Enterprise Risk Management and Capital Budgeting under Dependent Risks: an Integrated Framework

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Abstract

Risk management and capital budgeting are two critical components of the corporate decision process that often need to be considered jointly at the corporate level because of their natural interaction through the dependent risk exposures and other synergetic relationships within an intricate corporate structure in a dynamic business environment. This paper develops an integrated framework that aligns these two important corporate strategies across business divisions in a multi-period setting to optimize the streamlined enterprise strategic goal. The proposed integrated framework can serve as a practical guideline for corporate executives to optimally coordinate capital budgeting and risk management at the enterprise level.

Key words: Capital Budgeting, Enterprise Risk Management (ERM), Corporate Risk Management, Risk Dependency, Decisions under Uncertainty

1. Introduction

Risk management and capital budgeting are two critical components of the dynamic corporate decision process. In practice, they often need to be considered jointly as they are naturally connected by the dependent risk exposures and a variety of other synergetic relationships within an intricate corporate structure. As companies grow larger and more diverse, the interactive complexity of the organization increases and no component of the organization (such as a business division) can be easily isolated. As the recent financial crisis taught us again, complex and tightly coupled systems are highly susceptible to catastrophic breakdowns. Without an efficiently integrated decision process, corporations could be misled to suboptimal investment and risk management decisions, resulting in a permanent loss of the firm value.

In much of the current literature, the subjects of capital budgeting and risk management have been studied in separate fashions, hence a simplified setup neglecting the joint impact of these two business components. In the studies of the project ranking and selection criteria, Graham and Harvey (2001) and Graham et al. (2010) find that the net present value (NPV) is indeed one of the most popular selection criteria among corporate executives in both public and private firms in the U.S. Additionally, the capital budgeting literature in the recent decades focuses largely on agency problems that arise in the capital budgeting process rather than the decision process itself (e.g. Harris and Raviv 1996, Bernardo et al. 2004, Marino and Matsusaka 2005, Ozbas and Scharfstein 2010). We argue that the classic capital budgeting framework with an NPV-like ranking criterion to allocate capital to projects, either by a central planner or through delegation to division managers, might be an over-simplified environment for corporations to make optimal investment decisions and for researchers to study related problems (such as the agency costs). For many companies, risk dependency entails nontrivial interplay of corporate decisions across business divisions and time periods, and the resulted synergies cannot be easily separated out and attributed to every project and included in an NPV type of analysis.

Meanwhile, traditional corporate risk management literature addresses risks in silos too, ignoring their possible dependency and hence the collective contribution to the overall corporate performance. Such negligence could result in significant deviations from the optimal strategies from the enterprise-wide perspective, possibly lead to catastrophic outcomes (in the case of high positive dependency) and/or over-hedging (when "natural hedging" opportunities are ignored). Rather than being compartmentalized decisions, risk management should be contemplated at the enterprise level where other corporate functions (such as capital budgeting, financing, and performance assessment) usually take place. This concern gives rise to the development of enterprise risk management (ERM) (cf., COSO 2004; Ai et al. 2010), for which the most important goal and challenge is to fully encompass risk management into the overall corporate decision making.

Following the spirit of the pioneering work by Froot et al. (1993) and Froot and Stein (1998) where corporate risk management is necessitated by capital market frictions and where capital budgeting and risk management functions become connected, we propose in this paper an integrated framework that allows the corporations to design optimal investment and risk management strategies jointly and endogenously under dependent risks in a multi-period setting. Under our proposed approach, projects are first evaluated for its capital requirement, cash flow potentials, and risk exposures within the respective business divisions before they are integrated into a single optimization problem for the value–maximizing corporation. We model the risk dependency with copula and construct the optimization problem via the intuitive and visual interface of decision tree. We illustrate the proposed framework through a hypothetic financial services company. This framework can be easily adapted and internalized by corporate executives to obtain optimal solutions in an integrated decision making process.

This paper builds upon and contributes to several strands of literature. First, we contribute to the capital budgeting literature by studying optimal capital budgeting in a multi-divisional firm with dependent risk exposures. In order to fully capture these characteristics, we take a perspective different from most of the current literature to model the internal capital allocation process in the portfolio sense. We directly model the integration of risks and decisions to the firm level and incorporate specific risk management strategies in this course, rather than relying solely upon the use of an overall risk adjusted hurdle rate required in an NPV analysis (cf., Poterba and Summers 1995, Jagannathan and Meier 2002). Consequently, we are able to capture the dynamics in an optimal coordination between investment and risk management strategies, providing additional managerial insights not available under the traditional framework.

One other main stream of the capital budgeting literature in the recent decades studies the agency problems arising from an information gap between the central decision maker and the division managers. For example, Harris and Raviv (1996) find that the misaligned incentives of divisional managers with private information can lead to underinvestment or overinvestment problems. Similar studies include the choice of a centralized capital budgeting process vs. a delegated one determined upon agency costs (Marino and Matsusaka 2005); the optimal managerial compensation contract when divisional managers have valuable private information about other divisions and can exert efforts to enhance the value of other divisions (Bernardo et al. 2004); an agency-theory based explanation of the relative inefficiency of investment allocations in an unrelated segment of a conglomerate relative to a stand-alone firm using Q-sensitivity of investment (Ozbas and Scharfstein 2010); and more recently, a top-down approach of capital budgeting where the top executive signals the firm's prospects to stakeholders through the capital allocation decisions which can result in observed investment distortions (Almazan et al. 2011). Although our study does not directly focus on the agency problem, it does shed light on the benefits of fully accounting for interplays among divisions and hereinto reducing the information gap. Our proposed capital budgeting framework could also provide an improved environment to study these related problems in the future.

Second, we contribute to the corporate risk management (e.g., Froot et al. 1993, Froot and Stein 1998) and ERM literature (cf., Ai et al. 2010, COSO 2004) by operationalizing the concepts of ERM in incorporating risk management into corporate decision making. The seminal work of Froot et al. (1993) and Froot and Stein (1998) propose to study the problems of corporate risk management, capital budgeting, and corporate financing policies in conjunction. Froot et al. (1993) point out that optimal hedging policies depend on the nature of firms' investment and financing opportunities, and thus risk management should be used to "coordinate" these corporate policies. Froot and Stein (1998) further propose a framework for a financial institution where the three corporate functions are contemplated jointly to maximize shareholders' value. Our framework is in the same spirit and we specifically focus on characterizing the across-the-board risk dependency and other synergies that necessitate the linkages between different corporate functions.[1]

Finally, we also make technical contributions to the decision analysis literature by further developing the decision tools provided in Gustafsson and Salo (2005) and Wang and Dyer (2011) to handle the more managerially relevant problem of capital budgeting and enterprise risk management. Necessary technical details will be discussed in the next section when we present our model.

The rest of the paper is organized as follows. Section 2 presents our integrated framework of ERM and capital budgeting and describes in detail the construction of the framework for a generalized multi-divisional corporation with a multi-period planning horizon. Section 3 illustrates the framework with an example of a hypothetical financial services company in both banking and insurance businesses. Section 4 discusses some model insights and extensions of the framework. Section 5 concludes the paper.

2. The Integrated Capital Budgeting and Risk Management Framework

Our integrated capital budgeting and risk management model considers a multi-divisional corporation with a multi-year planning horizon. The framework is formulated by solving an optimization problem of the corporate decision maker, where she allocates capital to projects in different divisions in light of dependent risks within and across divisions and in this process, determines corporate risk management strategies. We assume that the utility maximization of the corporate decision maker, as obtained by solving the optimization problem, also achieves the corporation's strategic goals in shareholders' value maximization.

[1] Our current framework does not consider corporate financing policies except to acknowledge the same premises that the previous research has based on: costly external and internal financing along with other imperfections in the financial market make risk management relevant.

In constructing this optimization problem, we use the decision tree approach as an auxiliary step. Decision trees are commonly used in risk and decision analysis to identify the optimal dynamic strategy in the presence of uncertainties (Howard 1988, Clemen and Reilly 2000). With the advantage of a visual interface and natural backward dynamics, a decision tree represents all the possible paths that the decision maker might follow over time, including all possible decision alternatives and outcomes of risky events. While decision tree has become one of the fundamental tools for decision making under uncertainty in the operations research literature and in practice, its application to problems in finance and corporate risk management is limited, partly due to the computational difficulty and exponentially growing tree size involved in such complicated situations with multiple sources of dependent uncertainties and a series of sequential decisions. We overcome some of these difficulties by extending the most recent available techniques.

