Longevity and disability risk analysis in enhanced life annuities

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Abstract

The paper focuses on the analysis of mortality and disability risk in enhanced life annuities. This cover increases a life annuity once the insured becomes Long Term Care (LTC) disabled. As lifetime duration and disability are affected by demographic trends, we use both mortality and morbidity projections, after evaluating a set of different projected scenarios using Italian national data on health and mortality.
Risk analysis at portfolio level is carried out using risk reserve to quantify risk. Further, some Risk-Based Capital requirements are identified according to such a measure, considering different time horizons and confidence levels.
The paper shows how the effect of demographic risk on Risk-Based Capital requirements could be reduced through either safety loading and, or capital allocation strategies. Finally the effect of reinsurance is also considered.

Keywords: Demographic risks, Long Term Care covers, risk reserve, risk-based solvency requirements, stop-loss reinsurance.

1 Introduction

The term "Long Term Care" refers to the care and protection given to the elderly who are no longer able of looking after themselves. LTC insurance benefit usually consists in an annuity payable when the insured becomes disabled and as long as he remains in such a condition. LTC is frequently offered combined with a life insurance policy.
The paper deals with Enhanced Life Annuity (ELA), consisting in a deferred life annuity increased once the insured becomes LTC disabled. Both demographic risk (longevity and disability) and financial risk affect ELA. Demographic risk lies in estimating the remaining life expectancy when the insured is both either healthy or disabled, while financial risk involves the insurer and his capability to apply the correct discounting process. In this paper we concentrate on the former risk disregarding the one coming from investment.
Disability and lifetime duration are influenced by demographic trends requiring the use of mortality and morbidity projections. Projection models allow representing the systematic component in periodic deviations on demographic trend. Such deviations could arise from stochastic nature of life expectancy in healthy or LTC disabled state (process risk), uncertainty in the parameters values of the model's projection (parameter risk) or model's projection inefficiency to represent the mortality and disability trend (model risk). Our analysis focuses on process and parameter risk.
We firstly select a suitable model to describe the behaviour of insured mortality and disability. Therefore, we construct a set of different projected scenarios using both deterministic (scenario analysis) and stochastic approach. Some analyses on these topics have been performed by Ferri-Olivieri [FO00] and Olivieri-Pitacco [OP01], which consider health data coming from UK insurance experience.
In this paper projected scenarios are developed from Italian national data on health and mortality (see ISTAT [Ist00], [Ist02]). The risk extent is evaluated making use of the risk reserve to which we refer to calculate the Risk-Based Capital (RBC) requirements. The effects of demographic risks could be reduced by safety loading, capital allocation strategies and reinsurance, or by hedging (e.g. by longevity bonds or by writing insurance products with contrasting mortality dynamics). By lessening the level of uncertainty within the cash flows, the insurer should be able to reduce the RBC requirements strengthening the guarantees in the contract.
The paper is organized as follows. In Section 2 the probabilistic framework of an enhanced life annuity is defined consistent with the multiple state models, as well as the transition intensities estimate needed for pricing and reserving. Section 3 deals with the development of a set of projected demographic scenarios, while Section 4 concerns actuarial values of an enhanced life annuity. The analysis of longevity and disability risk, based on risk reserve, is illustrated in Section 5. Section 6 is devoted to quantify the solvency requirements, while Section 7 designs a stop-loss reinsurance, linking the reinsurer's intervention to the risk reserve. Finally, some concluding remarks are presented in Section 8.

2 Probabilistic framework

2.1 Multiple state model

LTC insurance is usually modelled according to a multiple state model (an extensive description of such models can be found in [HP99]). Let $S(t)$ represent the random state occupied by the insured at time t, for any $t \geq 0$, where t represents the policy duration and 0 the time of entry. The possible realizations of $S(t)$ are: 1 = "active" (or healthy), 2 = "LTC disabled", 3 = "dead". Only one level of disability is considered. We assume $S(0) = 1$. Moreover, the possibility of recovery from the LTC state is disregarded due to the usually chronic character of disability; transition $2 \rightarrow 1$ is therefore ignored. The graph in Fig. 1 illustrates the states occupied by the policyholder in a time instant and the possible direct transitions between states.
Let us assume that the stochastic process $\{S(t);\, t \geq 0\}$ is a time-continuous, time

