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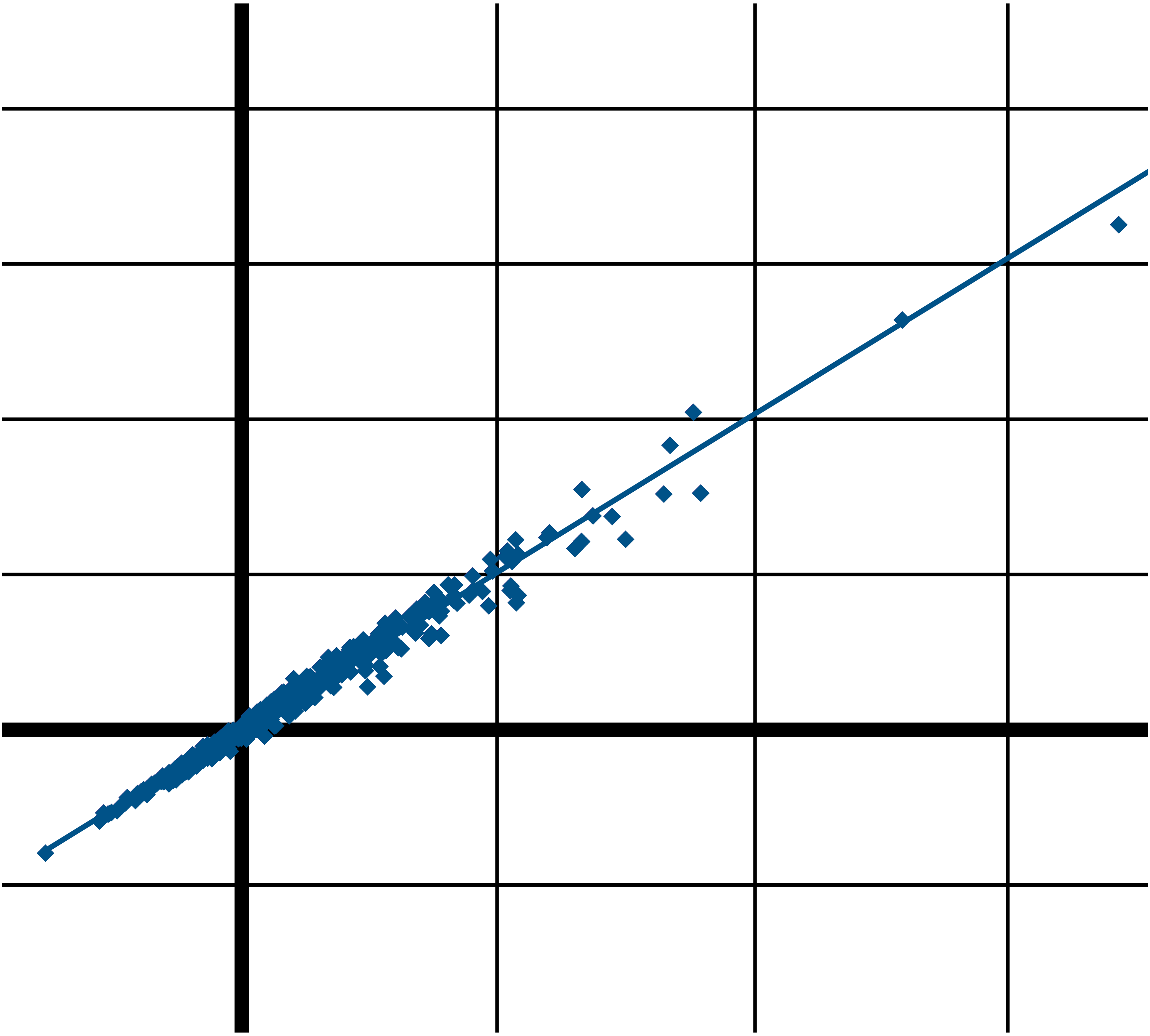
November 2009

Milliman

Replicating Portfolios

An Introduction: Analysis and Illustrations



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A replicating portfolio is a pool of assets designed to reproduce (replicate) the cash flows or market values of a pool of liabilities across a large number of stochastic scenarios.

Replicating portfolios may enhance the value of traditional actuarial modelling, most notably because they allow portfolio market values to be calculated very rapidly (often using closed-form solutions) after changes in market conditions.

OVERVIEW

In recent years many insurers have adopted increasingly complex stochastic models as part of their business-management processes. These models are used for a number of purposes, including:

- market-consistent embedded value
- economic capital
- enterprise risk management
- Solvency II
- fair-value reporting for U.S. GAAP

Traditional actuarial models can typically be developed to support much of this analysis. However, the complexity of these models makes it difficult to link them into a real-time financial reporting process. Consequently, a number of techniques have been developed to make such processes more practical. One such technique is the use of replicating portfolios.

A replicating portfolio is a pool of assets designed to reproduce (replicate) the cash flows or market values of a pool of liabilities across a large number of stochastic scenarios. Once set up, a replicating portfolio can be used to predict the behaviour or change in value of the liabilities across a range of other economic conditions.

A number of large European insurers have adopted replicating portfolios as part of their regular modelling and financial reporting process. In addition, we are seeing increased interest in several other jurisdictions, particularly in Asia.

For a number of reasons, replicating portfolios may enhance the value of traditional actuarial modelling, most notably because they allow portfolio market values to be calculated very rapidly (often using closed-form solutions) after changes in market conditions. However, as discussed below, there are also limitations with this approach. The wise modeller would be well advised to understand these limitations in order to reduce the risk of using the technique inappropriately, resulting in misguided and inaccurate management action. Key objectives associated with replicating portfolios are summarised in the table in Figure 1, along with caveats and limitations.

FIGURE 1: OBJECTIVES AND CAVEATS AND LIMITATIONS OF REPLICATING PORTFOLIOS

OBJECTIVES	CAVEATS AND LIMITATIONS
1. Replicating portfolios can reduce run time for the projection of liability cash-flow proxies and for liability valuations.	1. Replicating portfolios are not applicable to the measurement of non-financial risk, such as insurance risk.
2. Replicating portfolios facilitate sophisticated risk aggregation.	2. Determination of replicating portfolios requires a significant number of calibration scenarios, specific knowledge about universal assets, and a robust optimisation tool.
3. Replicating portfolios enable separation of investment and insurance business for management purposes.	3. There may not be actual assets or actively traded assets that replicate long-term or exotic features of insurance liabilities.

Replicating portfolios are also used for other purposes, including dynamic hedging and management of market risk. One other technique that has emerged is cluster modelling. Cluster modelling involves using a subset of the liabilities, with an appropriate scalar applied to each cell to represent the entire liability portfolio. This report discusses the strengths of each approach, along with caveats and limitations. In addition, it includes case studies for several insurance products.

REPLICATING PORTFOLIOS

Definition of replicating portfolios

A replicating portfolio is a proxy portfolio consisting of standard capital-market products that replicate the scenario-dependent payoffs of the insurance company's liability. It is determined across a wide selection of calibration scenarios by optimisation techniques. Because this replicating portfolio is composed of capital-market products, the valuation of liabilities is consistent with the valuation of the asset side of the balance sheet. Assets could include real assets on the market, imaginary assets, simple liabilities, or indeed any mathematical function.

Developed by the banking industry, the replicating portfolio technique has been used for several years. The great benefit of replicating portfolios lies in the speed of recalculating the effects of financial market developments. Banking asset-data systems have the ability to recalculate the value of assets in real time, often because closed-form solutions are available for determining asset market values. By using these systems, insurance companies have the ability to monitor and manage the financial risks at a much greater frequency.

The great benefit of replicating portfolios lies in the speed of recalculating the effects of financial market developments.

How the financial industry uses replicating portfolios

The concept of replicating portfolios serves a number of purposes. Broadly, there are two main applications of the replicating portfolio technique:

1. Economic-capital calculations. Because of their stochastic nature, economic-capital calculations require significant calculation power. The replicating portfolio as representation of insurance liabilities significantly reduces the run times for estimating the impact of economic changes on the value of the liability portfolio.

2. Hedging and management of financial risks.

Other applications of the replicating portfolio are in the areas of financial-risk management and financial reporting. All these applications are described in more depth in the following sections.

Economic-capital calculations

In recent years, regulators and rating agencies have turned to monitoring financial institutions using Value at Risk (VaR) and Tail Risk (TVaR) criteria. This focus has led to the development of internal economic-capital models within the financial industry. The introduction of Solvency II has brought with it a need for insurance companies to develop more stable, accurate, and auditable frameworks. Portfolio replication has become an important method for many insurance companies in the building of this next generation of economic-capital models.

