Equity Risk Premium
- Is there a magic number?

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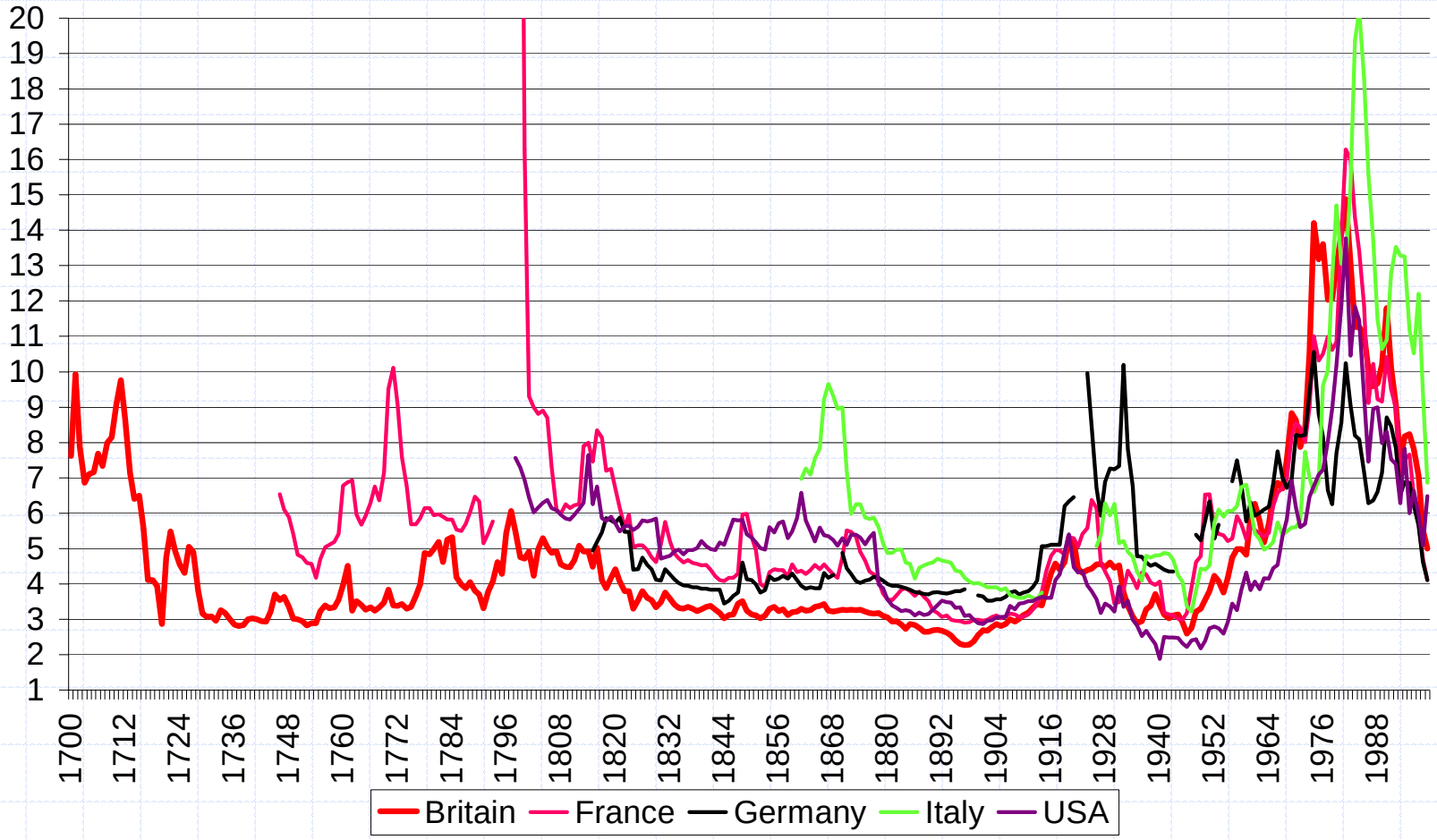
Outline of Talk

