

# What Makes a Good Economic Capital Model?

## Seminar on Reinsurance 2010

An interactive session for a lively discussion between audience members and panelists.

Economic Capital Models are being discussed frequently by rating agencies, regulators, boards of directors, and investment analysts.

What are the ingredients for "good" EC models - and their proper role in an ERM framework?

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# Agenda

## 1. Qualities of a good ECM

## 2. Basic model structure and key issues

- a) Reserve risk
- b) One year vs. ultimate
- c) Investment risk
- d) Bad debt risk
- e) Shock & Attritional
- f) Correlations
- g) Catastrophe
- h) Operational Risk

## 3. Principles of Internal Model Admissibility

# Qualities of a good ECM:

- Internal models should cover all material risks of the company and should be consistent across all risks.
- Internal models should reflect the nature, scale and complexity of the underlying businesses;
  - they should be proportional in sophistication to the materiality of the risks they cover.
  - Materiality levels should be determined by stakeholders based on the model's purpose.
- Practical considerations for models include usability, reliability, timeliness, process effectiveness, systems and cost efficiency.
- There should be an acceptable tradeoff between accuracy and the various practical constraints.

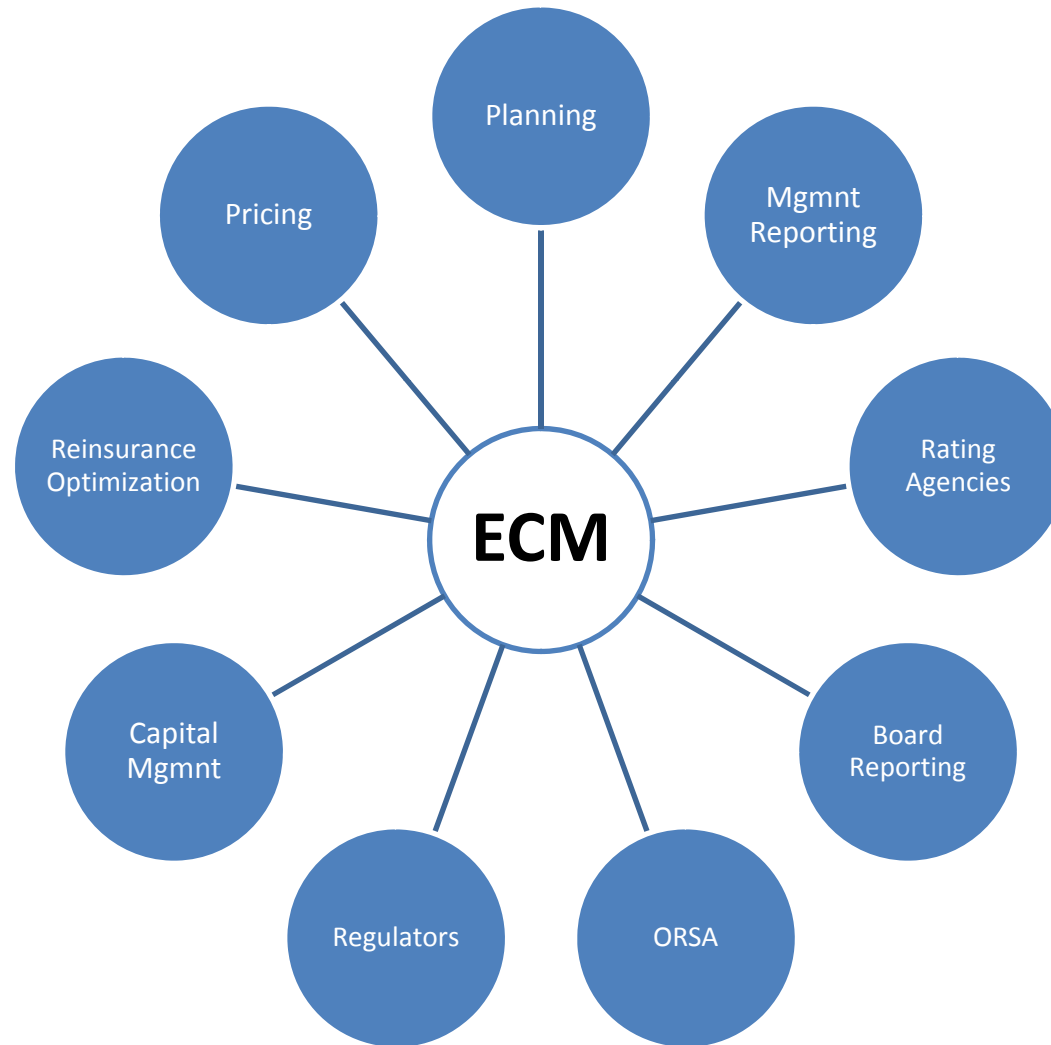
# Qualities of a good ECM

- **Principle 1 (Technical basis)** – Internal models should cover all material risks of the company in a consistent manner. They should adopt modeling techniques and approaches appropriate to the nature, scale and complexity of the business.
- **Principle 2 (Dynamic nature)** – Models should be dynamic and flexible in nature.
- **Principle 3 (Practicality and proportionality)** – Internal models must be practical in the context of the organization and the model's purpose.
- **Principle 4 (Transparency and Documentation)** – The insurer should document the governance, methodology and assumptions underlying the internal model and its development. Internal model results should be traceable and auditable.
- **Principle 5 (Use of the model)** – The insurer should ensure that the internal model, its methodologies and results, are fully embedded into the financial and risk strategy and operational processes of the insurer.
- **Principle 6 (Governance)** – The insurer should have adequate governance and internal controls in place with respect to the internal model.
- **Principle 7 (Independent review)** – Insurers should subject their models to suitable regular independent review - internal or external depending on materiality - to validate the appropriateness of the model and be able to demonstrate that the model remains fit for purpose in changing circumstances.