Replicating portfolios allow insurance companies to create an integrated view of assets and liabilities that can be used to perform a detailed analysis of asset-liability management (ALM) risks. Aggregation of non-market risks such as credit risk, operational risk, and mortality risk can be achieved using the more traditional correlation matrices or the more advanced copula approach.

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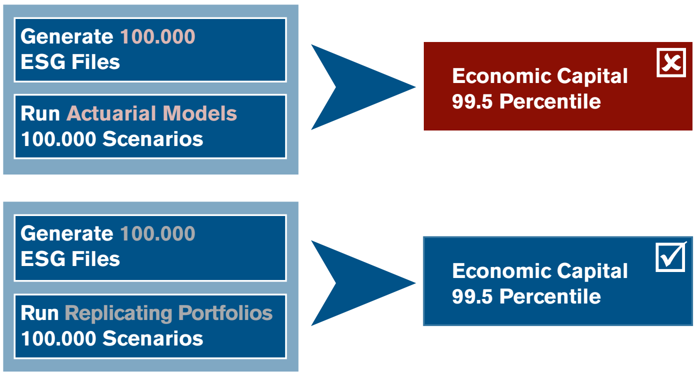
The main motivation for the use of replicating portfolios is speed. Insurance companies want to compute economic-capital figures quickly and accurately to use them in their business decision making, and portfolio replication offers one option for improving the speed of economic-capital calculations.

Speed

Economic-capital calculation requires internal models to support analysis of the behaviour of the extreme left tail of probability distributions. This requires internal models to run a large number of scenarios. Traditional actuarial models will not be capable of running these large sets of scenarios in a short time frame. With replicating portfolios, the run time can be reduced. Specifically, a subset of the scenarios can be used as a calibration input into the process of selecting the replicating portfolio. Then the replicating portfolio can be used in place of the liability portfolio to test a larger number of scenarios or shocks.

FIGURE 2



Accuracy

Insurance companies with more than one business unit have great difficulty accumulating economic-capital (EC) calculations across business units. Some large insurance companies use stress tests to approximate the economic capital. The aggregation techniques on the corporate level are usually highly simplified. Often, companies use predetermined correlation matrices that are hard to calibrate. In the replication method, an approximation is made by using the replicating portfolio instead of the 'true' liabilities. This allows the construction of integrated economic scenarios, facilitating a coherent approach to risk aggregation across business units. Each business unit will construct a replicating asset portfolio, which can be used as a proxy in economic-capital calculations. By adding up the different asset portfolios, the EC statistics on a total level can be determined, as well as the diversification effect.

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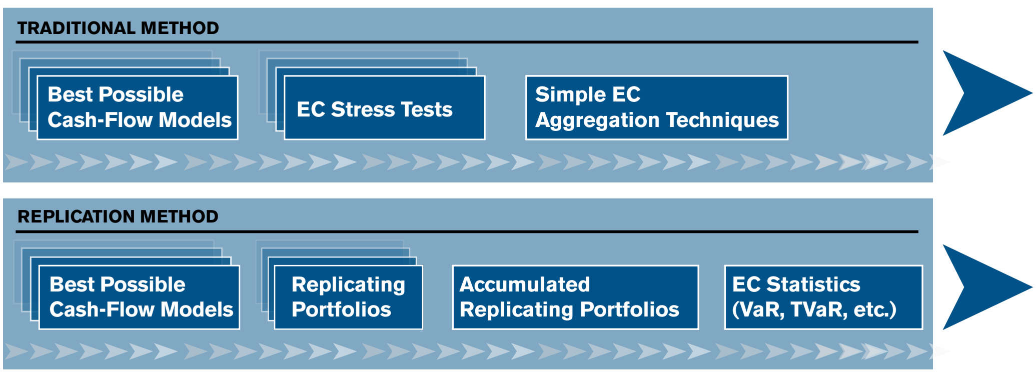
FIGURE 3

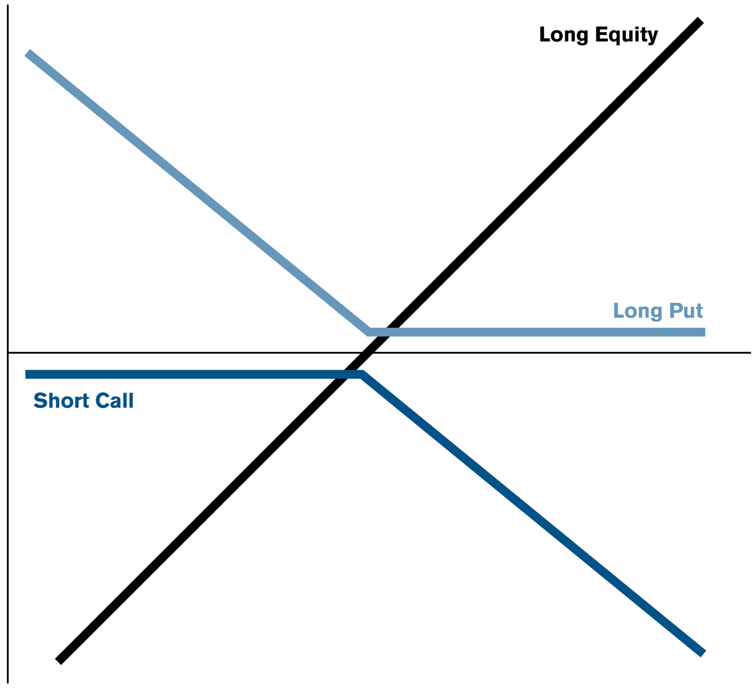


Quantification of the financial diversification effect of the liabilities
The financial characteristics of many types of assets are much more transparent than they are for most liabilities. Because replicating portfolios replace liabilities with assets, the result is greater transparency. In the aggregation of replicating portfolios, the diversification effects within the replicating portfolios are defined by combining assets. Examples of these combining assets are long/short positions, put/call parities, and offsetting/combining swaptions across product lines.
A simple example, put/call parity, illustrates this feature, as shown in Figure 4.

In the aggregation of replicating portfolios, the diversification effects within the replicating portfolios are defined by combining assets.

FIGURE 4

- Asset Portfolio 1: Long Equity Stock
- Asset Portfolio 2: Long Put Options
- Asset Portfolio 3: Short Call Option



Separate economic-capital calculations will show that Asset Portfolio 1 will lose value in the case of an equity drop, Asset Portfolio 2 will lose value in the case of an equity increase, and Asset Portfolio 3 will lose value in the case of an equity increase. The graph in Figure 4 shows that combining these three portfolios produces an asset portfolio that behaves like a cash portfolio. These financial diversification effects are much easier to measure from a replicating portfolio perspective. By monitoring financial-diversification effects, corporations can try to achieve and maintain the maximum diversification effects across business lines. These kinds of diversification effects could reduce the hedge costs of the insurance company.

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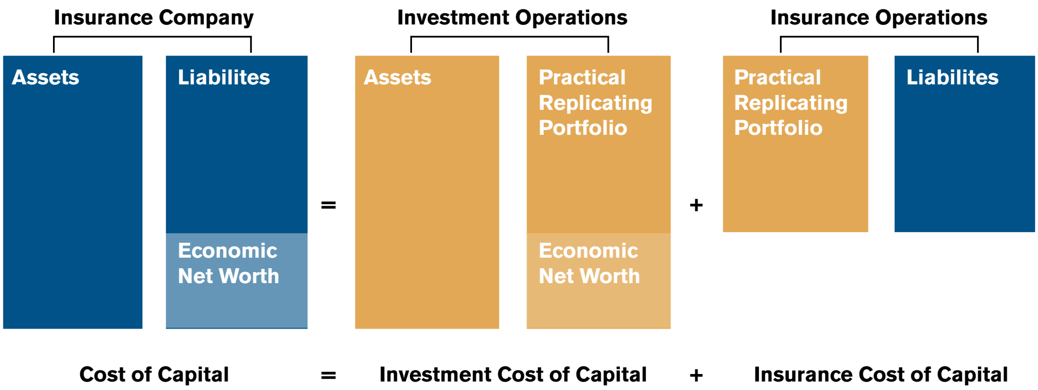
Financial risk management/hedging

Investment and insurance-operation balance sheets
In the banking industry, it is common practice to separate the balance sheet into a trading book and a banking book in order to separately measure the performance of banking and investment operations. The trading book within a bank is made up of all operations from the trading room business. The banking book of a bank is the sum of all the banking operations: loans to individuals, loans to corporations, deposits, etc.

This banking industry methodology can also be used within the insurance industry by replacing the banking book with an insurance book. The insurance company balance sheet can be split between investment operations and insurance operations. The replicating portfolio represents the insurance liabilities and transfers the financial risk from the insurance operations balance sheet to the investment operations balance sheet. Any residual financial risk on the balance sheet of the insurance operations results from imperfect replication. The 'real' assets and the related financial risks are managed in the investment operation. Generally, the economic net worth of the company is managed by an investment company. For this purpose, an ex-ante financial-risk budget—included in the risk appetite of the company—is established.

