A Practical Way to Estimate One-year Reserve Risk

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Abstract

The advent of Solvency II has sparked interest in methods for estimating one-year reserve risk. This paper provides a discussion of the one-year view of reserve risk and some of the methods that have been proposed for quantifying it. It then presents a new method that uses ultimate reserve risk estimates, payment patterns, and reporting patterns to derive one-year reserve risk values in a systematic fashion. The proposed method is a more refined version of the simplistic approach used in the Standard Formula. Yet, it is also practical and robust: triangles, regressions, or simulations are not required.

Keywords: Solvency II, One-year Reserve Risk, Best Estimate, Loss Reserves, Technical Provision.

Table of Contents

1. INTRODUCTION

One-year reserve risk is a relatively new concept, especially for actuaries in the United States. Historically, actuaries have been concerned with whether the reserve is adequate to cover ultimate loss. With the advent of Solvency II, actuaries have now also begun to consider the one-year perspective. Under Solvency II, the capital requirement for unpaid loss is defined as the amount sufficient to cover risk over a single year. Solvency II also features a market-consistent approach to the valuation of unpaid loss liabilities.[1] Under this approach to valuation, unbiased estimates of unpaid loss are discounted and then loaded with an explicit risk margin. This risk margin depends on the projected capital requirements over the run-off period. So, under Solvency II, one-year risk dictates not only the capital requirement, but also the valuation of the reserve.

What is one-year reserve risk and how is it computed? Conceptually, it is a measure of how much an initial unbiased mean estimate of the reserve might change in one year. Under European Insurance and Occupational Pensions Authority (EIOPA) regulations, such risk can be computed either with a carefully delineated Standard Formula or, alternatively, with an approved, enterprise-specific internal model.[2]

The Standard Formula assumes a lognormal distribution of one-year retrospective results for each EIOPA line of business. Each line is assigned a single coefficient of variation (CV) that applies

[1] There is no market in which loss liabilities are openly traded. So the market-based approach is really a mark-to-model approach. Not enough is disclosed about loss portfolio transfers to fit pricing on these deals to a model.
[2] Partial use of an internal model is also allowed subject to regulatory approval.

to all its unpaid losses and to each year of run-off. The CVs are promulgated by EIOPA.[3] The regulator also mandates a correlation matrix and prescribes algorithmic procedures for arriving at the all-lines aggregated estimate of one-year reserve risk.

An internal model tailored to the business written by a company should provide a more accurate estimate of its capital requirement. Yet, a firm may be reluctant to use an internal model. Building such a model is costly. The model must be supported by extensive documentation and it must pass validation checks. It must clear imposing regulatory hurdles. After all that, the model might well show the firm needs more capital than would be indicated by the Standard Formula.[4]

Even with an internal model, a company must still derive the required reserve capital based on one-year reserve risk. Several authors have presented methods for deriving one-year reserve risk.[5] This paper provides another technique. It is a bit more sophisticated than the Standard Formula, while being more practical and robust than many of the other proposed internal model approaches.

The paper will first provide an intuitive explanation of one-year reserve risk and outline the key conceptual factors that determine its magnitude. Then there will be a brief overview of how one-year reserve risk is used in computing Solvency II Solvency Capital Requirements (SCR) and the related Risk Margins in the Technical Provision for unpaid loss. Following that, the paper will summarize how one-year reserve risk is quantified in the Standard Formula. Next there will be discussion of the challenges faced in using Schedule P reserve tests or reserve ranges to derive Solvency II consistent one-year reserve risk values. Then, the paper will survey various methods that have been proposed to quantify one-year reserve risk in an internal model context. The paper will examine some of the difficulties in implementing such models and applying them to long-tailed lines of business with sparse data.

This will lead to a presentation of the proposed algorithm. It is very similar to the Standard Formula in that it uses lognormal distributions and CVs. Yet, it has two key features that distinguish it from the Standard Formula. First, it employs systematically derived CVs that vary based on the decomposition of the unpaid loss between IBNR and Case Outstanding (Case O/S)

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[3] In December 2011, the Joint Working Group (JWG)[5] recommended revisions in the proposed factors based on its calibration analysis.

[4] It is unclear whether Solvency II will increase or decrease in required funds relating to unpaid losses For long-tailed lines, the one-year view of risk may tend to produce a fairly small capital requirement, even if the ultimate risk is quite large. Discounting with an illiquidity premium, as dictated by Solvency II, also reduces the funds backing the unpaid loss liability.

[5] See Merz and Wuthrich [8], Ohlsson and Lauzeningks [10], and Rehmann and Klugman [11].

reserves. This leads to CVs that may change each year as the reserve runs off. Depending on the expected evolution of the mix of unpaid loss, the "Varying CV" model being proposed in this paper could arrive at capital requirements and risk margins higher or lower than the Standard Formula. The other key feature of the proposed algorithm is that it uses the expected change in ultimate reserve risk in order to derive one-year reserve risk. This is a natural approach that automatically reconciles ultimate reserve risk with the series of one-year views.

2. ONE-YEAR RESERVE RISK

What is one-year reserve risk? Intuitively, it is a gauge of how much the estimated ultimate loss might change over one year. Conceptually, it is equivalent to the variability in estimates of ultimate loss made one year later. In the context of Solvency II, the expected unpaid loss is called the undiscounted Best Estimate and it is assumed to be unbiased and have no built-in prudential margin. To restate with a bit more precision, one-year reserve risk is an assessment made at the current evaluation date of the variability that could exist in retrospective Best Estimate reserve valuations made one year later.

2.1 One-year Reserve Risk Illustrative Example

To clarify the concept, assume the set of scenarios and probabilities shown in Table 1. At the initial evaluation date, there is no way of knowing which scenario holds. What is known is that the mean unpaid is $100 over the four scenarios. Thus $100 is the initial undiscounted Best Estimate.

Table 1

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Scenario	Prob	Initial Case O/S	Initial IBNR	Initial Estimate Unpaid (3) + (4)	Yr 1 Paid	End of Yr 1 Case O/S	End of Yr 1 IBNR	End of Yr 1 Est'd Unpaid (7) + (8)	Retro Estimate Intial Unpaid (6) + (9)
1	25%	$40	$60	$100	$10	$45	$40	$85	$95
2	25%	$40	$60	$100	$10	$30	$35	$65	$75
3	25%	$40	$60	$100	$30	$45	$50	$95	$125
4	25%	$40	$60	$100	$30	$30	$45	$75	$105
Avg		$40	$60	$100	$20	$38	$43	$80	$100
Stnd Dev									$18

In each scenario, a retrospective (retro) estimate of the initial unpaid is obtained by adding the year one paid amount to the estimate of mean unpaid loss as of the end of year one. One-year reserve risk arises from the volatility of these retro estimates. The $18 standard deviation of the retro estimates is a quantification of the one-year reserve risk.

The example highlights the importance of the information to be gained over one year and the yearly movement in the distribution of the estimates of unpaid loss. It also demonstrates the importance of IBNR estimation. In this example, the calculation of the IBNR has been left deliberately vague. A different IBNR calculation might have produced IBNR estimates different from the ones shown in Column 8 of Table 1 and thus led to a different standard deviation value.

2.2 Conceptual Drivers of One-year Reserve Risk

There are three conceptual drivers of one-year reserve risk.

- First is the inherent volatility of the ultimate unpaid loss. Both the amount and timing will in general differ from current mean estimates. The difference can be due to random statistical fluctuation, systematic movement in underlying claims processes, and inherent estimation error in the initial undiscounted Best Estimate.
- Second is the amount of information that will be gained over one year. This information could include claim data such as paid loss, reported loss, claims closed, claims reported, and so forth, as well as external information such as a new judicial ruling or a medical treatment that could influence subsequent claims settlements. The information we gain is subject to statistical fluctuations.
- Third is the methodology and data used to derive an updated Best Estimate one year later. Actuaries often work up indications with a variety of methods and data. They may have a set of default weights for averaging the methods to get a final pick. Such weights would usually vary by accident year maturity.

