Risk based capital modelling for P&C insurers and financial sensitivity

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Abstract

In this contribution we implement a simulation model based on an Internal Risk Model approach, aimed to assess the default risk for Property & Casualty insurers over a short-term time horizon. The proposed framework includes a stochastic model for the financial market and dynamic portfolio strategies. Further, we analyse some risk-based capital requirements by means of risk measures such as VaR and the ruin probability, focusing in particular on the impact of different portfolio strategies, time horizons and levels of confidence. The paper aims to contribute to the current debate concerning the development of a general framework for solvency assessment, including the new EU capital requirements to be defined in the Solvency II phase.

Keywords: CIR process,geometric Brownian motion, non life insurance solvency requirements, Risk-based capital.

1 The insurance framework

Let $\tilde{U}_t$ be the stochastic risk reserve at the end of year t and let us assume that is given by

$$\tilde{U}_t = \left(1 + \tilde{j}_t\right)\tilde{U}_{t-1} + \left(\pi_t - \tilde{X}_t - E_t\right) + \tilde{j}_t L_{t-1} - TX_t - D_t, \tag{1}$$

where π_t is the volume of the gross premiums, $\tilde{X}_t$ is the amount of the stochastic aggregate claims, and E_t denotes general and acquisition expenses of the year. These expenses are calculated as a (constant) percentage c of the gross premiums, i.e. $E_t = c\pi_t$, where c is the expenses loading coefficient. Moreover, in equation (1) we denote with $\tilde{j}_t$ the stochastic annual rate of return on the company's financial investments; the financial model for the definition of $\tilde{j}_t$ is presented in the next section. Further, although we do not consider the claim reserving run-off, the stochastic risk reserve includes the return generated from investing the amount of the loss reserve as at the end of the previous year, L_{t-1}. Finally, also taxes, TX_t, and dividends, D_t (the latter are paid to stockholders at the end of each year) are considered in the model. It is worth noticing that for many general insurance lines (e.g. third-party liability) the run-off risk concerning the development of the initial estimate of the claim reserve is not negligible at all and in practice it is an additive source of risk.

More in details, the loss reserve in each year is assumed to be a constant percentage δ of the gross premiums, π, i.e.

$$L_t = \delta \pi_t. \tag{2}$$

However, the loss reserve at the end of year t depends also on the amount of claims deferred from the previous year, $\tilde{C}_t^d$, and paid in year t, and the amount of claims occurred in the current year which are settled during year t, $\tilde{C}_t^c$. Hence

$$L_t = L_{t-1} - \tilde{C}_t^d + \left(\tilde{X}_t - \tilde{C}_t^c\right). \tag{3}$$

Since the nominal gross premium volume increases every year by the claim inflation rate, i, and the real growth rate, g, which we assume constant over the given time horizon, although they might differ for different lines of business, then

$$\pi_t = (1+i)(1+g)\,\pi_{t-1}. \tag{4}$$

We observe that in this framework, the nominal gross premium is not affected by the level of the premia in the market. Equations (2)-(4) imply that the total amount of the claims paid in year t is

$$\begin{aligned} \tilde{C}_t^c + \tilde{C}_t^d &= \left(L_{t-1} + \tilde{X}_t\right) - L_t \\ &= \tilde{X}_t - \delta\pi_t\left[1 - \frac{1}{(1+i)(1+g)}\right]. \end{aligned} \tag{5}$$

Consequently, the annual net cashflows (ignoring taxes and dividends) originated by the insurance business are

$$\begin{aligned} \tilde{F}_t &= \pi_t - c\pi_t - \left(\tilde{C}_t^c + \tilde{C}_t^d\right) \\ &= \pi_t\left[(1-c) + \delta\left(1 - \frac{1}{(1+i)(1+g)}\right)\right] - \tilde{X}_t. \end{aligned} \tag{6}$$

Finally, we observe that the loss reserve in our framework is deterministic: equations (3) and (5) in fact imply that

$$L_t = L_{t-1} + \delta\pi_t\left[1 - \frac{1}{(1+i)(1+g)}\right].$$

The amount of the gross premiums is given by the risk premium $\mathbb{E}\left(\tilde{X}_t\right)$, the safety loading which is calculated as a (constant) quota φ of the risk premium, and the expense loading, i.e.:

$$\pi_t = \mathbb{E}\left(\tilde{X}_t\right) + \varphi\mathbb{E}\left(\tilde{X}_t\right) + c\pi_t.$$

Although the risk loading coefficient φ is kept constant over the whole time horizon, its value is calculated on the basis of the standard deviation premium principle for the overall insurance portfolio structure, in order to take account of both the explicit and implicit risk loading of the insurance business, i.e. the safety loading and the interests on claim reserves. Hence, the risk loading coefficient φ satisfies the following relationship:

$$(1 - tx)\left[\varphi \mathbb{E}\left(\tilde{X}_1\right) + L_0 \mathbb{E}\left(\tilde{j}_1\right)\right] = b\sqrt{\mathbb{V}ar\left(\tilde{X}_1\right) + L_0^2 \mathbb{V}ar\left(\tilde{j}_1\right)}, \tag{7}$$

where tx denotes the (constant) rate of taxation. In practice, the insurer may ask for a total loading amount (net of taxation) equal to b for each unit of standard deviation of the total risk of the overall insurer business. Note that we ignore the risk originated by the investment of the inital risk capital U_0. The benchmark value of $b = 35\%$ is considered here for the computation of the total risk amount.

Regarding the amount of the aggregate claims, we follow the collective approach and use a compound process:

$$\tilde{X}_t = \sum_{i=1}^{\tilde{k}_t} \tilde{Z}_{i,t},$$

where $\tilde{k}_t$ is the random variable representing the number of claims occurred in year t, and $\tilde{Z}_{i,t}$ is the random size of the i-th claim occurred in year t. Following Savelli (2003) and Rytgaard and Savelli (2004), we model the number of claims $\tilde{k}_t$ using a simple Poisson process with stochastic parameters $n_t\tilde{q}$, where $\tilde{q}$ is a random structure variable capturing the impact of short-term fluctuations on $\tilde{k}_t$, and $n_t = n_0(1+g)^t$. We ignore, instead, the effects of trends and long-term cycles. Consequently, the only restriction for the probability distribution of $\tilde{q}$ is that its expected value has to be equal to 1. In particular, we assume that $\tilde{q}$ is Gamma distributed with parameters (h, h). Thus, the moments of the structure variable are given by:

$$\mathbb{E}(\tilde{q}) = 1; \quad \sigma(\tilde{q}) = \frac{1}{\sqrt{h}}; \quad skew(\tilde{q}) = \frac{2}{\sqrt{h}} = 2\sigma(\tilde{q}).$$

It follows that the number of claims is Negative Binomial distributed.

