# RISK LABORATORY

BY PIERRE HEREIL AND THIERRY RONCALLI, GUEST COLUMNISTS

## Measuring the Risk Concentration of Investment Portfolios

Although the importance of risk considerations in asset allocation is widely admitted, the idea is often simplified to volatility minimization as described in classical portfolio theory. Mean-variance optimization, however, generally leads to portfolios concentrated in terms of weights. Slight differences in inputs can lead to dramatic changes in allocations and create portfolios heavily invested on very few assets. Therefore there is confusion between optimizing the volatility and optimizing the risk diversification that could be naively described by the general "don't put all your eggs in one basket" concept.

Sophisticated investors such as pension funds also pay significant attention to risk concentration, especially after the financial crisis of 2008. Seeing the importance of understanding risk concentration in portfolios, we reviewed and realized that a good way of defining it was to use the risk management work already done in connection with Basel II. In particular, the Gini coefficient, based on the famous Lorenz curve of inequalities, is one of the most widely used measures of concentration. In economics, the Lorenz curve is a graphical representation of the cumulative distribution function of the empirical probability of the distribution of wealth. The x-axis of the curve corresponds to the percentile of the population, ordered according to the statistic of interest (e.g. the income), and the y-axis to the cumulated value of the considered statistic. Applying this concept to the composition of an equity portfolio, the statistic of interest becomes the weight of stocks as the Lorenz curve represents the cumulated weights of the first x percent of the most important stocks. **In Fig. 1**, we provide an example of a Lorenz curve in which the 20 percent most weighted stocks represent 60 percent of the entire portfolio. In the case of no concentration, the Lorenz curve becomes a straight 45 degree line corresponding to a perfect equality of weights. Thus, perfect concentration corresponds to a case in which a single stock makes up 100 percent of the entire portfolio. Using the Lorenz curve, we can now define the Gini coefficient as a measure of dispersion which takes the value of 1 for a perfectly concentrated portfolio and 0 for an equally-weighted one.

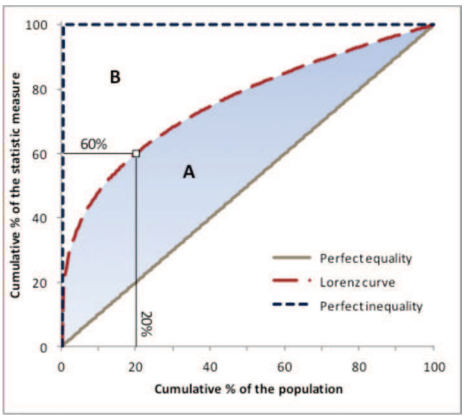



Fig. 1: The Lorenz curve

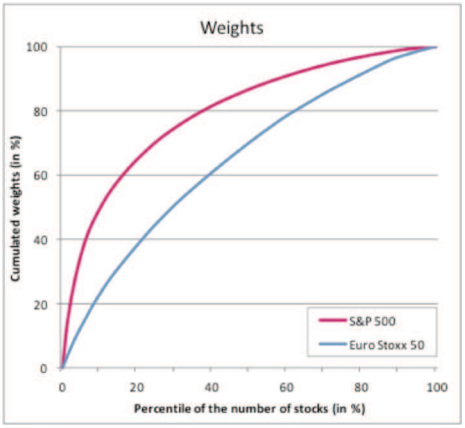
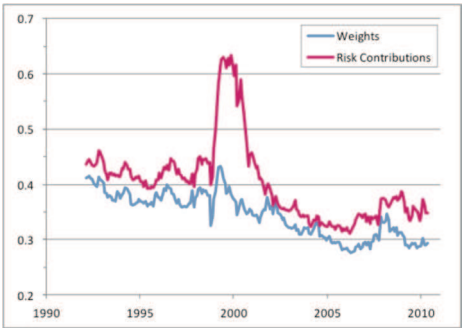



Fig. 2: Lorenz curves of the Euro STOXX 50 and S&P 500 indices (as of end of April 2011)




