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The Volatility of Low Rates

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Abstract

Traditional, fixed-income risk models are based on the assumption that bond risk is directly proportional to the interest rate, i.e. that the interest-rate distribution is "log-normal." Two corollaries would then follow. Firstly, nominal interest rates could never be negative. Furthermore, bond volatility vanishes when interest rates approach zero. These conclusions are obviously wrong, if we observe debt situation in a country such as Germany. Should we then infer that the traditional way to model fixed income is no longer valid? Our study, based on bond-data back tests, covering 40 years that encompass both periods of low rates and of high inflation, demonstrates that the traditional approach still applies. When interest rates are low, the bond risk is proportional to the rate, increased by 1%. As an example, when interest rates shift from 1% to 0%, the bond risk is divided by 2, instead of becoming null. The second conclusion is that the interest rate volatility behaves as if the absolute minimum level for a rate was not zero, but -1%.

Are Interest Rates "Normal" or "Log-Normal"?

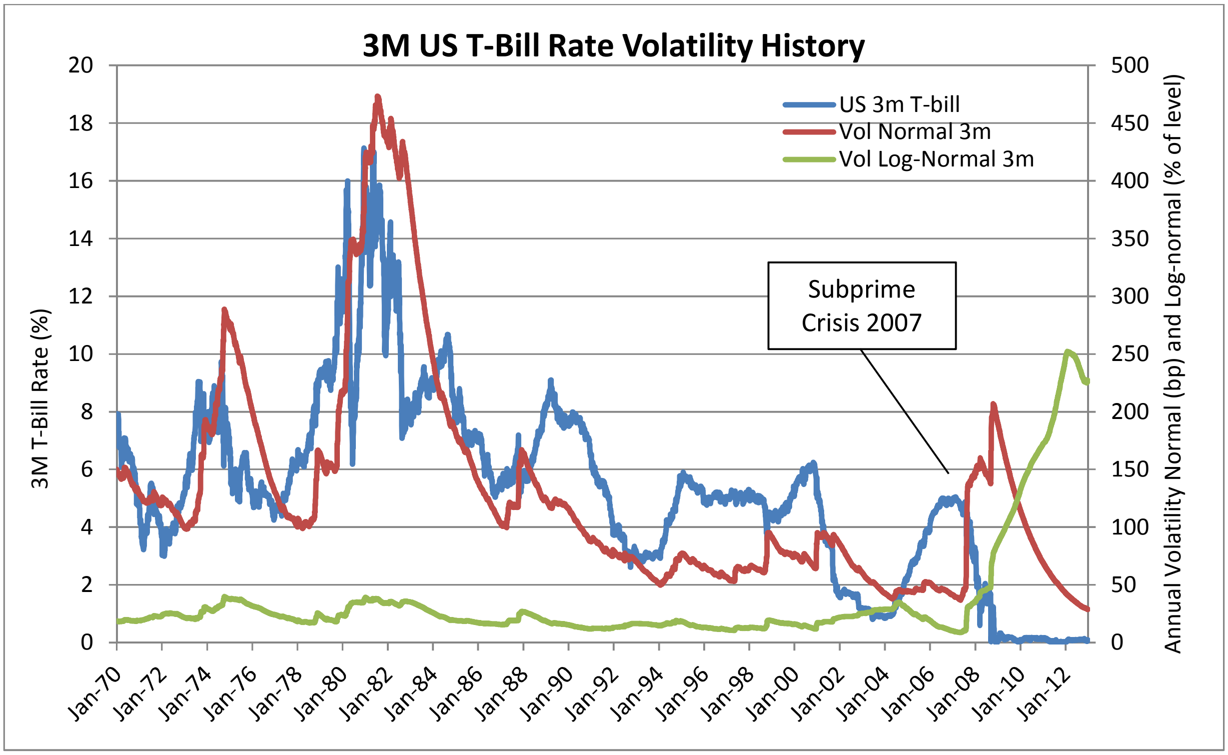
Since the collapse of Lehman Brothers in September 2008, the evolution of financial markets underwent deep and durable changes. While equity markets experience ups and downs, sovereign interest rates are at historically low levels, reaching sometimes a strict zero or even going negative. Prior to the crisis, only two countries knew such low rates: Switzerland and Japan. Since the financial crisis, quantitative easing, new regulations and flight to quality brought interest rates to bottom levels in all major western countries. The question arises of how to model their volatility. In 1997, Brace, Gatarek and Musiela (BGM) published their "*Market Model of Interest Rates Dynamics*" where the Libor is modeled as a (multi-factor) *log-normal* process. In particular, it cannot reach zero or negative levels and its volatility vanishes when rates approach zero. This model was published as an answer to Heath, Jarrow and Morton's (HJM) multi-factor modeling of the "*Term structure of Interest rates*" (1990-92), where the volatility of rates is assumed independent of their level (*normal* model). Not only was it commonly admitted that elementary arbitrage rules would prevent negative rates, an event not excluded by HJM model, but statistical evidence was showing, from the early 1980's until the subprime crisis in July 2007, a rather reliable proportionality rule between the level of rates and the standard deviation of their variations, whatever the country, the historical period and the horizon of variations. Exception was for the two countries Japan and Switzerland, who have been experiencing very low rates – even occasionally negative – for a long time. In these two countries, the *normal volatility* was seemingly uncorrelated to their levels. The reality is a bit more subtle, as we shall see, and the behavior of rates volatility was quite identical to that of western countries in current times.