- History of Capital Markets
  - Really just 20th Century and early 21st Century.
- Statistical Interpretation of History
  - "The problem with the past is that it is statistically insignificant: it is a sample of only one!"
  - No satisfactory models as yet for investment risk and reward
- Three Stylised Models of Market Returns
- Predictions from the Models of Extreme Events
- Concluding Remarks and Estimate

History – not that long

- Interest rates, in the modern sense, since about 1700.
- General Limited Liability allowed 1811 in NY, since 1850s in UK.
  - In 1844 just 10 companies listed on Irish equity market.
- Reasonable data on equity returns only since 1900, only on 19 surviving markets.
  - But for UK market prior see Gayer *et al.* (1940) and Grossman (2002), and for Irish market prior see the on-going project of Charles Hickson and John Turner of Queen's University, Belfast.
- A summary of the history…

(Long) Rate of Interest, 1700-2000



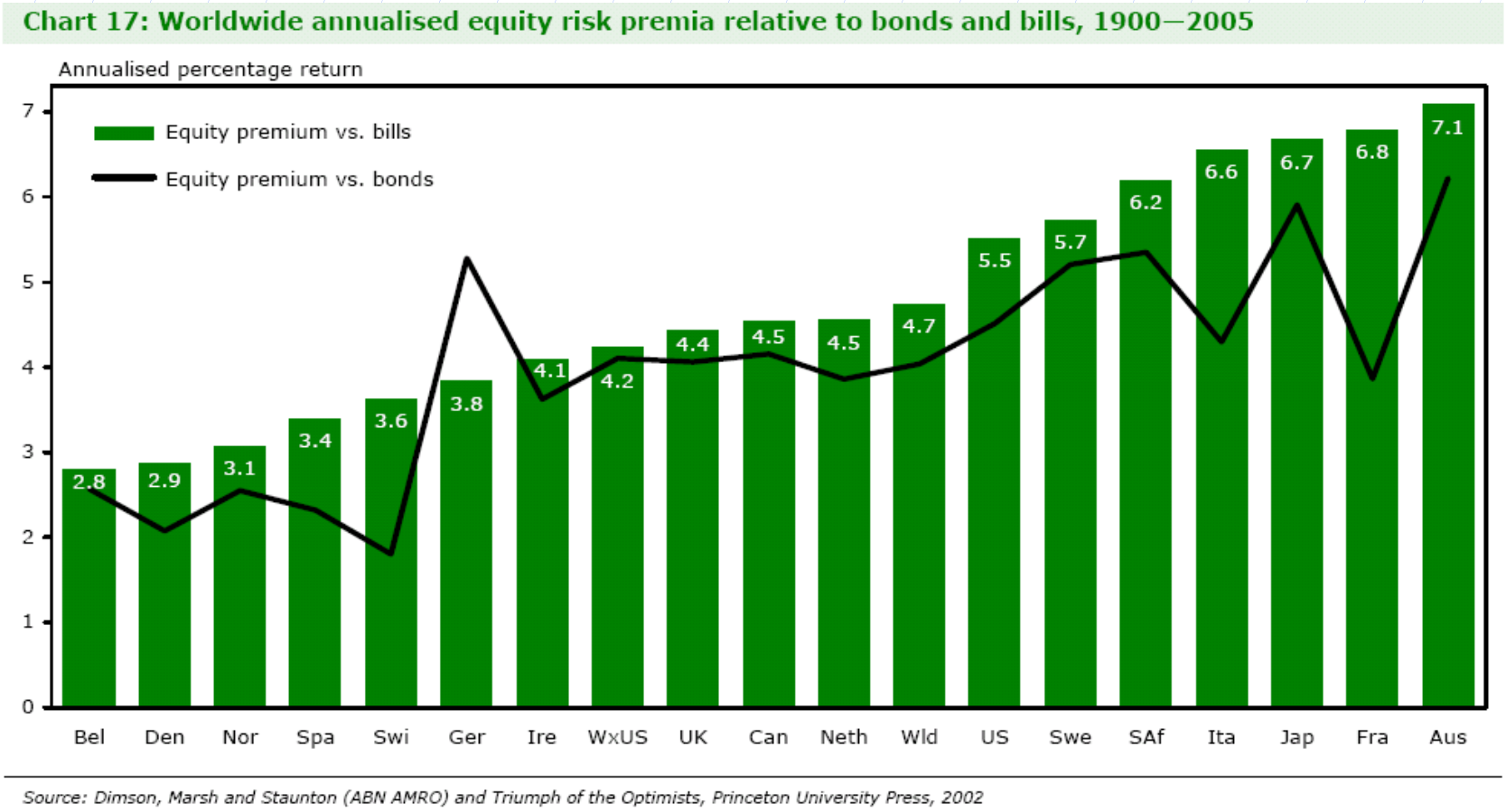
Sources of Data: A History of Interest Rates, Homer, S. & R. Sylla, Rutgers University Press, 1996; *Sylla, R.* website*; Global Financial Data* website; *British Historical Statistics*, Mitchell, B.R., Cambridge University Press, 1988. Note that the real return on gilts over the entire period since 1700 was 3.1% in the UK and 3.4% in US (*Source: Global Financial Data).*