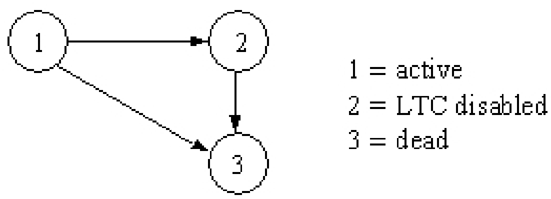

Fig. 1. A multiple state model for LTC insurance with one level of disability

inhomogeneous, three states Markov chain. Let us define transition probabilities:

$$P_{ij}(t,u) = \Pr\{S(u) = j \;|S(t) = i\} \qquad 0 \leq t \leq u, \qquad i,j \in \{1,2,3\} \tag{1}$$

and transition intensities:

$$\mu_{ij}(t) = \lim_{u \to t} \frac{P_{ij}(t,u)}{u-t} \qquad t \geq 0, \quad i,j \in \{1,2,3\}\,, \quad i \neq j \tag{2}$$

According to model depicted in Fig. 1, the following set of Kolmogorov forward differential equations holds, respectively for transition and permanent probabilities:

$$\frac{dP_{ij}(t,u)}{du} = \sum_{k;k \neq j} P_{ik}(t,u)\mu_{kj}(u) - P_{ij}(t,u)\mu_j(u) \tag{3}$$

$$\frac{dP_{ii}(t,u)}{du} = -P_{ii}(t,u)\mu_i(u) \tag{4}$$

where $\mu_i(u) = \sum\limits_{j;j \neq i} \mu_{ij}(u)$.

Solutions of equations (3) and (4) are given by the following probabilities:

$$P_{11}(t,u) = exp\left(-\int_t^u \mu_{12}(s) + \mu_{13}(s)\; ds\right) \tag{5}$$

$$P_{22}(t,u) = exp\left(-\int_t^u \mu_{23}(s)\ ds\right) \tag{6}$$

$$P_{12}(t,u) = \int_t^u P_{11}(t,s)\mu_{12}(s)P_{22}(s,u)\ ds \tag{7}$$

$$P_{13}(t,u) = \int_t^u P_{11}(t,s)\mu_{13}(s) + P_{12}(t,s)\ \mu_{23}(s)\ ds \tag{8}$$

$$P_{23}(t,u) = \int_t^u P_{22}(t,s)\mu_{23}(s)\ ds \tag{9}$$

2.2 Transition intensities estimate

In Italy LTC covers are recent products so insurance experience data about LTC claims are still inadequate for pricing and reserving. Population data collected from Italian national source represent, at that moment, the best available data.
In numerical computation we use Italian Life-Table (SIM 1999) and statistical data about people reporting disability (see [Ist00]).
Probabilities $P_{11}(t,u)$, $P_{12}(t,u)$ and $P_{13}(t,u)$ have been calculated for each integer (t,u) starting from prevalence rates of LTC disabled[3]. About LTC disabled mortality we assume that it is directly related to actives mortality (since specific data are lacking, such hypothesis seems quite reasonable):

$$P_{23}(t,t+1) = k(t)P_{13}(t,t+1) \tag{10}$$

Where values of the time-dependent coefficient, $k(t)$, come from experience data of an important reinsurance company. The intensities $\mu_{ij}(t)$ can be estimated from equations (5), (6) and (7) by the following approximation formulae:

$$\mu_{13}(t) = -\mu_{12}(t) - 0.5\,[lnP_{11}(t-1,t) + lnP_{11}(t,t+1)] \tag{11}$$

$$\mu_{23}(t) = -0.5\,[lnP_{22}(t-1,t) + lnP_{22}(t,t+1)] \tag{12}$$

$$\mu_{12}(t) = 0.5\left[\frac{P_{12}(t-1,t)}{P_{11}(t-1,t)} + \frac{P_{12}(t,t+1)}{P_{22}(t,t+1)}\right] \tag{13}$$

Let us consider $x = 50$ as age at policy issue. Numerical results suggest to approximate actives mortality with a Weibull law. Instead, transition intensity $\mu_{12}(t)$ shows an exponential behaviour suggesting the use of Gompertz law:

$$\mu_{13}(t) = \frac{\beta}{\alpha}\left(\frac{x+t}{\alpha}\right)^{\beta-1} \qquad \alpha,\beta > 0 \tag{14}$$

[3] Note that the ISTAT survey provides prevalence rates of LTC disabled by age groups, while probabilities estimation requires annual values. According to hypothesis of constant rates in each age group we parametrically adjust prevalence rates using the least squares method, obtaining an exponential function.

$$\mu_{12}(t) = \eta e^{\lambda(x+t)} \qquad \eta, \lambda > 0 \tag{15}$$

Mortality intensity of LTC disabled is expressed in terms of $\mu_{13}(t)$ by the time-dependent coefficient $K(t)$, well approximated by the function $\exp(c_0 + c_1 t + c_2 t^2)$:

$$\mu_{23}(t) = K(t)\mu_{13}(t) \tag{16}$$

3 Demographic scenarios

When LTC annuity benefits are concerned a key point in actuarial evaluations is to measure the risk coming from the uncertainty in disability duration, i.e. the time spent in LTC disabled state. To this purpose it is necessary to make some assumptions about the link between mortality and disability. As regards to such link three main theories have been formulated: pandemic, equilibrium and compression theory (for an overall review see [OP01]).