During the period from 1980 until the subprime crisis in July 2007, we observed both a time of very high rate (10Y T-note > 15%) at the beginning, following the oil shocks of 1977-79, and a period of very low rates (3M T-bills < 1%). The ratio between the rates level and their volatility, i.e. their *log-normal* volatility, has changed many times during this 27 years-long period of time, but these two extreme examples demonstrated that, as a broad behavior, the volatility tends to be proportional to the level, thus confirming the log-normal approach. However, since the subprime crisis and, even more so, since 2008 "credit crunch" following Lehman Brothers' default and the subsequent Fed's *Quantitative Easing*, sovereign borrowing rates of major countries – in particular in the USA – fell to near zero levels, while keeping their "normal" volatility. Mechanically, the log-normal volatility jumped to previously unknown levels (100+% on an annualized basis). This resulted from one of the most massive flight to quality financial markets ever experienced. Extreme demand for safe money heaven brought the value of T-bills above 100, hence interest rates to negative values, mathematically invalidating the log-normal approach.

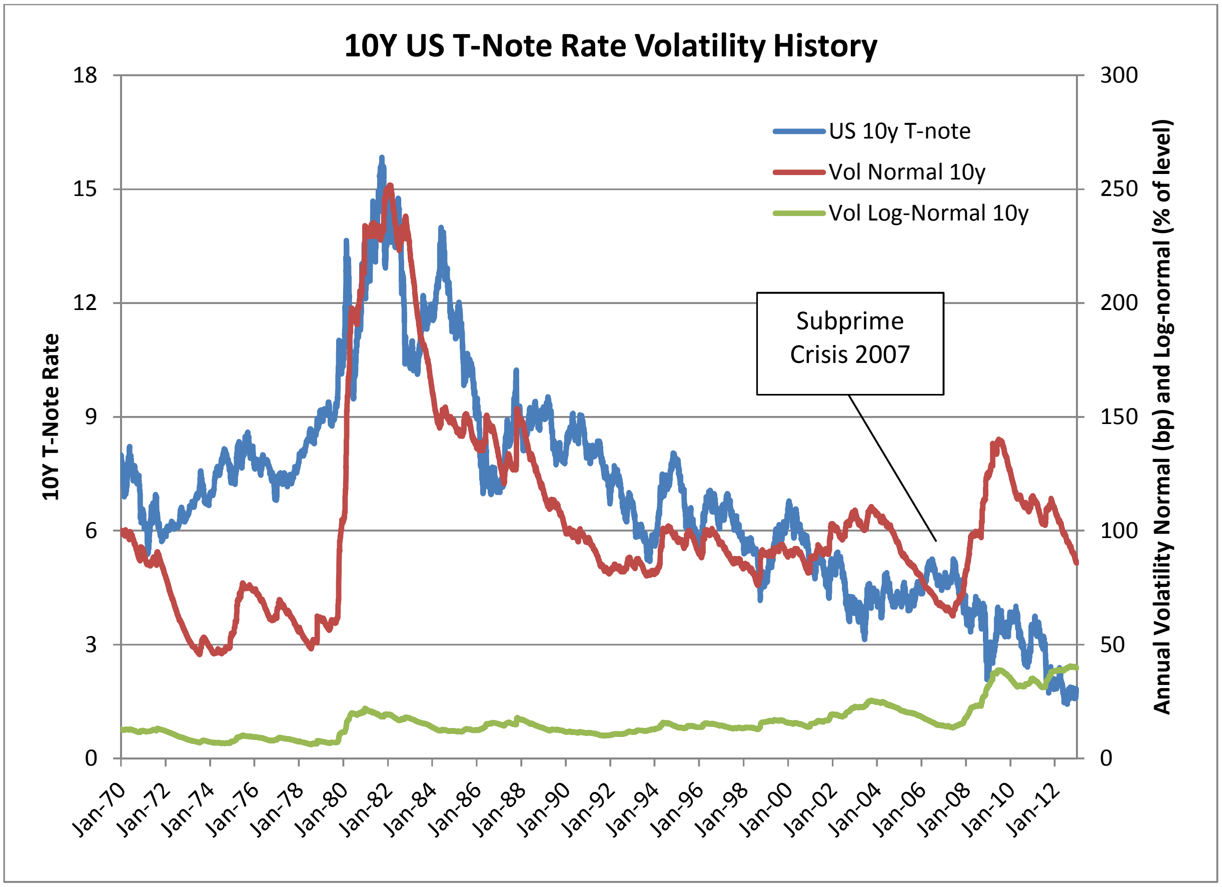
The following study aims at identifying where markets implicitly place their "real zero" level, in other words, what is the behavior of volatility when rates decrease and what is the (negative) level implicitly assumed not the be exceeded. Our findings show that this level is close to –1%, comparable across currencies and time periods, whether we consider Switzerland or Japan in the 90's or USA and Europe in 2008+.