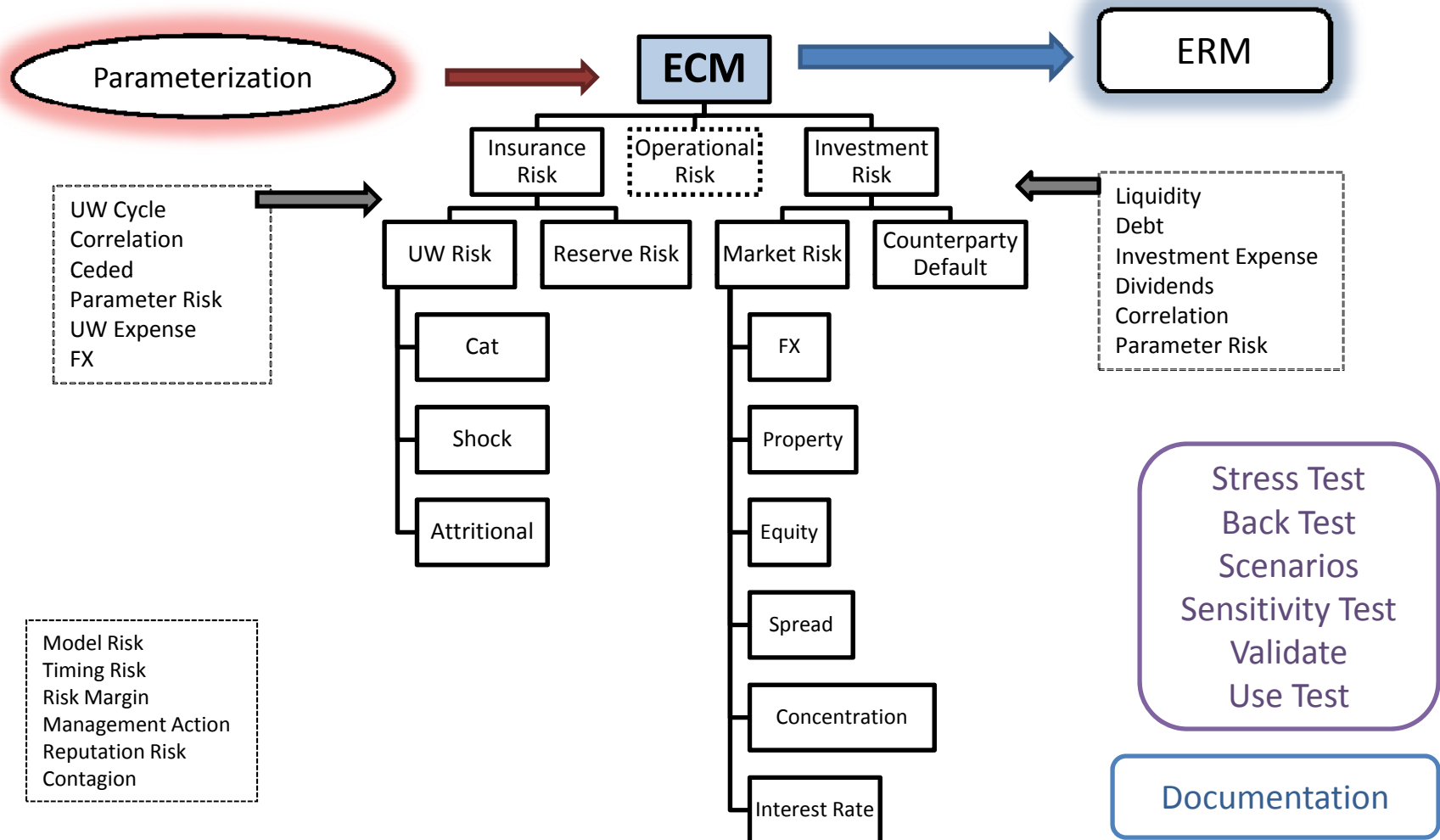
# Challenges of a Good ECM:

- Results used throughout company (planning, capital allocation, variable compensation setting, reinsurance purchases, capital structure, strategic decisions, etc.)
- Adds value, management buy-in.
- Model software: Flexible, not overly difficult to run, does not break-down easily, not a black box, fast enough to make mgmt decisions
- Reconciles with: experience, plan, stress tests, expectations.
- Documented, auditability, governance.
- Reporting capabilities: balance sheet; ultimate and/or one year; AY and/or UY; segment and sub-segment detail.