The uncertainty in future mortality and disability of the elderly suggests to adopt different projected scenarios in benefits evaluation. Therefore, taking into account the above mentioned theories we define some reliable scenarios, including projection of both mortality and disability. A similar method has been performed by Ferri-Olivieri [FO00]. Let us define the time expected to be spent in state j for a person in state i at time t:

$$\bar{e}_{ij}(t) = \int_t^\infty P_{ij}(t,u)du \tag{17}$$

In order to evaluate the time spent in state 1 and 2 we calculate $\bar{e}_{11}(t)$, $\bar{e}_{12}(t)$ and $\bar{e}_{22}(t)$. Since only one cohort of policyholders with same age at policy issue and same year of entry has been considered, the calendar year has been omitted in the life expectancy notation. Note that the total life expectancy for a healthy is denoted by: $\bar{e}_{1.}(t) = \bar{e}_{11}(t) + \bar{e}_{12}(t)$.

The theories above mentioned can be expressed in terms of life expectancies:

- Compression: $\bar{e}_{1.}(t)$ rises with time increase with a major contribution (in relative terms) from $\bar{e}_{11}(t)$;
- Equilibrium: both $\bar{e}_{11}(t)$ and $\bar{e}_{12}(t)$ rise with time increase, at similar rates;
- Pandemic: $\bar{e}_{1.}(t)$ rises with time increase, with a major contribution (in relative terms) from $\bar{e}_{12}(t)$.

The basic scenario, S_B, defined according to ISTAT data, has been considered as starting point to construct projected scenarios. Projected mortality has been modelled from ISTAT projections (low, main and high hypothesis, see [Ist02]), where total life expectancy increases anyhow. A different set of Weibull parameters (α, β) has been evaluated for each ISTAT projection. Furthermore, in the projections we suppose an unchanged ratio between disabled and actives mortality (i.e. a constant value of $K(t)$). About transition intensity, $\mu_{12}(t)$, three different set of Gompertz parameters (η, λ) have been defined starting from basic scenario. Only information available at the initial time 0 is taken into account, therefore shape parameter λ is supposed to be constant; while position parameter η has been determined to represent, regards to the basic scenario, a 40% decrease (Hp. a), a 10% decrease (Hp. b) and a 20% increase (Hp. c) in disability trend, respectively. By combining mortality and disability projections nine scenarios have been obtained (see Table 1), where $p(S_i)$ is the probability assigned to scenario S_i. In the paper scenarios probabilities are chosen assuming no correlation between parameters of both mortality and disability trends. Obviously, numerical results obtained in next sections are strongly affected by the assumptions formulated; different hypotheses could lead to different numerical outcomes in risk analysis, nonetheless general results should preserve their validity.

Table 1. Life expectancy at age 65

Scenario	$p(S_i)$	$\frac{\bar{e}_{11}(0)}{\bar{e}_{1.}(0)}$	$\frac{\bar{e}_{11}^{S_i}(0)}{\bar{e}_{11}^{S_B}(0)}-1$	$\frac{\bar{e}_{12}^{S_i}(0)}{\bar{e}_{12}^{S_B}(0)}-1$	$\frac{\bar{e}_{1.}^{S_i}(0)}{\bar{e}_{1.}^{S_B}(0)}-1$
S_{1a}	4%	91.68%	17.96%	−5.87%	15.53%
S_{1b}	12%	88.49%	9.26%	24.92%	10.86%
S_{1c}	4%	85.70%	2.09%	49.76%	6.96%
S_{2a}	12%	90.66%	22.91%	11.37%	21.73%
S_{2b}	36%	87.14%	13.35%	46.98%	16.79%
S_{2c}	12%	84.10%	5.56%	75.50%	12.71%
S_{3a}	4%	89.52%	27.80%	31.48%	28.18%
S_{3b}	12%	85.67%	17.34%	72.54%	22.97%
S_{3c}	4%	82.35%	8.90%	105.15%	18.74%

4 Actuarial values

Longevity risk and risk of needing care could be insured with a single product, called Enhanced Life Annuity (ELA). It consists in a life annuity whose level increases once LTC becomes necessary. We consider an ELA providing the following benefits:

1. a deferred annuity paid at an annual rate $b_1(t)$, when the insured is healthy;
2. a deferred enhanced annuity paid at an annual rate $b_2(t) > b_1(t)$, when the insured is LTC disabled.