Annualised Real Returns on Major Markets, 101 Years Ending 31st Dec. 2000

Country	Equity	Bonds	Cash	Inflation
	% p.a.	% p.a.	% p.a.	% p.a.
Ireland	4.7	1.0	0.7	4.5
UK	5.8	1.3	1.0	4.1
US	6.7	1.6	0.9	3.2
Japan	4.5	-1.6	-2.0	7.6
Netherlands	5.8	1.1	0.7	3.0
Germany	3.6	-2.2	-0.6	5.1
France	3.8	-1.0	-3.3	7.9
Italy	2.7	-2.2	-4.1	9.1
Spain	3.6	1.2	0.4	6.1

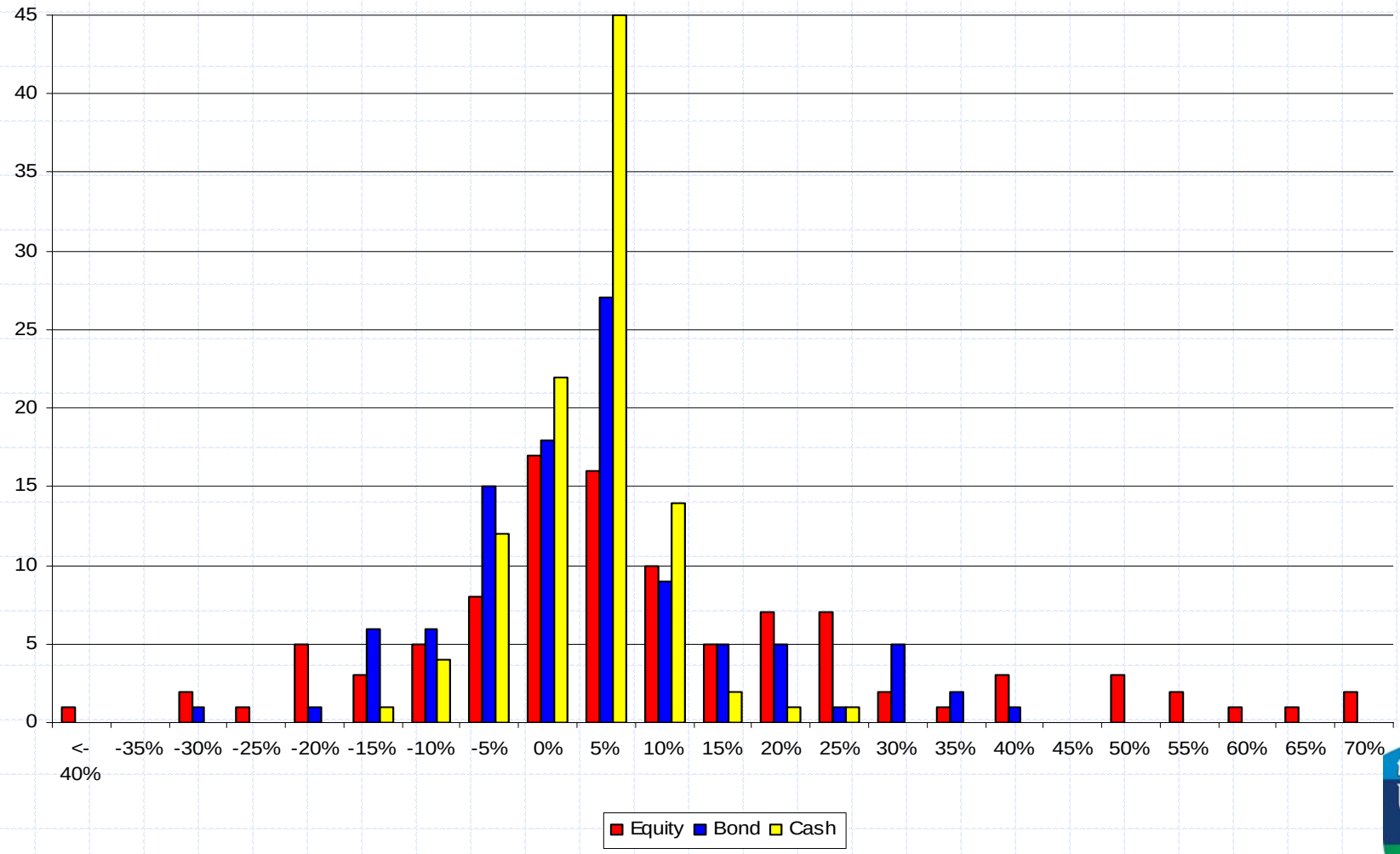
Sources: For Ireland see Whelan (2004), otherwise figures taken from Tables 4-1 and 5-1 in Dimson *et al.* (2002). Figures for Germany exclude the two-year hyperinflationary period of 1922-23. If this episode was included then German inflation would go up to an annualised rate of about 34%, cash returns fall to –19% real p.a, bond returns to –8.5%, and equities to 4.5% real p.a. (Dimson *et al.* (2000)).