# ECM uses in ERM



# Basic Model Structure





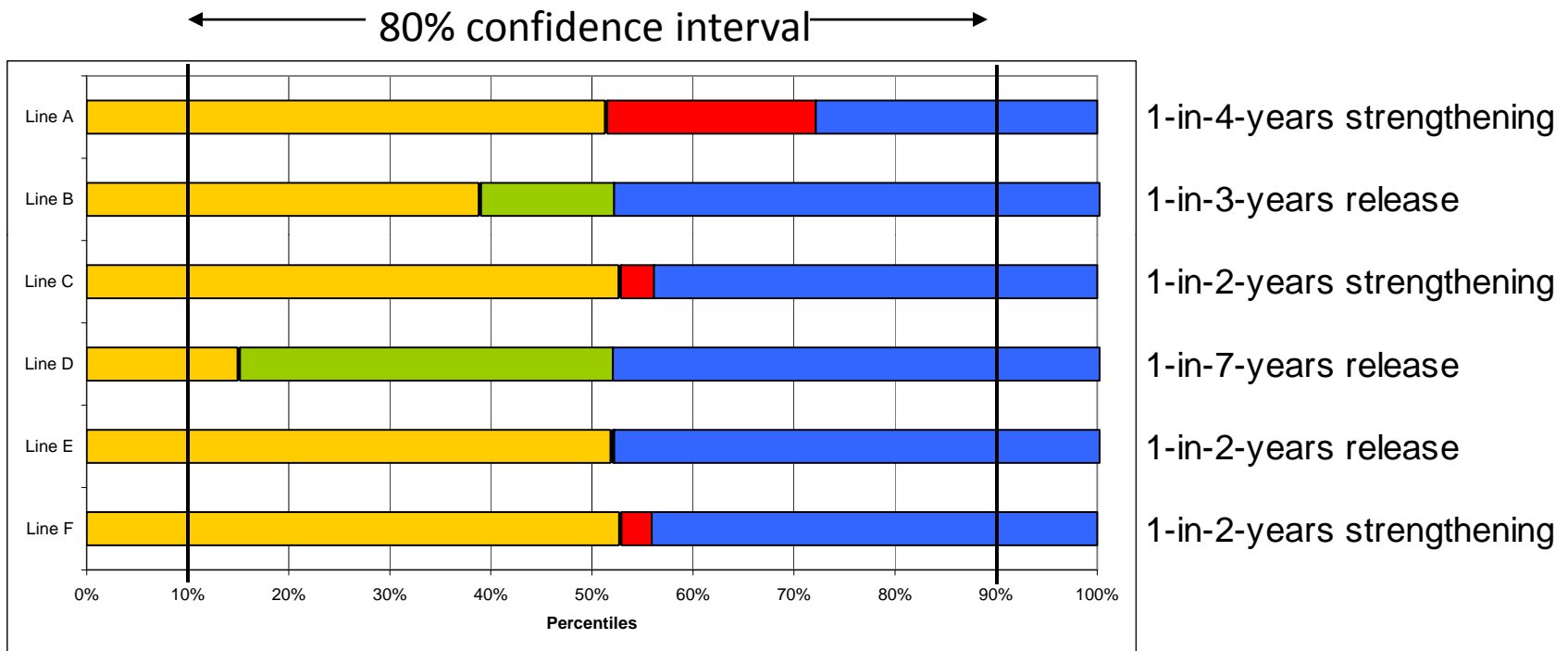
# Selecting Reserve Risk Distributions

- Consider multiple methods
  - Bootstrap, Mack, other
  - Paid vs Incurred
  - Historical reserve data
  - Industry data
- Consistency with current year pricing distributions / RDS / scenario tests
- How does the output compare with regulator and rating agency capital benchmarks?

# Selecting Reserve Risk Distributions

- Practical methods
  - How well does the reserve predicted by the model match the held reserve? If no, then can you trust the predicted standard deviation as well?
  - Do relationships across lines make sense?
  - Back-testing – Compare historical prior year development to model predictions
  - Changes to the mix of business or terms and conditions. May need to adjust the data or bear in mind when selecting results
  - Plot the output of model iterations (rather than just taking mean and standard deviation)
  - Benefit of fitting a distribution to simulated model output - more stable tail values
- Statistical methods - “Need to check the reasonableness of model assumptions”
  - Plot residuals – do they match assumed distributions?
  - Variance or Heteroskedasticity (can be a problem across development periods or underwriting years)
  - Outliers
  - Leverage – identify points with excessive influence on model results
  - Significance tests of model parameters
  - Compare goodness of fit with historical data

# Back-test reserve distributions



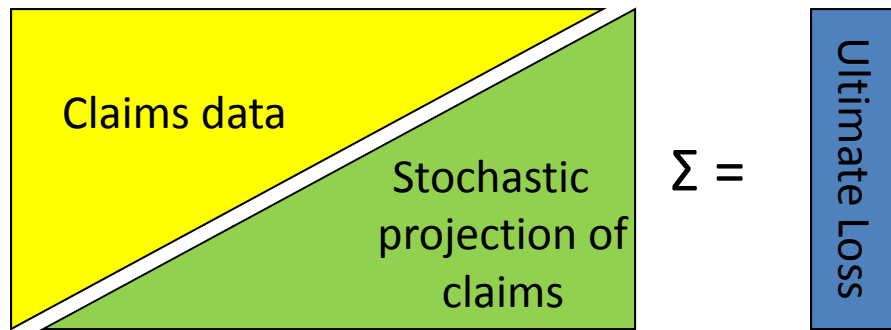
A back-test compares reserve development over a one-year period against the original estimate of a reserve range  
In this exhibit red indicates a strengthening of reserves, green a release

# One-Year vs Ultimate

- Ultimate view: “The risk that the current estimate of the fair value claims reserve is insufficient to cover the full run-off of the liabilities”
- Another perspective is the *one-year* view, which considers the claims development over a single annual time period
- Regulatory regimes have converged on the one-year view
  - Complete run-off of liabilities under the Solvency II regime is satisfied by additionally holding the present value of the cost of future one-year capital requirements to run-off the liabilities, otherwise known as a *market value margin*
- For many existing stochastic reserving models, generating one-year reserve distributions is more complex than it is for the ultimate perspective:
  - A one-year method needs to re-estimate the claims reserve at the end of the time period, using the new information gained
- **Key Issue: Timing of loss recognition is important in one-year models**