Finally, we assume that the claim sizes $\tilde{Z}_{i,t}$ are i.i.d. lognormal random variables; as the distribution has to be scaled by the inflation rate in each year, the moments from the origin are equal to:

$$\mathbb{E}\left(\tilde{Z}_{i,t}^j\right) = (1+i)^{jt}\,\mathbb{E}\left(\tilde{Z}_{i,0}^j\right) = (1+i)^{jt}\,a_{jZ,0}.$$

Further, $\tilde{Z}_{i,t}$ and $\tilde{k}_t$ are mutually independent in each year t. The expected claim size has been simply denoted by m_t. The distribution of the process $\tilde{X}_t$ for the parameter set used in this paper (and discussed in Table 2) is illustrated in Figure 1.

2 The model for the financial market

Consider a frictionless market with continuous trading, no taxes, no transaction costs, no restrictions on borrowing or short sales and perfectly divisible securities. Assume that,

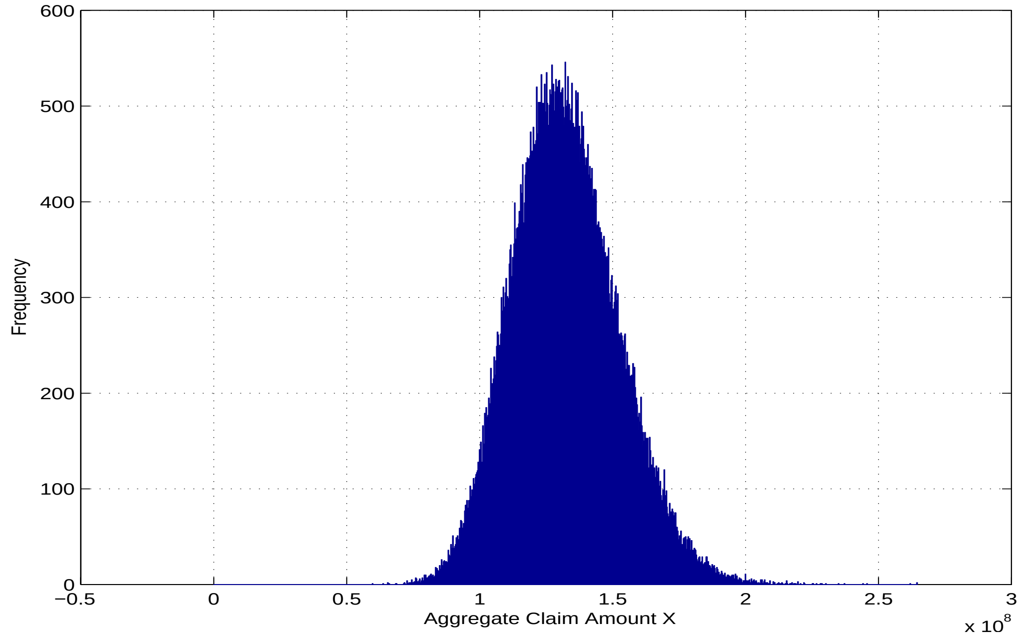

Figure 1: Simulated distribution of $\tilde{X}$ at time $t = 1$ (100,000 simulations). For this experiment $\mathbb{E}(\tilde{X}) = 132.3$ (ml); $\sigma(\tilde{X}) = 19.80$ (ml), whilst the variability coefficient is 15% (the parameters are given in Table 2).

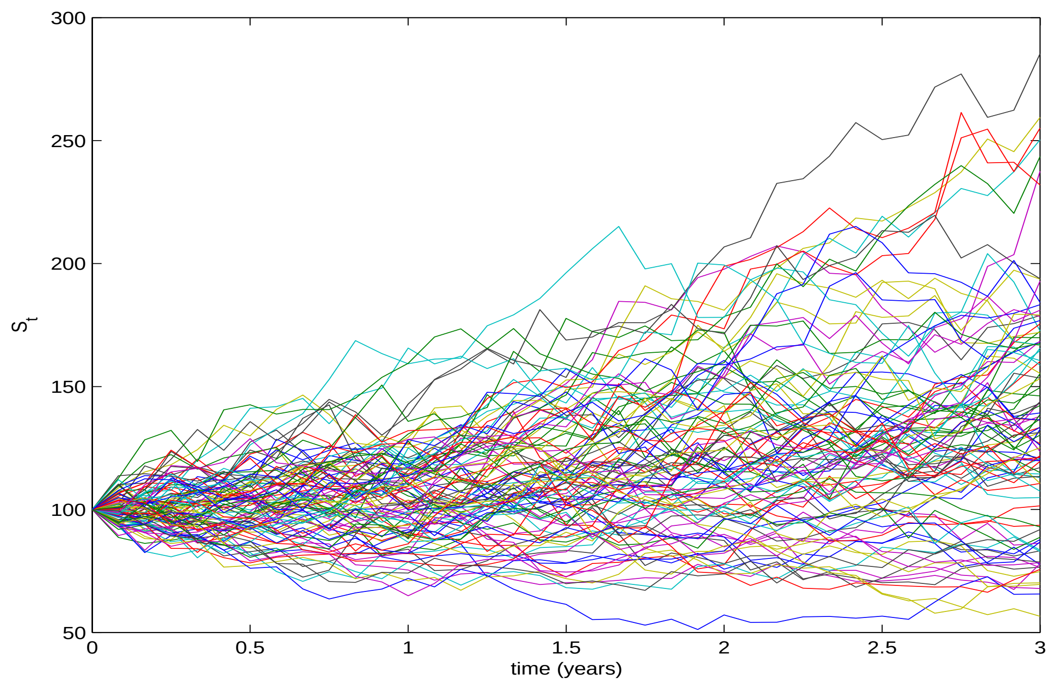

Figure 2: Sample of 100 possible trajectories of the stock index on monthly basis. Parameters: $\mu = 0.1$, $\sigma = 0.2$; $S_0 = 100$.