Across 19 Markets, 1900 to 2005



Source: Dimson, Marsh *et al.* (2006)

Histogram of Annual Real Returns from Irish Capital Markets, 1900-2001.



Risks in Other Markets

- **Europe**
  - France and Germany both post a run of over 50 years with a negative real return (1900-1952, 1900-1954 respectively) over the period 1900 to 2005. [Dimson, Marsh *et al.* (2006) p.31)]
- **US**
  - 1929 Crash. Fall of 71% from start 1929 to start 1933.
- **Japan**
  - Real return on equities –7.1% p.a in 1990s
  - Real return on bonds +5.4% p.a.
- **World**
  - From March 2000 to end 2002 – World equity markets (FTSE World) falls over 50%
  - From 1901 to 1920 World equity market posted negative real return.

Estimates of Prospective ERP, above Cash

	Arithmetic Mean Excess Return relative to Cash
Survey of 226 Fin Economists (Welch (2000)):	7.1%
Survey of 17 Experts (Welch (2000):	6.5%
Extended to 510 in Welch (2001) (both groups)	5 to 5.5%
Dimson, Marsh *et al.* (2002)	3.25% to 5.25%
Dimson, Marsh *et al.* (2006)	5%

[Note: Cash returns have averaged about 1% real around the world since 1900]

Careful!

- When estimating ERP in the past, should estimate arithmetic average (and variance of equities), not the geometric means.
  - Arithmetic mean is the superior estimator unbiased, the maximum likelihood estimator, and is, asymptotically normally distributed with variance the Cramér-Rao lower bound.
  - The geometric mean is significantly biased, the bias increasing with the tracking error and with term, (and may not, in fact, converge)
  - Also the expected geometric return of a portfolio is always higher than the weighted average of the geometric return of it constituents
  - Complex relationship between geometric mean and arithmetic mean, even when returns are normally distributed

$$E[r_n^G] = (1 + E[r_n^A]).\left(1 + \frac{Var[r_n^A]}{(1 + E[r_n^A])^2}\right)^{-\frac{1}{2}} - 1$$

(See, for instance McCulloch (2003) for a more comprehensive treatment).