All benefits are supposed to be constant with time. The difference $(b_2 - b_1)$ represents the disability benefit enhancement. Note that if $b_1 = 0$ the contract becomes a deferred LTC annuity (LTCA).

Let n be the deferment period and ω the maximum policy duration related to residual life expectancy at age x and let $v(s,t) = \prod_{h=s+1}^{t} v(h-1,h)$ be the value at time s of a monetary unit at time t; the actuarial value at time 0 of these benefits, $\Pi(0,\omega)$, is given by:

$$\begin{aligned}\Pi(0,\omega) = b_1 P_{11}(0,n)v(0,n)a_{11}(n,\omega) + b_2[P_{11}(0,n)a_{12}(n,\omega)\\ +P_{12}(0,n)a_{22}(n,\omega)]\ v(0,n)\end{aligned} \tag{18}$$

where: $a_{ij}(t,u) = \sum_{s=t}^{u-t-1} P_{ij}(t,s)v(s,t) \qquad \forall i,j \in 1,2$

Let π be the annual constant premium and π^T the annual constant gross premium, paid if the insured is active. Gross premium is defined as:

$$\pi^T = \frac{\pi}{1 - \frac{\alpha}{a_{11}(0,n)} - \beta - \gamma \frac{a_{11}(0,\omega)+a_{12}(0,\omega)}{a_{11}(0,n)}} \tag{19}$$

where α, β and γ represent the premium loadings for acquisition, premium earned and general expenses, respectively and $\pi = \frac{\Pi(0,\omega)}{a_{11}(0,n)}$.
Reserve for active lives is defined as:

$$V_1(t) = \begin{cases} \Pi(t,\omega) + \epsilon_1(t) - \pi^T a_{11}(t,n) & t < n \\ b_1 a_{11}(t,\omega) + b_2 a_{12}(t,\omega) + \epsilon_1(t) & t \geq n \end{cases} \tag{20}$$

where ϵ_1 is the expense loading for active lives:

$$\epsilon_1(t) = \pi^T \begin{cases} \alpha + \beta a_{11}(t,n) + \gamma \left[a_{11}(t,\omega) + a_{12}(t,\omega)\right] & t = 0 \\ \beta a_{11}(t,n) + \gamma \left[a_{11}(t,\omega) + a_{12}(t,\omega)\right] & 1 \leq t < n \\ \gamma \left[a_{11}(t,\omega) + a_{12}(t,\omega)\right] & t \geq n \end{cases} \tag{21}$$

While reserve for LTC disabled is given by:

$$V_2(t) = \begin{cases} b_2 P_{22}(t,n) v(t,n) a_{22}(n,\omega) + \epsilon_2(t) & t < n \\ b_2 a_{22}(t,\omega) + \epsilon_2(t) & t \geq n \end{cases} \tag{22}$$

where ϵ_2 is the expense loading for LTC disabled:

$$\epsilon_2(t) = \gamma a_{22}(t,\omega)\pi^T \qquad \forall t > 0 \tag{23}$$

In Table 2 we report the single premium rates for both LTCA and ELA under each scenario, assuming the following hypotheses: $x = 50$, ω=70, n=15, b_2=200, α=60%, β=2%, γ=8%. Since the paper focuses exclusively on the biometric risk analysis, we adopt a deterministic financial structure with $i(h-1,h) = \bar{i} = 3\%$ for each h.
Consistently with the product features, we assume the following relationship between ELA benefits: $b_1 = \frac{1}{2}\, b_2$. Table 2 shows the percentage variation of each premium rate compared to the one calculated under central scenario, S_{2b}, to which is assigned the highest probability of happening.

Table 2. Single premium rates under scenario S_i

	LTCA		ELA	
Scenario	$\frac{\Pi(0,\omega)\vert S_i}{b_2}$	$\frac{\Pi(0,\omega)\vert S_i}{\Pi(0,\omega)\vert S_{2b}}$ -1	$\frac{\Pi(0,\omega)\vert S_i}{b_2}$	$\frac{\Pi(0,\omega)\vert S_i}{\Pi(0,\omega)\vert S_{2b}}$ -1
S_{1a}	0.6365	−37.52%	4.4104	−5.14%
S_{1b}	0.8616	−15.42%	4.3452	−6.55%
S_{1c}	1.0486	2.95%	4.2830	−7.88%
S_{2a}	0.7546	−25.92%	4.6998	1.08%
S_{2b}	1.0186	0.00%	4.6495	0.00%
S_{2c}	1.2374	21.48%	4.6007	−1.05%
S_{3a}	0.8890	−12.73%	4.9968	7.47%
S_{3b}	1.1963	17.44%	4.9654	6.79%
S_{3c}	1.4503	42.38%	4.9335	6.11%

It is worth noting that the premium rates variability between different scenarios is higher for LTC stand alone rather than for ELA. Such variability demonstrates the greater biometric riskiness of LTCA. Next sections will be devoted to develop a risk model in order to define and measure such a risk.