In our model, the dependent corporate risks throughout the planning horizon are captured with a probability tree structure, i.e., the “state of nature” tree, reflecting both the individual risk characteristics and inter-relations of these risks within and across divisions and time periods, measured with a copula approach. The division-level investment decisions, specific risk management strategies, and the resulting cash flows of the corporate project portfolio are characterized with the help of this tree structure along with information on perceived investment opportunities, available capital resources, and other considerations. The corporate decision maker’s risk preference is modeled using an appropriate utility function and her optimization problem represented by the tree structure is solved to obtain the optimal decisions.

Building upon current techniques in the literature, we address several challenges to accomplish this task. First, we make use of the decision tree approach to characterize a holistic capital budgeting and risk management process as an optimization problem, accounting for dependencies across business divisions and time periods. Second, we adapt the statistical technique in Wang and Dyer (2011) to model a copula-based dependence structure among the risks and to increase the analytical tractability in a decision tree context. Finally, we overcome the computational challenges associated with the high dimensionality in our multi-layer setting and make a technical contribution to the decision analysis literature by extending a dimension reduction technique first proposed by Gustafsson and Salo (2005) to solve a more realistic and managerially relevant problem. While maintaining the reduced dimensionality, unlike in Gustafsson and Salo (2005), our model does not have to force different projects to share the same risks and accommodates a more flexible dependency and payoff structure. We next describe the general capital budgeting and risk management framework in detail.

2.1 The Objective Function

The corporate decision maker seeks to maximize the expected utility she derives from the corporation's future capital positions, i.e. max E[$u(X)$], where u is the von Neumann-Morgenstern utility function. We assume that the utility function can be approximated as a classic mean-risk model widely used in portfolio selection (cf., Markowitz 1952).

The expected capital position generated by project cash flows for the corporation at each state of nature at the end of the planning horizon T is

$$EV_T = \sum_{s \in S_T} p(s) V_s \tag{2.1}$$

where V_s is the capital position at state of nature s, $p(s)$ is the associated probability, and S_T is the set of all possible states at T.

While a broad range of risk measures is available in the literature and can be used in this general framework, we focus on the lower semi-absolute deviation measure (LSAD) (e.g., Eppen et al. 1989 and Fishburn 1977) to emphasize the downside risk that concerns investors more (Ang et al. 2006; Gustafsson and Salo 2005). Additionally, the LSAD risk measure has desirable theoretic properties such as its consistency with the first and second order of stochastic dominance (Fishburn 1977).

More specifically, the LSAD risk measure is given by

$$LSAD_T = \sum_{s \in S_T} p(s)|V_s - EV_T|. \qquad (2.2)$$

The objective function can now be written in the mean-risk form as

$$max[EV_T - LSAD_T], \qquad (2.3)$$

where EV_T is defined by Equation (2.1) and $LSAD_T$ is defined by Equation (2.2).

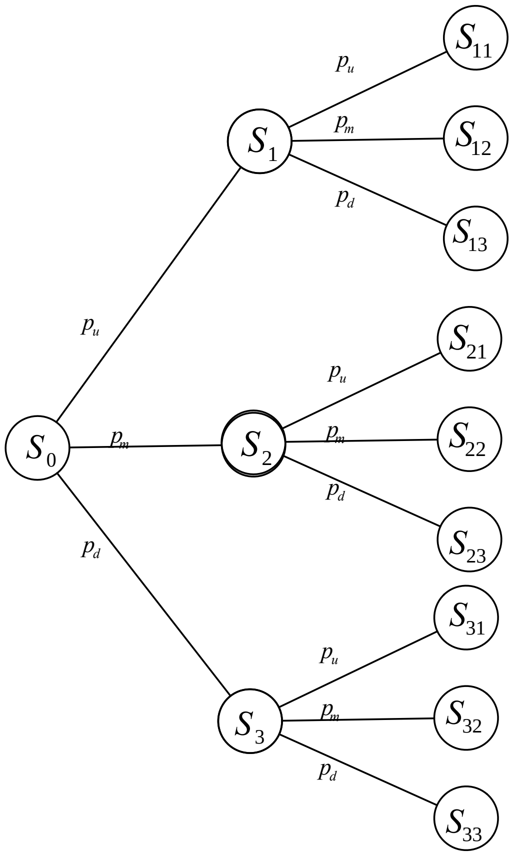
2.2 Corporate States of Nature

The expected future project cash flows are contingent on the corporate states of nature driven by multiple sources of risks across different business divisions. While it is crucial to properly model dependency among these risk exposures, these dependencies are often neglected in order to simplify the analysis (cf., Abbas, 2006, Bickel and Smith 2006), causing significant errors in decision making (cf., Smith, Ryan and Evans 1992).

We use copula to model the dependence structure of these uncertainties. Copulas transform the set of univariate marginal risk distributions into their multivariate joint distribution accommodating different types of dependence relationships by the choice of the specific copula function. A rich literature in finance and risk analysis uses copula models for dependency (e.g., Embrechts et al. 1999, Cherubini et al. 2004, and Biller 2009).

We adopt the dependent decision tree approach proposed by Wang and Dyer (2011) to formulate an optimization model for integrated capital budgeting and risk management. Using only information of marginal distributions and correlations, this copula-based approach allows multiple dependent uncertainties with arbitrary marginal distributions to be represented in a decision tree with a sequence of conditional probability distributions. Under this approach, each risk realization is represented with a trinomial discrete approximation using the extended Pearson-Tukey method ("EP-T," cf., Keefer and Bodily 1983). Following the EP-T approximation logic, the probabilities assigned to each of the three discrete points take a constant set of pre-determined value while the point realizations vary with the conditioning distributions describing the underlying risk scenarios. The primary advantage of the dependent decision tree approach over alternative approaches (e.g., Clemen and Reilly 1999) is that this approach provides more modeling flexibility by supporting the popular copula families, such as elliptical copulas and Archimedean copulas, and is more efficient computationally. A brief description of the normal-copulas based dependent decision tree approach is presented in the Appendix and the interested readers are referred to Wang and Dyer (2011) and Clemen and Reilly (1999) for detailed discussions of these approaches. We next briefly illustrate this process in Figure 1.

Figure 1 An Example of a State of Nature Tree



In Figure 1, the state of nature tree starts with a single base state S_0 in period 0, leading to a group of three states in period 1, $S_1, S_2, S_3,$ corresponding to the three possible outcomes of S_0 (i.e., “up,” “middle,” and “down,” per the trinomial approximation). Similarly in the future periods, a group of three subsequent states follows each possible outcome of the preceding state of uncertainties. The unconditional probability that is assigned to each state is computed recursively from the conditional probabilities of the preceding states.

In our framework, a state of nature tree is first constructed for each division/project and later combined into a single corporate state of nature tree. An example of a such constructed corporate state of nature tree will be presented in the next section.