US Interest Rate Volatility History

The graphs below compare the history of US interest rates and their volatility over the past four decades. Both the "normal" and the "log-normal" are displayed. Volatilities are computed as the square root of the exponential moving average of squared daily variations, with decay factor 0.004 (Riskdata's standard corresponding to 8 months half-life). One can see on the graph for 3M T-bill rates, that from the two oil crises of the 70's until the first decade of the XXIst century, the general movements of *normal* volatility levels followed that of interest rates: it surged during each peak and tamed down the 90's then again after the burst of the tech bubble in 2000. However, since 2007-08 crisis and the very low rates period we currently experience for T-Bills, the volatility doesn't reach 0 as fast as the rates, technically not preventing negative rates (which actually did occur). In such circumstances, the log-normal volatility reaches unprecedented levels because a non-zero figure is divided by one which is close to.

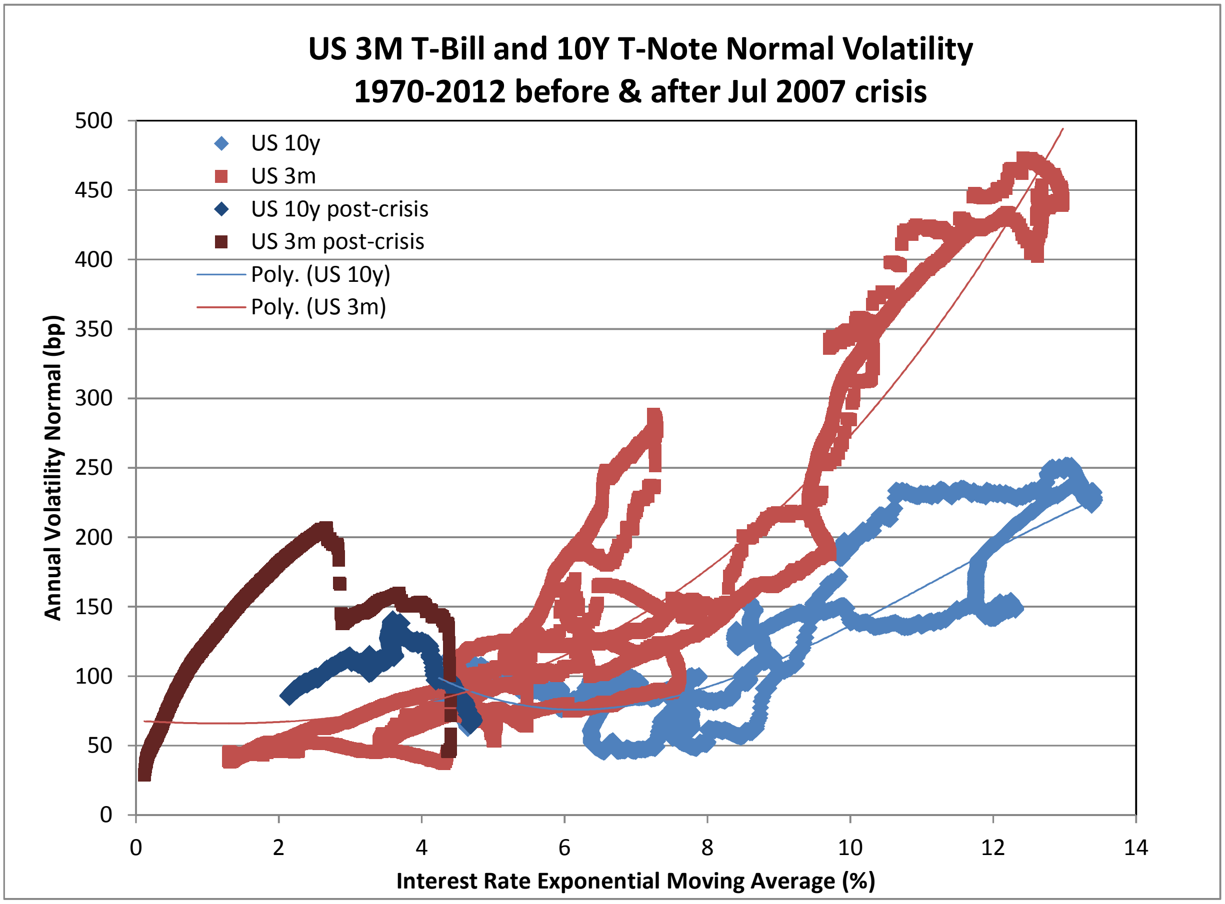


For 10Y T-note rates, the picture is similar but even a sharper surge occurred in 1980 when T-note rates increased to double digits, leading to a super-linear behavior: while rates doubled in level, the volatility was multiplied by 5. Then it went back down more or less proportionally to the rates in the 90's and kept a stable level in the 2000's, between the burst of the tech bubble and the subprime crisis, to, again surge during the subprime crisis, while a flight to quality was bringing rates even further down.