Source: *Lyxor*

Fig. 3: Evolution of the Gini coefficient of the Euro STOXX 50 index

The use of weight as the statistic of interest for the Lorenz curve is a first measure of risk concentration of a portfolio. This measure, however, may be improved by considering a statistic of interest that is directly linked to the risk (and not weight) components of the portfolio. For instance, risk may broken down as the sum of weights multiplied by the corresponding marginal risk contribution to the volatility of the portfolio. Risk contribution may also be used as the statistic of interest to compute the Lorenz curve, in which case, the Gini coefficient is an adequate measure of risk concentration.

Let us apply this concept to the Euro STOXX 50 index. To compute the volatility measure we consider the one-year empirical covariance matrix. Fig. 2 shows the Lorenz curve of concentration in which the statistic of interest is the weight. Computing the Gini coefficient, we obtain a value of 0.29 in terms of weight and 0.35 in terms of risk contribution. In Tab. 1, we report the Gini coefficients for several indices along with the value of the Lorenz curve for the 10, 25 and 50 percentiles, and arrive at the following conclusions: for the Euro STOXX 50 index, 10 percent (respectively 25 percent and 50 percent) of stocks represent 23 percent (respectively 45 percent and 70 percent) of cumulated weights and 26 percent (respectively 49 percent and 74 percent) of cumulated risks. Furthermore, it should be mentioned that a large difference between indices at any given date (**Tab. 1**) and across time **(see Fig. 3)** was noticed.

Capitalization-weighted indexation is the most common way to gain access to broad equity market performance. It is often backed by results of modern portfolio theory. Moreover, it provides two main advantages: simplicity of management (low turnover and transaction costs) and ease of understanding and replication. However, it also presents some drawbacks. For example, capitalization-weighted indexation is by definition a trend-following strategy where momentum bias leads to bubble risk exposure as weights of best performers increase. Moreover, as shown above, the absence of portfolio construction rules leads to concentration issues (in terms of sectors or stocks).

In this context, the concept of alternative-weighted indexation emerged after the dot.com bubble. An alternative-weighted index

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is defined as an index in which assets are weighted differently than in the market capitalization approach. We generally distinguish two forms of alternative-weighted indexation: fundamental and risk-based. Fundamental indexation defines the weights as a function of economic metrics like dividends or earnings, whereas risk-based indexation defines the weights as a function of individual and common risks. In January 2003, by launching the S&P 500 Equal Weight Index (S&P EWI), Standard & Poor's pioneered risk-based indexation. Indeed, the S&P EWI corresponds to a portfolio which is the least concentrated in term of weights as it minimizes the Gini coefficient when the statistic of interest is the weight. The same idea may be developed further by using the Gini coefficient when the statistic of interest is risk contribution. In this case, we obtain a portfolio which is called the equally-weighted risk contribution portfolio (ERC) or the risk parity portfolio. The properties of this portfolio have been extensively studied by Maillard et al. (2010). These authors show that this portfolio is unique, and lies between the minimum-variance and the equally-weighted portfolios. It is interesting to notice that minimizing risk concentration is thus a good compromise between minimizing volatility and maximizing weight diversification.

In order to illustrate the appealing properties of the ERC portfolio, we show in **Tab. 2** the simulation results of the ERC methodology applied to the Euro STOXX 50 universe. For comparison purposes we also added the Equally Weighted (EW) portfolio simulation and the Euro STOXX 50 index. **Tab. 2** shows a strong reduction of the ERC portfolio volatility and maximum drawdown in relation to both the market capitalization weighted and equally weighted indexes. With similar performance during bull markets and defensive behavior during crisis times, the ERC portfolio outperforms the Euro STOXX 50 over a complete market cycle.