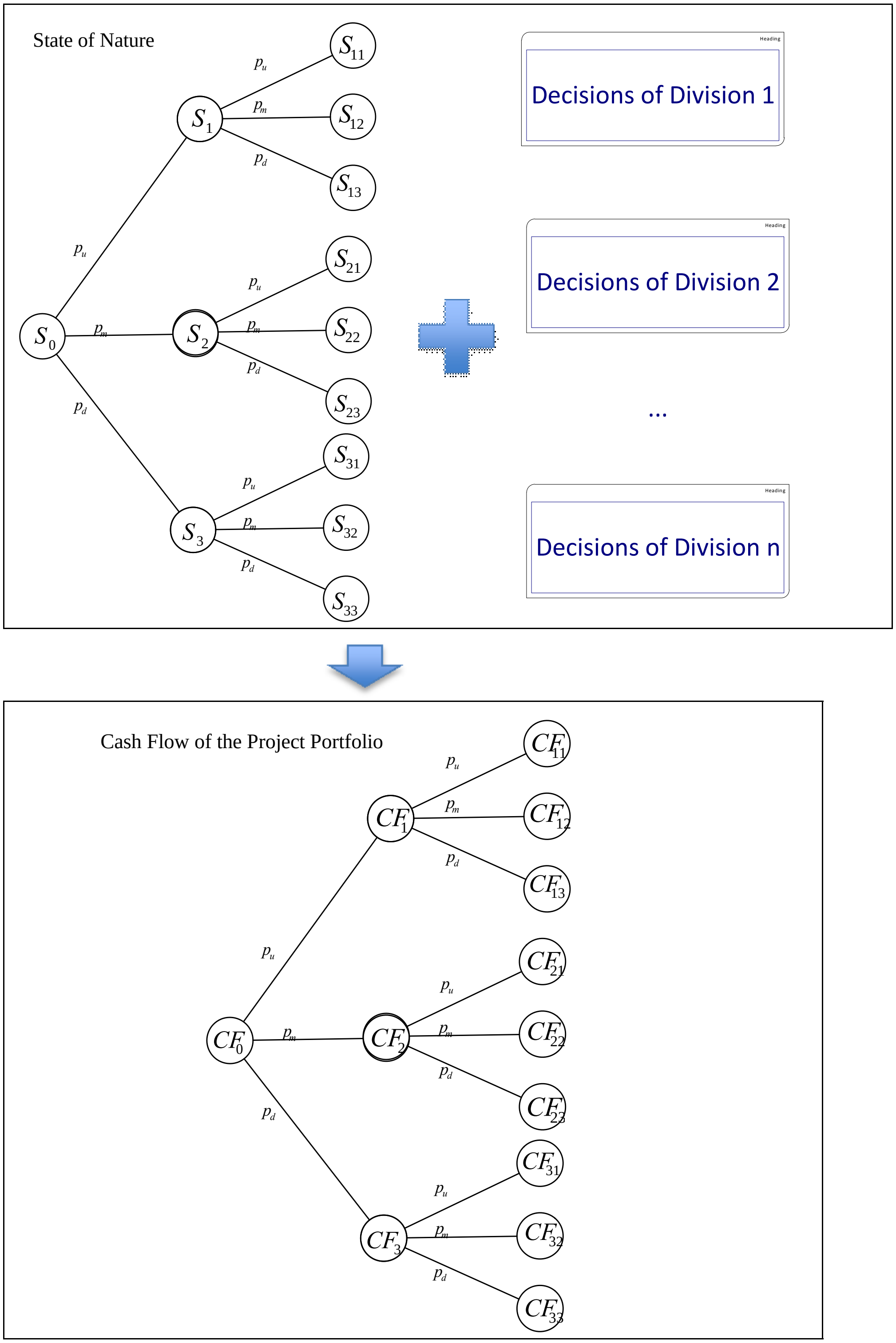
2.3 Cash Flow Characterization of the Corporate Project Portfolio

The corporate decision maker makes dynamic choices of the portfolio of projects contingent on the realized uncertainties and her previous decisions. These uncertainties and decisions will determine the future cash flows of the projects at each state of nature in each period.

We model the corporate decision making process in light of the dependent risk structure described in the corporate state of nature tree. Given the investment opportunities and the risk exposures, each division d's decision making is modeled by inserting its decisions $X_{d,s}$ into the given state of nature tree incorporating all possible states $s \in S$. We then combine the multiple division-level decision trees to derive corporate level optimal strategies. Figure 2 illustrates the process of characterizing future cash flows CF_s from the corporate project portfolio, for each state $s \in S_T$ of all possible states of nature at the end of horizon T.

One computational challenge of such a model is the quickly accumulating dimensionality associated with the multi-division, multi-dependent-risk, multi-period environment that we are working in. To reduce dimensionality, we adapt and further develop the approach in Gustafsson and Salo (2005). We map the decisions directly onto the corporate state of nature tree and characterize cash flows of the project portfolio with a set of logical consistency constraints and capital resource constraints. Therefore, the size of the decision tree stays the same as the state of nature tree rather than growing in an exponential manner with the added decision variables.

Figure 2 Characterizing State-Dependent Corporate Project Portfolio Cash Flows



2.4. The Constraints

Building on the cash flow characterization in the corporate tree structure, we write out the optimization of the decision process, with the objective function (2.3) and subject to three types of constraints: (1) logical consistency constraints, (2) capital resource constraints, and (3) risk constraints (cf., Gustafsson and Salo 2005).

2.4.1 Logical consistency constraints

The decisions of the project portfolio have to be consistent throughout the entire planning horizon. For instance, a project can either be selected or not be selected; and if the project is not started at the preceding period, the project cannot be continued at a later period. If a decision needs to be made for a given state s, the decision maker can take one and only one of the pair of actions ($X_{d,s,Y}$, $X_{d,s,N}$) only if the decision made in the preceding state is affirmative (i.e., $X_{d,s,Y} = 1$). Otherwise, the decision maker does not get to make a decision at state s, so neither of the two actions at s can be selected. These logical requirements imply the following consistency constraints:

$$X_{d,s,Y} + X_{d,s,N} = X_{d,pre(s),Y}$$

where $pre(s)$ is the preceding state.

The consistency constraints are useful in facilitating the characterization of cash flows in the capital resource constraints described below. In addition, they are essential in accomplishing the dimensionality reduction. As only an affirmative decision to make an investment will lead to uncertain capital position, we can eliminate all (unnecessary) decision nodes associated with a negative decision to conserve dimensionality. Technically this is achieved by utilizing the two sets of binary indicator variables for the affirmative and negative decisions respectively.

2.4.2 Capital resource constraints

Capital resource constraints are employed to ensure that capital is available and sufficient for future investments and risk management. To capture the capital positions, V_s, for the corporate project portfolio at each state s, each division's affirmative investment decision itself, $X_{d,s,Y}$, is combined with the cash flow (e.g., investment cost, risky returns, and losses) it entails in the cash flow tree characterization, leading to the capital resource constraints:

$$\sum_{d} CF(X_{d,s,Y}, s = 0) - V_0 + I_0 = 0$$

$$\sum_{d} CF(X_{d,s,Y}, s \in \{S\backslash 0\}) - V_s = 0$$

where $CF\left(X_{d,s,Y}, s\right)$ is the cash flow triggered by decision $X_{d,s,Y}$, V_s is the capital position at state s and $I_0$is the initial capital budget.

As such, the capital resource flows depend on the project returns subject to pertinent risk factor(s) reflected by the corresponding states of nature, the required investments, and the investment decisions subsequently made. The capital flows and the final capital positions of the corporation are consequently uncertain.

2.4.3 Risk constraints

Let ΔV_s^+ and ΔV_s^- be nonnegative deviation variables that measure how much the capital position in state s differs from a target value. ΔV_s^+ (ΔV_s^-) denotes when V_s exceeds (falls short of) the expected final capital position at time T, EV_T. Since we are using LSAD as the risk measure for the downside risk, we have additional logical requirements represented in these risk constraints for all $s \in S$:

$$V_s - EV_T - \Delta V_s^+ + \Delta V_s^- = 0$$

and the expected LSAD at the end of horizon T can be obtained as

$$LSAD_T = \sum_{s \in S_T} p(s) \Delta V_s^-.$$

2.5 Risk Management Strategies

In addition to accounting for dependency among projects and allowing for "nature hedges" among risks and divisions, the framework allows explicit risk management implementation. We consider hedging and other commonly used risk management strategies (e.g., (re)insurance and risk control). If a proportion $\alpha_{d,s}$ of the risk is hedged by division d at cost $c_{d,s}$ in state s, the baseline capital resource constraints are modified as follows to incorporate the hedging component:

$$\sum_d CF(X_{d,s,Y}, \alpha_{d,s}, s = 0) - X_{d,0,Y} * \alpha_{d,0} * c_{d,0} - V_0 + I_0 = 0$$

$$\sum_d CF(X_{d,s,Y}, \alpha_{d,s}, s \in \{S \backslash 0\}) - X_{d,s,Y} * \alpha_{d,s} * c_{d,s} - V_s = 0$$

and the future project cash flows are adjusted as

$$CF(X_{d,s,Y}, \alpha_{d,s}, s) = CF_O(X_{d,s,Y}, s) + (1 - \alpha_{d,s}) CF_I(X_{d,s,Y}, s) + \alpha_{d,s} E(CF_I(X_{d,s,Y}, s))$$

so that the future cash flows after implementing a hedging strategy equal to a hedged proportion of cash inflows from the hedged position and an un-hedged proportion of cash inflows still subject to the original state-dependent risk outcomes. This modeling approach is consistent with the existing literature on hedging policy.

Here we allow a different division cost $c_{d,s}$ for each risk and allow each division to choose the appropriate risk management method and their own $\alpha_{d,s}$. For example, a derivative contract might be used to manage the market and credit risk while reinsurance is purchased to mitigate the actuarial pricing risk for a financial services company having both banking and insurance businesses. We measure the *a priori* risk exposure by the expected value of the future risky outcomes (i.e., a proxy for market price of a hedging contract, or as is calculated in an insurance/risk control context).