In both cases, the *log-normal* volatility experienced variations within a range, which it exited from the upside in 2007, because of lowering rates, while the *normal* volatility was maintained.

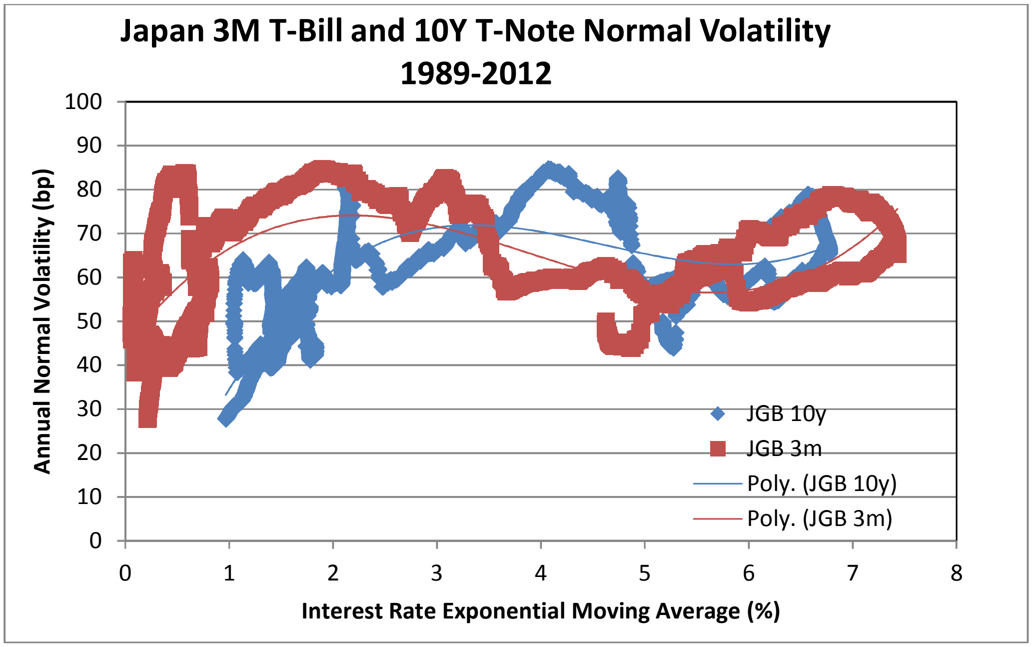
In order to better understand the actual behavior of interest rates volatility, let us observe a scatter plot of the normal volatility (annualized standard deviation of daily variations, as above) vs. the level of interest rates, of which we took an exponential moving average with the same parameters as for the volatility computation for consistency. The post-crisis period, starting in Jul 2007, is represented with darker dots. Two regimes are very visible: prior to the crisis, interest rates behavior is log-normal, with an even increased sensitivity to rate levels above 8% (this can be rather well represented by a quadratic formula of the volatility as a function of the interest rate level) whereas, after the crisis, it shifts to practically normal, as volatility moves up and down seemingly independently of rates level. In fact, for the part where 3m T-bill rates went down to practically 0%, one can observe a square-root type of shape, corresponding to Cox-Ingersoll-Ross model. Overall, this behavior of volatility as a function of rates level explains that, when we divide the *normal* volatility by the interest rate level, we get abnormally high figures for the *log-normal* volatility.