2.2.1 Long-Tailed Lines

When reserving long-tailed lines, actuaries generally opt for stability over responsiveness, at least for the first few years of development. This is entirely appropriate: wild swings in the valuation of reserves would justifiably undermine confidence in such valuations. However, one consequence is that long-tailed lines with the largest reserve risk at ultimate might have one-year reserve risk values that are relatively small in magnitude. It has been noted that the overall conceptual basis of one-year reserve risk could lead to a relatively low capital requirement for long-tail business, especially over the first few years of development.[6]

3. SOLVENCY II TECHNICAL PROVISION CALCULATION

The Solvency II Technical Provision (TP) as detailed in [4] is the sum of the Best Estimate (BE) plus a Risk Margin (RM).

$$TP = BE + RM \tag{3.1}$$

By definition under Solvency II, the "Best Estimate" is the discounted mean of possible scenarios.[7] The discounting is done using risk-free yield curves by currency as promulgated by EIOPA. The rates used for discounting are increased by "illiquidity" premiums that are also promulgated by EIOPA.[8]

$$BE = E[PV\ of\ Unpaid\ Loss] \tag{3.2}$$

The Risk Margin is the present value of Cost of Capital charges for the projected Reserve Solvency Capital Requirements (SCRs) over the run-off period.[9]

$$Risk\ Margin = \sum r_{COC} \cdot SCR_y \cdot v(y)^{y-1} \tag{3.3}$$

Here r_{COC} is the required cost-of-capital rate[10] and v(y) is the discount factor for year y.

The SCR each year is defined as the one-year reserve risk for that year. Thus computing the Technical Provision requires the actuary to project the series of one-year reserve risk values, year-by-year, over the run-off period.

4. STANDARD FORMULA RESERVE CAPITAL

Under EIOPA regulation [4], there are ten non-life Lines of Business (LOB). An SCR is

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[6] Ohlsson and Lauzeningks[10] observe, " … a problem with the one-year is that reserves for long tail business might …require less solvency capital than some short tail business…. This is a general problem with the one-year horizon".

[7] Most property and casualty actuaries find this terminology confusing and inconsistent with common usage in the profession. One, England [6], memorably noted the need to "retune your mind" in connection with the Best Estimate definition. As needed for clarity we will refer to Undiscounted Best Estimates and Discounted Best Estimates.

[8] Objections have been raised by property and casualty actuaries (See Schmidt [12]) to the use of Illiquidity Premiums. The effect of an Illiquidity Premium is to reduce the Best Estimate below the risk-free present value of unpaid loss. While Illiquidity Premiums may be used to explain market pricing of different investment instruments, there is no market of insurance liabilities with pricing data to validate whether this concept applies to property and casualty insurance liabilities.

[9] See SCR 9.2 in [4].

[10] Currently set by EIOPA at 6.0%.

computed for the combination of Premium Risk and Reserve Risk for the aggregated LOBs.

4.1 Standard Formula Premium and Reserve Risk

Under the Standard Formula, one-year reserve risk for each LOB is determined using a lognormal distribution with a CV as mandated by EIOPA. A lognormal assumption and a CV are also provided for Premium Risk for each LOB. Formulas are used to define Premium Volume and Reserve Volume measures. A Premium-Reserve covariance assumption is used along with these volume measures to arrive at a CV and a volume measure for the combined premium and reserve risk. This is done for each LOB. A combined lines CV is then derived using a correlation matrix supplied by EIOPA along with the individual LOB CVs and volume measures. An overall volume measure is also computed. This is done with a formula that gives credit for geographical diversity.[11] The SCR for premium and reserve risk is computed by multiplying the volume measure against the 99.5% percentile excess of the mean. The overall SCR is then used to generate the cost-of-capital and the overall risk margin. This extremely brief overview is intended to give the reader a general introduction to the Standard Formula reserve risk algorithm. This provides the context for understanding the computation of the standalone reserve SCRs.

4.2 CVs and Risk Margins by LOB

To allocate the overall risk margin by line, standalone SCRs at the LOB level are computed using the CVs provided by EIOPA. Then the guidance states, "The allocation of the risk margin to the lines of business should be done according to the contribution of the lines of business to the overall SCR during the lifetime of the business."[12] In Appendix A, we provide the derivation of a standalone SCR for reserves assuming the one-year distribution is lognormal as is done under the Standard Formula.

The original one-year CVs provided by EIOPA vary by LOB in a reasonable fashion as do the latest set of recalibrated factors produced by the JWG [5]. However, the use of one CV per line over the whole run-off period is a notable simplifying assumption. As reserves move from IBNR to a mix of IBNR and Case Outstanding and then to just Case Outstanding, it is not likely that the CV of the one-year development distribution would remain unchanged. However, the use of a single factor for each LOB is not uncommon in reserve capital requirement calculations. Rating

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[11] See SCR.9.2 in the QIS5 Technical Specifications [4] for more detail.
[12] See TP.5.26 in the QIS5 Technical Specifications [4].

agency reserve capital formulas typically use a single factor for each line of business, irrespective of the mix of reserves or the age of development. Perhaps the key advantage of a "one factor per line" approach is that it makes the calculations tractable. Further, if appropriately calibrated, it should yield reasonable indications for a company with an established book of business that has maintained an average pattern of growth over time.

4.3 Size Independent Formula

The Reserve SCR under the Solvency II Standard Formula is implicitly based on the assumption that all risk is parameter risk. This follows because the formula does not reflect the size of the reserve.

A size independent method is certainly practical and convenient. It dispels issues of fairness between large and small companies. Size independent methods have been used and are being used in other capital requirement calculations. In particular, rating agency capital requirements for reserves are also typically computed by applying factors to the line of business reserve balances. The factors typically do not depend on the volume of reserves.

While convenience and consistency are advantages in using a fixed factor, size independent approach, such an approach implicitly ignores process risk. Ignoring process risk is the only way the same factor can be used for all companies, large and small. Yet, the actual risk for any given company includes both process and parameter risk. Depending on the type of business, volume of business, and the limits involved, either parameter or process risk may predominate. With a large volume of high frequency-low severity business, process risk will approach zero. On the other hand, process risk can be huge for a relatively small volume of low frequency-high severity business.

4.4 Standard Formula Calibration to Average Size Portfolio

In calibrating factors for the Standard Formula, analysts have had to sidestep the contradiction in using a size-independent formula to model a type of risk that is partly size dependent. The latest EIOPA JWG report on calibration [5] noted, "… volatility factors for premium and reserve risks are typically impacted by the size of the portfolio (in the sense that with increasing size the volatility will typically decrease). However, the JWG was mandated to derive single factors for each of the individual lines of business (separately for premium and reserve risk), irrespective of portfolio size since this is consistent with the current design of the standard formula approach". The recommended factors are based on a portfolio of average size. The JWG recognized that any fixed factor "… will imply that the SCR will be too large for the larger portfolios and too small for the smaller ones".

5. ONE-YEAR RESERVE RISK DATA

Data from various countries and regulatory accounting paradigms have been examined by analysts [5] in deriving factors for the Solvency II Standard Formula. For possible use in internal models, we will briefly examine two sources of reserve volatility data that U.S. actuaries are familiar with.

5.1 Schedule P One-year Reserve Tests

In the United States, the one-year reserve test in the NAIC Statutory Annual Statement is a retrospective comparison providing information about current estimates of the adequacy of <u>Booked</u> Reserves one year ago. Results are shown by Schedule P line and by accident year. The one-year test would, in principle, provide an empirical measurement of undiscounted one-year reserve risk.

The problem is that Booked Reserves are not necessarily Best Estimates. Further, there may not be enough information disclosed to directly derive a Best Estimate. The Booked Reserves may include an implicit prudential margin. They may also be discounted at an undisclosed rate. As well, the adequacy of booked IBNR may vary over the underwriting cycle as companies build up and deplete reserve cushions in order to manage calendar year results.[13] If that is the case, it may be effectively impossible to disentangle inherently random statistical and projection error from systemic non-random error due to cycle management of the booked reserves.[14] This could also partly explain high correlations between different lines of business. Solvency II measures risk with respect to the one-year change in the mean estimate of ultimate. If posted estimates of ultimate are not equal to the mean, one could argue that risk estimates derived from posted reserve data might systematically overstate or understate the "true" amount of Solvency II risk.

5.2 Ranges

Actuaries in the United States have some considerable experience in estimating ranges for ultimate unpaid loss. The prior version of the relevant Actuarial Standard of Practice required an opining actuary to have such a range when judging whether a reserve was adequate, deficient, or

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[13] See Boor [2].