3. A Case Illustration for a Financial Services Conglomerate

Many financial services companies have now expanded and consolidated their businesses to achieve economies of scope and scale and to maintain competitiveness. A financial conglomerate, such as AIG, provides a variety of financial services ranging from commercial banking, retail banking, to insurance. These different lines of businesses entail inter-related risk exposures implicating an integrated corporate capital budgeting and risk management process.

We now present an example of such a hypothetical financial services company to illustrate our integrated framework. Without loss of generality, we will showcase the formulation and features of our model using a simplified, representative, and tractable setup. Generalizations and extensions concerning alternative scenarios can be naturally derived and some of these possibilities will be further discussed in the next section.

For this example, we consider a two-period planning horizon and two main business divisions, the loan division issuing loans to individuals and corporations, and the insurance division writing personal and commercial lines of insurance. For simplicity, each division has one potential project to invest in. In its planning process, the company needs to decide whether initial investments (e.g. marketing expenses and agent commissions, etc.) are made for one or both divisions in period 1, and further, if an initial investment is made, after observing the realizations of risks in period 1, whether continuing investments (e.g. expenses incurred in loan collections and claims adjusting, etc.) should be made in period 2.

To optimally allocate capital, each division has to consider a set of dependent risk exposures. In period 1 both divisions share the market risk, largely driven by the economy, which impacts both divisions' revenues from the amount of loans issued and the insurance premium written. In period 2 each division is faced with a division-specific risk exposure: credit risk for the loan division and actuarial pricing risk for the insurance division. The corporate decision maker allocates the initial corporate capital budget I_0 in neither, one, or both divisions/projects, by maximizing her expected utility from the corporation's final capital positions at the end of the planning horizon. We follow the modeling process described in Section 2 to build the optimization problem for this company.

3.1. Corporate States of Nature

In the two-period corporate "state of nature" tree, we denote the commonly shared market risk driven uncertain corporate revenues as R_A and represent it with chance/risk node A, and denote the two division-specific risks as R_B, the credit risk, for the loan division, R_C, the actuarial pricing risk, for the insurance division, and represent them as chance node *B* and *C*, respectively.

The three risk exposures are inter-connected across business divisions and time periods. Market risk R_A in terms of revenue is negatively correlated with credit risk as loan defaults tend to increase when the macroeconomic conditions deteriorate. Market risk is also negatively correlated with actuarial pricing risk as fraudulent activities tend to increase during periods of economic downturn (cf., Insurance Information Institute 2011). In addition, credit risk and actuarial pricing risk are positively correlated via shared customers. Actuarial science/insurance literature has long documented evidences of the strong predictive power of an individual's credit score in insurance losses. Brockett and Golden (2007) offered a psychological and biological based explanation for the linkage between financial behaviors and insurance claims, which in turn suggests potential positive dependency between the financial service company's credit risk and actuarial pricing risk from a shared clientele base.

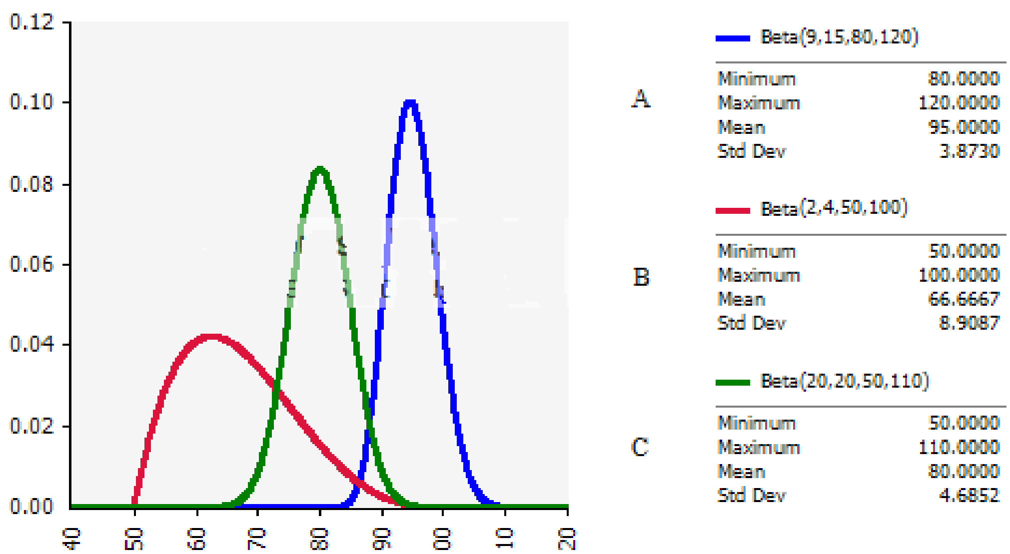
In this example, we specify the project cash flow payoff structure for each business division at the end of the planning horizon as "revenues minus losses," where revenues are generated from loan and insurance policies issuances, and losses are incurred from loan defaults and insurance underwriting results. Accordingly, we use the payoff functions $f(A, B) = f_1(A) - B$ for the loan division (division 1) and $f(A, C) = f_2(A) - C$ for the insurance division (division 2), where A, B, C are the uncertain outcomes of risks A, B, C and functions f_1 and f_2 describe how the market risk impacts the two divisions' revenues respectively. For simplicity, we assume $f_1(A) = f_2(A) = A$ for our illustration, i.e. the loan division and the insurance division generate the same revenue given the market conditions. Note that the specification of the payoff functions can be modified into a more generic or an alternative specific form for the problem context within the rationale of our formulation.

As described in Section 2, we follow the copulas-based dependent decision tree approach (Wang and Dyer 2011) to formulate the corporate state of nature tree. Because of the flexibility and analytical tractability of multivariate Normal copulas, we follow the literature to use the Normal copulas for our example setting. The following assumptions are made on the marginal distributions and the correlation structure of the three risks consistent with the finance and insurance literature (e.g., Rosenberg and Schuermann 2006). We first describe the risks using the scaled Beta distribution which provides modeling flexibility in capturing distributions with a broad set of shapes. The illustrative parameters are specified in Table 1 and the risks are graphed in Figure 3. From Figure 3 we can see that the market risk driven revenues and actuarial pricing risks are bell shaped distributions and the credit risk follows a right skewed loss distribution, consistent with the typical assumptions in the literature.

Table 1 Distributional Assumption for Marginal Distributions of Risks (in $million)

		Parameters		Range	
Risk	Distribution	Alpha	Beta	Lower bound	Upper bound
A (Market risk: revenues)	Scaled Beta	9	15	80	120
B (Credit risk: loan losses)	Scaled Beta	2	4	50	100
C (Actuarial pricing risk: underwriting losses)	Scaled Beta	20	20	50	110

Figure 3 Graphic Illustration of Marginal Distributions of Risks



We further assume the correlation structure among the three risks as shown in Table 2.[2] We follow Rosenberg and Schuermann (2006) to use a benchmark correlation of 0.5 between market risk and credit risk. As it is widely documented fraudulent activities tend to increase during the economic downturn and financial crisis (Insurance Information Institute 2011), we assume a relative high correlation of 0.7 between these two risk exposures. Note that as market risk negatively impacts revenues, risk *A* in our

[2] In this hypothetical example, we assume the correlation structure of the underlying copula directly. Wang and Dyer (2001) discussed how to derive this correlation structure of the underlying copula from the available historical data.

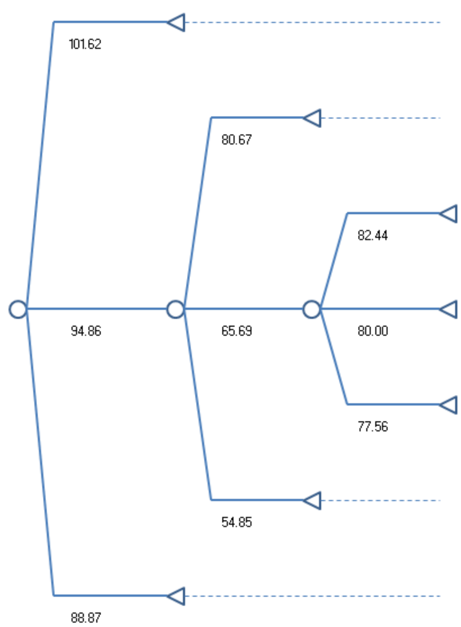
setting is actually negatively correlated with the other two risks (hence -0.5 and -0.7 in Table 2). Lastly, we assume a moderate percentage of shared clientele base between the loan and insurance divisions within the same financial services company, leading to a moderate positive correlation of 0.5.

Table 2 Correlation Structure among Risks

Correlation	A	B	C
A	1	-0.5	-0.7
B	-0.5	1	0.5
C	-0.7	0.5	1

Based on these assumptions and following the rationale described in Section 2, we create the overall corporate state of nature tree as shown in Figure 4. This is later used to frame and guide the decision making process.