The separation of both operations is depicted in Figure 5.

FIGURE 5



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The difference between the actual investment return and the return of the replicating portfolio is the reward for taking a financial risk. That reward is allocated to the investment operation.

An important advantage of the separation of insurance and investment operations is the simplicity of allocating the return of the replicating portfolio to the insurance portfolio. The difference between the actual investment return and the return of the replicating portfolio is the reward for taking a financial risk. That reward is allocated to the investment operation.

Hedging

Within this framework the replicating portfolio can be used for daily monitoring of the market risk between the liabilities and the assets backing the liabilities. By using a live feed of financial market information, common measures such as delta, vega, gamma, and rho can be monitored.

Another benefit of the separated insurance and trading-book framework is that the daily performance analysis can be split by type of financial risk, such as:

- non-market risks
- hedging risk
- non-hedgeable market risk
- strategic/tactical market risk

This type of financial risk measurement splits asset management into two types of asset management: hedging asset management and risk/return asset management.

The main focus of hedging asset management is on hedgeable market risk. This report will focus on a full financial de-risking of the liabilities under management. The replicating portfolio becomes the target portfolio. However, the theoretical replicating portfolio is not always practical because of the non-hedgeable market risk—for example, the extremely long-term interest-rate risk of some insurance liabilities.

Risk/return asset management focuses on strategic and tactical market exposures. This department targets financial opportunities within the markets. Risk/return asset management opens risk exposures based specifically on financial opportunities.

FIGURE 6

Valuation of insurance liabilities

Solvency II uses a market-consistent approach to value risks. The Solvency II directive (article 76, section 4) and QIS4 technical specification (II.A.8) give the option to value technical provisions via the replicating portfolio.

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> *Art. 76, sect. 4. Insurance and reinsurance undertakings shall value the best estimate and the risk margin separately. However, where future cash flows associated with insurance or reinsurance obligations can be replicated reliably using financial instruments for which a reliable market value is observable, the value of technical provisions associated with those future cash flows shall be determined on the basis of the market value of those financial instruments. In this case, separate calculations of the best estimate and the risk margin shall not be required.*
>
> *QIS4 TS.II.A.8. Separate calculations of the best estimate and the risk margin are not required, where future cash-flows associated with insurance obligations can be replicated using financial instruments for which a market value is directly observable. In this case, the value of technical provisions should be determined on the basis of the market value of those financial instruments.*

Generally, the cash flows cannot be perfectly replicated under all conditions with financial instruments for which a reliable market value is observable. The main reason is that underwriting and operational risks are considered to be non-hedgeable. For any adverse deviations in the best assumptions, a risk margin needs to be included in the technical provisions. Most companies utilising the replicating portfolio technique add a risk margin for the aforementioned risks to the market value of the replicating portfolio in order to determine the technical provision.

Fast-close financial reporting

The calculation of the market value of liabilities (excluding risk margin or cost of non-hedgeable risks) can be a time-consuming operation. Most of the time is expended in the evaluation of the time value of options and guarantees. The length of time it takes to calculate and analyse the results is incompatible with the ambition of companies to report figures within a few days after closing. One possible solution to this issue is to use replicating portfolios. The closing date is relevant for up-to-date information on economic scenarios and policy data. Using replicating portfolios allows quick adjustment of economic changes, as typically the replication process need not be refreshed in order

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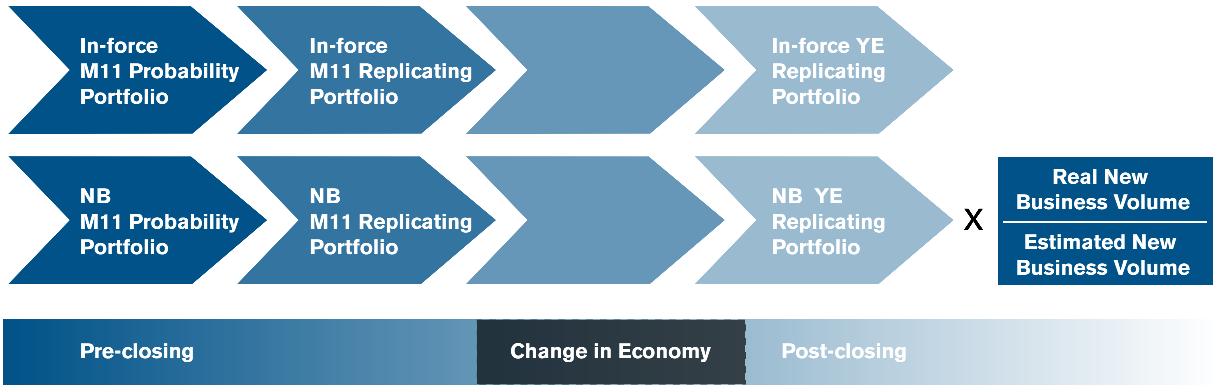
to reflect current market conditions. The illustration in Figure 7 shows the process of a fast close at the end of the year, where the fast close is based on the replicating portfolio of November of the reporting year.

FIGURE 7



The disadvantage of this reporting framework is the loss of liability details. Between the pre-reporting replication and the final reporting, changes may have happened to the liability development. For example, if the underlying fund value of a unit-linked product changes significantly, the replicating portfolio for the final reporting may be quite different from that for the pre-reporting. This framework can be used only when the liability development has been stable between pre-reporting replication and the final reporting date.

LINK WITH RISK MANAGEMENT

Monitoring of market-risk position/Quantification of market risk

In recent years, many insurance companies have been monitoring the market-risk exposures based on the Greeks. The main difficulty for insurance companies is to regularly update the Greeks. The banking industry calculates the Greeks of their assets and liabilities automatically, using live-feed information from the financial markets. These systems can also be used within the insurance industry to constantly monitor the Greeks of the liabilities. By using the replicating portfolio framework, the value of liabilities can be replicated using asset portfolios. These asset portfolios can use banking industry systems to continually monitor common measures such as delta, vega, gamma, and rho.

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Risk dashboard

A risk dashboard is a simple tool that presents the risk position of an insurance company. Using statistical numbers on a corporate level, the chief risk officer (CRO) and the complete board can view the risk exposures of the insurance company. Guided by the information on the risk dashboard, the CRO can monitor the risk development based on general financial market information. The information on the risk dashboard could include:

- market-value balance sheet
- economic capital (aggregate and per-risk type)
- the Greeks of the market-value balance sheet
- expected earnings
- economic data
- earnings at risk
- capital at risk

HOW TO DERIVE THE REPLICATING PORTFOLIO

There is a fairly generic process that can be followed for the determination of the replicating portfolio. Below is an outline of the process steps, followed by a more detailed description of the tasks.

1. Determination of the replication method
2. Selection of economic scenarios
3. Generation of calibration data
4. Definition of the universe of financial instruments
5. Definition of practical constraints
6. Determination of the optimisation of fit method and criteria

Determination of the replication method

Within the replicating-portfolio framework, there are different types of replication. The difference is in the optimisation target. The possible optimisation targets can be grouped into two types: market-value replication and cash-flow replication. Cash-flow replication is to replicate the future cash flows per time step under the different scenarios, while market-value replication attempts to replicate the market values of the liabilities.

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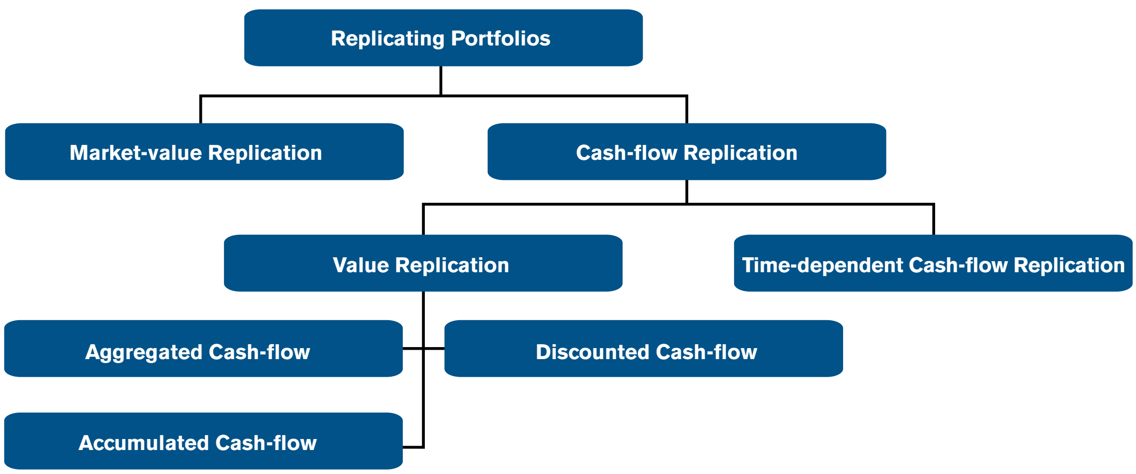
Within cash-flow replication there are two approaches:

1. Every cash flow at every time period will be replicated, This is
   - time-dependent replication

2. Cash flows of different time periods are expressed in one value and that one value is replicated. This one value could be
   - aggregated cash-flow replication
   - accumulated cash-flow replication
   - discounted cash-flow replication

The difference between these two types of optimisations can be compared with duration matching and cash-flow matching with respect to interest-rate risk. Duration matching gives a first-order optimal fit, but doesn't replicate the exact interest-rate exposure. Value replication fits the accumulated/discounted cash flows, but the timing of the cash flows does not have to match exactly. The cash-flow replication criterion is based on finding the exact timing of the liability cash flows, while value replication doesn't always require this type of fit.