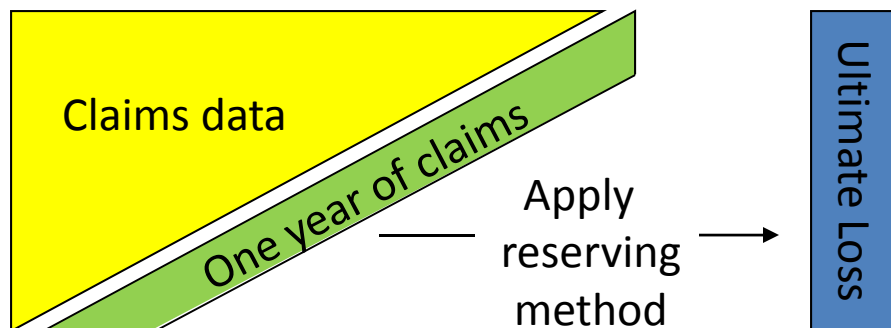
# One-Year vs Ultimate Risk Models

## Ultimate Risk Model



1. Simulate the completion of paid/incurred triangle
2. Sum these claims to get to an ultimate loss estimate
3. Compare with existing held reserve

## One-Year Risk Model



1. Simulate one year of new claims
2. Apply reserving method to get an estimate of ultimate losses one-year out
3. Compare with existing held reserve

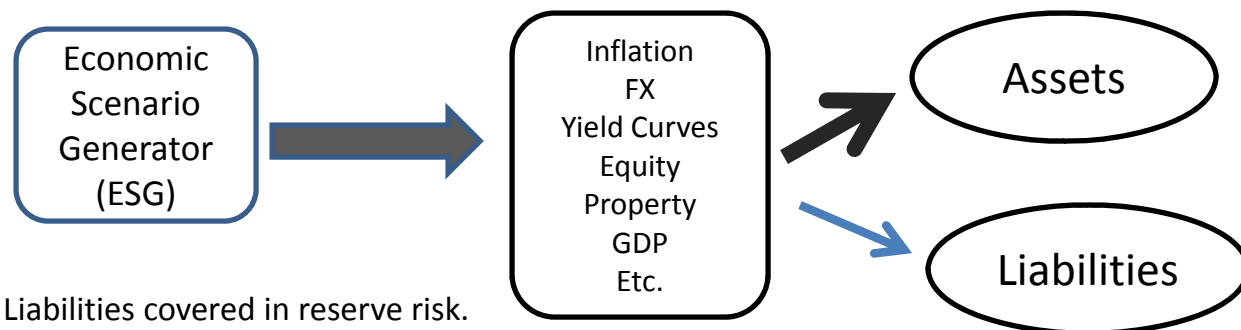
# Issues in One-Year Loss Parameterisation

- The reserving step in the one-year model is complex!
- Wacek\*: suggests two ways in which the estimate of the ultimate may vary as a result of the extra one year of claims information
  - The year end claims payments will generally have been different from those expected, and reapplying the same development factors will give rise to a new indication for the claims reserve
  - Secondly, the extra claims experience may also result in a different selection of development factors
- There is also a third: mechanically applied reserving methods do not reflect the reality. Actuaries will take into account information not contained in the triangle – this may result in bigger changes to ultimate loss estimates than the claims data would suggest.

\* Wacek, M.G., 2007, *The Path of the Ultimate Loss Ratio Estimate*. *Casualty Actuarial Society Forum*, Winter 2007, 339-370

# Investment Risk

Economic conditions impact assets and liabilities.



Liabilities covered in reserve risk.

Investment risk can be modeled:

- Stress-test/deterministic or stochastic
- In-house or outsourced
- Real-world or risk neutral (market consistent)

Most non-life insurance companies with ECM's use an outsourced, stochastic, real-world ESG due to the specialized expertise needed.

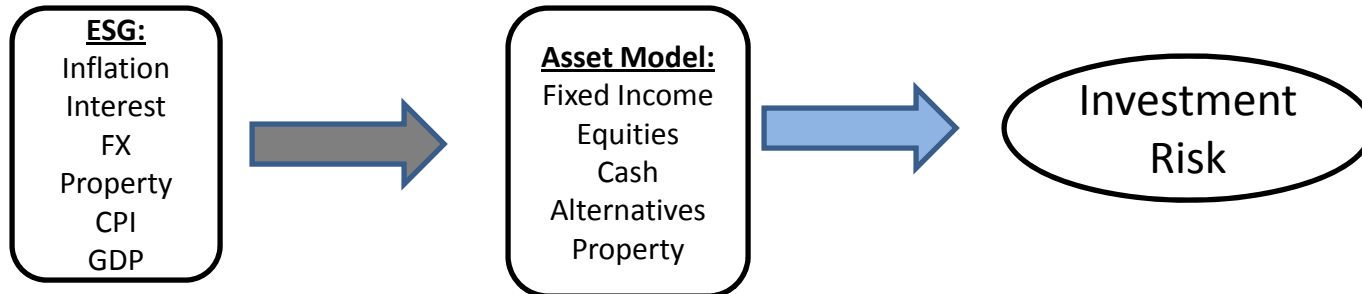
Real World: simulates target actual probabilities in the real world, from historical data and expert opinion.

Risk Neutral: trading or pricing methodology, assumes no arbitrage opportunities.

ESG usually initially outsource parameterization / calibration; however, reinsurer may be able to overlay own expectations.