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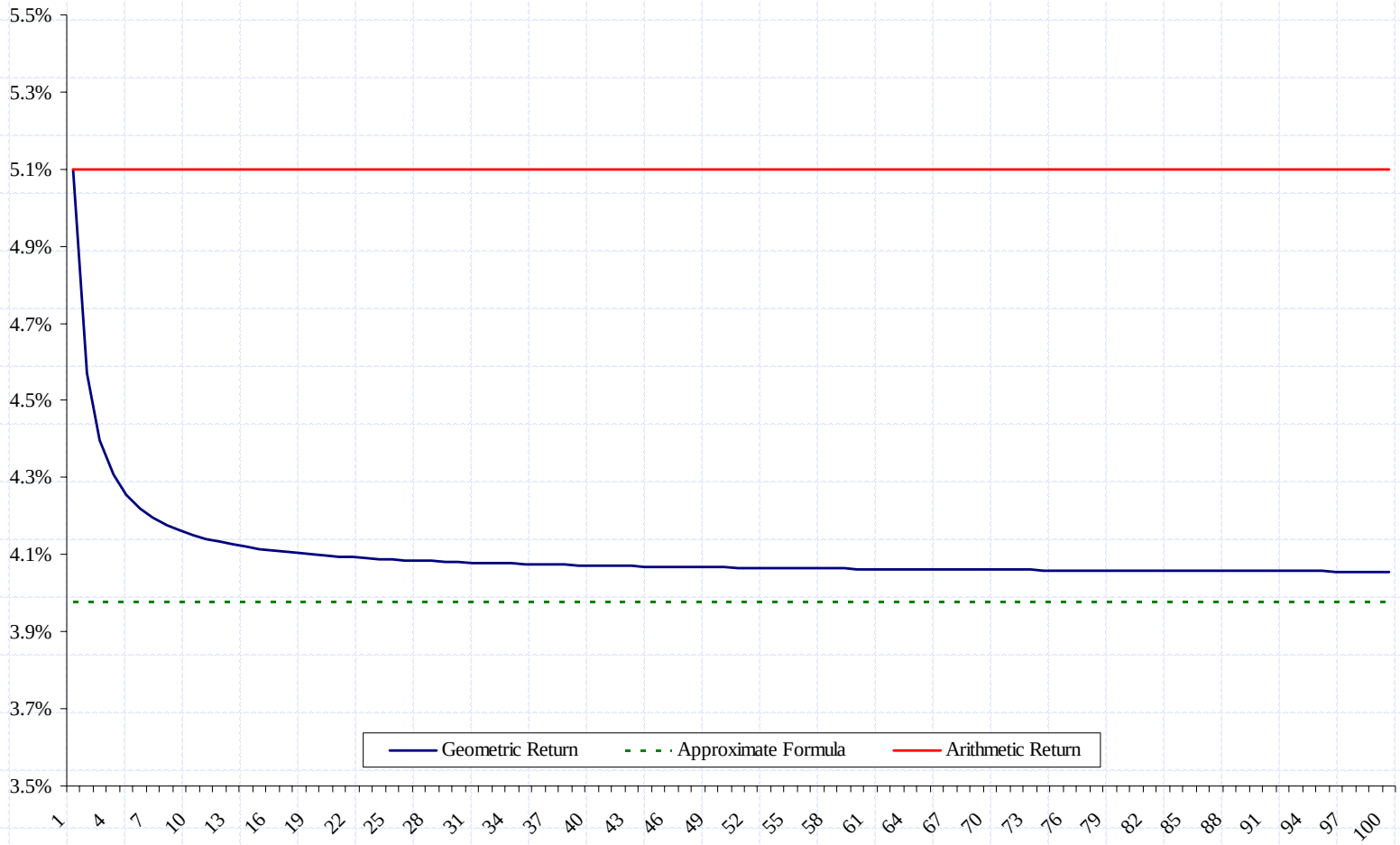
Relationship between Arithmetic Means and Geometric Means (Normal Model)[[Based on World Risk Premia Relative to Bonds, 5.1% AR, SD 15%, GM 4.0%, Table 11 in Dimson et al. (2006)]