5 Demographic risk analysis

With the aim of analysing risk deriving from uncertainty in mortality and disability trends, we consider the risk reserve or solvency margin at the end of each year t, which according to a well-known formula of risk theory (see [DPP94]), is defined as:

$$U(t) = U(t-1) + P^T(t) + J(t) - E(t) - B(t) - \Delta V(t) - K(t) \tag{24}$$

where $P^T(t)$ is the gross premiums income, $J(t)$ the investment returns on assets, $E(t)$ the expenses, $B(t)$ the outcome for benefits, $\Delta V(t)$ the increment in reserve and $K(t)$ the capital flows (if $K(t) > 0$ insurance company distributes dividend, if $K(t) < 0$ stockholders invest capital).

Risk reserve represents the insurer's ability to meet liabilities, therefore it can be considered a valid tool to evaluate the insurance company solvency and, more generally, for risk management assessment.

As the insurance company aims at preserving risk reserve positive, we focus on the left tail of the U-distribution. The risk analysis is performed in term of both deterministic and stochastic approach. In the first approach a certain scenario is selected and risk analysis is developed according to such scenario. On the contrary, in the stochastic one each scenario, S_i, is considered as a possible result with a given probability, $p(S_i)$.

Let us consider one cohort of policyholders with same age at policy issue, $x = 50$, same year of entry (time 0), same benefit amounts and same risk class. For maximum policy duration, deferment period, expenses loadings and benefits amount we adopt the values reported in the previous section. Premiums are calculated under central scenario, S_{2b}. Further, we assume $K(t) = 0 \; \forall t$.

In order to make data comparable all results are expressed in term of ratio between risk reserve and the actuarial value at time 0 of future benefits under central scenario, $\Pi(0, \omega)\,|S_{2b}$, for a portfolio of 1,000 insureds. Such a ratio is denoted by $u(t)$.

Figure 2 shows risk reserve results for a portfolio of 1,000 LTC annuities under both deterministic (left side) and stochastic approach (right side), assuming no safety loading. In case of stochastic approach, outcomes underline a great riskiness increase, as the model incorporates systematic deviations from central scenario.

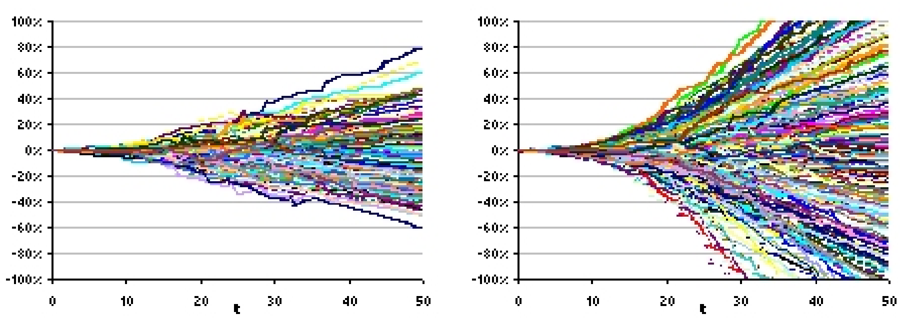

Fig. 2. $u(t)$ for 1,000 LTCA - deterministic and stochastic approach

Figure 3 shows risk reserve results for a portfolio of ELA (1,000 insureds) under both deterministic (left side) and stochastic approach (right side), assuming no safety loading.

By comparing Figure 2 and 3 we note that under both deterministic and stochastic approach ELA is less riskier than LTCA (see premium variability in Table 2). As a result, we can state that combining LTC insurance with a life annuity in a single policy is therefore more convenient for the insurer respect to the LTC stand alone.