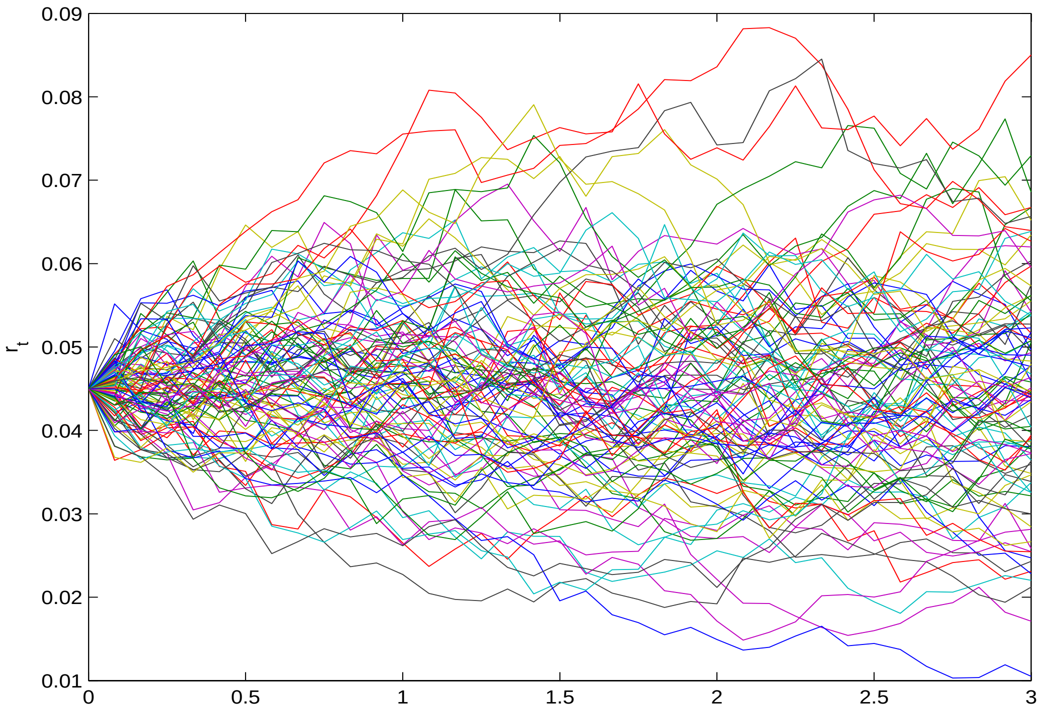

Figure 3: Sample of 100 possible trajectories of the short rate correlated to the corresponding trajectories of the stock shown in Figure 2. Parameters: $\kappa = 0.1$, $\theta = 0.04$; $v = 0.047$; $\rho = -0.2$; $r_0 = 0.045$.

under the real probability measure $\mathbb{P}$, the equity price process is described by the following stochastic differential equation:

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

where $(W_t : t \geq 0)$ is a standard one-dimensional $\mathbb{P}$-Brownian motion, $\mu \in \mathbb{R}$ is the expected rate of growth (or return) on the equity, and $\sigma \in \mathbb{R}^+$ is the stock volatility. A number of possible trajectories of the stock index are shown in Figure 2.

Further, we model the term structure of interest rates using a CIR process, so that it satisfies the following stochastic differential equation:

$$dr_t = \kappa\,(\theta - r_t)\,dt + v\sqrt{r_t}dZ_t, \tag{8}$$

where θ is the long-run mean interest rate level, κ is the speed of mean-reversion and $v \in \mathbb{R}^+$ is the volatility parameter. Moreover, $(Z_t : t \geq 0)$ is a standard one-dimensional $\mathbb{P}$-Brownian motion correlated with W, so that

$$dW_t dZ_t = \rho dt,$$

for any $\rho \neq 0$. Hence

$$Z_t = \rho W_t + \sqrt{1-\rho^2}X_t,$$

where $(X_t : t \geq 0)$ is a $\mathbb{P}$-Brownian motion independent of W_t. The resulting evolution of the short rate is shown in Figure 3. Under these assumptions, the price at time t of a

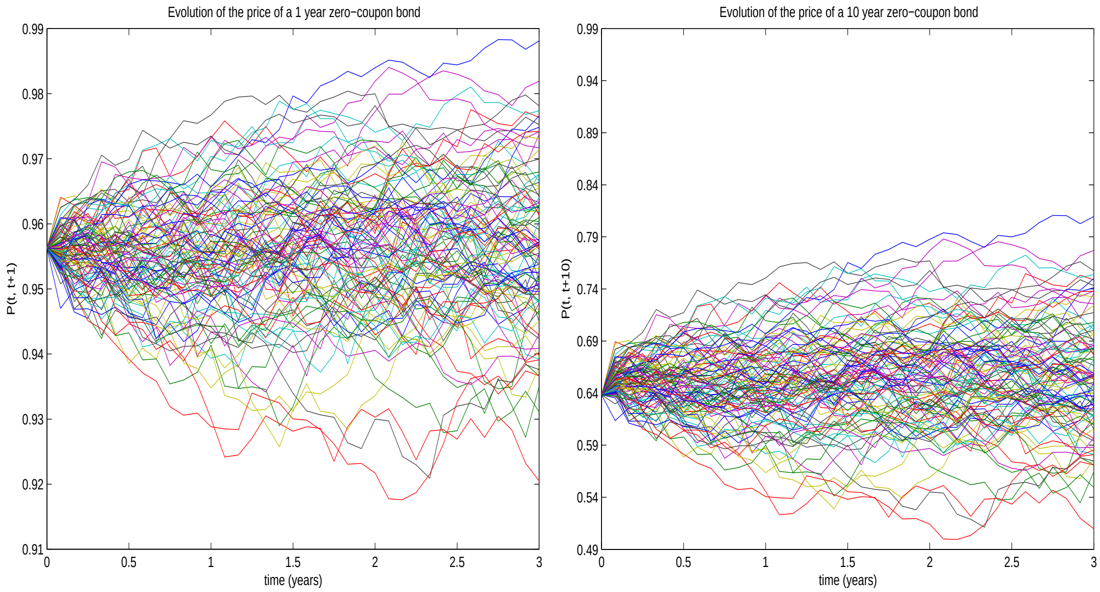

Figure 4: Sample of 100 possible trajectories on monthly basis of a 1 year zero-coupon bond and a 10 years zero-coupon bond. Parameters: $\kappa = 0.1$, $\theta = 0.04$; $v = 0.047$; $\rho = -0.2$; $\lambda = -0.005$; $r_1 = 0.0447$; $r_{10} = 0.0449$. r_1 and r_{10} are the yield to maturity at time $t = 0$ for maturity in 1 year and 10 years respectively.

