Reserving Uncertainty
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Purpose

- Review common stochastic reserving techniques
- Understand how the methods are applied
- Appreciate assumptions underlying the models
- Consider when it is appropriate to apply judgement
- Appreciate what the results tell us

Review of Method

- Stochastic reserving provides
  - Mean
  - Variance
- Mean = Ultimate cost of claims for origin period
- Variance = Measure of prediction error
- Prediction error = Estimation Error + Process Error
- Estimation Error (or Parameter Error) = The error due to the choice of model & parameters used to estimate the mean value
- Process Error = The error due to the actual claims generation & settlement process
- Other errors

Notation

$i = \text{Origin period index}$

$j = \text{Development period index}$

$n = \text{Number of data points in the triangle}$

$n_j = \text{Number of data points in development period } j$

$C_{ij} = \text{Cumulative payments at dev period } j \text{ in respect of origin period } i$

$D_{ij} = \text{Incremental payments at dev period } j \text{ in respect of origin period } i$

$\lambda = \text{Estimator for development factor}$

$\sigma = \text{Estimator for volatility}$

ODP Model

- Calculate development factors to determine mean value of ultimate claims
- MLE is equal to the volume weighted chain ladder factors
- Can choose different averaging periods
- Can remove development factors (but impact later on)
- Assume that claims are fully run off by latest development period in the triangle

ODP Model

- The sum of incremental amounts in any column must exceed 0

$$\sum_i D_{ij} \rangle 0$$

- All projected future incremental amounts are positive
- Origin periods are independent
- Incremental movements within the same origin period are independent

ODP Model

$$E\left[D_{ij}\right] = \lambda_j D_{ij-1}$$

$$\hat{\lambda}_j = \frac{\sum_i C_{ij}}{\sum_i C_{ij-1}}$$

- Equations above use all the information available, but you can use different averaging period & development factors

ODP Model – Paid Data Example

114	213	307	384	437	472	492	506	514	517
192	352	501	617	679	716	738	760	774	779
215	382	515	610	677	715	740	763	777	782
214	357	458	544	605	649	690	711	724	728
203	319	405	498	559	610	636	655	667	671
197	301	378	441	501	535	558	575	585	588
222	324	385	456	509	544	567	584	595	598
285	403	476	574	641	685	714	736	749	754
368	514	664	800	893	955	996	1,025	1,044	1,050
492	774	1,000	1,205	1,346	1,439	1,500	1,545	1,572	1,582
	1.573	1.291	1.205	1.117	1.069	1.042	1.030	1.018	1.006

Ult all years = 8,050

Ult 2 years = 7,717

ODP Model

- Variance is a measure of the prediction error
- Variance is proportional to the mean

$$V\left[D_{ij}\right] = \sigma^2 E\left[D_{ij}\right]$$

- Variance must be positive
- Variance is a function of the incremental amount
- Therefore the incremental amounts must be positive
- Variance is estimated from data by back casting

ODP Model

- The scale parameter can be calculated for the whole triangle (constant scale) or calculated for each development period (variable scale)

$$\sigma^2 = Bias\ Adj \times \sum_{i,j} \left(D_{ij} - \hat{D}_{ij} \right)^2 \Big/ (n-1)$$

$$\sigma_j^2 = Bias\ Adj \times \sum_{i} \left(D_{ij} - \hat{D}_{ij} \right)^2 \Big/ (n_j - 1)$$

ODP Model – Paid Data Example

	13.516	0.449	5.479	1.668	1.002	0.561	0.000	0.026	0.056	-
	10.312	3.140	12.571	2.449	0.805	1.692	2.175	0.005	0.037	
	3.166	5.297	4.092	0.326	0.163	1.271	0.823	0.041		
	0.739	1.496	0.099	0.695	0.219	0.020	6.154			
	0.167	0.110	0.981	0.551	0.014	3.287				
	1.126	0.008	0.667	2.182	0.911					
	7.098	0.282	6.975	0.539						
	11.026	2.091	10.799							
	5.315	9.267								
	-									
Constant	2.666									
Vary	5.723	2.717	5.844	1.376	0.612	1.677	2.995	0.035	0.091	

ODP Model - Judgement

- Selection of Mean
  - Choice of development profile
  - Removing outliers
- Volatility
  - Consider the pattern of volatility
  - Would you over write
  - Impact of over writing
- Tail
  - Adding a tail factor

ODP Model - Bootstrapping

- Bootstrapping is a technique to provide simulations
- Calculate residuals

$$\left(D_{ij} - \hat{D}_{ij}\right) \Big/ \sqrt{\hat{D}_{ij}}$$

- Assumes that incremental claims within an origin period are independent
- Claims are not "path dependent"