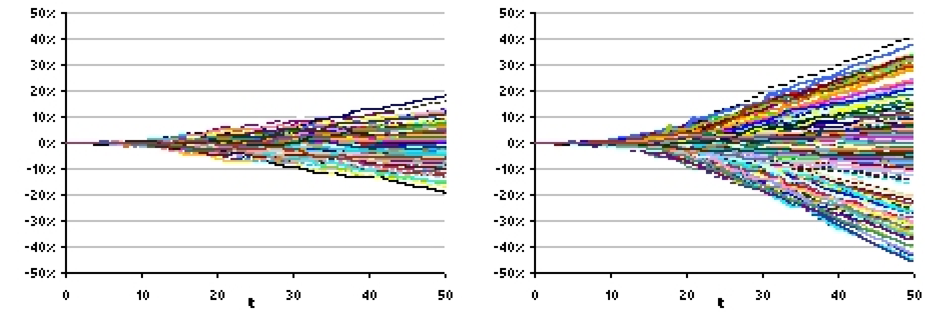

Fig. 3. $u(t)$ for 1,000 ELA - deterministic and stochastic approach

Figure 4 shows simulation results for the ELA under stochastic approach in term of ratio between risk reserve and reserve for a portfolio of both 1,000 insureds (left side) and 10,000 insureds (right side).
As a consequence of systematic risk, in Fig. 4 (right side) we can observe three branches of trajectories and a reduction of volatility within the trajectories. Since the mortality weight is stronger than disability, only three branches are visible instead of nine.
Therefore, increasing the number of insureds, risk of random fluctuations decreases (it is a pooling risk), while systematic deviations risk keeps on (it is a non pooling risk).

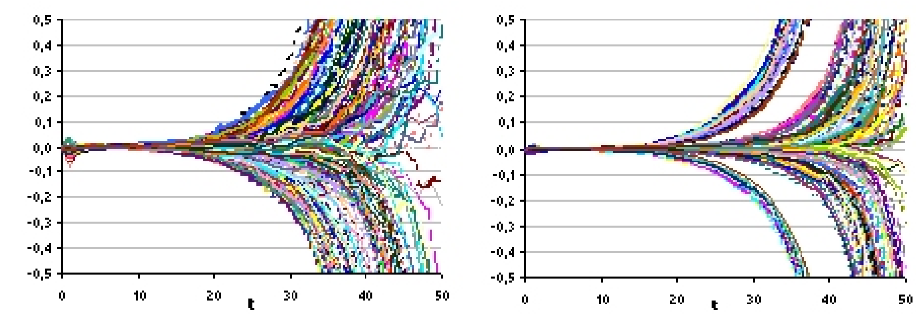

Fig. 4. $U(t)/V(t)$ for 1,000 and 10,000 ELA - stochastic approach

Let us introduce a safety loading on demographic pricing bases consisting in a 10% reduction of death probabilities that produces a premium increase of 4.6%.
As a result, risk reserve translates towards the top, reducing portfolio riskiness but not its variability (see Fig. 5).

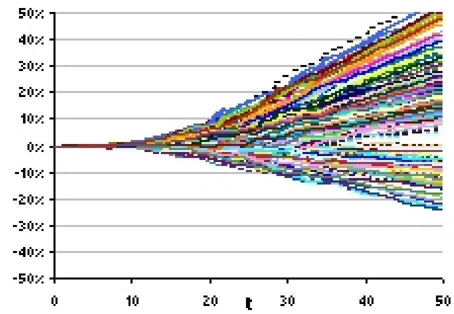

Fig. 5. $u(t)$ for 1,000 ELA - stochastic approach with safety loading

Now, let us suppose that the insurance company decides to realize the following capital flows policy:

$$K(t) = \begin{cases} U(t) - 0.08V(t) & \frac{U(t)}{V(t)} \geq 0.1 \\ 0.5[U(t) - 0.06V(t)] & 0.06 \leq \frac{U(t)}{V(t)} < 0.1 \\ 0 & 0.04 \leq \frac{U(t)}{V(t)} < 0.06 \\ U(t) - 0.04V(t) & \frac{U(t)}{V(t)} < 0.04 \end{cases} \tag{25}$$

According to (25) if the ratio $\frac{U(t)}{V(t)}$ is under 4% (this limit has been chosen consistently with the solvency margin fixed by EU Directive Solvency I) stockholders invest capital re-establishing company financial equilibrium. If the ratio exceeds 4% dividends rise with risk reserve enforcing financial equilibrium by autofunding.
Simulation results are presented in Figure 6. The right side shows the lower and the upper bounds of ratio $\frac{U(t)}{V(t)}$ stated in rule (25), while the left side gives evidence of risk reserve lower bound corresponding to the reserve behaviour.