FIGURE 8: CASH-FLOW REPLICATION



Time-dependent replication

This kind of replication involves matching cash flows for each time period independently. The overall replicating portfolio will consist of multiple smaller replicating portfolios, each representing a separate year. At each relevant time period, the cash flows are read in and the optimisation performed. The difference from the aggregate cash-flow method (see below) is that the cash flows from different years are assumed to be independently replicated.
Formulaically:

$$CF_{Liab}(s,t) \approx \sum_{p=1}^{P} w_p CF_{Asset}(p,s,t)$$

s = scenario
t = time step
w_p = weight for the asset in the universe of replicating assets
p = type of asset in the universe of replicating assets
$CF_{Liab}(s,t)$ is the aggregated cash flow of liabilities at time t in s-th scenario
$CF_{Asset}(p,s,t)$ is the cash flow of p-th asset at time t in s-th scenario

Aggregate cash-flow replication

Alternatively, the sum of the cash flows can be used for the replication. This method is mainly for short-term liabilities, where the time value of money has less impact.

Discounted cash-flow replication

This kind of replication involves matching a set of liability and replicating-portfolio cash flows discounted to a point in time.

$$\sum_{t=1}^{n} CF_{Liab}(s,t) * \prod_{j=0}^{t} (1+R(s,j))^{-1} \approx \sum_{t=1}^{n} \left[\sum_{p=1}^{P} w_p CF_{Asset}(p,s,t) * \prod_{j=0}^{t} (1+R(s,j))^{-1} \right]$$

R(s) is the one-year interest rate of scenario s.

Accumulated cash-flow replication

In this method, cash flows are rolled up using the forward risk-free rates, while taking into account their timing. The overall portfolio will consist of one replicating portfolio for all future years. The difference with the accumulated-cash-flow replication method is that the cash flows from the different years do not need to be independently replicated.

$$\sum_{t=1}^{n} CF_{Liab}(s,t) * (1+FR(t,n))^{n-t} \approx \sum_{t=1}^{n} \left[\sum_{p=1}^{P} w_p CF_{Asset}(p,s,t) * (1+FR(t,n))^{n-t} \right]$$

FR(t,n) is the forward rate from year t to n.

Market-value replication
This kind of replication involves matching the market value of liabilities and replicating-portfolio market values. However, the risk margin that is part of the market value of insurance liabilities cannot be fully replicated. The market value of liabilities in the formula below is marked with a star, to emphasise that the risk margin is not included.

This approach is also known as Greek-fitting. In this method, the market values under a number of scenarios are compared. Below is the equation for only one scenario.

$$MarketValue^{*}_{Liab} = \sum_{p=1}^{P} w_p MarketValue_{Asset}(p)$$

The difficulty with this type of replication lies in determining the market value of the liabilities for the different stress scenarios. For liabilities with options and guarantees, the values of those options and guarantees need to be determined for every stress scenario. Because the calculation of options and guarantees involves a substantial number of risk-neutral scenarios (say, 1,000), the total number of stochastic scenarios needed for the replication will grow rapidly. If there is a closed formula available for the calculation of the options and the guarantees, the calculation can be done quite easily, and only the stress scenarios will have to be calculated.

Advantages and disadvantages of the different methods
There are several advantages and disadvantages to each method, and neither method is clearly superior, although the cash-flow replication is employed most often. It is important to note that the choice of the type of replication is dependent on the insurance liability, its structure, the available resources, and time.

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The table in Figure 9 summarises the pros and cons of both methods. The advantages of one method are generally the disadvantages of the other method. In the table, advantages and disadvantages are not presented twice.

FIGURE 9

	CASH-FLOW REPLICATION	MARKET-VALUE REPLICATION (GREEK FITTING)
Advantages	1. For this method, it is not required to value options and guarantees for every node and every scenario as is necessary for discounted cash-flow replication. Consequently, there is no need for closed-form solutions or stochastic scenarios (risk-neutral scenarios for every replication scenario).	1. Closed formulas can be used to value the options and guarantees where those are available.
	2. Cash-flow replication supplies more information about the underlying structure of the liability.	.2. Market values are used for future financial reporting, and replication of the market value will be consistent with financial reporting.
	3. Generally, if there is a good fit for the cash flows, there is a good fit for the discounted cash flows, too. The results can be used to check the mark to model calculation of the insurance liabilities.	3. Greek fitting can be used for daily management of the financial risk.
	4. Under the condition of a stable insurance portfolio, the cash-flow information required for this method can be reused. Recalculation of the cash flow is not required and will give stable results.	
	5. Management actions and policyholder actions can be reflected in the cash flows, but whether replicating instruments can be found is heavily dependent on the complexity of the actions.	
	6. For multi-year economic capital purposes, the replication needs to be on a cash-flow level. By defining the replication of each cash flow separately, the future cash outflow can be taken into account	
Disadvantages	1. Cash-flow replication requires more detail, resulting in more scenarios and a more detailed optimisation.	1. Market-value replication requires the valuation of the options and guarantees for every node and every scenario of the replication. Consequently, risk-neutral scenario sets need to be generated for every scenario set that will be used for the replication. That is an arduous task.
	2. Constructing the replicating portfolio will take longer and requires more expertise.	2. The fit is optimised at one point in time. This will require a significant number of scenarios to be used in the replication.
	3. In some cases, few tradable financial instruments—or none at all—may be available to replicate the cash flows. Examples are equity instruments with durations longer than 10 years or market interest-rate instruments with durations longer than 30 or 50 years. If necessary, replication can be done using synthetic assets, but then the portfolio cannot be used for hedging purposes.	3. Depending on the purpose of the replication and the development of the insurance liabilities, frequent rebalancing may be required.

The replication process requires a large number of scenarios (typically around 1,000) to use as calibration scenarios. Liability models must be run across these scenarios to produce cash flows that can be used as input to the calibration process.

Selection of economic scenarios

The replication process requires a large number of scenarios (typically around 1,000) to use as calibration scenarios. Liability models must be run across these scenarios to produce cash flows that can be used as input to the calibration process. These scenarios should be chosen to reasonably represent a plausible range of economic conditions consistent with the stress tests that will be used with the replicating portfolio.

How many scenarios?

An important focus of replicating portfolios is to improve the speed of reporting and performance measurement. To be able to increase this speed, it is important to have a fast framework for identifying the replicating portfolios. The number of scenarios necessary for identification is highly dependent on the dynamics of the scenario-dependent cash flows. The number of scenarios can increase because of complications such as:

- book-value returns (which need modelling of extra accounting mechanisms for profit-sharing purposes)
- dynamic policyholder behaviour
- management actions

In general, 300 to 1,000 base scenarios are used, and 200 extreme scenarios, where the extreme scenarios depend on the type of liability. For example, for a traditional business with guaranteed interest rates and profit sharing, the required extreme scenarios mainly cover upside scenarios, because generally the cash flows don't change under negative stress scenarios. The base scenarios could be either real real-world or risk-neutral scenarios.

Distribution of scenarios

Pre-optimisation

The distribution of scenarios is a key driver of the scenario-dependent cash flows. By using a systematic set of scenarios that could be risk-neutral, real-world, or stress scenarios, the optimisation process will have a great range of scenario-dependent cash flows.

All three types—risk-neutral, real-world, and stress scenarios—are used because each shows a different type of behaviour.

All three types—risk-neutral, real-world, and stress scenarios—are used because each shows a different type of behaviour. Risk-neutral scenarios are used to try to replicate the behaviour of liability valuations. Real-world scenarios exhibit cyclical economic behaviour, which is missing in risk-neutral scenarios. Stress scenarios replicate extreme value-distribution effects in the tail of distributions. What models are used to generate the scenarios is not so important. It is much more important to use adequate types of scenarios consistent with the purpose of the replication. For example, if value replication is applied, the scenarios used to derive the values have to be market-consistent, and thus risk-neutral scenarios should be included. If the replicating portfolio will be simulated over real-world scenarios to derive economic capital, real-world scenarios as well as stress scenarios for the tail behaviour should be included in the optimisation.