*Pierre Hereil is a Quantitative Analyst and Thierry Roncalli is Head of Research & Development at Lyxor Asset Management in Paris.*
*This article is based on the Lyxor white paper "Risk-Based Indexation" available on the Lyxor website.*

## Measuring Portfolio Concentration

**TAB. 1 – GINI COEFFICIENT OF SEVERAL EQUITY INDICES (AS OF END OF APRIL 2011)**

| | GINI | | LORENZ CURVE OF THE WEIGHTS | | | LORENZ CURVE OF THE RISK CONTRIBUTIONS | | |
|---|---|---|---|---|---|---|---|---|
| INDEX TICKER | WEIGHTS | RISK CONTRIBUTIONS | 10 | 25 | 50 | 10 | 25 | 50 |
| SX5P | 0.27 | 0.28 | 21 | 44 | 69 | 21 | 43 | 70 |
| SX5E | 0.29 | 0.35 | 23 | 45 | 70 | 26 | 49 | 74 |
| INDU | 0.33 | 0.38 | 23 | 47 | 74 | 27 | 53 | 77 |
| BEL20 | 0.36 | 0.47 | 24 | 48 | 77 | 29 | 58 | 84 |
| CAC | 0.43 | 0.48 | 32 | 55 | 80 | 34 | 59 | 83 |
| DAX | 0.44 | 0.50 | 27 | 57 | 82 | 34 | 63 | 84 |
| AEX | 0.49 | 0.55 | 34 | 60 | 84 | 38 | 67 | 86 |
| HSI | 0.51 | 0.54 | 38 | 62 | 84 | 39 | 64 | 86 |
| NDX | 0.52 | 0.54 | 46 | 64 | 81 | 47 | 65 | 82 |
| HSCEI | 0.55 | 0.53 | 40 | 69 | 86 | 36 | 66 | 86 |
| SMI | 0.55 | 0.51 | 40 | 67 | 87 | 32 | 62 | 86 |
| NKY | 0.59 | 0.47 | 47 | 69 | 87 | 33 | 57 | 82 |
| SPX | 0.59 | 0.58 | 50 | 70 | 87 | 47 | 69 | 86 |
| UKX | 0.61 | 0.62 | 47 | 74 | 88 | 49 | 75 | 89 |
| SXXE | 0.62 | 0.67 | 50 | 75 | 89 | 55 | 78 | 91 |
| MEXBOL | 0.63 | 0.65 | 48 | 73 | 90 | 49 | 76 | 92 |
| IBEX | 0.64 | 0.71 | 53 | 77 | 89 | 62 | 83 | 93 |
| SXXP | 0.64 | 0.66 | 53 | 75 | 89 | 54 | 77 | 91 |
| SPTSX | 0.65 | 0.68 | 54 | 75 | 90 | 58 | 78 | 91 |
| TWSE | 0.79 | 0.80 | 72 | 86 | 95 | 73 | 87 | 95 |
| TPX | 0.82 | 0.82 | 74 | 90 | 97 | 74 | 90 | 97 |
| KOSPI | 0.87 | 0.90 | 81 | 94 | 98 | 86 | 97 | 99 |

**TAB. 2 - STATISTICS OF THE ERC, THE EW AND THE EURO STOXX 50 INDEXES**
*Data from 07/01/93 to 30/04/11; net total return performances*

| | ERC INDEX | EW INDEX | EURO STOXX 50 |
|---|---|---|---|
| CUMULATIVE RETURN | 746.3% | 582.6% | 361.1% |
| ANNUALIZED RETURN | 12.4% | 11.1% | 8.7% |
| VOLATILITY | 20.7% | 22.5% | 22.5% |
| SHARPE RATIO | 0.43 | 0.34 | 0.23 |
| TRACKING ERROR | 6.1% | 4.2% | NA |
| INFORMATION RATIO | 60.1% | 51.6% | NA |
| BETA | 0.89 | 0.98 | 1.00 |
| MAX DRAWDOWN | -55.1% | -60.4% | -64.6% |

Source: *Lyxor*

Sébastien Maillard, Thierry Roncalli and Jérôme Teiletche, The Properties of Equally Weighted Risk Contribution Portfolios, Journal of Portfolio Management, Summer 2010.