# Investment Risk

Asset model based on ESG with asset parameters selected by reinsurer.



ESG's can be used for:

- ALM
- ECM
- Dynamic hedging strategies

Areas needing improvement/research:

- Interest rate / current yield curve
- Fatter tail for equity – Most ESG's predicted 2008 and 2009 as greater than a 1-in-250 event.
- Tail correlation in market downturns
- Credit risk/spread on bonds
- Concentration Risk
- Currency Risk
- Asset/Liab correlation – Markets post an extreme catastrophe event. (EG. CA EQ, NYC S&T.)



# Bad Debt Risk

Credit Risk: (1) market credit risk on invested assets (mainly corporate bonds), (2) counterparty default risk on claim reserves, and (3) funds held by intermediaries.

(1) Covered in market / investment risk.

(2) Counterparty default risk largest source of bad debt for reinsurers.

(3) Usually limited for reinsurers.

Counterparty Default Risk exposure on:

- ✓ Prior years' ceded ultimate reserves (developed for possible future deterioration/improvement)
- ✓ Projected current / future year's ultimate ceded losses

Internal vs External reinsurance:

Bad debt charge on internal transactions. Bad debt charge for one segment would be offset by decreased assumed loss to the segment.

Methods:

- Haircut on recoverables (vary expected vs tail)
- Simple (freq/severity or aggregate severity) increasing rate % charge on ceded recoverables
- Individual (or grouped) reinsurer probability of default (or transition) matrix and LGD

Loss Given Default:

- Collateralized vs un-collateralized contracts
- Fully collateralized agreements still maintain a liquidity charge, in which the collateral may be tied up in bankruptcy proceedings for the intermediary holding company.
- Un-collateralized contracts LGD may increase with contagion.

# Bad Debt Risk

Risk Charge =  $\sum$  (if Default, 1 else 0) \* LGD \* Recoverable Exposure.

Where, calculated at the reinsurer level takes into account reinsurers' exposure on both prior and current year exposures.

Probability of default function of:

- A. Base probability of default / downgrade
- B. Willingness to pay / dispute
- C. Economic conditions
- D. Industry insurance results for current year
- E. Industry reserve adequacy
- F. Duration of contract
- G. Contagion / domino effect / correlation
- H. Other

A. Base probability of default based on idealized default matrix from rating agencies. This could be a one-year probability of default, or a transition matrix taking into account default over time, for the length of each contract. (Timing risk.) Issues: What about variability about the default matrix due to rating agency improperly rating?

C. Economic conditions: ESG can give current year conditions. AM Best's Impairment Rate and Rating Transition Study, April 2008, shows relationship between real GDP growth and downgrade/upgrade ratio.

D. Industry results for current year: multiple very large losses will affect multiple reinsurers at the same time. This factor may account for some contagion. We can assume our own loss levels are similar to industry (basis risk) or we can bring in industry losses (AIR/RMS.)

E. Industry reserve adequacy: Reserve risk largest factor in insurer default.

# Bad Debt Risk

Example: (values are for example only)

- Company C has an AM Best FSR rating of A+. Base probability of default within one year 0.12%.
- Willingness to pay selected as 0.05% for all external contracts.
- Economic condition is considered Recovering. Scale factor 1.10.
- Simulation # x year loss is between the 95% and the 99% VaR. Scale factor 2.0
- Industry reserves are considered adequate. Scale factor 1.0

Total probability of default for company c within one year for simulation # x:

Function (base 0.12%, willingness 0.05%, economy 1.10, industry 2.0, reserves 1.0)

If you assumed additive for base probability of default and willingness to pay, and multiplicative for other loads (which is purely for ease of example), then probability of default for the current simulated year is:

$$(0.12\% + 0.05\%) * 1.10 * 2.0 * 1.0 = 0.374\%$$

Other issues:

Downgrade risk: bad debt charge for downgrade (not just default.) Not many people currently do this.

When do you recognize the risk charge?

# Shock and Attritional

- Variety of options
  - Frequency/severity versus attritional loss picks
  - Option taken depends on availability of data or materiality
- Particular issue is the dependence structures chosen between lines of business and risk types
- Correlation with cat losses?

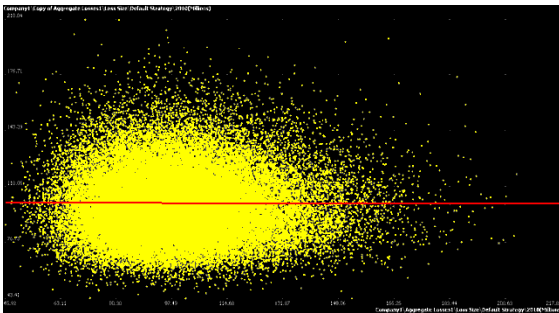
# Correlation Issues

- Linear correlation
  - $\rho(X,Y) = \text{Cov}(X,Y) / \sigma_X \sigma_Y$
  - One commonly used measure of stochastic dependence
  - Easy to calculate and works fine for jointly elliptically distributed distributions (e.g. multivariate Normal)
    - But we don't live in a "Normal" world
  - Example:  $X \sim \text{Ln}(0,1)$  and  $Y \sim \text{Ln}(0,\sigma^2)$ 
    - As  $\sigma$  approaches infinity,  $\rho(X,Y)$  approaches zero regardless of dependence
  - Spearman rank correlation or Kendall's Tau better measures of correlation