[14] In calibrating Standard Formula risk factors, the EIOPA JWG [5] noted "the possible existence of an underwriting cycle but did not find it practicable to incorporate or embed an explicit recognition of such cycles into the calibration methodology."

redundant.[15]

Several problems need to be solved in order to use ranges to derive one-year reserve risk values. First an assumption is needed about how to translate a reserve range into a statement about statistics of the ultimate unpaid loss random variable. Sometimes, ranges are derived by looking at the range of estimates resulting from different reserving methodologies or different sets of parameters. For Solvency II applications the reserve range needs to be related to statistics of the unpaid loss distribution. For example, the range might be defined as two standard deviations under and over the mean or it might be the interval from the 25th percentile to the 75th percentile. Even after the range is related to a statement about the statistics of the ultimate loss random variable, additional significant assumptions may be needed to arrive at the 99.5% percentile. One common assumption, for instance, is that the distribution is lognormal. The next major problem is to figure out how to use the ultimate view to derive the series of one-year views. The variability at ultimate should lead to variability in the series of annual results over the run-off period. Volatility in the estimation process may add additional year-by-year movement. Another key challenge is practical: how to produce a consistent set of ranges in fine enough detail. Depending on the level of detail in an internal model, ranges might be needed by line, business unit, or by accident year. Usually ranges are not produced at such a level. Even if an actuary has a method for producing ranges at a high level of aggregation, an approach is needed to ensure ranges at a more granular level are consistent.

6. ONE- YEAR RESERVE RISK FROM AN INTERNAL MODEL

Solvency II regulations allow for partial or complete use of an internal model, subject to approval by supervisory authorities. Our focus is on use of an internal model to quantify one-year reserve risk. With an internal model, a firm may cut data in categories different those proscribed under the Standard Formula. It may also employ algorithms different from those used in the Standard Formula.

6.1 Size Dependence and Modeling Refinements

An internal model may allow for consideration of process and parameter risk and it may be implicitly or explicitly dependent on the volume of reserves. For large companies this may legitimately produce a relative capital requirement lower than that produced by the Standard Formula.

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[15] Under the latest ASOP #43 [1], ranges are not required if an actuary judges the reserves to be adequate.

An internal model might incorporate finer line of business breakouts than the Solvency II defaults. Such finer breakouts should result in a model that more closely matches the actual organizational and line of business divisions of a company. This is important if the internal model is ever to be employed for anything beyond computing regulatory capital requirements. Other uses need to be found if an internal model is to pass the "Use Test," a requirement for approval of under Solvency II.[16]

An internal model can also reflect different levels of risk by accident year within a particular line of business. As was true for line of business refinement, accident year refinement should provide a more accurate model of the Best Estimate reserves.

With each refinement, the size of individual reserve cells gets smaller. A size-dependent internal model would assign each cell a relatively larger amount of process risk. However, after being added together, the aggregated result may have a lower amount of risk than if it had been left as an undivided whole. It all depends on the correlation assumptions.

6.2 One-year Reserve Risk Estimation Methods

Several general ways have been proposed for estimating one-year reserve risk.

6.2.1 One-year Variance in Chain Ladder Projection Ultimate

Merz and Wuthrich [8] derive estimates for the variance in the one-year claims result[17] based on the Distribution-Free Chain-Ladder framework. They built on work done by Mack on estimating variance at ultimate in projections made with the Chain-Ladder method. A key assumption is that unbiased estimates of ultimate losses can be obtained by applying age-to-latest age factors to the latest diagonal of cumulative paid losses. The age-to-latest age factors are derived from the triangle of paid loss data. Merz and Wuthrich arrive at closed-form estimators of the one-year prediction error with terms that depend only on the actual triangle of data. This work was ground-breaking and showed that results from a one-year perspective could be obtained with a standard reserving methodology.

However, the method does not directly generate the 99.5% percentile needed for Solvency II

[16] Since an internal model is inappropriate or too cumbersome for either pricing business or estimating unpaid losses, the Use Test may be difficult to pass. Evaluating capital required for different business units may be a "use", but that would vanish if the internal model is not done at the business unit level.
[17] The one-year claims result is defined by Merz and Wuthrich as the difference between the retrospective estimate and the initial estimate of unpaid loss.

calculations. An additional assumption is needed. For example, as is often done in Standard Formula derivations, one could assume the distribution is lognormal. With the variance the method does generate and a lognormal assumption for the one-year claims result, the computation is straightforward. Another serious concern is that the method does not handle tail factors. Therefore it may not work on very long-tailed lines. In addition, it does not readily generalize beyond the Chain-Ladder framework.

6.2.2 Triangle Regression Analysis

Rehman and Klugman [11] analyze triangles of estimated ultimate losses. They assume the age-to-age ratios of the estimated ultimates are lognormal. They define the natural logs of the ratios as error random variables, $e_i^j = \ln(U_i^{j+1}/U_i^j)$ where U_i^j is the estimate of ultimate for accident year, i, as of calendar year j. The development year is d = j- i +1. Under the lognormal assumption and assuming the lognormal parameters depend only on the development period (column), it follows that $e_i^j \sim N(\mu_d, \sigma_d^2)$. If an estimator is unbiased, one would have: $E\left[U_i^{j+1}/U_i^j\right] = 1$. For an unbiased estimator it would follow that $\mu_d = -.5 * \sigma_d^2$. However, the method does not require the estimators be unbiased. The "μ" parameters are estimated by taking the average of error random variables in a column. The "σ" parameters are estimated by computing the sample variance (with bias adjusted denominator) in a column. Using the lognormal assumption one can compute the 99.5% percentile of the one-year error distribution as is needed for Solvency II. Rehman and Klugman [11] also compute overall error for an accident year and for a calendar year diagonal using empirical covariance estimates from the triangle.

The method of Rehman and Klugman is an analysis of results produced by an algorithm, but it does not require that the algorithm be specified. Of course, it is required that the same algorithm be used throughout the historical triangle and it is assumed the same algorithm will be used for the projection. Because it does not require the analyst to know just what algorithm is being used and because it is focused solely on the results that have been obtained, the methodology can be fairly described as a general and solidly empirical approach.[18] However the method does require as many evaluations as are needed for at least a few years to be fully developed. Otherwise the later evaluation age parameters may be very erratic.

Miccolis and Heppen [9] applied this approach to data from a number of insurance groups and

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[18] As was noted by Rehman and Klugman [11], it can even be applied to paid or reported data as well as to the projected ultimates.

obtained good results for most lines. However, they noted problems could arise when data was sparse or subdivided into small business units. They suggested combining data for variance analysis.

6.2.3 Simulation of Next Diagonal and Actuary-in-a-Box Revaluation

Ohlsson and Lauzeningks [10] outline a simulation methodology that starts with an estimated unpaid amount that is assumed to be the actuary's undiscounted Best Estimate. They further assume it is derived from a specified algorithm. This algorithm does not need to be a simple formula. It may encompass use of different particular methods that can vary by accident year maturity.[19] The reserve computation algorithm is called the "actuary-in-a-box".[20] Under the Ohlsson and Lauzeningks framework, a simulation model is then used to generate the next diagonal. Ohlsson and Lauzeningks did not specify distributional assumptions or forms: they left that to the modeler. All the simulation needs to do is to produce what the actuary-in-a-box requires to arrive at the updated Best Estimate. Then the model computes the retrospective Best Estimate of the initial unpaid. After running the simulation thousands of times, one will obtain a simulated distribution of one-year claim development results and the 99.5% percentile of this distribution is the initial capital requirement. This is a very general framework. By embedding it within a simulation model context, it allows the developer of an internal model to simulate correlations between accident years and between lines of business.

While the Ohlsson and Lauzeningks framework makes sense as a constructive way to estimate one-year reserve risk based on given assumptions supplied by the modeler, the user needs to be aware that the answer is based on those underlying assumptions.