zero coupon bond with redemption date $\tau > t$ is (see Cox, Ingersoll and Ross, 1985, and Hull and White, 1990):

$$P(t,\tau) = A(t,\tau)\, e^{-B(t,\tau) r_t}, \tag{9}$$

with

$$\begin{aligned} B(t,\tau) &= \frac{2\left(e^{\gamma(\tau-t)}-1\right)}{(\gamma+\kappa+\eta)\left(e^{\gamma(\tau-t)}-1\right)+2\gamma}, \\ \gamma &= \sqrt{(\kappa+\eta)^2+2v^2}, \\ A(t,\tau) &= \left[\frac{2\gamma e^{\frac{(\gamma+\kappa+\eta)(\tau-t)}{2}}}{(\gamma+\kappa+\eta)\left(e^{\gamma(\tau-t)}-1\right)+2\gamma}\right]^{\frac{2\kappa\theta}{v^2}}. \end{aligned}$$

η represents the "market risk" parameter; following Hull and White (1990), it can be shown that the corresponding market price of interest rate risk (i.e. the Girsanov exponent) is $\lambda(t;r) = \eta\sqrt{r_t}/v$. In Figure 4, we show the dynamic of a 1 year zero coupon bond and a 10 year zero coupon bond corresponding to the trajectories of the short rate presented in Figure 3.

In this framework, the zero yield is given by

$$R(t,\tau) = \frac{B(t,\tau)\, r_t - \ln A(t,\tau)}{\tau - t}.$$

3 The asset portfolio

Given an initial capital, $c_0 = L_0 + U_0$, the insurance company invests the available funds in a portfolio, A, composed by $\alpha\%$ equity and $(1-\alpha)\,\%$ bonds of different maturities. The asset allocation is then kept constant over time.

As said in the previous section, the equity dynamics is driven by a geometric Brownian motion; let us assume that the stochastic differential equation of the bond price is

$$P\left(t,\tau\right) = a^{(\tau)}\left(t,r\right) P\left(t,\tau\right) dt + b^{(\tau)}\left(t,r\right) P\left(t,\tau\right) dZ_t,$$

where $a^{(\tau)}\left(t,r\right) = r_t + \lambda\left(t;r\right)$ is the expected rate of return on the bond maturing at time τ. From equation (9), it follows that the diffusion coefficient is $b^{(\tau)}\left(t,r\right) = -B\left(t,\tau\right) v\sqrt{r_t}$. As mentioned above, we assume that $(1-\alpha)\,\%$ of the available funds is used to purchase gilts of different maturity. More in details, we make the assumption that the insurance company invests $\beta^{(i)}\%$ of the funds in zero-coupon bonds with time to maturity $i = 1, 2, 3, 5, 10$ years. The allocation is again kept constant over time. Let B_t be the value at time t of the investment in this bond portfolio; Itô's lemma implies that

$$\begin{aligned} dB_t &= (1-\alpha)\,A_t \sum_{i\in N} \beta^{(i)} \left(a^{(t+i)}\left(t,r\right) dt + b^{(t+i)}\left(t,r\right) dZ_t\right) \\ &= (1-\alpha)\,A_t a^{(t+i)}\left(t,r\right) dt + (1-\alpha)\,A_t \Sigma\left(t\right) dZ_t, \end{aligned}$$

where $N = \{1, 2, 3, 5, 10\}$ and

$$\Sigma\left(t,r\right) = \sum_{i\in N} \beta^{(i)} b^{(t+i)}\left(t,r\right).$$

Consequently, the stochastic differential equation of the asset portfolio A price process is

$$\begin{aligned} dA_t &= \left(\alpha\mu + (1-\alpha)\,a^{(t+i)}\left(t,r\right)\right) A_t dt + \left(\alpha\sigma + (1-\alpha)\,\rho\Sigma\left(t,r\right)\right) A_t dW_t \\ &\quad + (1-\alpha)\sqrt{1-\rho^2}\,\Sigma\left(t,r\right) A_t dX_t. \end{aligned} \tag{10}$$

Equation (10) describes the evolution over time of the asset portfolio net of the cash-flows generated by the insurance business as discussed in section 1. In order to take these additional financial resources into account, we define P_t to be the current price of the bond part of the portfolio, then the value of the portfolio A at time $t > 0$ is

$$\begin{aligned} A_t &= \alpha\left[A_{t-1} + F_{t-1}\right] \frac{S_t}{S_{t-1}} + (1-\alpha)\left[A_{t-1} + F_{t-1}\right] \frac{P_t}{P_{t-1}} \\ A_0 &= c_0, \end{aligned} \tag{11}$$

where F is the net cashflow generated by the insurance business and defined in section 1, equation (6). Since the value at year $t > 0$ of the investment in the bond portfolio, B_t, is given by

$$\begin{aligned} B_t &= (1-\alpha)\left[A_{t-1} + F_{t-1}\right] \sum_{i\in N} \beta^{(i)} \frac{P\left(t,t-1+i\right)}{P\left(t-1,t-1+i\right)} \\ B_0 &= (1-\alpha)\,A_0, \qquad N = \{1, 2, 3, 5, 10\}; \end{aligned}$$

the corresponding unit market price is

$$P_t = \frac{B_t P_{t-1}}{(1-\alpha)\left[A_{t-1} + F_{t-1}\right]}.$$

This implies that equation (11) can be simplified to[1]

$$A_t = \alpha \left[A_{t-1} + F_{t-1}\right] \frac{S_t}{S_{t-1}} + B_t; \tag{12}$$

therefore, the annual rate of the return on the asset is

$$\tilde{j}_t = \frac{A_t - (A_{t-1} + F_{t-1})}{(A_{t-1} + F_{t-1})}.$$

4 The numerical experiment

In this section, we use the framework set up in sections 1-3 to analyze the solvency profile of general insurers with different financial portfolios. The Monte Carlo procedure is based on 100,000 simulations; the dynamic of the stock and the short interest rate are produced with monthly steps. The annual rate of return on the insurer's portfolio are then considered.

More in details, we consider four general insurers with different types of asset allocation; the base example is a general insurer that invests 15% of its financial resources in equity (Standard Insurer); we then consider insurers with a higher equity component in their investment portfolio, specifically, we set $\alpha = 30\%$ (Insurer A); $\alpha = 50\%$ (Insurer B); and $\alpha = 100\%$ (Insurer C). The benchmark set of parameters of the financial market is given in Table 1, whilst the parameters of the general insurers considered are presented in Table 2.