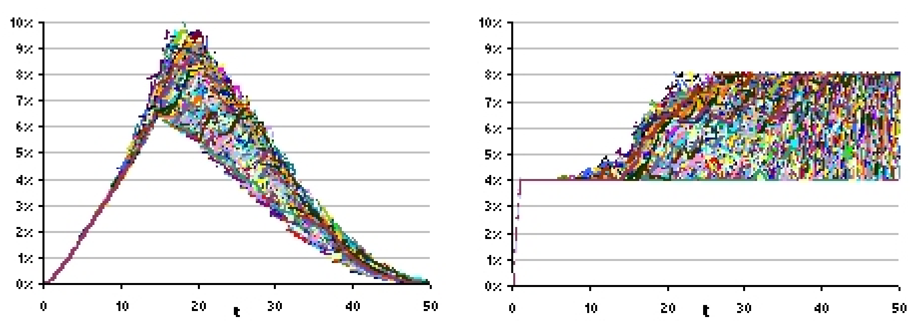

Fig. 6. $u(t)$ and $U(t)/V(t)$ for 1,000 ELA - stochastic approach with capital flows policy

Now, in order to quantify the capital provision from the investors, we calculate the random present value at time 0 of future capital flows:

$$Y^K(0,T) = -\sum_{s=0}^{T} K(s)v(0,s) \tag{26}$$

Further, we evaluate the present value of capital that shareholders should invest to assure the insurer solvency with a $(1-\epsilon)$ confidence level, i.e. the $(1-\epsilon)$ percentile of $Y^K(0,T)$ (see Table 3). Results are expressed as a ratio of $Y^K_{1-\epsilon}(0,T)$ to the actuarial value at time 0 of future benefits under central scenario for 1,000 insured: $y^K_{1-\epsilon}(0,T)$.

Table 3. $y^K_{1-\epsilon}(0,T)$ for 1,000 ELA

$y^K_{1-\epsilon}(0,T)$	T=15	T=30	T=50
$\epsilon = 0.005$	6.70%	8.93%	9.99%
$\epsilon = 0.01$	6.54%	8.28%	9.63%
$\epsilon = 0.02$	6.34%	7.89%	8.93%

It should be remarked that $y^K_{1-\epsilon}(0,T)$ can be compared with the RBC requirements on the time horizon $(0,T)$ reported in the next section.
As shown in Table 3 the required capital investment is included in the range 6%-10% of single premium income. It can be noticed that $y^K_{1-\epsilon}(0,T)$ shows high values for T=15, although longevity risk occurs after deferment period. This behaviour is a consequence of the capital retaining needed to achieve 4% of reserve (see (25)).

6 Risk-based requirements

Risk-Based Capital (RBC) is a method for assessing the solvency of an insurance company by calculating capital requirements reflecting the size of overall risk exposures of an insurer.
Let us consider RBC requirements based on risk reserve distribution. They provide monetary values summarizing the information contained in the left tail of such a distribution. In the paper we analysed solvency requirements from two different points of view.

6.1 RBC requirements at year *T*

RBC requirements can be defined taking into account both VaR (Value at Risk) and TVaR (Tail VaR) of risk reserve distribution.
Let $U_\epsilon(t)$ be the ϵ-th quantile of the U-distribution at time t, assuming no initial capital $(U(0) = 0)$, the VaR and TVaR for the horizon time $(0,T)$ with a $(1-\epsilon)$ confidence level are, respectively, defined as:

$$VaR_{1-\epsilon}(0,T) = -U_\epsilon(T) \tag{27}$$

$$TVaR_{1-\epsilon}(0,T) = E[-U(T)| - U(T) \geq VaR_{1-\epsilon}(0,T)] \tag{28}$$

The parameter ϵ can be interpreted as the risk tolerance probability, therefore it is often selected to be a small value such as less than 5%. TVaR is the expected value of all possible realization of $-U(T)$ above the VaR.
Hence, considering the investment return coming from the initial capital, RBC requirements are given by:

$$RBC_{1-\epsilon}^{VaR}(0,T) = VaR_{1-\epsilon}(0,T)\, v(0,T) \tag{29}$$

$$RBC_{1-\epsilon}^{TVaR}(0,T) = TVaR_{1-\epsilon}(0,T)\, v(0,T) \tag{30}$$

If an initial capital $U(0)$ is given, all the requirements increase of the amount $U(0)$. RBC requirements as stated in (29) and (30) are computed according to stochastic approach investigating the impact of different time horizons for three levels of confidence: 98%, 99%, and 99.5%.
Simulation outcomes are reported as a percentage of the actuarial value at time 0 of future benefits under central scenario (see Table 4). They are denoted by $rbc_{1-\epsilon}^{VaR}(0,T)$ and $rbc_{1-\epsilon}^{TVaR}(0,T)$.
In numerical application three time horizons are considered: the deferment period, $T = 15$; a medium term horizon near to insured life expectancy, $T = 30$; a long term horizon, $T = 50$ (for $T = \omega$ results are omitted because very close to the ones obtained for $T = 50$). Results shown in Table 4 quantify the highest riskiness of LTCA compared with ELA: *rbc* values of LTCA are about five times greater than ELA.