6.2.4 Bootstrapping and Extended Simulation Results

Boumezoued, Angoua, Devineau, and Boisseau [3] describe various general models within the simulation framework. One of particular interest is a bootstrapping simulation method that yields one-year (expected) simulated variance equal to the Merz and Wuthrich variance formula. However, their simulation does more than provide a way to approximate the variance. It also provides a direct way to estimate the 99.5% percentile.[21] Boumezoued, Angoua, Devineau, and Boisseau also extend the one-year recursive bootstrap method to include a tail factor.[22]

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[19] Ohlsson and Lauzeningks mention for example a development factor and regression extrapolation method for older years and Generalized Cape Cod for the latest years.
[20] The phrase "actuary-in-a-box" has been attributed to Ohlsson.
[21] One could make a lognormal assumption and use the variance computed via the method of Merz and Wutrich to derive the CV. With the CV, one could then calculate the 99.5% percentile.
[22] Recall the Merz and Wuthrich algorithm does not explicitly contemplate a tail factor.

Boumezoued, Angoua, Devineau, and Boisseau also analyze process error and perform simulations using different copulas to capture dependence between different accident years and lines of business. In addition, they compute the distribution for each year of the run-off period until ultimate. This set of computations for each of the years is needed for the Risk Margin calculation under Solvency II.

6.2.5 Recognition Factor Methods

A class of popular methods[23] starts with ultimate volatility and then uses "recognition factors" to estimate the one-year risk. Perhaps the simplest variant of this approach is to start with an estimate of the variance at ultimate and then apply a one-year recognition factor to estimate the variance recognized after one year. If the mean unpaid was $100 and the ultimate variance was 400, then with a first year recognition factor of 40%, the recognized variance after year one would be 160.

The idea can also be applied in a simulation context. First, an ultimate value of unpaid is simulated. Then a fraction of the deviation of the simulated ultimate from the initial mean unpaid is recognized as dictated by the first year recognition factor. If the mean unpaid was $100 and a simulated unpaid was $150, then with a first year recognition factor of 40%, the recognized retrospective estimate of unpaid after one year would be $120 [=$100+ 40%* ($150-$100)].

There are a few different ways to employ a recognition factor approach beyond the first year. In one approach, there are a set of factors by run-off year and the factors sum to unity. If the factor for a particular run-off year is 15%, then 15% of the initially estimated variance would be recognized in that year. An alternative is to apply the factors to the remaining unrecognized variance as of the end of the prior year. With this alternative, the factors would not need to sum to unity. With run-off factors of 60% and 50% for the first two years, 60% of the initial variance would be recognized the first year and 20% the second year. The 20% is obtained as 50% of the 40% remaining after the 60% has been recognized the first year.

Other variations utilize beta distributions to model recognition factors or employ more sophisticated year-by-year sequential simulation algorithms.

An advantage of the recognition factor methods is that they connect directly to the estimated distribution of ultimate unpaid. However, if the recognition factors are not in some way connected to the reserve run-off, one could well end up with CVs that vary erratically by year. Partly to

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23 The Lloyds Solvency II workshop slides [7] stated that "Most approaches … fall into one of two categories" and then listed "Recognition pattern" methods as the second of the two.

prevent such anomalies, recognition factors chosen in practice vary between long-tail and short-tail lines. Property recognition factors tend to be fairly large the first year or two. They then decline sharply so that the run-off of unpaid loss does not outpace the run-off of the variance. Casualty recognition factors are typically modest the first few years. Then they increase and finally decline. While sensible ad hoc rules ameliorate the potential mismatch between the remaining unpaid loss and the remaining unrecognized variance, they do not always eliminate it. A better approach would be to tie the recognition factors to the reserve run-off pattern so the resulting one-year risk CVs are always reasonable.

While recognition factor approaches have some intuitive appeal, it is not clear how to obtain them from data. Discussion of recognition factor methods can sometimes become confused since the word "recognition" is subject to misinterpretation. From one perspective, it seems to imply that the ultimate is already known to management and that management has decided it will recognize in financial reports only a portion of what is known. This is not a correct interpretation of "recognition" in the context of computing Solvency II one-year reserve risk. In that context, the concept of "recognition" describes all that can reasonably be known and projected, given the inherent lack of knowledge at the evaluation date.

After the possible confusion from terminology is dispelled, there still remains the question of how to compute a recognition factor from data. Historic booked reserves reflect a complex mix of prudential margins, implicit discounting, systematic trends, noise, biased methods, and cycle management. So, the movement of booked reserves alone does not provide data on recognition in the Solvency II context.

6.3 Comparative Summary of One-year Reserve Risk Methods

Surveying the field, we see a variety of methods with different strengths and weaknesses.

Basing a model on a triangle of loss data, whether by making Chain-Ladder projections or fitting natural logs of ratios of estimates of ultimate, is a fine approach when there is enough data, when that data is well-behaved, and when there is no tail. A key advantage is that no exogenous parameters or assumptions are needed: the data dictates the answer. However, triangle-based models often become erratic with sparse long-tailed data or on low-frequency, high-severity businesses. Combining data from several lines could temper volatility and thus produce less erratic parameter estimates. However, the practice of combining nonhomogeneous lines is questionable. While ostensibly leading to better-behaved risk estimates, it may also implicitly underestimate the risk for particular segments of the combined business and also for the combined total. This tradeoff between stability and accuracy needs to be carefully considered when combining data.

The actuary-in-a-box technique is more robust. It can be applied to businesses for which the triangle is not complete. However, it depends on assumptions that may or may not be reasonable. Bootstrapping can work fairly well and connects with actual data by construction, but it may give a misleading picture if there is insufficient data to work from or if data does not go to ultimate.

Calibration is an issue with simulation methods. One suggestion is to follow Boumezoued, Angoua, Devineau, and Boisseau and run the simulation out for every diagonal until run-off is complete. Then the modeler can gauge the variability at ultimate and calibrate accordingly.

Recognition Factor methods are practical and they do tie to ultimate, but how the factors are chosen is unclear. Further, unstable CV patterns can result if the recognition factors are not appropriately related to the run-off of reserves.

Many of the concerns are compounded when looking at any particular company and line. There may not be a full history: the business may be new and the actuary-in-a-box method may not work well on a bootstrap of available data.

One possible idea to solve a host of problems is to fit models to industry data triangles and then use the results to estimate risk for individual companies. However, it is not clear what adjustments are needed to translate industry risk estimates to risk estimates for a particular line and company. Due to process risk, an adequate solvency requirement for the industry as a whole might lead to serious solvency problems if applied to individual companies.

In summary, we have a mixed picture. With a full triangle of data for a well-behaved and relatively short-tailed line of business, the triangle methods should work quite well. These methods are not as simple as the Standard Formula, but they are not extraordinarily complicated. Yet, for long-tailed business, for low frequency, high-severity businesses, or for new businesses, it may be necessary to use simulation or recognition factors or other methods.

7. PROPOSED FORMULA

The formula proposed in this paper produces CVs for one-year reserve risk by LOB. In that sense, it yields the same output as the Standard Formula. However, it arrives at the CV for one-year reserve risk in a systematic fashion based on estimates of ultimate risk. Ultimate risk in this context is the 99.5% percentile of unpaid loss in excess of the mean of unpaid loss. It is also assumed that unpaid loss follows a lognormal distribution and that the CV of the ultimate unpaid loss has already been previously derived. The proposed method differs from the Standard Formula in that it produces CVs that vary by year over the run-off period. Recall that each year's reserve risk capital is needed in calculating the Risk Margin component of the Technical Provision. The proposed method can also be recast as a form of a recognition factor approach with a built-in systematic way of deriving the recognition factors based on the reserve run-off. The proposed method produces results that directly depend on the mix of Case O/S and IBNR. It is a conceptual advantage of this approach that it differentiates levels of risk based on the relative amount of Case O/S versus IBNR. Since the split between Case O/S and IBNR can be projected by standard actuarial techniques, the method is also eminently practical.

7.1 CV for Ultimate Unpaid

The proposed formula starts with the selection of a CV for undiscounted unpaid loss for a line of business in the internal model. This could be done using ranges or any other method the user feels is appropriate. Note this is <u>not</u> the CV for one-year risk.

Actuaries have experience dealing with ultimate risk. Also many models produce estimates of the variance of the unpaid loss. The other key advantage of dealing with ultimate is that it mitigates much of the concern about biases in the booked reserves.