The relevant index of profitability that we consider is the so-called capital ratio

$$\tilde{u}_t = \frac{\tilde{U}_t}{\pi_t};$$

for solvency purposes, instead, we use the minimum risk-based capital ratio

$$rbc_{1-\varepsilon}(0,t) = \frac{U^{Req}_{1-\varepsilon}(0,t)}{\pi_0}, \tag{13}$$

which expresses the minimum amount of capital (per unit of initial premium paid) required by regulators and legislators to ensure that the company is solvent in year t with a certain interval probability $1-\varepsilon$, which we choose to be 99.0% and 99.9%. Note that we express the risk-based capital as a percentage of the total initial premiums in order to make sensible comparisons between different insurers. In equation (13), $U^{Req}(0,t)$ represents the minimum amount of capital required at time $t = 0$ to ensure that the maximum

[1]Further details on the derivation of equations (11) and (12), are offered in the Appendix.

Portfolio inputs
$T = 3$ years; $\beta^{(1)} = 40\%; \beta^{(2)} = 25\%; \beta^{(3)} = 15\%; \beta^{(10)} = 10\%; \beta^{(15)} = 10\%$
Parameters for the equity dynamic
$\mu = 10\%; \sigma = 20\%; S_0 = 100$
Parameters for the CIR model
$\kappa = 10\%; \theta = 4.00\%; v = 4.70\%; \rho = -0.2; \lambda = -0.005; r_0 = 4.5\%$
Initial yield curve
$r_1 = 4.47\%; r_2 = 4.38\%; r_3 = 4.39\%; r_5 = 4.43\%; r_{10} = 4.49\%$

Table 1: Benchmark set of parameters for the stock index, the dynamic of the short rate of interest and the bond portfolio. The initial yield curve corresponds to the UK market as at 31/12/2004 (source: Bank of England). For the market price of interest rate risk, λ, we consider the approximation provided by Stanton (1997). The remaining parameters are calibrated accordingly.

		Insurer			
Parameters		**Standard**	**A**	**B**	**C**
Time horizon (years)	$T = 3$				
Initial solvency ratio	$u_0 = 0$				
Initial expected number of claims	$n_0 = 20000$				
Variance structure variable q	$\mathbb{V}ar\left(\tilde{q}\right) = 0.02$				
Initial expected claim size	$m_0 = 6000$				
Variability coefficient	$\frac{\sigma(\tilde{Z})}{\mathbb{E}(\tilde{Z})} = 7$				
Loss - Reserves ratio	$\delta = 120\%$				
Expenses loading coefficient	$c = 25\%$				
b coefficient	$b = 35\%$				
Safety loading coefficient	φ	-1.42%	-2.08%	-2.59%	-2.92%
Real growth rate	$g = 5\%$				
Claim inflation rate	$i = 5\%$				
Taxation rate	$tx = 0$				
Dividend rate	$dv = 0$				
Asset allocation					
Equity component	α	15%	30%	50%	100%
Bond component	$1 - \alpha$	85%	70%	50%	0%
Expected rate of return	$\mathbb{E}\left(\tilde{j}\right)$	4.89%	5.79%	6.99%	10.00%
Volatility of the rate of return	$\sigma\left(\tilde{j}\right)$	3.85%	6.49%	10.28%	20.00%

Table 2: Parameters of the general insurers. The (theoretical) expected rate of return on the asset portfolio and its volatility are calculated using the governing stochastic differential equation of the price process A.

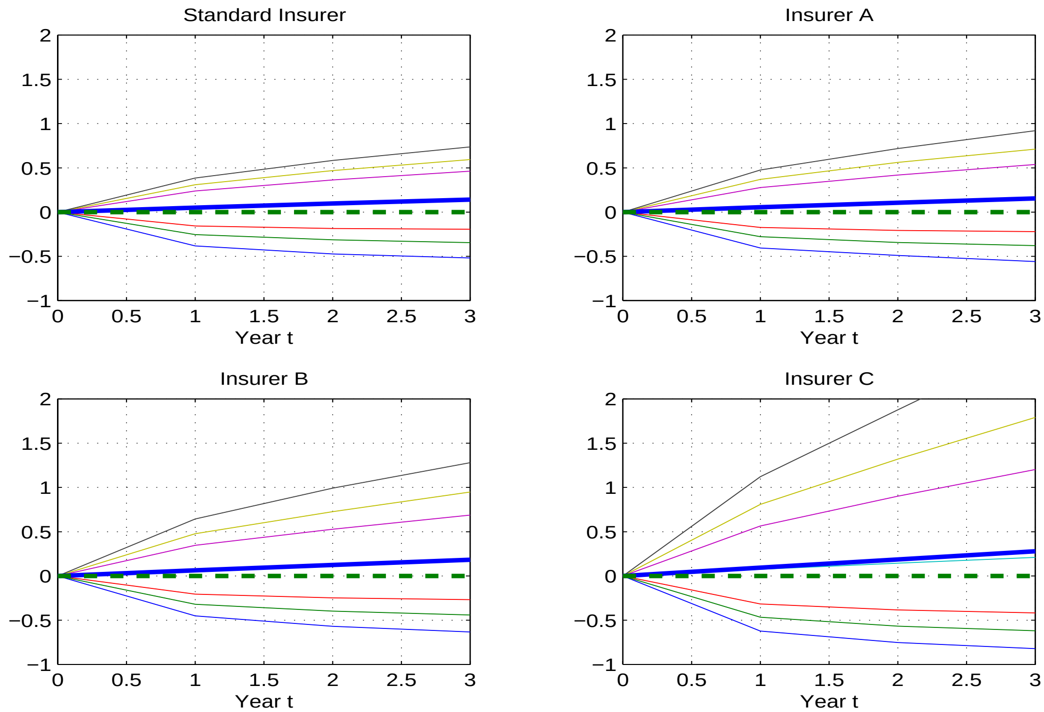

Figure 5: Mean and quantiles of simulated $\tilde{U}/\pi$ (confidence level: 0.1%,1%, 5%, 95%, 99%, 99.9%).