Figure 4 Two-Period Corporate State of Nature Tree



3.2. Cash Flows of the Corporate Project Portfolio

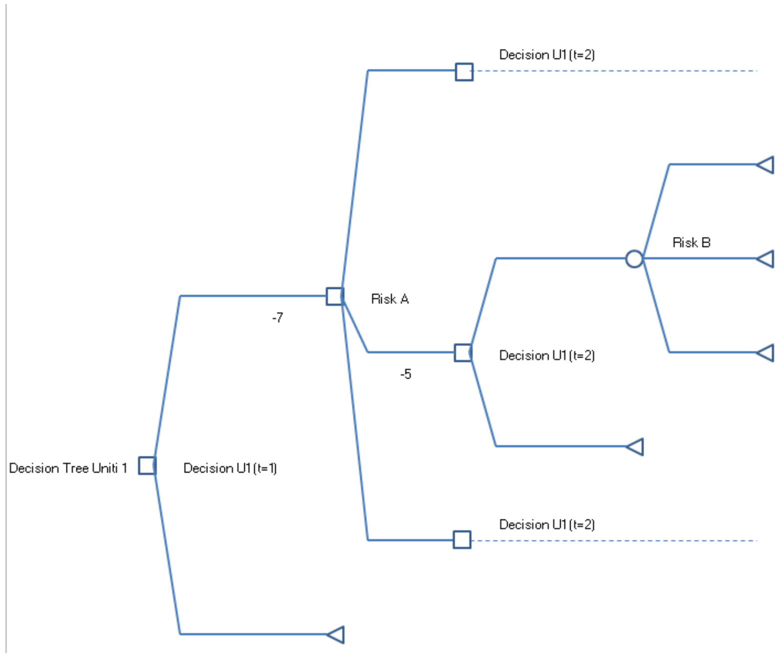
To construct cash flows, we first discuss investment opportunities and other assumptions. Assume at the beginning of the planning horizon (t=0), the company has an initial budget $30 million to allocate to any potential projects in either or both divisions. At t=1, the company decides if to continue invest in the project(s) that were previously invested in. Table 3 presents the required investments for division/project (i) (i =1, 2) at t=0 and t=1 and other assumptions for this example.

Table 3 Investment Requirements and Other Assumptions

For	Investment ($ millions)	Notation
Division 1 (t=0)	7	$Inv_{1,0}$
Division 1 (t=1)	6	$Inv_{1,1}$
Division 2 (t=0)	7	$Inv_{2,0}$
Division 2 (t=1)	6	$Inv_{2,1}$
Initial Resources (I_0) ($ millions)	30	
Risk Free Interest Rate (r_f)	6%	

Given the investment opportunities and the risk factors it faces, each division's decision making process can be depicted as a decision tree. Figure 5 shows the decision making process of division 1. Division 2 has a decision tree similarly constructed reflecting its own investment opportunities and risk factors.

Figure 5 Division 1 Decision Tree



Next we combine the two division-level decision trees to a single corporate level decision tree describing cash flows and derive corporate level optimal strategies, with the help of a set of constraints. In our illustrative set-up with two divisions, three risks, and two time periods, the size of the corporate level decision tree should amount to $2^4 * 3^3 = 423$ decision nodes. By following the Gustafsson and Salo (2005) to reduce dimensionality, the size of our corporate decision tree is significantly reduced to $3^3 = 27$ decision nodes, i.e., the size of the decision tree stays the same as the state of nature tree rather than growing in an exponential manner with the added decision variables. This will greatly improve computational efficiency and thus managerial relevance.

3.3. Constraints

3.3.1 Logical consistency constraints

We now describe the logical constraints to ensure consistency in decisions. As explained in Section 2, we use a binary indicator $X_{1,0,Y} = 1$ (or 0) to indicate whether an affirmative decision is made for division 1 to start its project (or not) at t=0 and another binary indicator $X_{1,0,N}$ =1 (or 0) to indicate whether a negative decision is made. A constraint $X_{1,0,Y}$ + $X_{1,0,N}$ =1 is added to make sure that one and only one of the two variables takes the value 1 to be logically consistent. Similarly, a set of two indicator decision variables $X_{2,0,Y}$ and $X_{2,0,N}$ are created to describe division 2 decisions whether to make an initial investment on project 2 at t=0. Again, $X_{2,0,Y} + X_{2,0,N}$ =1.

Table 4 shows all such binary decision variables and the associated consistency constraints. Note that we have three such decision variables for each division in period 2 for the corresponding states (denoted $X_{1,1,Y}$ for division 1 in the "up" state, and in the same manner others) in the state of nature tree, as under the Wang and Dyer (2011) approach a trinomial discrete approximation is used to construct the probability trees leading to three possible corresponding states for each risk. Obviously, the additional consistency requirement here is that a period 2 investment on one project is possible only if this project is invested in at t=0.

Table 4 Consistency Constraints

$X_{1,0,Y}+X_{1,0,N}=1$
$X_{2,0,Y}+X_{2,0,N}=1$
$X_{1,1,Y}+X_{1,1,N}=X_{1,0,Y}$
$X_{1,2,Y}+X_{1,2,N}=X_{1,0,Y}$
$X_{1,3,Y}+X_{1,3,N}=X_{1,0,Y}$
$X_{2,1,Y}+X_{2,1,N}=X_{2,0,Y}$
$X_{2,2,Y}+X_{2,2,N}=X_{2,0,Y}$
$X_{2,3,Y}+X_{2,3,N}=X_{2,0,Y}$

3.3.2 Capital resource constraints

We now discuss the set of capital resource constraints to reflect the capital flows at each decision node and to map out the entire decision process. Given that an investment of $7 million is needed for this project (as in Table 3), the capital flow contributed by division 1/project 1 at the initial node of the corporate state of nature tree is -7($million)*$X_{1,0,Y}$. Similarly, a capital flow of -7($million)*$X_{2,0,Y}$ is attributed to division 2 decision at t=0. Therefore, the total capital flow for the entire corporation at the initial node (t=0) of the corporate state of nature tree is -7($million)*$X_{1,0,Y}$-7($million)*$X_{2,0,Y}$. This completes our mapping at the initial stage of the decisions to combine the two separate division-level decision trees into the single corporate state of nature tree. We continue this process in the same manner to map out the rest of the tree structure.

As a result, one such resource constraint is used to describe the resource position at each node of the corporate state of nature tree. At the initial node, the resource position is denoted by the resource surplus V_0 and the resource constraint is formulated as

$- Inv_{1,0}*X_{1,0,Y} – Inv_{2,0}(1)* X_{2,0,Y} + I_0 - V_0 = 0$, i.e., period 1 investment made by one or both divisions ($Inv_{1,0}$ and $Inv_{2,0}$) should equal to the difference between the total amount of initial resource available (I_0) and the amount of resource left at t=0 (V_0). The resource surplus is deposited at the risk free rate. Recall also that there is a payoff for each division given by A-B and A-C, respectively, if a project is invested in both periods. Ultimately, three such resource constraints are used corresponding to the three nodes at t=1 (three discrete approximation points for risk R_A) and twenty seven such resource constraints are used at t=2 corresponding to the twenty seven nodes (three points for R_A *three points for R_B *three points for R_C) in the tree. The specific resource constraints are presented in Table 5.

Table 5 Resource Constraints Used in the Case Example

$- Inv_{1,0}*X_{1,0,Y} – Inv_{2,0}(1)* X_{2,0,Y} + I_0 - V_0 = 0$
$- Inv_{1,1}* X_{1,k,Y} - Inv_{2,1} * X_{2,k,Y} +(1+r_f)*V_0-V(\mathrm{k})=0$
$(A(k)-B(i,j))* X_{1,k,Y} +(A(k)-\mathrm{C}(\mathrm{i,j}))* X_{2,k,Y} +(1+ r_f)*V(k)-V(i,j)=0$

All resource surplus variables V>=0, i.e., the company cannot run out of cash because of the budget constraint	
k=1, 2, 3 for the decision nodes at the end of period 1, i=1,...,9 and j=1,…,3 for decision nodes at the end of period 2	

Note that the payoff of each investment is uncertain as it is impacted by how the risks play out during the course of the two periods. By capturing the dependence among these risks, we allow the company to address risk considerations by exploiting any "natural hedge" opportunities and staying alert to any "catastrophic" risk exposures. For example, we capture in our model the company's much increased risk exposure coming from both divisions/projects in a bad economy as we incorporate the positive dependence between the credit risk and the actuarial pricing risk, which will help the company make overall optimal investment decisions.