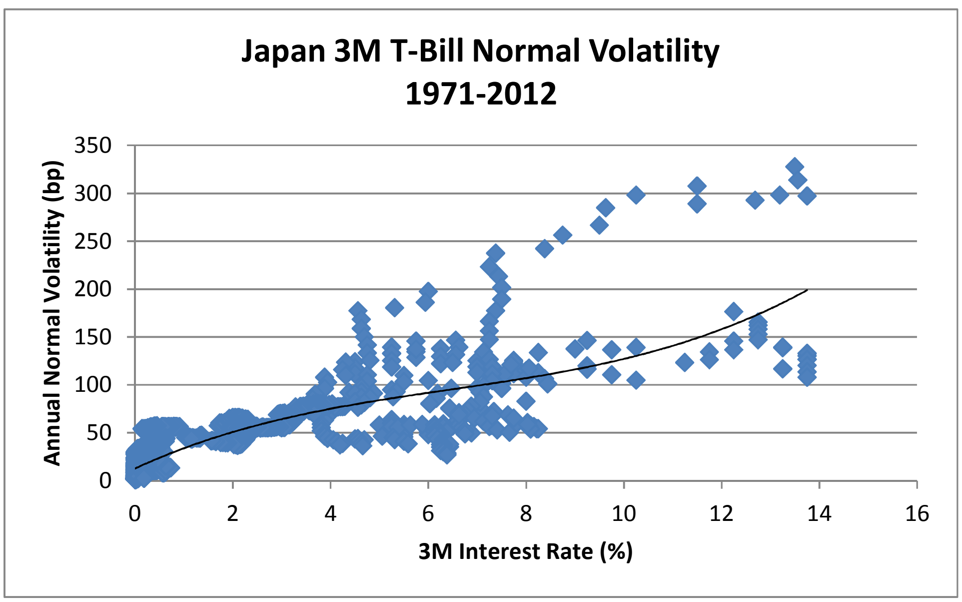
Japanese and Swiss Rates

Japanese and Swiss rates have experienced very low – even sometimes negative – levels for a very long time. Economic reasons for this fact are different. Japanese economy faced recession or low growth for the past 20 years at least, leading the Bank of Japan to progressively ease its monetary policy to the 0% floor level. Switzerland is known for being the "safe of the world". By supply and demand effects, money inflow, as well as a rather weak growth, rates are driven to 0% and even negative levels. In any case, explaining low rate levels in these two countries is not the topic of this note. Let us simply observe the behavior of their volatility.