7.2 CV of Case O/S Reserves vs. IBNR

A key aspect of the proposed method is that it differentiates risk between Case O/S and IBNR. Most actuaries would agree that IBNR is more variable than Case O/S. For example, if $1,000 is the mean estimate of unpaid loss and the entire amount is IBNR, the variance of unpaid loss will be greater than if the entire $1,000 was due to Case O/S. In the proposed method, an assumption is made relating the CV of ultimate loss per dollar of IBNR and the CV of ultimate loss per dollar of Case O/S. To illustrate this, it might be assumed that the CV of IBNR is 125% of the CV of Case O/S. With such an assumption and with the split of reserves into IBNR and Case O/S components, one can derive how much of the variance in the estimate of ultimate is due to IBNR and how much is due to Case O/S. Further, if one assumes these CVs by reserve type stay constant over the run-off period, and if projections of the run-off of IBNR and Case O/S have been separately derived, then one can also project how the variance of ultimate loss will evolve over time. To summarize, our initial goal is to arrive at a robust way of estimating the variance of estimated unpaid loss year by year as a function of the projected IBNR and projected Case O/S. Since actuaries often make projections of IBNR and Case O/S run-off for various business units, the resulting method will provide a practical way of estimating year-by-year variance for whatever lines of business are used in an internal model.

In pursuit of this goal, we may make whatever mathematical assumptions are needed to arrive at a cogent formula. We will then observe the method can be used in a wide variety of cases, even if it has been proved valid only in more limited circumstances. We note that the fundamental idea that there is a clean split of variance into Case O/S and IBNR related components is debatable on theoretical grounds. Since some of the IBNR may be related to development on known cases, there is some conceptual overlap between the risk associated with IBNR and the risk associated with Case O/S. Our approach will be to ignore all complexities and simply focus on the goal of writing total variance of unpaid loss as the sum of a term related to IBNR and a term related to Case O/S. In the end, this approach will be more intuitively appealing and theoretically superior to the Standard Formula and to methods that utilize judgmentally selected recognition patterns.

To begin the mathematical development, let R(t) be the ultimate unpaid loss at the end of evaluation year t. Let COS be the Case O/S. The undiscounted Best Estimate is then given as

$$E[R(t)] = COS(t) + IBNR(t). \tag{7.2.1}$$

Let CV_{COS} denote the CV of the unpaid associated with Case O/S and CV_{IBNR} the corresponding CV associated with IBNR. Suppose these CVs do not vary with the evaluation date. To simplify notation, we will now suppress the dependence of the reserves on the evaluation year, t, but later reintroduce it as needed

Assume we can decompose the ultimate variance in unpaid so it is valid to write

$$Var[R] = (CV_{COS} \cdot COS)^2 + (CV_{IBNR} \cdot IBNR)^2. \tag{7.2.2}$$

This is a key assumption. It says total variance is the sum of the variance on Case O/S and the variance on IBNR. The lack of a cross-term in Equation 7.2.2 implicitly indicates IBNR and Case O/S are assumed to be independent. This is one of those assumptions made to ensure the formula is simple and robust.

Now let κ be the ratio of the CVs:

$$\kappa = \frac{CV_{IBNR}}{CV_{COS}}. \tag{7.2.3}$$

For example, if we select κ =1.50, then when the CV of Case O/S is 0.10, the CV of IBNR will be 0.15. Given that we have already determined the variance in our estimate of ultimate and given that we have made a selection of κ, we can solve for the various CV parameters. Consider

$$Var[R] = (CV_{COS} \cdot COS)^2 + (\kappa \cdot CV_{COS} \cdot IBNR)^2. \quad (7.2.4)$$

It follows that

$$ {CV_{COS}}^2 = \frac{Var\,[R]}{COS^2 + \kappa^2 IBNR^2}. \quad (7.2.5)$$

For example, suppose Case O/S is $400 and IBNR is $200 and assume the ultimate unpaid has a 25% CV. So the ultimate unpaid has a mean of $600, a standard deviation of $150, and a variance of 22,500. Now assume κ =1.50 so that each unit of IBNR has 150% of the CV as a corresponding unit of Case O/S. Then the square of the CV for Case O/S is equal to 0.09= 22,500/(160,000 + 2.25*40,000) =22,500/250,000. So the CV for Case O/S is 0.30 =$.09^{.5}$ and the CV for IBNR is .450 = 1.50 * 0.30. Based on those CVs, the total variance of 22,500 can be decomposed into a portion related to Case O/S equal to 14,400 ($(.3*400)^2$) and a component related to IBNR equal to 8,100 ($(.45*200)^2$).

The point is that with the Case O/S and IBNR at the current evaluation and an estimate of the variance of ultimate unpaid loss, one can decompose ultimate variance into a portion related to Case O/S and a portion related to IBNR. As part of the derivation, one obtains CVs for each type of unpaid loss. We can then project the run-off of these different categories and use the CVs to arrive at a consistent year-by-year series of ultimate variance estimates for total unpaid loss.

7.3 Projected Evolution of Case O/S and IBNR

Actuaries often project the run-off of Case O/S and IBNR. This can be done with an Accident Year breakout of Case O/S and IBNR and with accident year paid and reported patterns. Such data would usually be included in a reserve review. Exhibits 1-3 provide an example of how this can be done. Exhibit 1 shows Premium and Loss data by accident year and includes the breakout of Case O/S and IBNR.

Exhibit 1
Premium and Loss Data

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
AY	Eval Age (Years)	Prem	Loss Paid to Date	Case O/S	Reptd to Date (4)+(5)	Reptd LR to Date (6)/(3)	IBNR	Current Estd Ult (6) +(8)	Estd Ult LR (9)/(3)	Expected Unpaid Loss (9)-(4)
2002	10	**1,995**	**2,125**	**55**	2,180	109%	**-**	2,180	109%	55
2003	9	**2,005**	**1,250**	**132**	1,382	69%	**25**	1,407	70%	157
2004	8	**1,950**	**800**	**50**	850	44%	**65**	915	47%	115
2005	7	**2,000**	**1,550**	**277**	1,827	91%	**93**	1,920	96%	370
2006	6	**2,250**	**550**	**395**	945	42%	**148**	1,093	49%	543
2007	5	**3,800**	**2,500**	**605**	3,105	82%	**361**	3,466	91%	966
2008	4	**3,200**	**900**	**530**	1,430	45%	**446**	1,876	59%	976
2009	3	**3,750**	**750**	**650**	1,400	37%	**1,000**	2,400	64%	1,650
2010	2	**4,250**	**150**	**750**	900	21%	**1,750**	2,650	62%	2,500
2011	1	**4,000**	**25**	**250**	275	7%	**2,000**	2,275	57%	2,250
Total				**3,694**			**5,888**		69%	**9,582**

Not atypically, the latest accident years have reserves that are mostly IBNR and their current estimated ultimate loss ratios are within a relatively narrow band. On the other hand, the more mature years have reserves that are predominantly Case O/S and their ultimate loss ratios display larger variations.

Exhibit 2 shows paid and reported loss development factors (LDFs) and how these are used to derive one-year age-to-age reserve decay factors. These are defined, for example, so that an 80% decay factor implies the reserve declines on average by 20% from one age to the next.

Exhibit 2
Development Patterns to Decay Factors

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Age	Reptd ATU LDF	Paid ATU LDF	Cumul Reptd	Cumul Paid	Increm Reptd	Increm Paid	Unreptd PCT	Unpaid PCT	IBNR -1 year Decay Factor	Unpaid - 1 yr Decay Factor
			1.0/(2)	1.0/(3)	Δ(4)	Δ(5)	1.0-(4)	1.0-(5)	Col (8) Row ratios	Col (9) Row ratios
11	**1.000**	**1.000**	100.0%	100.0%	0.1%	1.0%	0.0%	0.0%		
10	**1.001**	**1.010**	99.9%	99.0%	0.4%	1.0%	0.1%	1.0%	0.000	0.000
9	**1.005**	**1.020**	99.5%	98.0%	1.5%	2.8%	0.5%	2.0%	0.201	0.505
8	**1.020**	**1.050**	98.0%	95.2%	1.9%	6.0%	2.0%	4.8%	0.254	0.412
7	**1.040**	**1.120**	96.2%	89.3%	4.4%	9.3%	3.8%	10.7%	0.510	0.444
6	**1.090**	**1.250**	91.7%	80.0%	4.8%	13.3%	8.3%	20.0%	0.466	0.536
5	**1.150**	**1.500**	87.0%	66.7%	7.0%	33.3%	13.0%	33.3%	0.633	0.600
4	**1.250**	**3.000**	80.0%	33.3%	13.3%	13.3%	20.0%	66.7%	0.652	0.500
3	**1.500**	**5.000**	66.7%	20.0%	33.3%	10.0%	33.3%	80.0%	0.600	0.833
2	**3.000**	**10.000**	33.3%	10.0%	16.7%	6.7%	66.7%	90.0%	0.500	0.889
1	**6.000**	**30.000**	16.7%	3.3%	16.7%	3.3%	83.3%	96.7%	0.800	0.931

Exhibit 3 shows the standard run-off triangles and the derivation of projected paid losses, Case O/S, and IBNR by calendar year. To explain the sequence of the calculation, we first use the Unpaid Decay factor for an accident year to figure out how much should be unpaid on average as of the next evaluation. This is shown in Exhibit 3-Table 1, while Exhibit 3-Table 2 shows the resulting estimates of paid loss by year.