loss accumulated by time $t > 0$ can be offset by the risk premiums, the risk loading and the capital with probability $1-\varepsilon$. Since in our analysis the initial capital is set equal to zero, this amount corresponds to the CaR (Capital-at-Risk) of the insurer over the period $(0,t)$ with confidence level equal to $1-\varepsilon$, once the investment returns over $(0,t)$ have been considered. Hence

$$U_{1-\varepsilon}^{Req}(0,t) = CaR\,(0,t)_{1-\varepsilon}\left[1+\mathbb{E}\left(\tilde{j}\right)\right]^{-t},$$

with

$$CaR\,(0,t)_{1-\varepsilon} = -U_{\varepsilon}(t)\,,$$

and $U_{\varepsilon}(t)$ is the ε-th quantile of the risk reserve $\tilde{U}$ at time t. Equation (13) can be expressed in terms of capital ratio $\tilde{u}_t$, so that

$$rbc_{1-\varepsilon}(0,t) = -\frac{u_{\varepsilon}(t)}{\bar{r}^{t}}, \tag{14}$$

where

$$u_{\varepsilon}(t) = \frac{U_{\varepsilon}(t)}{\pi_t}; \qquad \bar{r} = \frac{1+\mathbb{E}\left(\tilde{j}\right)}{(1+g)(1+i)}.$$

4.1 Analysis of the capital ratio and the risk based capital

In Figure 5 we present the quantiles of the capital ratio of the four insurers considered for different confidence levels. From the comparison between the four insurers, it emerges

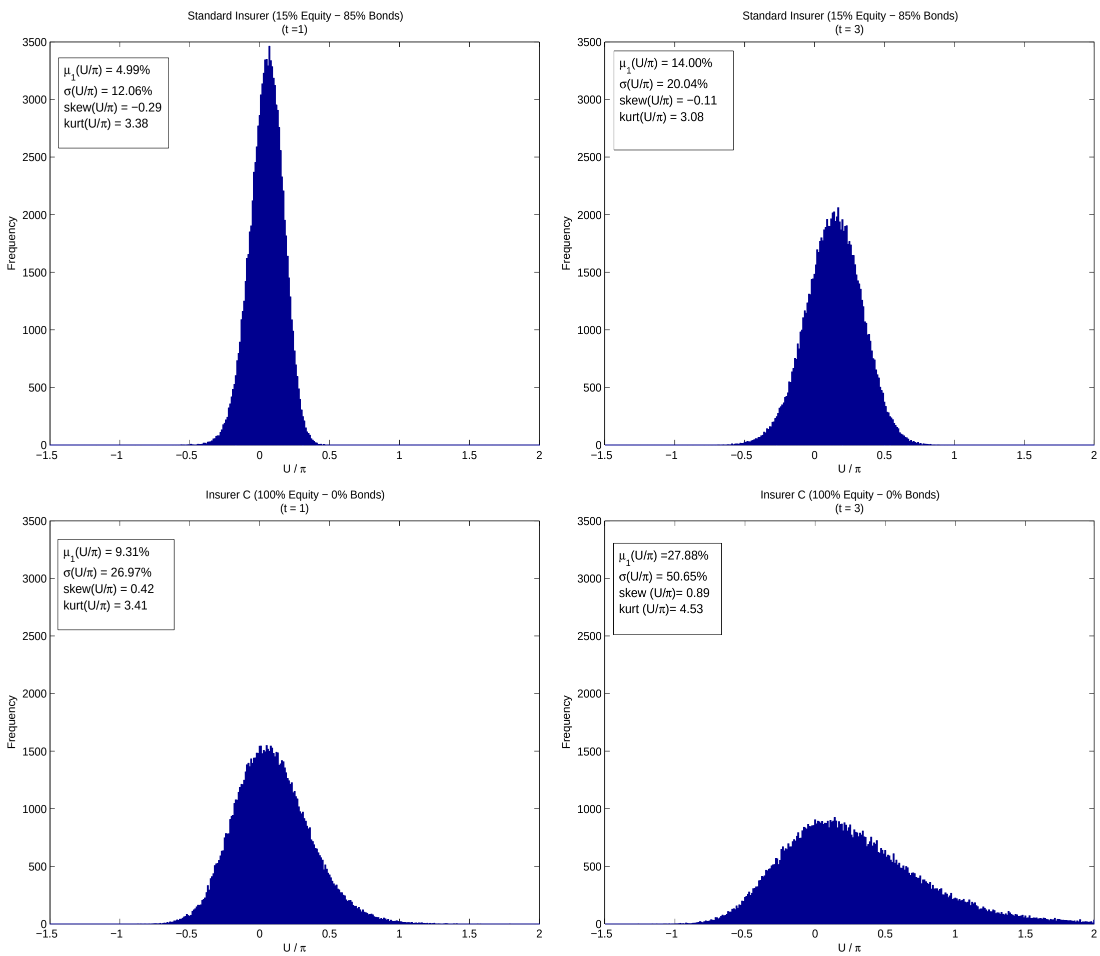

Figure 6: Simulated probability distribution of the capital ratio u_t at time $t = 1$ and $t = 3$ for the case of the Standard Insurer and Insurer C.

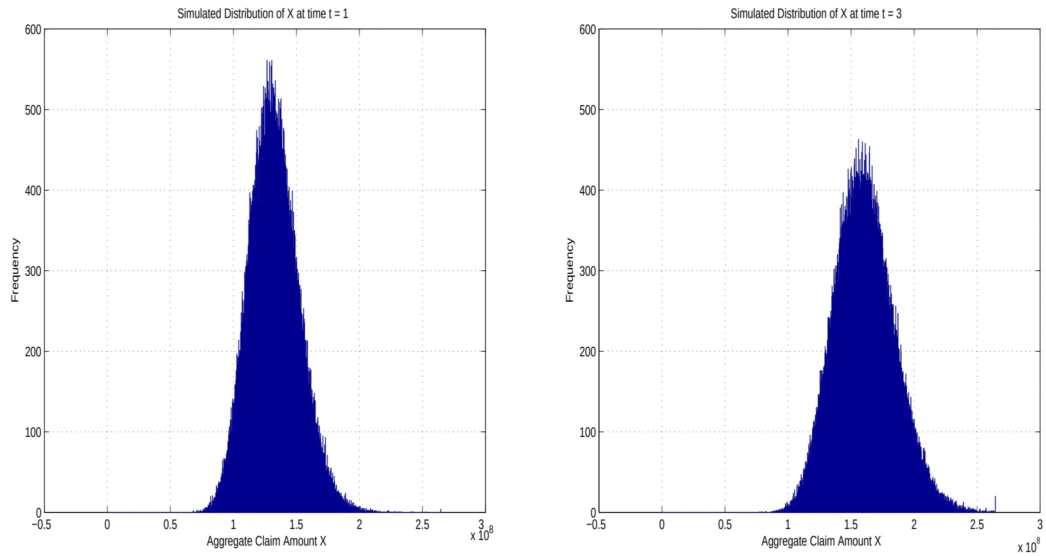

Figure 7: Simulated probability distribution of the process $\tilde{X}$ at time $t = 1$ and $t = 3$.