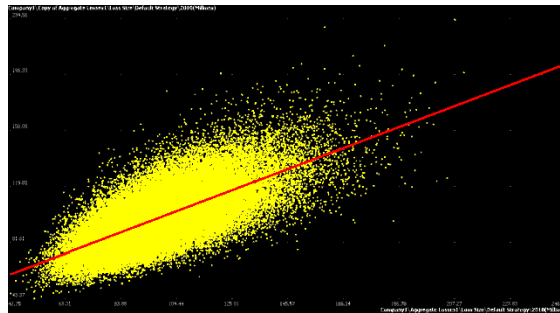
# Correlation Issues

- Copulas
  - Joint distribution of  $U(0,1)$  variables
  - These unit uniform variates can be viewed as probabilities from some other distribution
    - Allows us to simulate correlated variables with various different relationships
    - Defines the complete dependence structure
- Normal Copula
  - Easy to work with
  - BUT, the Normal copula yields asymptotic independence
    - We are mainly concerned with correlation in the tail, so this is a serious problem
- Other possible copulas
  - Frank and Clayton – Also independent in upper tail
  - Gumbel – Tail dependence and asymmetric (more weight in right tail).
    - Upper tail dependence =  $2 \cdot 2^{-1/\theta}$
  - Heavy Right Tail Copula – High correlation in right tail, much less in left
  - Student t Copula – Equal dependence in both tails
- References
  - “Modelling Dependence with Copulas and Applications to Risk Management”, Embrechts, et al
  - “Dependence Models and the Portfolio Effect”, Mango and Sandor
  - “Correlation”, Thomas Strupeck
  - “Tails of Copulas”, Gary Venter
  - “Understanding Relationships Using Copulas”, Frees and Valdez

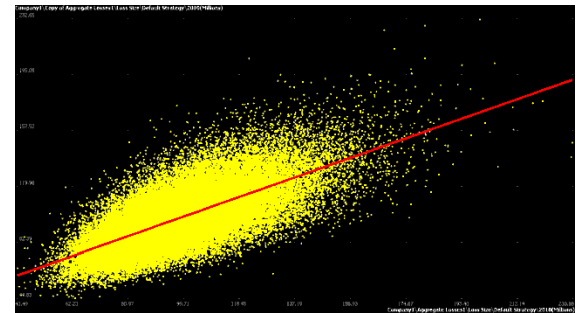
# Correlation Issues



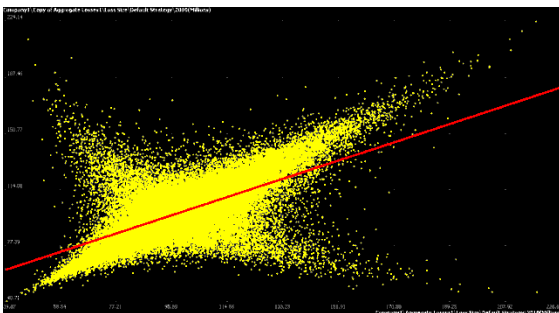
Independent  
Lognormal Mean 100 and CV=20%



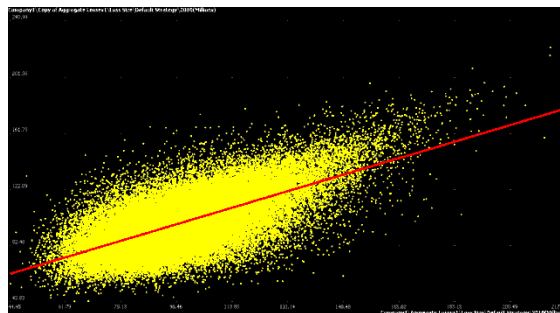
Normal Copula (0.7)



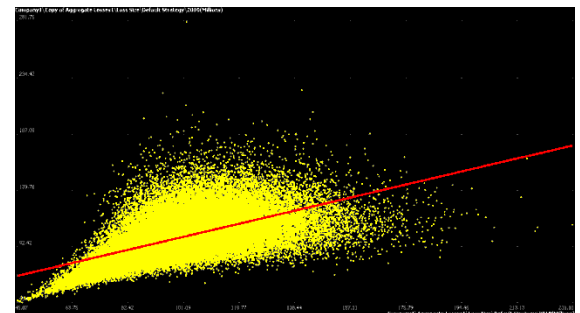
Student t Copula (0.7)  
100 degrees of freedom



Student t Copula (0.7)  
1 degree of freedom



Gumbel Copula (1.8)



Clayton Copula (1.8)

# Correlation Issues

- Correlation between lines of business (projected prospective loss ratio)
  - Examining historical data (e.g. Schedule P data for many companies over many years) will lead one to conclude that lines loss ratios across lines are highly correlated
    - BUT, what are the drivers of this correlation
      - Mainly due to point in pricing cycle and inflationary forces that are similar across lines
      - Also systematic bias in loss reserving practices
    - Need to pay careful attention to what is being modeled