3.3.3 Risk constraints

For the illustrative example, we model the corporate decision maker's risk preference by using the certainty equivalent (CE), where CE = expected final capital resource position (EV_T) – Risk. Risk here is measured by applying a risk aversion coefficient, assumed to be 0.5 in this example, to the expected lower semi-absolute deviation (LSAD) from the expected final resource position, i.e., EV_T - 0.5*$E(\Delta V_s^-)$. A set of risk constraints are also entailed in the form of $V(i,j) - EV_T - \Delta V_s^+ + \Delta V_s^-$ =0 by definitions of the risk measures ΔV_s^+ and ΔV_s^-, where $\Delta V_s^+ \geq 0$ and $\Delta V_s^- \geq 0$ and only one of them in each state (i,j) can be positive, i = 1,..., 9 and j = 1,…, 3.

3.4. Risk Management Strategies

Now we incorporate specific risk management strategies in the decision framework as a further step of risk management. For this illustration, we assume that the RM cost C takes the functional form $C(\alpha)$ = c*exposure*α, where c is the division cost of RM of a specific risk exposure and α is the hedge ratio. Essentially for risk R_A in period 1, the cost of managing the market risk C_A (α) is taken out of the total resource available for period 1 if risk management is used. Consequently at the end of the planning horizon, the payoffs from the project investments are not of the original functional form $f = A - B$ (or C), as now risk management activities have changed the impact of the risks on the project cash flows. We then construct the new cash flows reflecting the hedged payoff and the unhedged payoff taking into account the risk management strategies as we described in Section 2. For ease of illustration, we only consider linear hedging or pro rata reinsurance policy, as incorporation of nonlinear risk management strategy will likely add to the computational difficulty of our formulation but does not necessarily improve the merits of our modeling approach in a fundamental way.

In this example, we design the division RM cost c such that it varies depending on the characteristics of these risks (i.e., more "skewed" R_B has a higher c than R_A and R_C as shown in Figure 3) and show them in Table 6 below.

Table 6 Division Risk Management Cost for Each Risk

Cost of R_A: c_A	0.006
Cost of R_B: c_B	0.01
Cost of R_C: c_C	0.005

We now present the final capital resource constraints in Table 7 taking into account risk management.

Table 7 Capital Resource Constraints with Risk Management Strategies

$- Inv_{1,0} * X_{1,0,Y} - Inv_{2,0} * X_{2,0,Y} - X_{1,0,Y} * c_A * \alpha_{1,0} - X_{2,0,Y} * c_A * \alpha_{2,0} + I_0 - V_0 = 0$
$- Inv_{1,1} * X_{1,k,Y} - Inv_{2,1} * X_{2,k,Y} - X_{1,k,Y} * c_B * \alpha_{1,k} - X_{1,k,Y} * c_C * \alpha_{2,k} + (1+r_f) * V_0 - V(k) = 0$
$(A(k1) - B(i,j)) * X_{1,k,Y} + (A(k2) - C(i,j)) * X_{2,k,Y} + (1+r_f) * V(k) - V(i,j) = 0$
Where $A(k1) = (1 - \alpha_{1,0}) * A + \alpha_{1,0} * E(A)$
$A(k2) = (1 - \alpha_{2,0}) * A + \alpha_{2,0} * E(A)$
$B(i,j) = (1 - \alpha_{1,k}) * B + \alpha_{1,k} * E(B)$
$C(i,j) = (1 - \alpha_{2,k}) * C + \alpha_{2,k} * E(C)$

All resources surplus variables V>=0, i.e., the company cannot run out of cash because of the budget constraint	
k=1, 2, 3 for the decision nodes at the end of period 1, i=1,...,9 and j=1,…,3 for decision nodes at the end of period 2	

3.5 Discussion of Results

Finally, the corporate decision maker's expected utility according to her risk preferences is maximized over the set of state dependent final corporate resource positions, subject to the set of logical consistency constraints, capital resources constraints, and risk constraints as described above.

Solving the corporate decision maker's optimization problem, we obtain streamlined decisions for capital investment and risk management. Table 8 details these optimal decisions taken by each division in each period.

Table 8 Optimal Decisions

$X_{1,0,Y}$	1	$\alpha_{1,0}$	0
$X_{1,0,N}$	0	$\alpha_{2,0}$	0
$X_{1,1,Y}$	1	$\alpha_{1,1}$	0
$X_{1,1,N}$	0	$\alpha_{1,2}$	0.6301
$X_{1,2,Y}$	1	$\alpha_{1,3}$	1
$X_{1,2,N}$	0	$\alpha_{2,1}$	0
$X_{1,3,Y}$	1	$\alpha_{2,2}$	0.5759
$X_{1,3,N}$	0	$\alpha_{2,3}$	1
$X_{2,0,Y}$	1		
$X_{2,0,N}$	0		
$X_{2,1,Y}$	1		
$X_{2,1,N}$	0		
$X_{2,2,Y}$	1		
$X_{2,2,N}$	0		
$X_{2,3,Y}$	1		
$X_{2,3,N}$	0		

The results are rather intuitive under the specifications of our example. The optimal decisions show that the company will invest in both the loan and the insurance businesses in both periods. The loan division (division 1) manages a very small percentage of its risk exposure in period 1 while the insurance division manages a rather large portion. In period 2 both divisions choose to manage their risk exposures to a significant extent in the bad states by incurring the necessary cost of risk management. These investment and risk management decisions have already taken into account the dependence across divisions, risks, and time periods. The optimal value of the certainty equivalent is \$47.93 million, which is much higher than investing the resources in risk-free assets: $30*(1+6\%)^2$ = \$33.71 million.

Note that the optimal decisions obtained above are aligned with the corporate decision maker's strategic objective and risk preferences, have streamlined the investment decisions and the risk management considerations, and have fully accounted for the risk dependencies. Should one of these features be overlooked, the decision making process would not have resulted in the desired optimal capital budgeting and risk management strategies for the corporation. We will illustrate this and offer more discussions in the next section.

3.6 Sensitivity Analysis

As there might be errors in the correlation assessments, we examine the robustness of the optimal decisions to the correlation estimates. A set of one-way sensitivity analysis is conducted with different correlation matrices as inputs into the copula model, calculating means and standard deviations of the resulting risk profiles.

Clemen & Reilly (1999) and Clemen et al. (2000) discuss subjective correlation assessment methods including estimating the probability of concordance and conditional fractile estimates, and reported ±0.2 as the average Mean Absolute Deviation for correlation assessment. Following the literature, we examine the effect of each individual correlation assessment by perturbing them one at a time in the ±0.2 range. The interested readers are referred to Clemen & Reilly (1999) and Clemen et al. (2000) for detailed discussions of practical correlation assessment methods.

When the correlation between market and credit risk, Corr (A, B) = -0.5, is perturbed from -0.7 to -0.3, the optimal decisions stay the same. The optimal certainty equivalent is decreasing and both hedging ratio $\alpha_{1,2}$ and $\alpha_{2,2}$ are increasing due to the less natural hedging benefit.

Table 9 Sensitivity Analysis for Correlation between Risk A and B

Corr (A, B)	-0.7	-0.6	-0.5	-0.4	-0.3
$\alpha_{1,0}$	0	0	0	0	0
$\alpha_{2,0}$	0	0	0	0	0
$\alpha_{1,1}$	0	0	0	0	0
$\alpha_{1,2}$	0	0	0.630	0.869	0.904
$\alpha_{1,3}$	1	1	1	1	1
$\alpha_{2,1}$	0	0	0	0	0
$\alpha_{2,2}$	0	0	0.576	0.561	0.585
$\alpha_{2,3}$	1	1	1	1	1
Optimal CE	48.500	48.164	47.926	47.736	47.559

Similary, when Corr (A, C) is perturbed from -0.9 to -0.5, the optimal decisions stay the same. The optimal certainty equivalent is decreasing and both hedging ratio $\alpha_{1,2}$ and $\alpha_{2,2}$ are increasing due to the less natural hedging benefit.