Analysis

After the optimisation of the replicating portfolio, it is necessary to assess the goodness of fit using an out-of-sample set of scenarios. The goodness of fit should generate a fit of the same quality as the calibration scenarios. Performance validation by an out-of-sample test is necessary to verify that the replication is not specific to the optimised scenario set. The out-of-sample scenario set uses the same type of sets as within the optimisation—risk-neutral, real-world, and stress scenarios—but is different from the scenario set used to determine the optimal portfolio. Based on the performance across the types of sets and the purpose of the replicating portfolio, a choice can be made to adjust the weighting across these sets. For economic-capital calculations, for example, the optimisation will overweight negative-stress scenarios because the tail of the distribution is most important.

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Generation of calibration data

The first step in the replication process is the generation of liability cash flows across a large number of calibration scenarios. Such cash flows will usually be generated from a seriatim run of a traditional actuarial model. These models should generally reflect appropriate dynamic policyholder or company behaviour that is consistent with the economic scenarios.

Definition of the universe of financial instruments

It is important to define beforehand what the main goal of the replicating portfolio will be. Whether the replicating assets will actually be traded by the insurance company is important, as that constrains the liquidity of the replicating assets. Accordingly, the universe will be more limited than for the situation where an optimal fit is more important. For the latter, all kinds of over-the-counter (OTC) and synthetic products will be part of the universe.

One of the advantages of traded assets is that the market value of any specific asset is available at any time; consequently, the determined replicating portfolio can be used for hedging purposes. In the case of non-traded assets, a market-consistent valuation of the assets will be required.

Starting from theoretical replication, the universe of financial instruments should include all types of assets that could replicate the scenario-dependent payoffs of the liabilities. Most of the scenario dependencies are explained by the movement of liability cash flow according to economic conditions, such as interest rates, equity, property, and foreign-exchange rates. In the list of assets presented below, all such characteristics are included. OTC exotic assets could also be included, but those asset classes aren't always functional for live feed of financial markets and include higher trading expenses.

FIGURE 10: SUMMARY OF A REPLICATING PORTFOLIOS ASSET UNIVERSE

ASSET TYPE	DESCRIPTION
Zero-coupon Bonds	Fixed maturity proceeds, no coupons
Coupon-bearing Bonds	Fixed coupons and maturity proceeds
MBS	Mortgage-backed securities, allowing for partial prepayment over the term of each security based on prevailing interest rates
FRNs	Floating-rate notes, receiving coupons, with maturity linked to movement in short rates
Swaps	Receive or pay fixed coupons in exchange for floating coupons. Forward-start swaps, which allow for a delayed start of the swap, available as the replicating instrument
CMS	Constant maturity swaps, receiving or paying a swap rate in exchange for the short rate
Swaptions	Option to enter into payer or receiver swaps (both cash-settled and physically settled variations available)
Barrier Swaptions	Same as above, but knock-in or knock-out (up or down) path-dependent option (forward-start functionality available)
Interest Rate Caps/Floors	Series of call options on a specified interest rate
Index Assets	Range of equity or property total-return index-based assets
Index Derivatives	European calls or puts on index assets
Indexed-linked Bonds	Coupons and redemption linked to inflation index
FX Options	Options on the foreign-exchange index

Definition of practical constraints

A number of practical constraints will be applied if the assets of the replicating portfolio are effectively held in the portfolio and are not considered only for the calculation of a theoretical replicating portfolio. Four such practical constraints are listed below.

Trading activity and trading expenses

The 'exotic' behaviour of insurance liabilities has the effect of allowing a theoretical replicating portfolio to include illiquid and expensive-to-trade assets. The trading expenses are mostly dependent on the tradability/liquidity—for example, exotic OTC derivative contracts. By applying constraints on hedge cost and illiquidity, the replicating portfolio could decrease the exposure to these types of contracts. Such constraints could lead to a decrease in hedge effectiveness, but decrease the overall hedge cost. The constraints depend to a great degree on the expense/risk ratio of insurance companies.

Frequency of rebalancing

The replicating portfolio will be determined on a frequent basis. The assets that constitute the replicating portfolio may yet change significantly. For example, during the previous replication, 500 million zero-coupon bonds with a duration of 8.5 years were bought, while in the current replication (after three months) there is a better fit with zero-coupon bonds that have a duration of nine years. If the company uses the frequency of rebalancing as a constraint, then the original portfolio will be held.

Short and long positions
Products with regular premiums tend to have a replicating portfolio with short positions. If the replicating portfolio will be used for true hedging, short positions may be undesirable. In this case the replication should be done with a restriction on the amount of short positions. This may result in a less optimal but more applicable replicating portfolio.

Strategic/tactical asset mix

Concentration of assets
If the replicating portfolio will be used for hedging purposes, the company should be aware of a possible concentration risk. The replicating portfolios could lead to a high concentration of assets in liquid markets. From a hedging point of view, this is a logical effect. From an investment point of view, the replicating portfolio leads to a decrease in the diversification effect. A practical constraint can be to limit replicating assets to a predefined concentration level.

If the replicating portfolio will be used for hedging purposes, the company should be aware of a possible concentration risk. The replicating portfolios could lead to a high concentration of assets in liquid markets.

Short-term view financial markets
Asset-management departments are usually well developed within insurance companies and will have tactical views on the financial markets. Deviations from the strategic-asset mix need to be monitored on a daily basis.

Determination of optimisation of fit method and criteria
A variety of metrics can be used to evaluate the quality of the replication. This section gives a brief overview of two possible metrics.

Quality of the replication

Least squares
The quality of the replication is measured via several diagnostic measures. The easiest-to-apply measure is the mean-squared error.

$$\sum_{t=1}^{T}\sum_{s=1}^{S} q(s)\left[CF_{liab}(s,t) - CF_{Repl}(s,t)\right]^2$$

s = scenario
t = time step
q(s) = weight of the scenario

The result of the sum should be minimised.
A high correlation coefficient is required for successful hedging and economic-capital calculations. The first step is the calculation of the average cash flow of the liabilities across time and scenarios.

$$\overline{CF}_{liab} = \sum_{s=1}^{S}\sum_{t=1}^{T} q(s) CF_{liab}(s,t)$$

With constraint

$$\sum_{s=1}^{S} q(s) = 1$$

This is followed by the determination of the R^2.

$$R^2 = 1 - \frac{\sum_{s=1}^{S}\sum_{t=1}^{T} q(s)\left[CF_{liab}(s,t) - CF_{repl}(s,t)\right]^2}{\sum_{s=1}^{S}\sum_{t=1}^{T} q(s)\left[CF_{liab}(s,t) - \overline{CF}_{liab}(s,t)\right]^2}$$

It is common to have equal weights for q(s) for every scenario. However, for economic-capital calculations there is more interest in the results in the tail of the distribution. To take this into account, scenarios where the present value of the cash flows of the liabilities are high will have a greater weight than those with a low present value.

It is common to have equal weights for q(s) for every scenario. However, for economic-capital calculations there is more interest in the results in the tail of the distribution. To take this into account, scenarios where the present value of the cash flows of the liabilities are high will have a greater weight than those with a low present value.

The R^2 is not the only measure for optimising the replicating portfolio. The purpose of the replication, as well as the question of under which scenarios the fit is good or poor, should be evaluated in the decision-making process. For instance, if the replicating portfolio is used for economic-capital calculations, the emphasis is on the extreme scenarios and less on the scenarios around the mean. Taking that restriction into consideration may change the replicating portfolio and the fit.

Residuals

In order to get more insight into the replication for every time step of the projection, the relative or normalised residuals can be derived. The residual is the difference between the cash flow of the liabilities and the replicating portfolio. The graph below can be used to gain insight as to where the mismatch of cash flows reaches a relatively high level.

Mean normalised residuals can be used to compare different replicating portfolios and to review the evolution of the residuals over time.

$$r(s,t) = \frac{CF_{liab}(s,t) - CF_{repl}(s,t)}{CF_{liab}(s,t)}$$

$$\bar{r}(t) = \frac{1}{S}\sum_{s=1}^{S} r(s,t)$$

LIMITATIONS OF REPLICATING PORTFOLIOS

Replicating portfolios can help insurance companies improve the speed and accuracy of certain calculations. However, replicating portfolios have some limitations.

Insurance risk

Replicating portfolios can be very useful for quantifying and managing financial risk, but they add no value when measuring or managing insurance risk. For example, if an analyst wishes to quantify the impact of a swine flu pandemic, replicating portfolios are of no value. Furthermore, if there are material changes in insurance assumptions, the replication calibration process must be repeated.