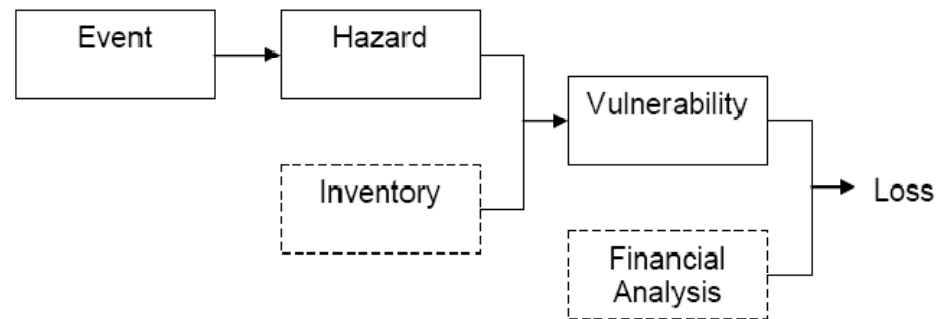
# Correlation Issues

- Rather than generating multivariate distributions we could directly model the underlying causes of correlation
  - Misestimation of loss trend
    - Loss trend is an embedded assumption (either explicitly or implicitly) in the loss ratio estimate
      - Can use inflation from ESG as a proxy for loss trend
        - » Perhaps regression fit to CPI or other inflation index
        - » Creates a natural link to asset prices
        - » “Variance and Covariance Due to Inflation,” David R. Clark
        - » “Correlation and the Aggregation on Unpaid Loss Distributions,” Paul J. Brehm
  - Misestimation of rate change and prior year’s loss ratios
    - Anticipated rate change and estimate’s of prior year ultimate loss ratios also assumptions underlying the prospective loss ratio estimate
      - Can use a collective risk model approach to add parameter uncertainty and induce correlation at same time
        - » Simulate RVs with mean one and Variance= $V_i$ . These, in turn could be correlated. These RVs are then used to scale frequency, severity or aggregate loss distributions. Yields natural contagion across lines and geographies.
        - » “The Aggregation and Correlation of Insurance Exposure,” Meyers, Klinker and Lalonde

# Correlation Issues

- How do we know if we have the “right” amount of correlation
  - You don’t
  - Can analyze your own companies data (assuming it is sufficiently large and has been in existence long enough)
  - Rely on publicly available data
    - Aon Benfield Insurance Risk Study
    - Schedule P data

# Basic Structure of a Catastrophe Model



- The two dotted boxes represent the data inputs into the catastrophe model, the rest portray the engine of the catastrophe model
- Need to understand:
  - Completeness and quality of data received, and any assumptions made for incomplete data
  - Assumptions inherent within the catastrophe model and modifications made
- The A+ student versus the B student
  - Everyone has access to the catastrophe models (i.e. been to the same classes and given the same materials)
  - It's the ones who make best use of it (and do their homework!) that prosper
- Reference: 'Report of the Catastrophe Modelling Working Party - Presented at GIRO (Vienna) September 2006
- Available: [http://www.actuaries.org.uk/\\_\\_data/assets/pdf\\_file/0007/30121/Fulcher.pdf](http://www.actuaries.org.uk/__data/assets/pdf_file/0007/30121/Fulcher.pdf)

# Catastrophe Model Issues

- How to split catastrophe losses from underwriting risk parameterisation, and avoid double counting
  - Truncate catastrophe model distributions or exclude natural catastrophe losses from underwriting risk data
- Data
  - Location data – varies widely by location and peril, and is typically far more granular in ‘peak zones’ such as California earthquake or East coast windstorm
  - Availability of risk characteristics such as occupancy, construction
  - What is done when detail is lacking, what are the default or standard assumptions?
- Frequency Trends – key model assumption in the aggregate modelling done for economic capital models
- Severity Calculations – damage ratio, demand surge
- Mathematical approximations E.g.
  - Assumptions in correlation between lines of business when applying secondary uncertainty
  - Combining distributions for damage caused by earthquake shaking and fire following
  - Generating secondary uncertainty distributions post Excess of Loss reinsurance
- Unmodelled Elements
  - Unmodelled pieces of modelled contracts (e.g. multi-location contracts or business interruption)
  - Classes that often have significant unmodelled exposures such as retrocession or aspects of marine
  - Unmodelled elements of a modelled loss – e.g. loss expenses, storm surge or underinsurance

# Operational Risk

- What is it?
  - Many definitions – Basel II definition:
    - Risk of loss resulting from inadequate or failed internal processes, people, and systems or from external events...
- Distinction from other risks of an insurance enterprise
  - Consciously decide to take investment and insurance risk because perceived returns
  - Operational risk a necessary evil
    - Act to reduce it to extent mitigation efforts cost effective
    - Impossible to eliminate entirely
    - Some operational risks necessary to accomplish company's objectives – Operational Risk Appetite

# Operational Risk

- ECM perspective
  - Any risk that is not otherwise modeled
  - Examples:
    - Poorly managed growth in new products
    - Regulatory compliance
    - Data integrity
    - Succession planning / key employee risk
    - Legal compliance (e.g. antitrust)
    - Outsourcing
    - Data access and availability
    - Disaster recovery, business continuity
    - Employee fraud
    - Revenue recognition, earnings manipulation
    - Tax compliance
  - Need to first identify all such risks of the company

# Operational Risk

- Risk Indicators – A Starting Point “Quantifying Operational Risks in General Insurance Companies”, GIRO Working Party
  - Exposure Related
    - Processing Volume (claims, policies, endorsements)
    - Premium Growth
    - Outsourcing costs
    - Number of IT projects underway
  - Loss-Related
    - Number of complaints
    - Budget overruns
  - Cause Related
    - Number of unresolved internal audit issues
    - Employee Turnover
    - Training budget per employee
    - Hours of overtime per employee
- Stress testing and scenario analysis
  - Does not require strict estimation of associated probabilities
  - Useful method for thinking about operational risk

# Operational Risk

- Careful no overlap with risks modeled elsewhere
  - E.g. lack of controls on reserving:
    - Reserve risk or operational risk
- Correlated with other business risks
  - E.g. economic downturn leads to operational risks and impacts asset values and insurance liabilities
- Extreme Value Theory
  - Operational risk losses extremely skewed
  - Traditional statistical techniques do not work well
  - EVT focuses on tail – severity of loss given that it exceeds threshold
- Loss distribution analysis requires data
  - May have internal data, but certainly not enough to parameterize tail of distribution
    - Often would need hundreds of data points to properly estimate the quantiles of concern
      - “Quantifying Regulatory Capital for Operational Risk”, Embrechts, Furrer and Kaufmann
  - External data
    - Need to “on-level” the losses
      - Requires some measure of exposure
    - Basel Committee Loss Data Collection Exercise for banks
      - Applicability to an insurance enterprise



## **Principle 2 (Dynamic nature)**

- Models should be dynamic and flexible in nature.