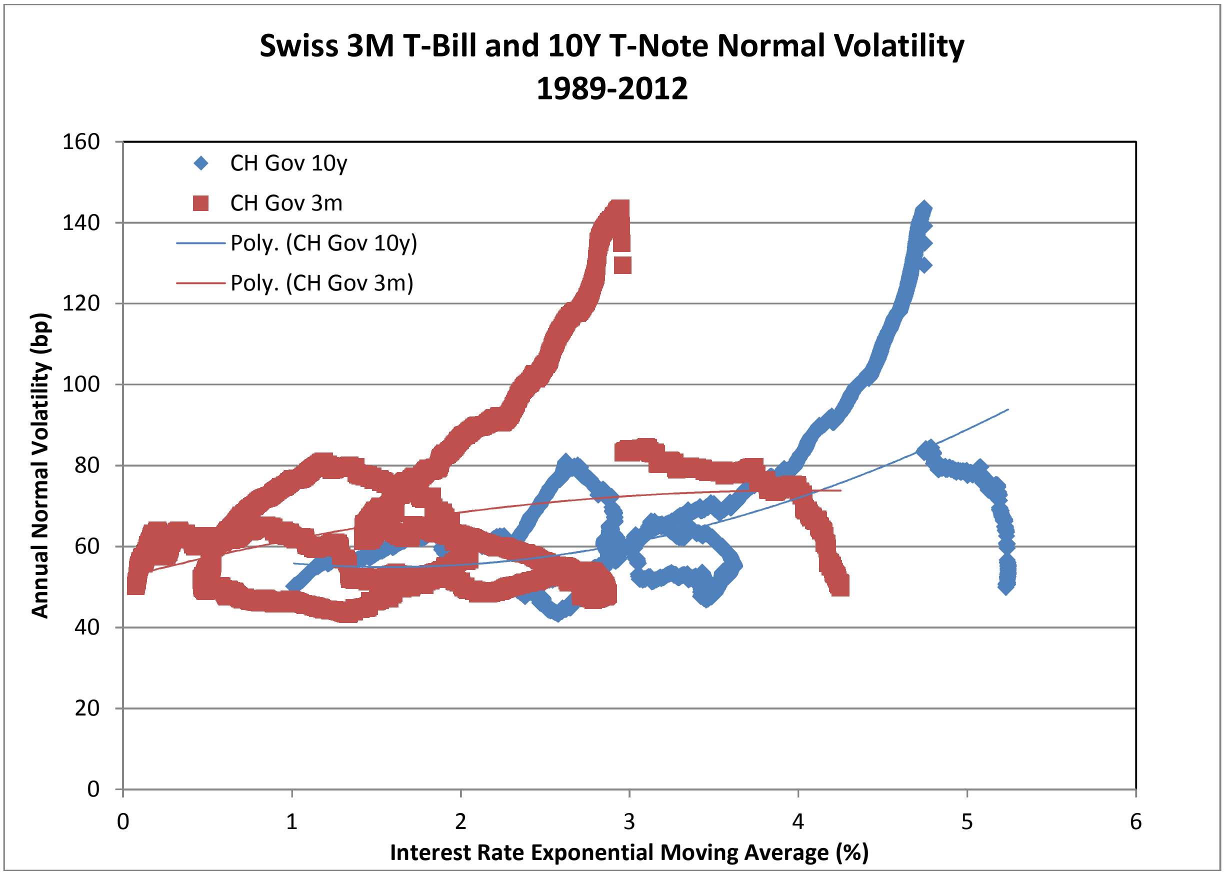
Data on Japanese rates over the period 1989-2012, during which interest rates didn't exceed 8%, but were often close to 0% or even negative, it appears that the volatility is rather independent of the rates level, showing that a normal model looks appropriate. However, when we try to match this picture with that of the USA, we see that this behavior is not incompatible with the previous observations. We even see the start of a rise of the volatility when rates exceed 5%, which is consistent with US data.



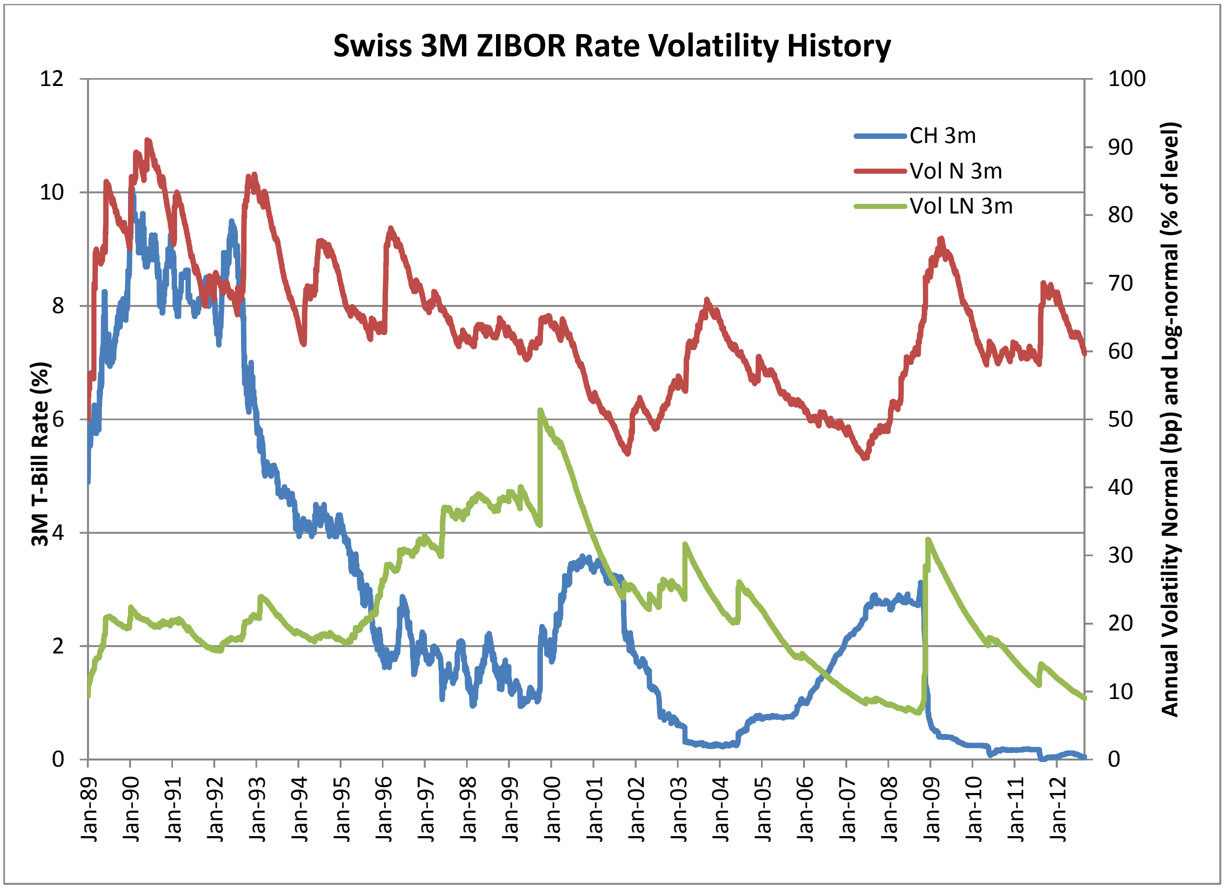
The same analysis on monthly 3M rates data since 1971 shows that, like in the USA, the volatility of interest rates is proportional to them when they exceed a threshold of the order of 4 to 5%.