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Scenario risk

The replication process involves using assets to represent liabilities, and it generally requires a large number of scenario sets (calibration scenarios) in order to capture dynamic policyholder behaviour that is embedded in liability cash flows. There is no guarantee that a good fit across calibration scenarios will produce a good fit across all scenarios unless the calibration scenarios are carefully selected. This is easy to see via a thought experiment. Suppose that 1,000 scenarios are used to derive the replicating portfolio. Now imagine the cash flows that this replicating portfolio will produce on scenario 1,001. Whatever they are, it is possible that the liabilities will produce entirely different cash flows across that scenario. Users have to realise that this can lead to a significant risk. This might particularly be a problem in any of the following situations:

There is no guarantee that a good fit across calibration scenarios will produce a good fit across all scenarios unless the calibration scenarios are carefully selected.

- replicating assets with cuspy behaviour (even if scenario 1,001 is very similar to one of the first 1,000 scenarios, there is no guarantee that the replicating portfolio will produce similar cash flows in the two scenarios; in contrast, liabilities rarely exhibit cuspy behaviour with respect to scenarios)
- leveraged portfolios with combinations of short and long positions
- liability portfolios with significant dynamic policyholder or company behaviour

Inadequate pools of real replicating assets

Some types of insurance liabilities may behave in a way that is inconsistent with real traded assets. Examples may include very long-tailed liabilities or inflation-indexed liabilities. This may require use of synthetic assets for replication. While this is not a theoretical problem as a modelling exercise, it makes the use of these portfolios of minimal value in the development of investment strategy.

Education

Replication work requires thorough knowledge of assets and liabilities and of the way they change in different economic situations. Actuarial modellers may need to be educated in the aforementioned area before they can fulfil the task of deriving the replicating portfolio without guidance. Choices for constraints need to be made, and the optimisation of these constraints may be more art than science. Actuaries, working with replicating portfolio tools, need to have a thorough understanding of assets and liabilities and how they react under different economic situations.

CLUSTER MODELLING:

AN ALTERNATIVE TO REPLICATING PORTFOLIOS?

With replicating-portfolio techniques, a portfolio of assets is chosen to represent the liabilities. An alternative to this, known as cluster modelling, involves using a subset of the liabilities, with an appropriate scalar applied to each cell, to represent the entire liability portfolio. To accomplish this, policies are grouped into a relatively small number of fairly homogeneous clusters, based on their similarity, along a set of variables considered appropriate by the modeller. Once each cluster of policies is determined, the most representative policy in the cluster is scaled up to represent the entire cluster.

Some of the variables used to measure similarity can be based on the liability in-force file (such as the distribution of account value across fund types); others can be based on policy-level results from one or a few calibration scenarios (such as the present value of future cash flows on each policy under a specified scenario). The variables should be chosen with an eye to the intended use of the model, which may include matching across economic scenarios, or may include matching with changes to mortality and lapse assumptions, for example.

Like replicating portfolios, clustering is an optimisation process. However, rather than using a pool of assets to represent the liabilities, a cluster-modelling algorithm will choose a small subset of the liabilities to represent all the liabilities.

Like replicating portfolios, clustering is an optimisation process. However, rather than using a pool of assets to represent the liabilities, a cluster-modelling algorithm will choose a small subset of the liabilities to represent all the liabilities. As with replicating portfolios, cluster modelling can greatly reduce model run time, often by three orders of magnitude or more. And cluster modelling offers the following additional advantages:

- By modelling liabilities using the original characteristics of the liabilities, the modeller can be more confident that the model will replicate well scenario 1,001, if it does the first 1,000.
- Cluster liabilities are real liabilities simply scaled up to represent the cells mapped into the model points. This reduces the risk of unexpected model behaviour.
- Cluster models can produce accurate regulatory, IFRS, or US GAAP income statements and balance sheets–not simply replicate market values or cash flows.
- Cluster models can be used to measure and model insurance risk–not just market risk.
- Cluster models can be used to model company capital needs across stochastic scenarios and model solvency risk.
- Cluster models commonly require far fewer calibration scenarios (usually less than five) and are easily and automatically refreshed when models are refreshed.

For further information on cluster modelling, see the Milliman report "Cluster analysis: A spatial approach to actuarial modeling" (available online at http://www.milliman.com/expertise/life-financial/publications/rr/pdfs/cluster-analysis-a-spatial-rr08-01-08.pdf).

Compared to replicating portfolios, cluster modelling is less suitable for hedging and risk management of market risks.

Other disadvantages of cluster modelling that are comparable to those of replicating portfolios include:

- Cluster modelling requires a certain amount of faith to say that it will work for 'out' scenarios.
- Because models change at every valuation date, some noise in period-to-period results is introduced that is difficult to quantify.

THE HOLISTIC RELATIONSHIP OF REPLICATING PORTFOLIOS AND CLUSTER MODELLING

Replicating portfolios and cluster modelling do not necessarily complete each other as a technique for efficient calculations. As we explained in the prior sections, there nevertheless exist several limitations of replicating portfolios that may be supplemented by cluster modelling. For example, if we postulate the work of a Solvency II solvency capital requirement (SCR) calculation with the standard formula approach, a replicating portfolio can almost instantaneously calculate market risk by using a closed formula for liability value. However, the replicating portfolio is useless in measuring life underwriting risk, for which cluster modelling would work as a strong tool. Although cluster modelling may reduce the time to calculate market risk as well, it still requires stochastic projection and would be slower than a replicating portfolio for that purpose.

Replicating portfolios and cluster modelling do not necessarily complete each other as a technique for efficient calculations. As we explained in the prior sections, there nevertheless exist several limitations of replicating portfolios that may be supplemented by cluster modelling.

FIGURE 11

RISK MODULE		CALCULATION OF CAPITAL CHANGE UNDER QIS4
Market Risk	Interest-rate Risk	Change of net asset value when interest-rate term structure is changed upward or downward. Shock rate is defined by maturity and multiplied to current yield curve.
	Equity Risk	Change of net asset value when equity price drops immediately. Shock rate is defined by type of equity market. Please note that this affects not only asset value, but also liability value of equity-linked products.
	Property Risk	Similar to equity risk, and shock rate is multiplied to property price.
	Currency Risk	Similar to equity risk, and shock rate is multiplied to foreign-currency-denominated asset value.
	Spread Risk	Change of asset value when credit spread changes. Generally not relevant to liability value.
	Market Risk Concentration	Capital change is calculated for an asset that has significant exposure to a particular issuer.
Life Underwriting Risk	Mortality Risk	Change of net asset value when mortality rate is increased by 10%.
	Longevity Risk	Change of net asset value when mortality rate is decreased by 25%.
	Disability Risk	Change of net asset value when disability rate is increased by 35% for the next year and 25% for the following years.
	Lapse Risk	Change of net asset when lapse increases (Up) or decreases (Down). Down and Up are 50% and 150% of base lapse rate respectively where such surrender increases liability value. Up is floored by 'Mass' lapse.
	Expense Risks	Change of net asset when unit expense is increased by 10%. Inflation is increased by 1% per year.
	Revision Risks	Change of net asset when annual amount payable for annuity is increased by 3%.
	Catastrophe Risks	0.15% of capital is at risk.

EXAMPLES

In this section the results for a variety of products are presented:

1. Single-premium variable annuity in the United States
   - Policyholder's dynamic behaviour is reflected according to in-the-moneyness of account value.
2. Premium-paying endowment product in the Netherlands with an interest guarantee of 3.5% and profit sharing based on the u yield return.
   - The u yield return is a return standard that is used for profit sharing within the Netherlands.
   - The u yield return is based on the yield-to-maturity of a package of government bonds with a term of between two and 15 years.

Single-premium variable annuity

Product description
Variable annuities (VA) offer a choice of guaranteed living benefits (GLBs):

- **Guaranteed Minimum Accumulation Benefits (GMAB)** - GMABs typically guarantee that the account value will be no less than a specified percentage of the premiums paid after a designated number of years. The simplest form is a 100%-of-return-of-premium guarantee after 10 years. More complicated types offer bonuses or the option to renew the guarantee for future terms upon expiration of the initial term.
- **Guaranteed Minimum Income Benefits (GMIB)** - GMIBs allow for a minimum amount of income in the form of an annuity. The guaranteed value is converted to a payout annuity utilising guaranteed purchase rates that are based on conservative interest and/or mortality assumptions, provided that the policyholder has been in force for a certain period, typically seven to 10 years. The latter is ordinarily referred to as a waiting period. The base guaranteed value prior to the annuitisation usually utilises roll-up and/or ratchet features. This benefit requires the policyholder to annuitise the contract.