## **Principle 3 (Practicality and proportionality)**

- Internal models must be practical in the context of the organization and the model's purpose.

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- The insurer should document the governance, methodology and assumptions underlying the internal model and its development. Internal model results should be traceable and auditable.

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## Publications / Resources:

“Internal Models Benchmarking Study Summary Results”, CRO Forum

<http://www.croforum.org/assets/files/publications/crofintmodelbms30jan09.pdf>

“Benchmarking Study of Internal Models”, CRO Forum

<http://www.vif.ac.at/filipovic/PAPERS/BMSReportfinal.pdf>

“Internal Model Admissibility: Principles and Criteria for Internal Models”, CRO Forum

[http://www.croforum.org/publication/internal\\_model\\_admissibility/](http://www.croforum.org/publication/internal_model_admissibility/)

“The Ultimate and One-Year Views of Reserving Risk with Respect to Solvency and Risk Margins”, by Andrzej Czernuszewicz. GIRO Conference.

“Solvency II Non-Life Modeling Issues: Calculating Economic Capital Under Solvency II”, by John Charles and Andrew Gray. GIRO Conference

“Integrating Reserve Risk Models into Economic Capital Models”, by Stuart White. CLRS Conference.

“Reserving risk, risk margins and solvency: Re-tuning your mind”, by Peter England. EMB

[http://www.solvency-2.com/Solvency-2/PDF/EMB%20Briefings\\_scr%20risk%20margins.pdf](http://www.solvency-2.com/Solvency-2/PDF/EMB%20Briefings_scr%20risk%20margins.pdf)

Lloyds. Solvency Guidance.

[http://www.lloyds.com/Lloyds\\_Market/Solvency\\_II/](http://www.lloyds.com/Lloyds_Market/Solvency_II/)

“Principle-Based Risk Management to Solvency Requirements.” SCOR.

<http://www.scor.fr/www/index.php?id=433&L=2>

“Presentation to Investors and Analysts on Capital Management.” Royal & SunAlliance.

<http://www.rsagroup.com/rsa/uploads/reports/FinalCapitalDaypresentation.pdf>

## Publications / Resources:

“Specialty Guide on Economic Capital”, by the EC Subgroup of the SOA RMTF

[http://rmtf.soa.org/rmtf\\_ecca.html](http://rmtf.soa.org/rmtf_ecca.html)

“Economic Scenario Generation for Financial Institutions”, by Tony Dardis

<http://www.soa.org/library/content/cc/mtg-2009-econ-scenario.pdf>

“Simulation Modeling: Pitfalls and Best Practices”, by Tom Hettinger

<http://www.casact.org/education/reinsure/2009/handouts/CS13-hettinger.pdf>

“Actuarial Aspects of Internal Models for Solvency II”, by Brooks et al. FSA.

[http://www.actuaries.org.uk/\\_data/assets/pdf\\_file/0009/146664/sm20090223.pdf](http://www.actuaries.org.uk/_data/assets/pdf_file/0009/146664/sm20090223.pdf)

“Practical Implementation Challenges of Internal Models under Solvency II”, by Austin et al. SIAS (Staple Inn Actuarial Society)

“Economic Capital Modeling: Practical Considerations”, by Finkelstein et al. Milliman.

<http://europe.milliman.com/perspective/special-reports/pdfs/economic-capital-modeling-practical-SR14-12-06.pdf>

“Insurance Risk Study: Modeling the Global Market.” Aon Benfield

“Economic Scenario Generators and Solvency II”, by E.M. Varnell. Institute of Actuaries

[http://www.actuaries.org.uk/\\_data/assets/pdf\\_file/0015/162150/sm20091123.pdf](http://www.actuaries.org.uk/_data/assets/pdf_file/0015/162150/sm20091123.pdf)

“Practical issues in ALM and Stochastic modeling for actuaries”, by MacNamara and Gibbs. Institute of Actuaries

[http://www.actuaries.asn.au/IAA/upload/public/1.b\\_Conv07\\_Paper\\_Gibbs%20McNamara\\_Practical%20Applications%20in%20ALM.pdf](http://www.actuaries.asn.au/IAA/upload/public/1.b_Conv07_Paper_Gibbs%20McNamara_Practical%20Applications%20in%20ALM.pdf)

“The Modeling of Reinsurance Credit Risk”, by R.A.Shaw. GIRO Conference.

<http://www.actuaries.org.uk/?a=127497>

**“Two Approaches to Calculating Correlated Reserve Indications Across Multiple Lines of Business”, by Kirschner, Kerley and Isaacs**

[http://www.actuaries.org.uk/\\_data/assets/pdf\\_file/0006/18717/Kirschner.pdf](http://www.actuaries.org.uk/_data/assets/pdf_file/0006/18717/Kirschner.pdf)