different time periods. The potential investor can get a significantly better idea of the risk of the investment from such a table than is currently typically available. The distribution takes the form shown in Table 10 below.

Table 10
Irish Equity Fund
Distribution of Returns Net of Fund Management Fees
Data Sample: Daily Return Data for the Period 1st October 1996 to 21st November 2007

	1% of returns less than	5% of returns less than	25% of returns less than	Mean return	25% of returns greater than	5% of returns greater than	1% of returns greater than
Calendar Month	-10.5%	-7.1%	-2.3%	1.0%	4.3%	9.3%	13.0%
Calendar Year	-25.6%	-16.2%	-0.8%	12.9%	25.0%	47.4%	65.5%
Five Calendar Years *Annualised*	-6.9%	-1.9%	5.7%	12.9%	17.2%	26.2%	33.0%
Five Calendar Years ***Total***	-30.1%	-9.3%	31.8%	83.7%	121%	220%	316%

The daily data set runs from 30th March 2005 to 21st November 2007.

Table 7 shows investors that while there is a 1 in 20 chance that the investor might gain 47.4% or more in a calendar year, there is also a 1 in 20 chance that she might lose 16.2% or more of her capital over the same time horizon. If the investor could not bear a loss of capital of that magnitude with that level of probability, then the table has been particularly useful in identifying to her both magnitude and associated probability of loss. The table is also useful as a benchmark against which an investor might judge the performance of the investment.

The bootstrap re-sampling technique incorporates 'fat-tailed events' which are never satisfactorily allowed for when a manager's daily return distribution is modelled using a normal or lognormal distribution. As the data sample spans the bursting of the technology bubble in 2000, the bear market in equities in 2001 and 2002 and the falls in the Irish market up to 21st November 2007, fat tailed events are included in the data set and work their way through to the distribution of returns.

When a large sample size is used, the bootstrap re-sampling technique gives a much more accurate estimate of value-at-risk or VaR.

While the bootstrap re-sampling technique and its presentation in a table like that shown in Table 10 above have many technical advantages, and may be of benefit to potential consumers of investment products in illustrating the risk of different products, they have at least one disadvantage that ought to be brought to the attention of readers.

That disadvantage is that the approach assumes that daily returns are independent over time or, put another way, do not exhibit *serial correlation*.

Put simply, the presence of serial correlation means that the daily returns for time periods not too far apart are not independent of each other. Serial correlation may be positive or negative. Positive serial correlation arises when the daily returns for time periods which are not too far apart show a positive correlation with each other whereas negative serial correlation arises when the daily returns for time periods not too far apart show negative correlation with each other.

Positive serial correlation will tend to underestimate risk so that downside returns are underestimated while negative serial correlation will tend to overstate the risk of downside returns. Corrections can be made for serial correlation but there is no universally agreed method of doing so that is likely to meet with universal approval.

Despite this disadvantage, the authors believe that the illustration of risk for customers is a very significant improvement on what is currently available in the Irish and UK markets. Currently, while fund analysis systems can often calculate the annualised standard deviation of returns, explaining the risk implications of different annualised standard deviation of returns to most customers will not in general give the customer the same kind of information that we present in Table 10 above.

The paper also argues that investors might consider adjusting the returns of investment managers for risk when comparing the performance of investment managers

Finally, the paper illustrates the benefits of putting together funds that don't have their periods of positive and negative returns at the same time so as to reduce the risk of the combined portfolio without, where possible, reducing returns. The paper illustrates the meaning of the lower risk using a table similar to Table 10 above for a combination of funds and in terms of the probability of suffering a peak-to-trough fall in value of 20% and 25% over a ten-year time horizon.

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How risky is my investment?

A paper presented to the Society of Actuaries in Ireland by John Caslin and Damian Fadden on 28th November 2007.

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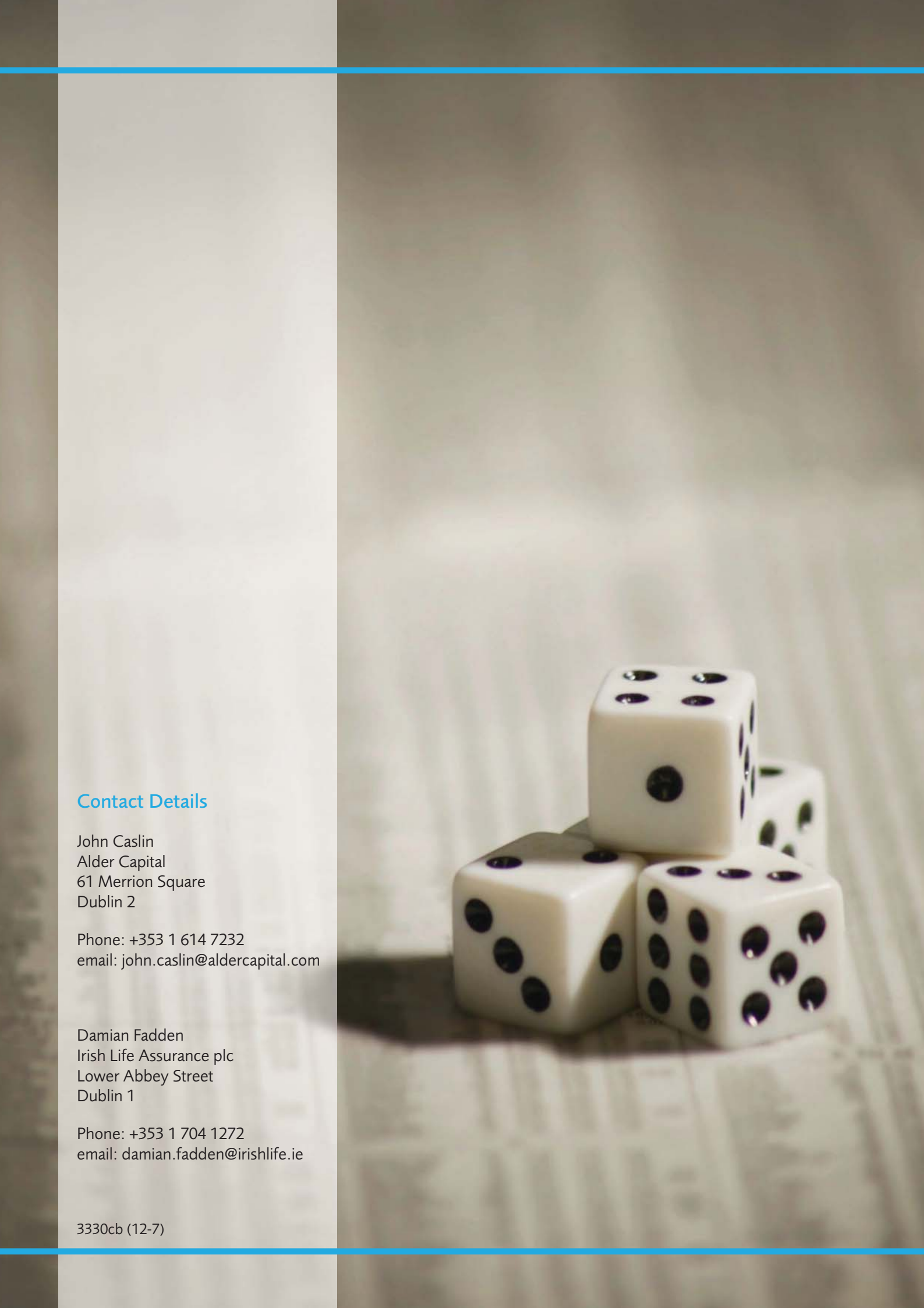

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