that increasing the percentage of the financial resources invested in the equity component, increases the expected value of $\tilde{u}_t$; moreover the upper percentiles (i.e. the 75%, 99% and 99.9% percentiles) increase, whilst the lower percentiles (i.e. the 0.1%, 1% and 25% percentiles) decrease. However, over the long run, the shape of the upper percentiles suggests a higher probability mass in the upper tail of the distribution for Insurer C than for the Standard Insurer. This fact can be better observed in Figure 6, in which we show the simulated distributions and the first four moments of the capital ratio at time $t = 1$ and $t = T = 3$ for the Standard Insurer and Insurer C. We also note that a riskier asset portfolio implies a higher volatility of the capital ratio. The contribution over time to the capital ratio from the claim process and the asset return process is shown in Figures 7-9. In particular, we note the change in the shape of the distribution of $\tilde{j}$ for the different asset allocations: at time $t = 1$, in fact, the probability distribution of $\tilde{j}$ for the case of the Standard Insurer is very peaked, with low volatility; moving from the Standard Insurer to Insurers A, B and C we can observe the increased variance, skewness and kurtosis of the distribution. This phenomenon is even more accentuated for the (cumulated) return process $\tilde{j}$ at time $t = 3$. Finally, we point out that in the case of Insurer C, the process $\tilde{j}$ is lognormal; the estimated moments of the distribution as shown in Figures 8-9 match the exact moments.

Given how a more aggressive asset allocation strategy affects the capital ratio, we expect more demanding risk based capital requirements in correspondence of a higher equity component in the investment portfolio. This is in fact the case, as shown in Table 3, which contains the capital requirements for the four insurers, together with the corresponding safety loading coefficient and the total amount of the premiums that they have to pay at inception. In particular, we observe that the safety loading coefficient φ decreases as α increases, consequently the level of the inital gross premiums decreases

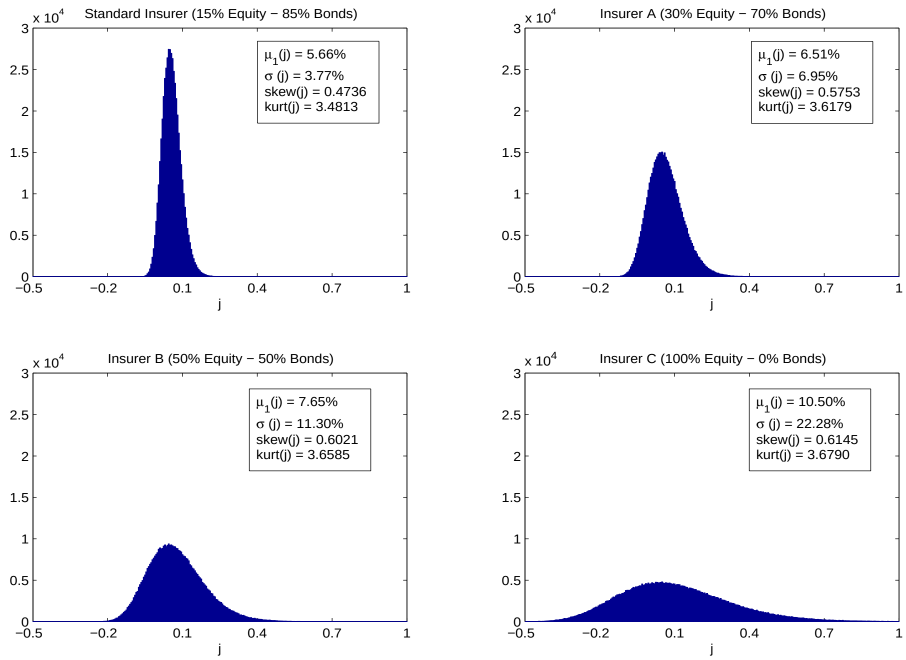

Figure 8: Simulated probability distribution at time $t = 1$ of the rate of return on the asset portfolio, $\tilde{j}$, for each asset allocation considered in this paper.

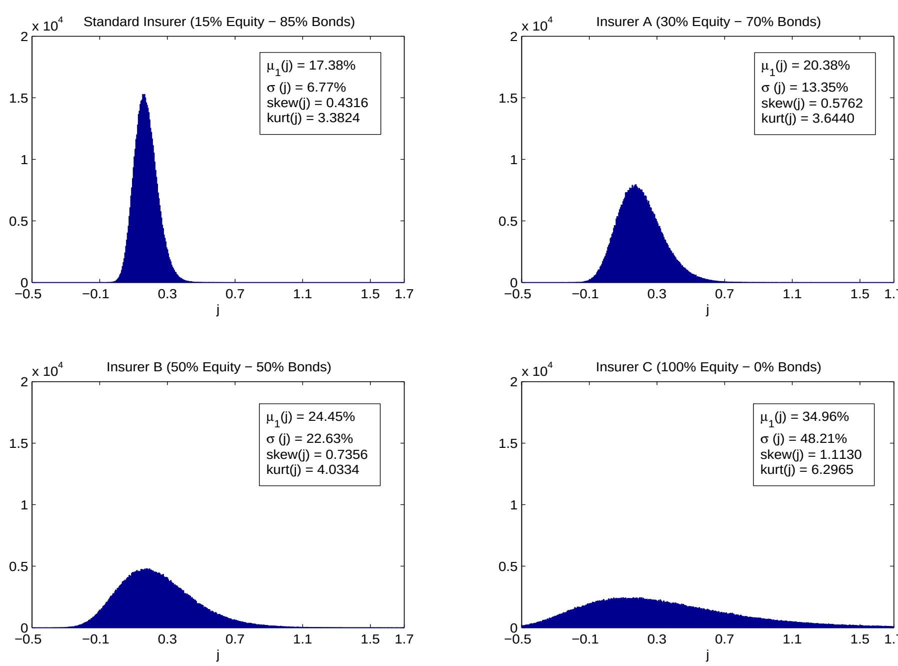

Figure 9: Simulated probability distribution at time $t = 3$ of the (cumulated) rate of return on the asset portfolio, $\tilde{j}$, for each asset allocation considered in this paper.