GMWBs guarantee a specified amount of partial withdrawals regardless of the account-value performance. GMWB designs include a guarantee equal to a certain dollar amount (referred to as the benefit base).

- **Guaranteed Minimum Withdrawal Benefits (GMWB)** - GMWBs guarantee a specified amount of partial withdrawals regardless of the account-value performance. GMWB designs include a guarantee equal to a certain dollar amount (referred to as the benefit base). A commonly known feature is an annual 7% withdrawal that guarantees the policyholder 7% of benefit base for a minimum of 14.2 years. These benefits usually allow for step-ups every three to five years, when investment performance is good.
- **Guaranteed Lifetime Withdrawal Benefits (GLWB)** - GLWB designs guarantee withdrawals for life, rather than a certain dollar amount. As with other guarantees, benefit base usually exhibits some form of roll-up and/or ratchet features.

GLBs are regarded as a policyholder option provided by an insurance company. Policyholders may exercise their options by annuitising (GMIB), withdrawing (GMWB/GLWB), or keeping the policy for a certain period (GMAB) when the individual policyholder believes it is beneficial. The policyholder may give up the option by surrendering the policy when it becomes less valuable, such as when the account value goes up and a GLB is not likely to come into play. In order to reflect such policyholder behaviour, it is common to assume dynamic lapse, dynamic annuity election rates, and dynamic withdrawal rates in projecting future cash flows for VA.

Portfolio
For the replication, we chose a portfolio with in-force contracts with the following characteristics:

- 210,000 in-force policies of single-premium variable annuity (SPVA) with various types of minimum guarantees, including guaranteed death benefit (GMDB), guaranteed accumulation benefit (GMAB), and guaranteed minimum withdrawal benefit for life (GMWB)

- premiums and benefits denominated in U.S. dollars, with accessibility to various funds, such as U.S. equity funds, U.S. bond funds, and international equity funds

Replicating portfolio results
Based on 1,000 scenarios of cash-flow projections for the portfolio, the replicating-portfolio tool tries to find a replicating portfolio using the annual cash-flow replication method, with the following restrictions:

- Mean across the scenarios of the yearly cash flows that should replicate.

- Mean across the scenarios of the discounted cash flows that should replicate.

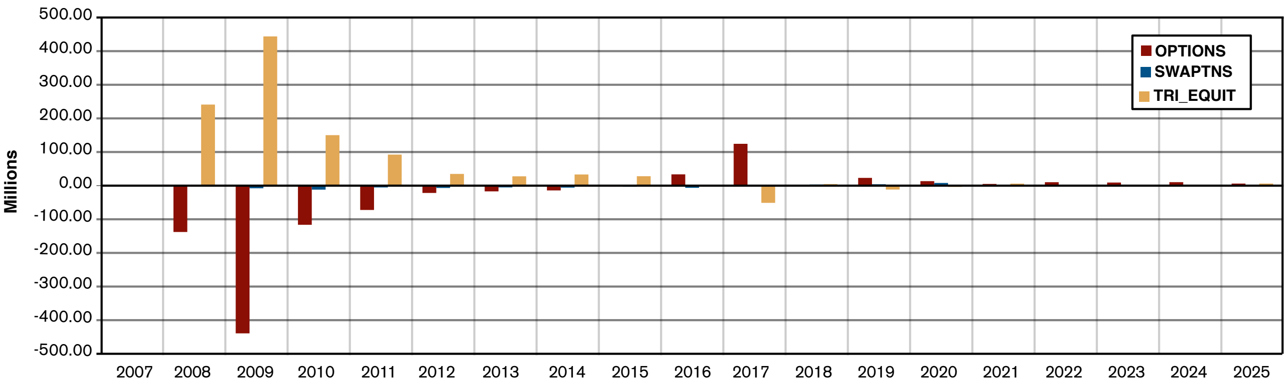
The replication resulted in the replicating portfolio shown in Figure 12.

Based on 1,000 scenarios of cash-flow projections for the portfolio, the replicating-portfolio tool tries to find a replicating portfolio using the annual cash-flow replication method.

FIGURE 12

ASSET	AMOUNT
Options	-493.8 M
Swaptions	-31.4 M
Total Return Index Equity	1021.2 M

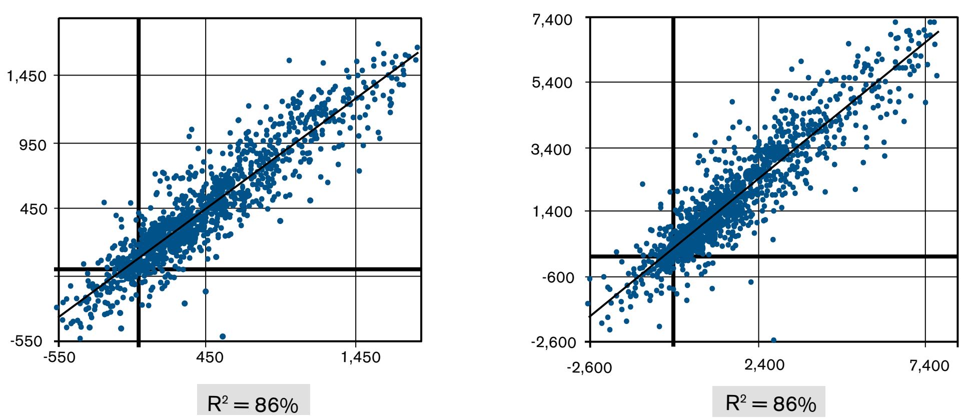
Breakdown RP by Year



The graphs in Figure 13 compare the replicating-portfolio discounted value and the replicating-portfolio accumulated value with both compared to the true discounted and accumulated cash flows. The graphs show that the replicating portfolio has an R^2 of 86% for both discounted and accumulated values.

FIGURE 13

Replicating Portfolio Discounted Value | **Replicating Portfolio Accumulated Value**

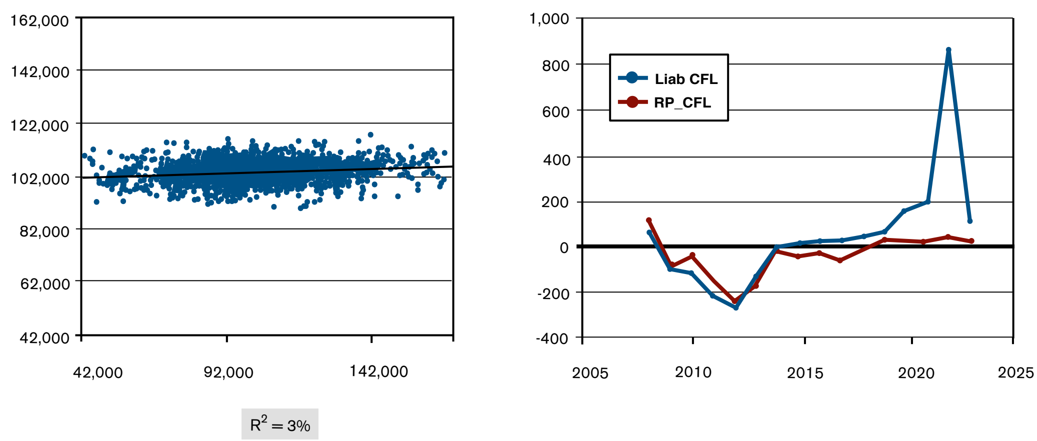


Although the replication was based on an annual cash-flow replication method, the R^2 of the replicated annual cash flows is only 3%. With the available assets, it is not possible to find a replicating portfolio with a high R^2 for the yearly cash flows. This can be explained by the fact that the cash-flow projections contain policyholder behaviour that leads to extreme cash flows. For example, if fund values increase rapidly, then the guarantee is far out of the money and policyholders will surrender their policies, which results in one very high cash flow in a certain period that is due to an economic event. Such movements could be replicated with Barrier options, but those options are currently not part of the replication tool.

The graphs in Figure 14 show the R^2 of the cash-flow replication of the first projection year and the cash flows of one simulation with an extreme cash flow in one projection year.

FIGURE 14

Replicating Portfolio Cash Flows | **Simulation Cash Flows**



Clustering results

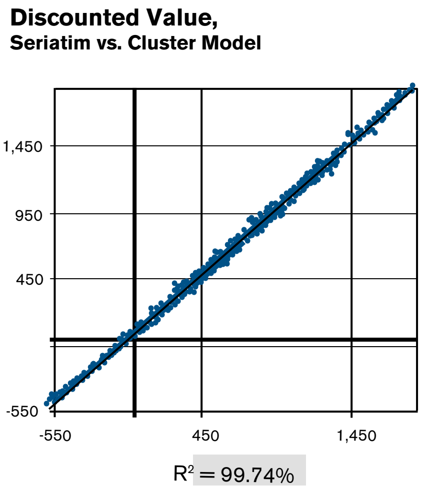
Cluster modelling can be more effective at reproducing liability results, particularly for challenging products to model, such as variable annuities with complex or extreme policyholder behaviour. In contrast to replicating portfolios, clustering is not constrained by the pool of assets available for use. Clustering employs a subset of the liabilities to represent all the liabilities, thus such behaviour is usually easier to reproduce.