For Swiss rates, we could only get data on 3m and 10y sovereign rates starting in 1994, but they still show the same pattern. Note that in this case, the volatility doesn't fall when rates approach 0%. This is consistent with the fact that government rates fell negative (up to –70 bp's), hence allowing rates near 0% remain volatile.



More importantly, when rates rise above 2% for the 3M and 3% for the 10y, the volatility may stay unrelated to the rates level, but also may jointly rise with them. We observed an instance of the first kind in 2000 by the burst of the Tech Bubble as well as in 2007 with the Subprime Crisis, while we had an instance of the second kind in the early 1990's by the time of the GBP crisis.



Consequence on Risk Computation

Accurately monitoring the behavior of interest rate volatility as a function of their level has strong implications for both derivatives pricing and risk computation. In this note, we primarily focus on risk computation. In this purpose, uncertainty must be taken from the conservative side. For instance, if, as observed with Swiss rates, the volatility may, or may not, rise with interest rate, sound risk measurement will assume the most volatile hypothesis.

Watching not enough a lengthy time period may erroneously lead statisticians to conclude that, in the case of Japan and Switzerland, interest rates are more “normal” than “log-normal”. Looking deeper into the history of interest rates, we find that at times where they rose at significantly higher levels than those we know today, their volatility did rise jointly. It is in fact rather intuitive that, when rates are above 10%, a 1% shift is almost as likely as a 10-20bp shift when their level is around 1%. People who experienced the oil shocks in the late 1970’s and the subsequent high inflation period remember this highly volatile environment. It is in fact worthy of note that the behavior of interest rate volatility depends less on the particular country, its economic structure or political regime than merely on the level of interest rates. Certain economic periods, such as today, display very low interest rates and inflation and, in this case, their volatility reaches a floor with a strictly positive value – of the order of 50bp on an annual basis – while interest rates occasionally turn negative. During agitated periods with high inflation, interest rates are consequently high and so is their volatility.

Modeling such a behavior can be achieved in two ways. One way is to select a log-normal model[1], in which we shift the 0 to a negative level, which seems to hover around –1%. In other words, we model the standard deviation of rate changes as being proportional not the level itself, but to the level +1%. When interest rates approach 0%, their volatility is still of the order of the proportionality coefficient x 1%. However, would rates rise to 5% or above, their volatility would jointly rise with them.

A second way, which provides similar numerical results, is to assume that interest rate volatility results from the sum of two sources of noise, one being log-normal and rising up with interest rates when those do, and one being normal, preventing the volatility to fall to 0 when rates approach this level. Here, the "normal" source of noise should be of the order of the interest rates volatility when they are of the order of +1%, in order to achieve a similar behavior as the "shifted log-normal" model.

This graph shows the volatility as a function of interest rates level in both cases. Comparing with actual data, the second model (normal noise + log-normal noise) seems to better fit the true behavior of volatility.

Value-at-Risk Back-tests

In practice, selecting the correct VaR model comes down to its performance for regulatory purposes. Tables below show the number of exceptions for the VaR 99% 10 days under 3 models: the normal model, the shifted log-normal – where the volatility is assumed to be proportional to (rate + 1%), and the log-normal with noise – where an artificial 50 bp per annum standard deviation noise is added to the unshifted log-normal volatility.

Tests have been made on the 3 months and 10 years rates in the USA, over two reference periods: 2000-2013 and 1970-1999. While the most recent period corresponds to a usual test period for back-tests, the older one, which contains the oil shocks of the 1970's, is meant to anticipate the results of possible crises to occur in the future.