Table 10 Sensitivity Analysis for Correlation between Risk A and C

Corr (A, C)	-0.9	-0.8	-0.7	-0.6	-0.5
$\alpha_{1,0}$	0	0	0	0	0
$\alpha_{2,0}$	0	0	0	0	0
$\alpha_{1,1}$	0	0	0	0	0
$\alpha_{1,2}$	0.512	0.460	0.630	0.657	0.684
$\alpha_{1,3}$	1	1	1	1	1
$\alpha_{2,1}$	0	0	0	0	0
$\alpha_{2,2}$	0	0	0.576	0.633	0.672
$\alpha_{2,3}$	1	1	1	1	1
Optimal CE	48.237	48.063	47.926	47.814	47.707

When Corr (B, C) is perturbed from 0.3 to 0.7, the optimal decisions stay the same. The optimal certainty equivalent is decreasing due to the higher positive correlation among two divisions. The change of hedging ratio is more complicated since it is determined by the profound influence of the correlations. Hedging ratio $\alpha_{1,2}$ is increasing due to the higher positive correlation among two divisions. Hedging ratio $\alpha_{2,2}$ first increases and then decreases due to the trade-off between the need to hedge and the relatively high negative correlation between A and C.

Table 11 Sensitivity Analysis for Correlation between Risk B and C

Corr (B, C)	0.3	0.4	0.5	0.6	0.7
$\alpha_{1,0}$	0	0	0	0	0
$\alpha_{2,0}$	0	0	0	0	0
$\alpha_{1,1}$	0	0	0	0	0
$\alpha_{1,2}$	0.281	0.355	0.630	0.666	0.713
$\alpha_{1,3}$	1	1	1	1	1
$\alpha_{2,1}$	0	0	0	0	0
$\alpha_{2,2}$	0	0	0.576	0.544	0.485
$\alpha_{2,3}$	1	1	1	1	1
Optimal CE	47.990	47.945	47.926	47.911	47.896

The sensitivity analysis results suggested that the optimal decisions are robust to assessment errors.

4. Discussions and Extensions

4.1 Discussions of Dependence and Risk Management Assumptions

We now discuss the importance of dependency modeling and risk management by examining three cases: (1) the hedging strategy is not taken into account, (2) the dependency is not taken into account, and (3) neither the dependency nor the hedging strategy is taken into account. We investigate the impact of these ignorance in modeling on the optimal capital budgeting decisions and the optimal values of the corporation.

4.1.1 Ignoring the risk dependency

We first discuss the importance of dependency modeling by investigating the case where the dependency is not taken into account, i.e., assuming now the three risks faced by the two divisions, R_A, R_B, and R_C, are mutually independent. All other specifications remain the same as before. Solving the new optimization problem, we obtain the new optimal investment and risk management decisions and present them in Table 12. Now only the loan division gets the investments in both periods and it ends up not managing the market risk exposure but controlling heavily the credit risk exposure even in the good state in period 2. The optimal value for the certainty equivalent is consequently reduced to \$46.60 million. The difference in expected final resource position for the two cases is significant (\$47.90 million vs. \$46.60 million).

Table 12 Optimal Decisions under Independent Risks

$X_{1,0,Y}$	1	$\alpha_{1,0}$	0
$X_{1,0,N}$	0	$\alpha_{2,0}$	0
$X_{1,1,Y}$	1	$\alpha_{1,1}$	0.5808
$X_{1,1,N}$	0	$\alpha_{1,2}$	0.9804
$X_{1,2,Y}$	1	$\alpha_{1,3}$	0.5383
$X_{1,2,N}$	0	$\alpha_{2,1}$	0
$X_{1,3,Y}$	1	$\alpha_{2,2}$	0
$X_{1,3,N}$	0	$\alpha_{2,3}$	0
$X_{2,0,Y}$	0		
$X_{2,0,N}$	1		
$X_{2,1,Y}$	0		
$X_{2,1,N}$	0		
$X_{2,2,Y}$	0		
$X_{2,2,N}$	0		
$X_{2,3,Y}$	0		
$X_{2,3,N}$	0		

This example highlights the importance of acknowledging and properly modeling the inter-relations among risks across different business divisions in the corporate decision making process. Suboptimal or even catastrophic consequences can occur if capital budgeting decisions are made without fully accounting for the dependent risk exposures.

4.1.2 No hedging considered

Now we consider the no hedging case. All other specifications remain the same as before except now a hedging strategy is no longer part of the decision process. Solving the new optimization problem, we obtain the new optimal investment decisions and present them in Table 13. The optimal capital budgeting decision is different for $X_{2,3,N}$. The optimal value for the certainty equivalent is reduced to $45.87 million, a difference of $47.90-$45.87=$2.03 million from the original model. This illustrates that appropriately chosen hedging strategies, even at a cost, can add value to the firm.

Table 13 Optimal Decisions when No Hedging is Considered

$X_{1,0,Y}$	1	$\alpha_{1,0}$	0
$X_{1,0,N}$	0	$\alpha_{2,0}$	0
$X_{1,1,Y}$	1	$\alpha_{1,1}$	0
$X_{1,1,N}$	0	$\alpha_{1,2}$	0
$X_{1,2,Y}$	1	$\alpha_{1,3}$	0
$X_{1,2,N}$	0	$\alpha_{2,1}$	0
$X_{1,3,Y}$	1	$\alpha_{2,2}$	0
$X_{1,3,N}$	0	$\alpha_{2,3}$	0
$X_{2,0,Y}$	1		
$X_{2,0,N}$	0		
$X_{2,1,Y}$	1		
$X_{2,1,N}$	0		
$X_{2,2,Y}$	1		
$X_{2,2,N}$	0		
$X_{2,3,Y}$	0		
$X_{2,3,N}$	1		

4.1.3 No hedging or risk dependency considered

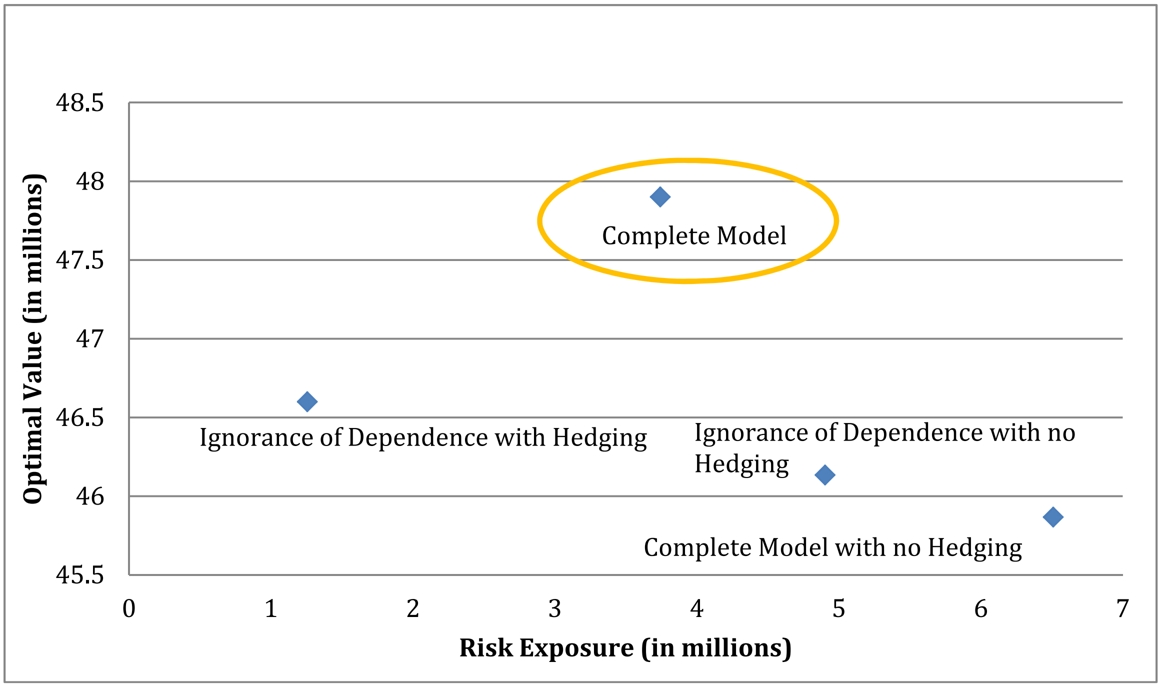
Lastly, if we ignore the risk dependency and forego any risk management strategy, the optimal value for the certainty equivalent is reduced to $46.13 million with a set of suboptimal capital budgeting decisions as reported in Table 14.