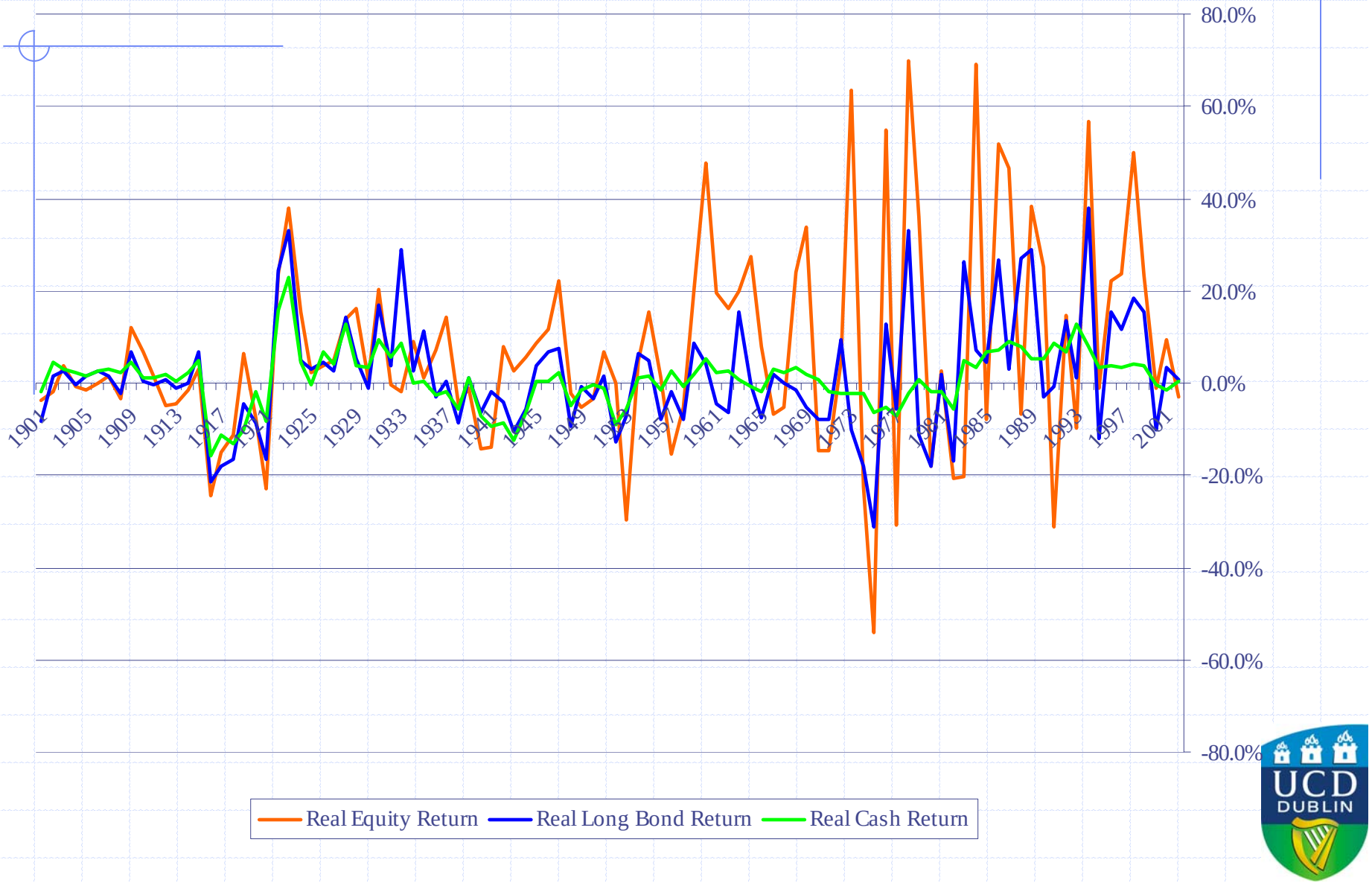
Careful!

- The ERP it is the excess returns of equities over an assumed riskless asset.
  - The riskless asset can be taken as cash, or nominal or real bonds of a certain duration.
  - Note that the duration of the bond must be specified – returns from different durations are markedly different and duration mismatch can be as significant a risk as equity risk (Whelan (2004)).
  - To calculate the an unbiased estimate of the ERP, one calculates the arithmetic mean of the excess return and the standard deviation of the excess return (i.e, the tracking error).
  - This prescription is not pedantic - Derrig & Orr (2004) show using long-term US data that the ERP can vary from 5% to 8.4% per annum depending on definition and calculation method.
- Implications for actuarial guidance on the ERP are:
  - ERP should be stated relative to (a) cash, (b) nominal bonds of different durations, (c) real bonds of different duration
  - The historic arithmetic average with associated tracking error should be stated.
  - Dimson, March & Staunton do not do this, they employ geometric means (subtracting them) and duration of bond varies with time and across countries.
- Perhaps, better than ERP, is to talk simply of the long run returns from equities, from bonds (of different durations) and cash.