ODP Model - Bootstrapping

- Standardise residuals

$$Bias\ Adj \times \left[\left(D_{ij} - \hat{D}_{ij} \right) \Big/ \sqrt{\hat{D}_{ij}} \right] / \sigma$$

- Sample from residuals with replacement
- Back forecast from leading diagonal to create pseudo incremental data triangle
- Projected increments have to be positive

Mack Model

- Calculate development factors to determine mean value of ultimate claims (use cumulative data)
- LSE is equal to the volume weighted chain ladder factors
- Can choose different averaging periods
- Can remove development factors (but impact later on)
- Assume that claims are fully run off by latest development period in the triangle

Mack Model

- Negative increments allowed
- Origin periods are independent
- Claims are path dependent
- Mack Model is distribution free (?)

Mack Model

$$E\left[C_{ij} / C_{ij-1}\right] = \lambda_j C_{ij-1}$$

$$C_{ij} = \lambda_j C_{ij-1} + \sigma_j \sqrt{C_{ij-1}} \varepsilon_{ij}$$

$$Where,$$

$$\varepsilon_{ij} \sim i.i.d.\, N(0,1) and$$

$$\hat{\lambda}_j = \frac{\sum_i C_{ij}}{\sum_i C_{ij-1}}$$

Mack Model – Example

417	514	549	555	555	552	580	574	547	546
279	442	567	659	708	736	752	769	779	778
350	503	605	677	717	739	757	773	763	762
359	480	559	619	662	690	720	729	720	719
372	458	513	571	603	638	659	667	659	658
367	439	477	506	535	553	572	579	572	571
433	492	524	541	570	590	609	617	609	608
515	581	604	657	693	716	740	750	740	739
625	683	769	837	882	912	942	954	942	941
800	989	1,113	1,210	1,276	1,319	1,363	1,380	1,363	1,361
	1.235	1.126	1.088	1.054	1.034	1.034	1.012	0.988	0.998

Ult all years = 7,682
Ult 2 years = 7,407

Mack Model

- Variance calculation

$$\sigma_j^2 = \left[\frac{\sum_i C_{ij-1} \times \left(f_{ij} - \hat{\lambda}_j \right)^2}{n_j - 1} \right] \times Bias \quad Adj$$

$$Where \quad ,$$

$$f_{ij} = \frac{C_{ij}}{C_{ij-1}}$$

Mack Model – Example

	0.003	1.684	3.164	1.641	0.860	0.141	0.309	0.688	-	-
	33.990	11.074	3.095	0.282	0.019	0.097	0.067	0.514		
	14.109	3.013	0.553	0.020	0.005	0.068	0.052			
	3.683	0.741	0.219	0.156	0.044	0.065				
	0.008	0.006	0.297	0.002	0.363					
	0.591	0.623	0.373	0.009						
	4.311	1.839	1.561							
	5.779	4.318								
	12.559									
Sigma^2		7.712	2.737	1.269	0.347	0.265	0.102	0.176	0.988	

Mack Model - Judgement

- Selection of Mean
  - Choice of development profile
  - Removing outliers
- Volatility
  - Consider the pattern of volatility
  - Would you over write
  - Impact of over writing
- Tail
  - Adding a tail factor

Mack Model - Bootstrapping

- Bootstrapping is a technique to provide simulations
- Calculate residuals

$$\left(C_{ij} - \hat{\lambda}_j C_{ij-1}\right) \Big/ \sqrt{C_{ij-1}}$$

- Incremental movements within an origin period are correlated
- Claims are path dependent

Mack Model - Bootstrapping

- Standardise residuals

$$Bias\ Adj \times \left[\left(C_{ij} - \hat{\lambda}_j C_{ij-1} \right) \Big/ \sigma_j \sqrt{C_{ij-1}} \right]$$

- Sample from residuals with replacement
- Project forward from the previous (actual) development period to create pseudo cumulative data triangle
- Projected increments do not have to be positive

Scaling

- Additive
- Multiplicative
- Question scaling

	Paid €m	Incurred €m
2 year	7,717	7,407
AIII	8,050	7,682

Process

Select Model

Select development factors to determine mean

Determine weighted (Actual – Expected)

Calculate volatility / variance

Calculate residuals

Simulation

Process with Judgement

Select Model – What model

Select development factors to determine mean – What averaging period / dev factors, tail factor

Determine weighted (Actual – Expected)

Calculate volatility / variance – Does the variance look reasonable, would you overwrite

Calculate residuals

Simulation – Prevent negative results & SCALING