Table 14 Optimal Decisions under No Hedging Assumption

$X_{1,0,Y}$	1	$\alpha_{1,0}$	0
$X_{1,0,N}$	0	$\alpha_{2,0}$	0
$X_{1,1,Y}$	1	$\alpha_{1,1}$	0
$X_{1,1,N}$	0	$\alpha_{1,2}$	0
$X_{1,2,Y}$	1	$\alpha_{1,3}$	0
$X_{1,2,N}$	0	$\alpha_{2,1}$	0
$X_{1,3,Y}$	1	$\alpha_{2,2}$	0
$X_{1,3,N}$	0	$\alpha_{2,3}$	0
$X_{2,0,Y}$	1		
$X_{2,0,N}$	0		
$X_{2,1,Y}$	1		
$X_{2,1,N}$	0		
$X_{2,2,Y}$	1		
$X_{2,2,N}$	0		
$X_{2,3,Y}$	1		
$X_{2,3,N}$	0		

Figure 6 summarize the comparison of these incomplete models with the original complete model. It shows that the optimal value would be reduced significantly if we ignore the risk dependence, and foregoing hedging strategies would further reduce the optimal value with or without the modeling of dependence. Clearly, ignoring the risk dependency will underestimate the risk and the optimal hedging strategies can significantly reduce the risk taking of the corporation.

Figure 6 Comparison of Optimal Values and Risk Measure LSAD for the Four Cases



Many assumptions imposed in the formulation of our decision modeling approach presented with the simplified illustrative example can be extended to more general cases. These modifications should not change the rationale and applicability of our modeling approach. They may lead to minor revisions of the problem specification and may require more involved solution techniques which are largely available in the operations research literature. We next explore some of the more important extensions.

4.2 Extensions

In this paper, we have focused on presenting the conceptual set-up of the integrated capital budgeting and risk management framework. Under the same conceptual framework, a few interesting extensions are in place.

First, we used an overall utility function to model the corporate decision maker's risk preferences. Any valid forms of utility functions can be used in our model. More importantly, the decision maker's risk appetite can alternatively be articulated and directly built into the decision model, as suggested in the ERM literature (cf., Ai et al. 2010). In this way, she is allowed to have different appetites and prioritizations for different risk types, divisions, projects, and time periods. The risk appetite modeling could also reflect her preference for anticipated information problems associated with potential investment opportunities.

Second, we focused on linear risk management methods and assume all risk management activities are planned as the planning process progresses. Alternatively, we could explore and incorporate non-linear risk management strategies (such as options) which might be more effective for certain types of risks. It will also be relatively easy to allow all risk management decisions to be planned at the beginning of the planning horizon by modifying associated constraints.

Third, we use the copula approach to model dependency among risks. The copula model can accommodate a large class of dependence structure (e.g., for fat tails). Alternatively, we could consider other dependency measures, such as linear correlations, rank correlations, and other advanced models. Depending on the specific choice of the dependence measure, our model structure needs to be adjusted.

Other extensions include choosing a different form of the payoff function and incorporating corporate financing decision into the current integrated framework. These extensions are topics of our ongoing research.

5. Conclusion

In this paper, we propose an integrated capital budgeting and risk management framework under dependent risks in a multi-division, multi-project, multi-period environment. We formulate the model as an optimization problem of the corporate decision maker and construct the model via a decision-tree method. We illustrate our framework using a financial services company example.

Rather than maintaining two separate functions for risk management and capital budgeting as traditionally done in a company, we allow the risk management considerations to be aligned with the capital allocation decisions. As the business divisions are intrinsically connected by the risk exposures that are dependent on each other, the integrated framework will promote efficiency in both types of corporate decisions. In accomplishing these goals, we propose a prototype modeling framework while overcoming several modeling and computational challenges, such as the increased dimensionality and dependence modeling.

As one of the first papers to prescribe such an integrated corporate decision framework in the presence of dependent risks for inter-related business divisions, a challenging problem especially for financial conglomerates, our paper makes contributions to the capital budgeting literature, the enterprise risk management (ERM) literature, and the decision analysis literature. Our paper also has significant managerial relevance for capital allocation and risk management in practice. Decision makers can adapt our model efficiently for their own specific setting. Future research includes extensions on better modeling risk appetites and prioritization, incorporating corporate financing policies in the framework, as well as studying other relevant capital budgeting issues (such as the agency problems) under our proposed framework.

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Appendix

In the illustrative example presented in the paper, we use the normal copulas-based dependent decision tree for its simplicity and that its properties are appropriate for our dependency modeling problem. The proposed framework also provides efficient accommodations for *t* - copulas and Archimedean copulas-based decision trees to capture tail dependence when extreme events occur. A brief introduction to the dependent decision tree approach is presented below.

A multivariate normal copula C_N is given by $C_N(u_1, \dots, u_n) = \Phi_\Sigma(\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_n))$. It is derived from a multivariate normal cumulative distribution function Φ_{Σ_Z} with mean zero and correlation matrix Σ by transforming the marginals by the inverse of the standard normal distribution function Φ.

To build the multivariate normal copula-based decision tree for $(X_1, \dots X_n)$, we first construct a discrete approximation for the unconditional uniform variable u_1, then recursively compute the dependent uniform variables $\mathrm{u_k}$ $(\mathrm{k} = 2, \dots, \mathrm{n})$, conditioning on each of the point realizations of the previous discrete approximations for $(\mathrm{u_1}, \dots \mathrm{u_{k-1}})$.

$$\mathrm{u_{n=}}\, \Phi\,(\mathrm{A_{n1}}\, \Phi^{-1}(\alpha_1) + \cdots + \mathrm{A_{n(n-1)}}\, \Phi^{-1}(\alpha_{n-1}) + \mathrm{A_{n(n)}}\, \Phi^{-1}(\alpha_n)) \quad \text{(A1)}$$

where $\mathrm{A_{ij}}$ is the element of the Cholesky factorization that decomposes the covariance matrix Σ as $\Sigma = \mathrm{AA^T}$ to give the lower triangular matrix $\mathrm{A} = (\mathrm{A_{ij}})_{\mathrm{i,j=1}}^{\mathrm{n}}$ and α_{i} is the percentile of the conditional distribution $\mathrm{X_i}|\mathrm{X_1}, \dots, \mathrm{X_{i-1}}$ according to the extended Pearson Tukey method (Keefer and Bodily, 1983). The discrete approximations of X_i are obtained by applying the inverse of the target marginal distribution function for each realization of u_i , $X_i = F_i^{-1}(u_i)$. Taking the point-to-point inverse marginal transformation, each realization of $(X_1, \dots X_n)$ is discretely approximated using the extended Pearson-Tukey method.

We now illustrate this approach in the context of our example in Section 3. To create the probability tree for the standardized uniform variables, we first generate the extended Pearson-Tukey discretization for u_A. It is a three point discrete approximation for the standard normal distribution with probabilities 0.185, 0.630, and 0.185 assigned to the percentiles 0.05, 0.5 and 0.95. The three branches of the uncertainty A are therefore 88.87, 94.86 and 101.62, the inverse of the Beta distribution of uncertainty A for 5^{th}, 50^{th} and 95^{th} percentile respectively.

The subsequent discrete chance nodes are contingent on the outcomes of the precedent nodes. We apply the Cholesky factorization to decompose Σ_Z into the lower triangular Cholesky matrix shown in Table A1 to assist the calculation of the dependent uniform variables.

Table A1 Decomposed Lower Triangular Cholesky Matrix

Cholesky		
1	0	0
-0.5	0.866	0
-0.7	0.173	0.693

There are two dependent uniform variables to calculate:

(1) The dependent uniform u_B given the outcomes of u_A. Using formula (A1) for the bivariate case, we can calculate u_A as follows:

$$u_A = \Phi(-0.5\Phi^{-1}(\alpha_1) + 0.866\Phi^{-1}(\alpha_2))$$

(2) The dependent uniform u_C given the outcomes of u_A and u_B. Using formula

(A1) , u_C is calculated as follows:

$$u_C = \Phi(-0.7\Phi^{-1}(\alpha_1) + 0.173\Phi^{-1}(\alpha_2) + 0.693\Phi^{-1}(\alpha_3))$$

For instance, if the outcome of u_A is 0.5, then the conditional chance node for $u_B|u_A = 0.5$ is calculated for the 5th, 50th, and 95th percentiles, and the three contingent outcomes are determined to be 0.077, 0.5, and 0.923 respectively and the three branches of the uncertainty B given A=94.86 are therefore 54.85, 65.69 and 80.67, the inverse of the Beta distribution of uncertainty B for 5th, 50th and 95th percentile respectively.

Similarly, we create the contingent tree for each successive node until we generate the complete multivariate standard decision tree.

We refer the interested readers to Wang and Dyer (2011) for detailed discussion of the dependent decision tree approach. For an introduction to the theory of copulas and the discussions of different copulas, see Nelsen (1999).