Statistical challenges to Modelling Market Returns

- **Returns series are non-stationary.**
  Secular changes in (unconditional) covariance structure of returns.

Irish Asset Classes – Real Returns, 1900-2001



Statistical challenges to Modelling Market Returns

- **Returns series are non-stationary.**
  Secular changes in (unconditional) covariance structure of returns.
- **Return series are heavy-tailed distributions**
  - Similar shape irrespective of Δt
  - Not in domain of attraction of stable distribution
  - So thick that 4th moment is unlikely to exist
  - Still evident when volatility clustering removed (by ARCH models, etc) but now less heavy
- Volatility Clustering
  - Positive correlation of volatility measures with time
  - Power-law decay with increasing time
- Volume-Volatility Correlation high
- Others...
  - asymmetry between large positive and negative movement (latter more frequent)
  - 'leverage effect', where the correlation of the return to future (instantaneous) volatility is negative decaying to zero.

3 Stylised Models of Equity Returns

1. Returns are Normally Distributed
2. Returns come from a Stable Distribution
3. Returns come from a Mixed Distribution of the form

$$(1\text{-}p).(-100\%) + p.\,X$$

where X is distributed as 1 or 2.

Model 3 : Mixed Distribution

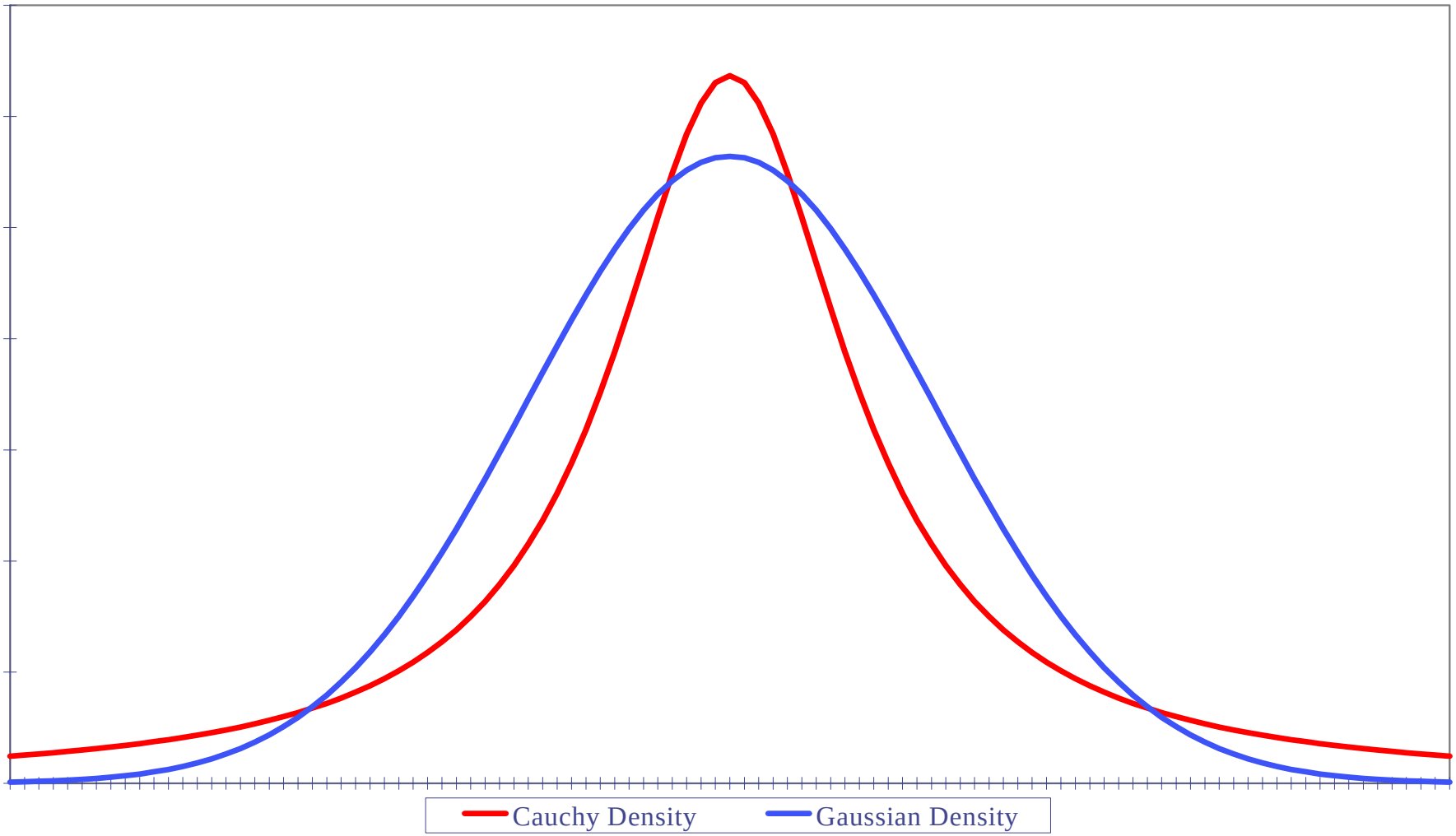
- This simply says that the average return and risk of model 1 and model 2 are too high and must be lowered to take into account the survival bias of our data
  - i.e., we must allow for markets like Russia, Greece, Romania, Hungary, Egypt, etc., that failed in the 20th century, giving investors nearly a -100% return.