Exhibit 3 - Table 1

Projected Unpaid Loss												
			Evaluation Lag									
AY	Eval Age	Current Unpaid	1	2	3	4	5	6	7	8	9	10
2002	10	**55**	-	-	-	-	-	-	-	-	-	-
2003	9	**157**	79	-	-	-	-	-	-	-	-	-
2004	8	**115**	47	24	-	-	-	-	-	-	-	-
2005	7	**370**	164	68	34	-	-	-	-	-	-	-
2006	6	**543**	291	129	53	27	-	-	-	-	-	-
2007	5	**966**	580	311	138	57	29	-	-	-	-	-
2008	4	**976**	488	293	157	70	29	14	-	-	-	-
2009	3	**1,650**	1,375	688	413	221	98	40	20	-	-	-
2010	2	**2,500**	2,222	1,852	926	556	298	132	54	28	-	-
2011	1	**2,250**	2,095	1,862	1,552	776	466	249	111	46	23	-
CY total		**9,582**	7,342	5,226	3,272	1,706	919	437	186	73	23	-

Exhibit 3 - Table 2

Projected Incremental Paid Loss												
			Evaluation Lag									
AY	Eval Age		1	2	3	4	5	6	7	8	9	10
2002	10		55	-	-	-	-	-	-	-	-	-
2003	9		78	79	-	-	-	-	-	-	-	-
2004	8		68	23	24	-	-	-	-	-	-	-
2005	7		206	97	34	34	-	-	-	-	-	-
2006	6		252	162	76	26	27	-	-	-	-	-
2007	5		386	269	173	81	28	29	-	-	-	-
2008	4		488	195	136	87	41	14	14	-	-	-
2009	3		275	688	275	192	123	58	20	20	-	-
2010	2		278	370	926	370	258	165	78	27	28	-
2011	1		155	233	310	776	310	216	139	65	23	23
CY total		**-**	2,240	2,116	1,953	1,567	787	482	251	113	50	23

Next we project IBNR by applying the IBNR decay factors to current IBNR. The resulting projections for our example are shown in Exhibit3 -Table 3.

Exhibit 3 - Table 3

Projected IBNR												
			Evaluation Lag									
AY	Eval Age	Current IBNR	1	2	3	4	5	6	7	8	9	10
2002	10	**-**	-	-	-	-	-	-	-	-	-	-
2003	9	**25**	5	-	-	-	-	-	-	-	-	-
2004	8	**65**	16	3	-	-	-	-	-	-	-	-
2005	7	**93**	47	12	2	-	-	-	-	-	-	-
2006	6	**148**	69	35	9	2	-	-	-	-	-	-
2007	5	**361**	229	106	54	14	3	-	-	-	-	-
2008	4	**446**	291	184	86	44	11	2	-	-	-	-
2009	3	**1,000**	600	391	248	115	59	15	3	-	-	-
2010	2	**1,750**	875	525	342	217	101	51	13	3	-	-
2011	1	**2,000**	1,600	800	480	313	198	92	47	12	2	-
CY total		**5,888**	3,732	2,057	1,221	704	372	161	63	15	2	-

Taking differences we arrive at projections of incremental reported loss as shown in Exhibit 3-Table 4.

Exhibit 3 - Table 4

Projected Incremental Reported Loss												
			Evaluation Lag									
AY	Eval Age		1	2	3	4	5	6	7	8	9	10
2002	10		-	-	-	-	-	-	-	-	-	-
2003	9		20	5	-	-	-	-	-	-	-	-
2004	8		49	13	3	-	-	-	-	-	-	-
2005	7		46	35	10	2	-	-	-	-	-	-
2006	6		79	34	26	7	2	-	-	-	-	-
2007	5		132	122	52	40	11	3	-	-	-	-
2008	4		155	107	98	42	33	9	2	-	-	-
2009	3		400	209	144	132	57	44	12	3	-	-
2010	2		875	350	183	126	116	49	38	10	3	-
2011	1		400	800	320	167	115	106	45	35	10	2
CY total		**-**	2,156	1,675	836	517	333	211	98	49	12	2

Then we take differences to get the projected Case O/S (COS) using the formula

$$COS = Unpaid - IBNR. \tag{7.3.1}$$

Exhibit 3 - Table 5

Projected Case OS Loss												
			Evaluation Lag									
	Eval	Current										
AY	Age	Case OS	1	2	3	4	5	6	7	8	9	10
2002	10	**55**	-	-	-	-	-	-	-	-	-	-
2003	9	**132**	74	-	-	-	-	-	-	-	-	-
2004	8	**50**	31	21	-	-	-	-	-	-	-	-
2005	7	**277**	117	56	32	-	-	-	-	-	-	-
2006	6	**395**	222	94	44	25	-	-	-	-	-	-
2007	5	**605**	351	204	84	43	26	-	-	-	-	-
2008	4	**530**	197	109	71	26	18	12	-	-	-	-
2009	3	**650**	775	296	165	106	39	26	17	-	-	-
2010	2	**750**	1,347	1,327	584	339	197	81	41	25	-	-
2011	1	**250**	495	1,062	1,072	463	267	157	64	34	21	-
CY total		**3,694**	3,609	3,168	2,051	1,001	547	276	123	59	21	-

Note Case O/S in some cases can reasonably be projected to increase the first few years during the run-off period. On the other hand, IBNR will typically decrease year by year.

7.4 Projecting Ultimate Risk by Year

To decompose the ultimate risk into Case O/S and IBNR components, selections are made for the ultimate CV and the κ parameter. Then with the initial Case O/S and IBNR balances, the respective CVs for Case O/S and IBNR may be derived as shown in Exhibit 4.

Exhibit 4
CV Coefficient Derivation

	Item	Value	Source
(1)	CY	**2011**	
(2)	Mean of Full Value of Ultimate Unpaid Loss	9,582	Ex 3 Tbl 1
(3)	Case O/S	3,694	Ex 3 Tbl 5
(4)	Mean IBNR	5,888	Ex 3 Tbl 3
(5)	CV of Ultimate Unpaid Loss	**20.0%**	User selection
(6)	k = CV of IBNR versus CV of Case O/S	**150.0%**	User selection
(7)	Stnd Dev of Ultimate Unpaid	1,916	(2)*(5)
(8)	Case OS CV Coefficient	0.200	sqrt{$(7)^2$)/[$(3)^2$+ $((6)*(4))^2$]}
(9)	IBNR CV Coefficient	0.300	(8)*(6)

Note that Equation 7.2.5 is used in Row 8 of Exhibit 4 and the selected κ is used to obtain Row 9.

Next, assume the respective CVs for Case O/S and IBNR are applicable for each year over the whole run-off period. Recall the Standard Formula uses a single CV for one-year reserve risk for each line of business and that this same CV applies for each year of the run-off period. The CVs under our method will evolve over the run-off period because the mix of Case O/S and IBNR will evolve. Because it reflects the changing mix of reserves, the proposed method should result in more accurate reserve risk estimates in any particular year than that produced using the single CV method of the Standard Formula.[24] The calculation of the year-by-year variances is shown in Exhibit 5.