	Standard Insurer (15% equities - 85% bonds)			Insurer A (30% equities - 70% bonds)		
$rbc_{1-\varepsilon}(0,t)$	$\varphi = -1.42\%; \pi_0 = 157.73$			$\varphi = -2.08\%; \pi_0 = 156.67$		
(equation (14))	$T = 1$	$T = 2$	$T = 3$	$T = 1$	$T = 2$	$T = 3$
$1 - \varepsilon = 99.0\%$	26.75	34.71	40.14	28.93	37.27	42.89
$1 - \varepsilon = 99.9\%$	40.13	52.28	60.28	42.34	53.19	63.30

	Insurer B (50% equities - 50% bonds)			Insurer C (100% equities - 0% bonds)		
$rbc_{1-\varepsilon}(0,t)$	$\varphi = -2.59\%; \pi_0 = 155.86$			$\varphi = -2.92\%; \pi_0 = 155.33$		
	$T = 1$	$T = 2$	$T = 3$	$T = 1$	$T = 2$	$T = 3$
$1 - \varepsilon = 99.0\%$	32.95	42.17	48.24	46.69	57.00	62.41
$1 - \varepsilon = 99.9\%$	46.59	60.39	69.24	62.43	75.58	82.77

Table 3: The risk based capital for the four insurers considered in this analysis. The table also shows the safety loading coefficient and the corresponding initial premiums for each insurer.

as well. On the other hand, the capital requirements for example at 99.0% confidence level increase from 26.75% (Standard Insurer) to 46.69% (Insurer C) over the first year, reaching the 62.41% level over the 3 years time horizon, as shown in Table 3. Therefore, the raise in the capital requirements induced by a riskier asset allocation strategy implies more financial pressure on the shareholders of the companies, as they have to provide further capital injections.

References

[1] Bank of England (2005). Government liability yield curve. http://www.bankofengland.co.uk/.

[2] Cox, J., J. Ingersoll, and S. Ross (1985). A theory of the term structure of interest rates. *Econometrica*, 53, 385-408.

[3] Hull, J. H. and A. White (1990). Pricing interest rate derivative securities. *The Review of Financial Studies*, 3, 573-592.

[4] Rytgaard, M. and N. Savelli (2004). Risk-based capital requirements for property and liability insurers according to different reinsurance strategies and the effect on profitability. XXXVth International Astin Colloquium, Bergen.

[5] Savelli, N. (2003). A risk theoretical model for assessing the solvency profile of a general insurer. XXX GIRO Convention, Cardiff.

[6] Stanton, R. (1997). A nonparametric model of term structure dynamics and the market price of interest rate risk, *The Journal of Finance*, 52, 1973-2002.

Appendix: the dynamic of the asset portfolio

Assume that the insurer enters time t with available financial resources A_t. This resources are then used to settle the claims of the year and, at the same time, they are also increased by the premiums received. Hence, the total resources available for investment purposes are $A_t + F_t$, where F is the net cashflow generated by the insurance business including claims, premiums and expenses, as defined in section 1. The insurer invests this amount in the financial market by acquiring a portfolio composed by $\alpha\%$ equity and $(1-\alpha)\,\%$ bonds of different maturity. Hence, the portfolio is given by

$$A_t + F_t = x_t^S S_t + x_t^P P_t$$

where x_t^S and x_t^P are respectively the number of shares in equity and bonds purchased. In order to preserve the asset allocation, we need

$$\begin{aligned} x_t^S S_t &= \alpha\,(A_t + F_t) \\ x_t^P P_t &= (1-\alpha)\,(A_t + F_t)\,. \end{aligned}$$

Therefore, the insurer enters time $t+1$ with a portfolio's value

$$\begin{aligned} A_{t+1} &= x_t^S S_{t+1} + x_t^P P_{t+1} \\ &= \alpha\,[A_t + F_t]\,\frac{S_{t+1}}{S_t} + (1-\alpha)\,[A_t + F_t]\,\frac{P_{t+1}}{P_t}. \end{aligned}$$

As described in section 3, the insurance company invests every year $(1-\alpha)\,\%$ of its resources in a mix of zero-coupon bonds with different maturities. Specifically, the insurers invests $\beta^{(i)}\%$ of the funds in zero-coupon bonds with time to maturity $i = 1, 2, 3, 5, 10$ years, and the mix is kept constant over time. Therefore, similarly to the asset portfolio, the bond share is given by

$$\begin{aligned} (1-\alpha)\,(A_t + F_t) &= x_t^{(1)} P\,(t, t+1) + x_t^{(2)} P\,(t, t+2) + x_t^{(3)} P\,(t, t+3) \\ &\quad + x_t^{(5)} P\,(t, t+5) + x_t^{(10)} P\,(t, t+10)\,, \end{aligned}$$

where $P\,(t, \tau)$ is the price at time t of a zero coupon bond with redemption date $\tau > t$, and $x_t^{(i)}$, $i = 1, 2, 3, 5, 10$ years, are the units of zero-coupon bonds with time to maturity i in the portfolio at time t. In order to maintain the mix unchanged over time, we require

$$x_t^{(i)} P\,(t, t+i) = \beta^{(i)}\,(1-\alpha)\,(A_t + F_t)\,, \quad \forall i = 1, 2, 3, 5, 10 \text{ years.}$$

Consequently, the value at year $t+1$ of the investment in the bond portfolio, B_{t+1}, is given by

$$\begin{aligned} B_{t+1} &= (1-\alpha)\,[A_t + F_t]\left[\beta^{(1)}\frac{1}{P\,(t, t+1)} + \beta^{(2)}\frac{P\,(t+1, t+2)}{P\,(t, t+2)}\right. \\ &\quad \left. + \beta^{(3)}\frac{P\,(t+1, t+3)}{P\,(t, t+3)} + \beta^{(5)}\frac{P\,(t+1, t+5)}{P\,(t, t+5)} + \beta^{(10)}\frac{P\,(t+1, t+10)}{P\,(t, t+10)}\right]. \end{aligned}$$

Since

$$B_{t+1} = x_t^P P_{t+1},$$

then the unit price of the bond portfolio is

$$P_{t+1} = \frac{B_{t+1}}{x_t^P} = B_{t+1} \frac{P_t}{(1-\alpha)(A_t + F_t)}.$$