Model 1 : Normal Distribution (Poor Fit)

Calibrated from Returns of Irish Equity Market over	Periodicity of Market Fall of x% in one year, where x =		
	-10%	-20%	-50%
10 Year Daily Returns	94	663	2,700,000
70 Year Monthly Returns	7.8	27.3	12,000
200 Year Annual Returns	8.1	27.1	8,800

For details, see Whelan (2003).

Normal & Symmetric Stables



Model 2 : Stable Distribution (better fit)

Calibrated from Returns of Irish Equity Market over	Periodicity of Market Fall of x% in one year, where x =		
	-10%	-20%	-50%
10 Year Daily Returns	12.7	24.6	99
70 Year Monthly Returns	5.7	10.0	28
200 Year Annual Returns	11.4	26.3	86

For details, see Whelan (2003).

Little Confidence in Predictions

- History shows the ERP varies over time
- History shows that equity risk varies over time
  - this variability is not random variation – it is of a sort that cannot yet be modelled adequately (see Whelan (2005))
- Using stylised models we showed the wide variability of future equity returns

A World Without a Crystal Ball

"Everything should be made as simple as possible, but not simpler."

A. Einstein

The ERP, in almost all applications, must be forecast with the associated risk

and,

there must to be a management programme to cater for the inevitable merging differences (i.e., an 'actuarial control cycle')

If I had to Predict Equity Returns

- Reasonable to assume be above real return on long index-linked stock
  - So greater than 2% real (at the current time).
- History of surviving equity markets, 1900-2005, gives arithmetic mean return of 7.2%, with standard deviation of 17.2%
- Reduce, for survivorship bias, and allow for greater integration of markets
  - So real arithmetic mean of, say, 6.5% (see Jorion & Goetzmann (20000) which justifies reduction of 0.4% from 1920) with a standard deviation of 20% (equal to UK and US market over 106 years)
  - Translates to a geometric return of about 4.7%.
  - Other factors – current rating of markets, aging population, size of markets relative to economies, etc., might suggest to take above figure as towards upper bound.
  - So best estimate range of say, 3% to 5% real. Central best estimate of 4% to 4.5% real p.a. (geometric).
- Remember must model risk associated with this return using non-normal distribution
  - Including high probability of 20 year period with negative real returns, as three-quarters of the 19 national stock markets experienced a negative real return lasting more than twenty years. [Dimson, Marsh *et al.* (2006) p.32)]

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