Exhibit 5
Projection of Year by Year Variance of Ultimate Unpaid

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
CY	Eval Lag	Case O/S	IBNR	Total Unpaid	Stnd Dev from Case O/S	Stnd Dev from IBNR	Variance	Stnd Dev	CV
				(3)+(4)	(3)*CV_{COS}	(4)*CV_{IBNR}	$(6)^2+(7)^2$	$(8)^{1/2}$	(9)/(5)
2011	0	3,694	5,888	9,582	739	1,768	3,672,589	1,916	0.200
2012	1	3,609	3,732	7,342	723	1,121	1,777,969	1,333	0.182
2013	2	3,168	2,057	5,226	634	618	783,872	885	0.169
2014	3	2,051	1,221	3,272	411	367	303,081	551	0.168
2015	4	1,001	704	1,706	200	212	84,925	291	0.171
2016	5	547	372	919	109	112	24,451	156	0.170
2017	6	276	161	437	55	48	5,380	73	0.168
2018	7	123	63	186	25	19	962	31	0.167
2019	8	59	15	73	12	4	157	13	0.171
2020	9	21	2	23	4	1	18	4	0.182
2021	10	-	-	-	-	-	-	-	0.000

7.5 From Ultimate Risk to One-year Risk

Next we derive one-year variance estimates by taking the difference between successive ultimate variance projections. Figure 1 depicts the idea.

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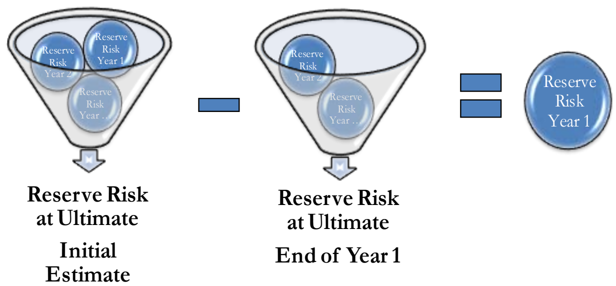
24 An even more sophisticated model could be developed in which the CVs of Case O/S and IBNR also evolve over time.

Figure 1



The differencing formula is based on three major assumptions:

- First, it presumes the estimates of mean unpaid loss subsequent to each evaluation do not change as the result of the intervening observations. This is behavior of unpaid loss estimates derived using the Bornheutter-Ferguson method, when LDFs and expected loss ratios (ELRs) are frozen.
- Second, it assumes the incremental paid losses from separate run-off years have no covariance with one another. This could likely be derived from the first assumption.
- Third, it assumes there is no change in the estimate of variance of paid loss for any year of run-off.

With these assumptions, differencing of the variances between ultimate unpaid for two consecutive year-end valuations produces the one-year variance during the year. A mathematical derivation is provided in Appendix B.

Exhibit 6
Projection of One -Year Variance and SCRs

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
CY	Ultimate Variance	One-Year Variance	One-Year Stnd Dev	One-Year CV	σ	μ	Mean	Full Value 99.50th percentile	Full Value SCR	SCR as % of reserve
		$\Delta(2)$	$(3)^{1/2}$	(4)/E[R]					(9) -(8)	(10)/(8)
2011	3,672,589	1,894,620	1,376	0.144	0.14	9.16	9,582	13,706	4,124	43.0%
2012	1,777,969	994,097	997	0.136	0.14	8.89	7,342	10,305	2,963	40.4%
2013	783,872	480,790	693	0.133	0.13	8.55	5,226	7,280	2,054	39.3%
2014	303,081	218,156	467	0.143	0.14	8.08	3,272	4,670	1,398	42.7%
2015	84,925	60,473	246	0.144	0.14	7.43	1,706	2,443	737	43.2%
2016	24,451	19,071	138	0.150	0.15	6.81	919	1,335	416	45.3%
2017	5,380	4,419	66	0.152	0.15	6.07	437	637	201	46.0%
2018	962	805	28	0.153	0.15	5.21	186	271	86	46.2%
2019	157	139	12	0.161	0.16	4.28	73	109	36	49.1%

Percentage for SCR Percentile	**99.5%**
Standard Normal Percentile	2.576

Calculation notes

(6) $\sigma = [\ln(1+CV^2)]^{1/2}$

(7) $\mu = \ln(E[R]) - 1/2\ \sigma^2$

(8) Mean $= E[R] = \exp(\mu + 1/2\ \sigma^2)$

(9) 99.5th percentile $= \exp(\mu + 2.576\ \sigma)$

One-year variance calculations for our example are shown in Exhibit 6. The first one-year variance is 1,894,620, which is the difference between the initial variance of 3,672,589 and the year-end variance of 1,777,969. With the variance and the mean, it is straightforward to derive the CV and other parameters of the associated one-year reserve risk lognormal as is done in columns (6) and (7) of Exhibit 6. In this table, the notation E[R] in column 5 stands for the expected total unpaid displayed in Exhibit 5. After the CV is calculated in column 5 of Exhibit 6, the lognormal parameters, μ and σ, are found separately for each year using the formulas shown in the calculation notes. Please see Appendix A for more detail. With the parameters, the 99.5th percentile may be readily computed and it is then straightforward to compute the amount excess of the mean as shown in column 10 of Exhibit 6. This is the standalone undiscounted SCR. It is useful to express the SCR as a percentage of the mean reserve as is done in column 11 of Exhibit 6. For any particular year, the calculations are similar to what would be done using the Standard Formula. The major difference is that the Standard Formula SCR calculation uses the same CV for all years of run-off whereas the proposed approach has CVs that vary by year because the mix of Case O/S and IBNR changes by run-off year.

7.6 Discounted SCR and Technical Provision

We compute the discounted mean unpaid loss for each year of run-off and then the associated standalone SCR by applying the undiscounted SCR factor. This is similar to the approach taken in the Standard Formula where a fixed CV is used to get factors that are applied to discounted reserves. With the SCRs we compute the cost of capital amounts by year and discount those to get the standalone risk margin in the Technical Provision. Exhibit 7 shows these calculations.

Exhibit 7
Calculation of Discounted Reserve and Standalone Risk Margin

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
CY	Paid Loss from Ex 3 Table 2	Full value Unpaid Loss from Ex 3 Table 1	Discounted Unpaid Loss (3) * Ex 8 Col 5	SCR Factor from Ex 6	SCR (5)*(4)	Cost of Capital CocRate*(6)	Discounted Cost of Capital (7) * Ex 8 Col 5
2011	-	9,582	9,056	43.0%	3,897	234	231
2012	2,240	7,342	7,020	40.4%	2,834	170	166
2013	2,116	5,226	5,042	39.3%	1,982	119	114
2014	1,953	3,272	3,175	42.7%	1,357	81	75
2015	1,567	1,706	1,659	43.2%	717	43	38
2016	787	919	897	45.3%	407	24	21
2017	482	437	428	46.0%	197	12	10
2018	251	186	182	46.2%	84	5	4
2019	113	73	72	49.1%	35	2	2
2020	50	23	23	56.6%	13	1	1
2021	23	-	-	0.0%	-	-	-
Total	9,582				7,613	457	429

Cost of Capital Rate	6.00%

Exhibit 8 shows the interest rates used in discounting. They are derived by summing the risk-free-rate and the illiquidity premium. While the rates were loosely taken from EIOPA charts, they are meant to be used here only for illustrative purposes. They should not be used in real applications. However, they do provide a rough idea of the magnitudes and shape of yield curve and the impact of the illiquidity premium.