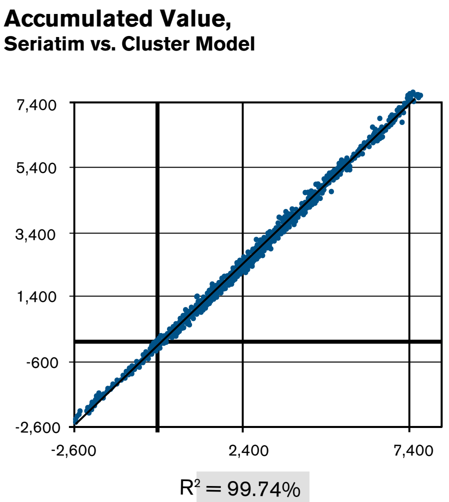
Cluster modelling can be more effective at reproducing liability results, particularly for challenging products to model, such as variable annuities with complex or extreme policyholder behaviour.

We created a cluster model to represent the same portfolio of variable annuities as those in Figure 14, using two calibration scenarios; in one, all funds steadily increased, while in the other, all funds steadily decreased. In fitting the cluster model, we primarily used the present value of key cash flows under the two scenarios, though we also used the account value by fund at the projection start date. The cluster process employs an automated algorithm to select a subset of the policies that closely reproduce the calibration data of the seriatim model. Because we are using a subset of the liabilities rather than a pool of assets to represent the liabilities, typically only a handful of calibration scenarios is needed, rather than the 100-500 scenarios that are commonly employed for replicating portfolios. We have found that one scenario is usually sufficient for traditional business, and two to three scenarios are usually sufficient for interest-sensitive or separate-account business. As noted, we used two calibration scenarios in this application.

We then ran the cluster model, which contained 250 cells, through the 1,000 stochastic scenarios and compared the present value of cash flows. The R^2 was 99.7%, as shown in the graphs in Figure 15.

FIGURE 15





By comparing this graph to the replicating-portfolio results, we see that cluster results compare quite favourably. Furthermore, the cluster algorithm makes it very easy to adjust the degree of compression to allow the user to select the desired trade-off between run time and precision. In the example above, we employed a compression ratio of 840-1, meaning that the average cluster cell represents 840 seriatim policies. This contrasts with traditional actuarial models for variable annuities, which typically have compression ratios on the order of 5- or 10-1. If we desire a greater R^2, we can increase the number of cluster cells, and if we need a faster run time, we can reduce the number of cells, likely at the cost of decreases in R^2.

By comparing this graph to the replicating-portfolio results, we see that cluster results compare quite favourably. Furthermore, the cluster algorithm makes it very easy to adjust the degree of compression to allow the user to select the desired trade-off between run time and precision.

With the compression ratio of 840-1, we would expect huge decreases in model run time relative to using the seriatim data. Typically, since liability run time is a substantial portion of the overall run time in a model such as this, the total run time will decrease by a ratio similar to the cell compression ratio.

However, because, unlike for replicating portfolios, there is not a closed-form solution for the cluster cells, the run time will still be materially greater than that of the replicating portfolio.

Premium-paying endowment with profit sharing

Product description
In this example, the replicating portfolio was determined for an in-force portfolio of Dutch endowment products with a guaranteed interest rate of 3.5% and profit sharing based on the 'u-return.'

For the replication, we chose a portfolio with the following characteristics:

- 5,500 in-force policies with a total mathematical reserve of € 7.9 million

Replicating portfolio results
Based on 1,000 scenarios of cash-flow projections for the portfolio, the replicating portfolio tool tried to find a replicating portfolio using the annual-cash-flow replication method, with the following restrictions:

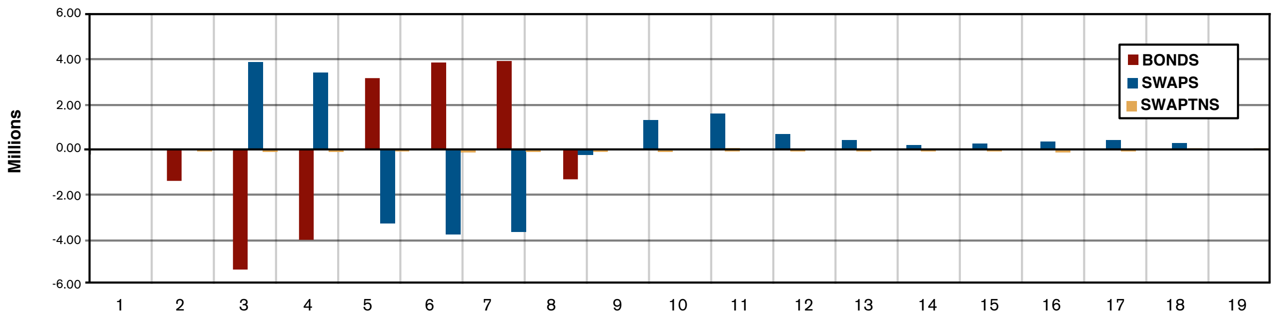
- Mean across the scenarios of the yearly cash flows that should replicate.
- Mean across the scenarios of the discounted cash flows that should replicate.

The replication resulted in the replicating portfolio shown in Figure 16.

FIGURE 16

ASSET	AMOUNT
Bonds	-1.4 M
Swaps	1.5 M
Swaptions	0.1 M

Breakdown RP by Year

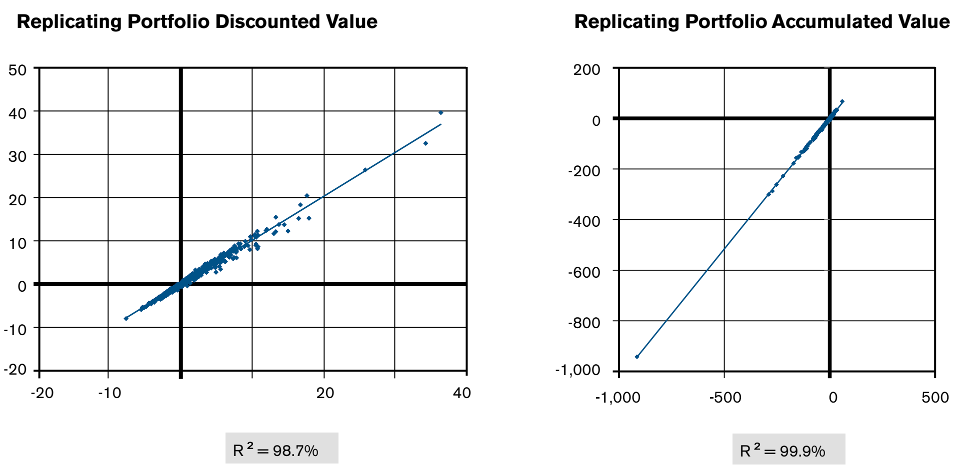


The optimal replicating portfolio has big short and long positions. A product with regular premiums commonly has short positions. The long position is to match the future benefit (outgo), and the short position is to match the future premiums. This can be changed to use restrictions on short positions. Generally, the fit will be poorer.

The graphs in Figure 17 compare the replicating-portfolio discounted value and the replicating-portfolio accumulated value, with both compared to the true discounted and accumulated cash flows. The graphs show that the replicating portfolio has an R^2 of 98.7% for discounted values and 99.9% for accumulated values.

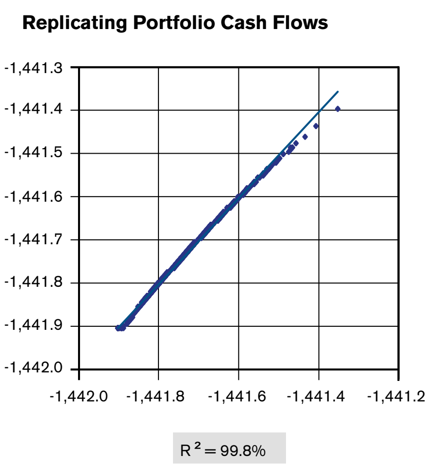
The optimal replicating portfolio has big short and long positions. A product with regular premiums commonly has short positions. The long position is to match the future benefit (outgo), and the short position is to match the future premiums.

FIGURE 17



The graphs in Figure 18 show the R^2 of the cash flow replication of the first projection year. For this product, the replication is also suitable for the year-by-year cash flows.

FIGURE 18



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