[1] We don't mean here the actual Gaussian distribution of the log of the rates, there could be – and there are! – fat tails, but primarily the fact that the standard deviation of rate changes is proportional to their level.

3m Rate 10 days 99% VaR Exceptions 1970-1999			
	Normal	Log-normal shift 1%	Log-normal + Noise 50bp
Up	2.75%	2.60%	1.89%
Down	2.29%	2.81%	2.14%

10y Rate 10 days 99% VaR Exceptions 1970-1999			
	Normal	Log-normal shift 1%	Log-normal + Noise 50bp
Up	2.17%	2.39%	1.34%
Down	2.33%	2.69%	1.53%

3m Rate 10 days 99% VaR Exceptions 2000-2012			
	Normal	Log-normal shift 1%	Log-normal + Noise 50bp
Up	0.92%	1.77%	0.27%
Down	2.04%	1.74%	2.01%

10y Rate 10 days 99% VaR Exceptions 2000-2012			
	Normal	Log-normal shift 1%	Log-normal + Noise 50bp
Up	1.15%	0.92%	1.06%
Down	1.24%	2.04%	1.24%

One can observe the very visible reduction of exceptions when using the third model – log-normal with noise. The normal model and the shifted log-normal produce similar results. The fact that the number of exceptions exceeds 1% is due to the presence of fat tails in the distribution of rates changes over 10-days periods. These fat tails are acceptable and, in the end, fully controlled even with a Gaussian model, with the log-normal + noise model over the recent period 2000-2012. In the previous 30 years period, which contained high inflation times, fat tails are still very strong, but better controlled with the third model.

It could be conceived that a reactive enough normal model would better perform when back-testing the Value-at-Risk than a less reactive one. Indeed, when rates are low, it would automatically calibrate to the correct volatility level, and when rates are high, its reactivity would allow it adapt to the high volatility context. Only when rates rise too fast with respect to the model reactivity, would it potentially underestimate the risk.

The table below shows back-test results for a normal model with decay factor 0.97, used by RiskMetrics.

3m and 10y Rate 10 days 99% VaR Exceptions				
	3M Rate 1970-1999	3M Rate 2000-2012	10Y Rate 1970-1999	10Y Rate 2000-2012
Up	3.17%	3.44%	1.06%	1.51%
Down	3.84%	3.25%	2.72%	1.42%

Test results show that a more reactive decay factor doesn't improve the number of VaR exceptions.

Our preference goes for a log-normal model with mild decay and an artificial noise to handle low rates. First it immediately reacts if rates blow up. In the current context of interest rates artificially maintained at low levels by Central

Banks interventions, the occurrence of a monetary crisis could induce short term rates to suddenly rise from below 1% to above 5% in a period of time during which a normal model doesn't have time to adapt to new market conditions. Second, too reactive a risk model is very hard to manage, because even with fixed positions, a manager can see his/her margin vary significantly. This leads us to use a decay factor very close to 1 in exponential moving averages and volatility computations (namely 0.996 i.e. 8 month half-life). Such a decay factor produces VaR figures that are stable enough for portfolio management purposes, but which need to automatically adapt to interest rate levels.

Conclusion

Observing the true behavior of interest volatility under several economic configurations, from high inflation periods to times of very low rates as we know today, across several countries, we find that the model that best fit the actual behavior of interest rate volatility is a log-normal model in which we insert a source of noise which is independent of the level of interest rates. The standard deviation of this source of noise is the "floor" level of volatility when interest rates approach 0%. It is of the order of 50bp per annum – to be scaled by the square root of the risk horizon, for instance 3bp for 1 day, 10bp for 10 days, 14bp for one month. Above 4-5%, the volatility of interest rates can be considered as proportional to their level.

Various models have been tested to account for this behavior: a normal model, such as Heath-Jarrow-Morton, with different decay factors – one very reactive 0.97 and one more stable 0.996; a shifted log-normal model, where the volatility of interest rates is proportional to (rate + 1%) in order to allow rates to fall to as low as –1%; and an unshifted log-normal model to which an artificial noise of 50bp per annum is incorporated. VaR exceptions have been counted on two periods: the past 13 years 2000-2012 and the last 30 years of the XX^{th} century 1970-1999, which contains an epoch of high interest rates and inflation following the oil shocks. This test shows that reactivity doesn't help the normal model to adapt fast enough to the level of interest rates when these rise. It also shows that the unshifted log-normal model with noise produces better VaR back-test results on any period of time.