Exhibit 8
Yield Curve, Illiquidity Premiums, and PV Factors

(1)	(2)	(3)	(4)	(5)
time yrs	Risk-free yield	Illiquidity Premium	Rate for Discounting (2)+(3)	PV Factor (1.0+ (4))^-(1)
1	**0.331%**	**0.710%**	1.041%	0.9897
2	**0.385%**	**0.710%**	1.095%	0.9785
3	**0.773%**	**0.710%**	1.483%	0.9568
4	**1.220%**	**0.710%**	1.930%	0.9264
5	**1.678%**	**0.710%**	2.388%	0.8887
6	**2.090%**	**0.710%**	2.800%	0.8473
7	**2.441%**	**0.710%**	3.151%	0.8048
8	**2.721%**	**0.710%**	3.431%	0.7635
9	**2.953%**	**0.710%**	3.663%	0.7234
10	**3.128%**	**0.710%**	3.838%	0.6862
11	**3.384%**	**0.710%**	4.094%	0.6432

Exhibit 9 shows the derivation of the final standalone Technical Provision for unpaid loss.[25]

Exhibit 9
Derivation of Standalone Technical Provision for Unpaid Loss

	Item	Value	Source
(1)	Mean of Full Value Ult Unpaid Loss	9,582	Ex 7 Col 3
(2)	Mean of Discounted Unpaid Loss	9,056	Ex 7 Col 4
(3)	Effect of Discount	(526)	(2) - (1)
(4)	Risk Margin	429	Ex 7 Col 8
(5)	**Technical Provision**	**9,485**	(1) + (3) + (4)

Note that in this example that the effect of discounting more than offsets the explicit inclusion of a risk margin. In other examples, such as those for short tail lines, the risk margin often exceeds the magnitude of the discount. Stepping back, the overall impact is generally to arrive at a Technical

[25] The impact of reinsurance has been omitted in this discussion.

Provision not far off from the original mean of undiscounted unpaid losses. However, this result depends highly on the interest rate. Currently, interest rates are at historic lows. If they move up a few points, the Technical Provision for many long-tail lines could fall well below the undiscounted mean unpaid loss.

8. CONCLUSION

Our proposal is a very practical refinement of the Standard Formula. It is focused on finding one-year CVs that can be directly related to estimates of ultimate risk and to the types of reserves and how they evolve. In that sense it is a bridge between various known variables about which actuaries have some intuition and a new quantity, one-year reserve risk, about which actuaries know little. It provides a coherent framework within which recognition can be projected in a systematic and logical manner. Other methods do not use the information about risk contained in knowing the split between Case O/S and IBNR: this one does.

The method is also applicable in a wide range of circumstances as it employs user-selected patterns that need not be derived from data. For new businesses such data may not yet exist, but reserving actuaries may have selected paid patterns and reporting patterns to be used in reserving analysis. Another plus is that the method works well for long-tailed lines of business. Note that the proposed method is flexible, as it can be used at the level of business at which the enterprise is managed. There is no need to aggregate the data to make the algorithm work. In conclusion, this is a practical way to compute one-year reserve risk in an internal model. It is one of several methods to consider when deciding on how to quantify one-year reserve risk for Solvency II requirements.

APPENDIX A –LOGNORMAL STANDARD FORMULA CALCULATIONS

For a lognormal, X, with parameters (μ, σ), it is well known that

$$E[X] = e^{\mu+\frac{1}{2}\sigma^2} \quad and \quad E[X^2] = e^{2\mu+2\sigma^2} \quad . \tag{A.1}$$

Following the standard derivation we have

$$CV^2 = e^{\sigma^2} - 1. \tag{A.2}$$

Thus we can derive:

$$\sigma = \sqrt{\ln(1 + CV^2)}. \tag{A.3}$$

To get the 99.5% percentile, π_p , we evaluate

$$Prob(X < \pi_p) = .995. \tag{A.4}$$

Taking natural logs we see

$$Prob\left(\frac{\ln(X)-\mu}{\sigma} < \frac{\ln(\pi_p)-\mu}{\sigma}\right) = .995. \tag{A.5}$$

The left hand side of the probability is the standard unit normal, so we have

$$z_p = \Phi^{-1}(.995) = 2.576 = \frac{\ln(\pi_p)-\mu}{\sigma}. \tag{A.6}$$

Therefore

$$\ln(\pi_p) = \mu + 2.576 \cdot \sigma, \tag{A.7}$$

from which it follows that

$$\pi_p = e^{\mu+2.576\cdot\sigma}. \tag{A.8}$$

Therefore the Standalone Solvency Capital Requirement (SCR) is

$$\begin{aligned} SCR = \pi_p - E[X] &= e^{\mu+2.576\cdot\sigma} - e^{\mu+\frac{1}{2}\sigma^2} \\ &= E[X](e^{2.576\cdot\sigma-.5\sigma^2} - 1). \end{aligned} \tag{A.9}$$

Note the model breaks down for large CV, where σ >5.152.

APPENDIX B –ONE-YEAR RESERVE RISK FORMULAS AND DERIVATIONS

Let $X_y(t)$ denote the paid loss in year t after the end of calendar year y for claims incurred as of the end of calendar year y. It is the payout in the t^{th} year of run-off. Use t=0 to indicate the balance at the end of calendar year y. This is the start of the run-off period.

Write $C_y(t)$ for the cumulative payments in the run-off period up to and including the t^{th} year. Set $R_y(t) = C_y(\omega) - C_y(t)$ so that R is the remaining run-off payments subsequent to the t^{th} year. We will suppress the subscript y to simplify notation.

The initial undiscounted Best Estimate is the mean of the unpaid loss, $E[R(0)|t=0]$.

At the end of the first year of run-off, we will be able to make a Retrospective Estimate of the initial unpaid. We will denote this as $E[R(0)|t=1]$. It is equal to the sum of the paid over the first year plus the mean unpaid as of the end of the first year:

$$E[R(0)|t = 1] = X(1) + E[R(1)|t = 1] \tag{B.1}$$

The one-year variance is equal to:

$$One - year\ Variance = \ E\ [\ (E[R(0)|t = 1] - \ E[R(0)|t = 0])^2\ ] \tag{B.2}$$

Under the Bornheutter-Ferguson (BF) method, the expected value at a given evaluation data of unpaid loss beyond a given subsequent date is independent of the evaluation date. In particular:

$$E[R(1)|t = 1] = E[R(1)|t = 0] \tag{B.3}$$

This implies:

$$One\ Year\ Variance = \ E\ [\ (X(1) - \ E[X(1)|t = 0])^2\ ] = Var(X(1)) \tag{B.4}$$

Now consider the ultimate variance of the initial unpaid run-off is:

$$Var(R(0)|\ t = 0) = \tag{B.5}$$

$$\sum\nolimits_{s=1}^{\omega} Var(X(s)|t = 0) + \sum\nolimits_{r \neq s} Cov(X(r), X(s)|t = 0)$$

Similarly, the ultimate variance of the unpaid run-off at the end of year one is:

$$Var(R(1)|\ t = 1) = \tag{B.6}$$

$$\sum\nolimits_{s=2}^{\omega} Var(X(s)|t = 1) + \sum\nolimits_{r \neq s, r>1, s>1} Cov(X(r), X(s)|t = 1)$$

Now assume all the covariances in B.5 and B.6 are zero. This is a generalization of the Bornheutter-Ferguson assumption. Subtracting B.6 from B.5 and using this vanishing covariance assumption, we obtain:

$$Var(R(0)|\ t=0) - Var(R(1)|\ t=1) = \tag{B.7}$$

$$\sum\nolimits_{s=1}^{\omega} Var(X(s)|t=0) - \sum\nolimits_{s=2}^{\omega} Var(X(s)|t=1)$$

Finally, we suppose that the variances of the incremental unpaid amounts do not change from one evaluation to the next. Under these admittedly stringent assumptions we have:

$$Var(R(0)|\ t=0) - Var(R(1)|\ t=1) = Var(X(1)). \tag{B.8}$$

Comparing B.8 to B.4 leads to the result shown in Figure 1.

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Disclaimers

This paper is solely the work of the author and the opinions expressed herein are solely those of the author. This paper contains no express or implied presentation or endorsement of the views of the author's prior employers. No liability whatsoever is assumed for any losses, direct or indirect, that may result from use of the methods described in this paper.

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Abbreviations and notations

Collect here in alphabetical order all abbreviations and notations used in the paper

BE, Best Estimate
BF, Bornheutter-Ferguson
COC, Cost-of-Capital
DFCL, Distribution Free Chain Ladder
IBNR, Incurred But Not Reported
RM, Risk Margin
SCR, Solvency Capital Requirement
SF, Standard Formula
TP, Technical Provision

Biography of the Author

Ira Robbin has held positions with Endurance, Partner Re, CIGNA PC, and INA working in several corporate and pricing actuarial roles. Ira has an undergraduate degree in Math from Michigan State and PhD in Math from Rutgers University. He has written papers on risk load, development patterns, IBNR formulas, ROE, Coherent Capital and other topics.