Table 4. $rbc_{1-\epsilon}^{VaR}(0,T)$ and $rbc_{1-\epsilon}^{TVaR}(0,T)$ for 1,000 LTCA and ELA: stochastic approach

	LTCA			ELA		
$rbc_{1-\epsilon}^{VaR}(0,T)$	T=15	T=30	T=50	T=15	T=30	T=50
$\epsilon = 0.005$	12.43%	37.95%	51.11%	2.34%	7.50%	9.98%
$\epsilon = 0.01$	10.76%	35.26%	47.29%	2.17%	7.08%	9.62%
$\epsilon = 0.02$	9.53%	30.90%	43.55%	1.97%	6.54%	8.91%
$rbc_{1-\epsilon}^{TVaR}(0,T)$	T=15	T=30	T=50	T=15	T=30	T=50
$\epsilon = 0.005$	13.10%	39.42%	52.64%	2.67%	8.03%	10.37%
$\epsilon = 0.01$	12.33%	37.77%	50.87%	2.47%	7.65%	10.09%
$\epsilon = 0.02$	11.26%	35.45%	48.28%	2.25%	7.25%	9.65%

Including a safety loading on demographic pricing bases RBC requirements strongly decrease (see Table 5). Such an effect increases with time.
Note that the reduction becomes more evident for ELA; e.g. $rbc_{0.99}^{VaR}(0,30)$ achieves a 27% reduction for LTCA against a 42% for ELA (compare Table 4 with 5).

Table 5. $rbc_{1-\epsilon}^{VaR}(0,T)$ and $rbc_{1-\epsilon}^{TVaR}(0,T)$ for 1,000 LTCA and ELA: stochastic approach with safety loading

	LTCA			ELA		
$rbc_{1-\epsilon}^{VaR}(0,T)$	T=15	T=30	T=50	T=15	T=30	T=50
$\epsilon = 0.005$	10.70%	28.28%	34.85%	1.80%	4.47%	5.10%
$\epsilon = 0.01$	9.11%	25.87%	31.52%	1.59%	4.09%	4.76%
$\epsilon = 0.02$	7.97%	21.97%	28.13%	1.38%	3.56%	4.07%
$rbc_{1-\epsilon}^{TVaR}(0,T)$	T=15	T=30	T=50	T=15	T=30	T=50
$\epsilon = 0.005$	11.38%	29.68%	36.28%	2.05%	5.04%	5.46%
$\epsilon = 0.01$	10.65%	28.16%	34.73%	1.88%	4.66%	5.21%
$\epsilon = 0.02$	9.63%	26.03%	32.39%	1.68%	4.25%	4.78%

6.2 RBC requirements on a time horizon (t, T)

The previous requirements refer to time T considering the solvency of an insurance company at a specific date. As alternative approach the insurer solvency can be verified if the risk reserve is not negative for all integer $s \in (t,T)$. Therefore, RBC requirements for the time horizon (t,T) with a $(1-\epsilon)$ confidence level are defined as:

$$RBC_{1-\epsilon}(t,T) = \inf \left\{ U(t) \geq 0 \, \middle| \, \Pr \left\{ \bigcap_{s=t}^{T} U(s) \geq 0 \right\} \geq 1-\epsilon \right\} \tag{31}$$

RBC requirements as stated in (31) are computed under stochastic approach with safety loading, investigating the impact of different time horizons for three levels of confidence: 98%, 99%, and 99.5%. As in the previous tables, results are expressed in relative terms, in this case they are represented as a percentage of the actuarial value at time t of future benefits according to central scenario (see Table 6 for LTCA and Table 7 for ELA). When LTC insurance is concerned RBC requirements reduce with time decreasing. The trend is different for ELA where requirements decrease until $t = 15$ and rise subsequently.
In $t = 0$ the values of $rbc_{1-\epsilon}(t,T)$ are equal or slightly greater than $rbc_{1-\epsilon}^{VaR}(0,T)$. For $t > 0$ a comparison with $rbc_{1-\epsilon}^{VaR}$ is not possible but it emerges that LTCA requirements are even higher than ELA ones.

Table 6. $rbc_{1-\epsilon}(t,T)$ for 1,000 LTCA: stochastic approach with safety loading

	$rbc_{0.995}(t,T)$			$rbc_{0.99}(t,T)$			$rbc_{0.98}(t,T)$		
t	T=15	T=30	T=50	T=15	T=30	T=50	T=15	T=30	T=50
0	10.70%	28.28%	35.53%	9.23%	25.87%	32.31%	7.97%	21.97%	28.27%
5	9.71%	27.54%	34.75%	8.44%	25.15%	30.86%	6.92%	21.43%	27.99%
10	7.12%	24.62%	31.83%	6.49%	23.29%	28.71%	5.36%	19.75%	26.36%
15	–	20.87%	28.06%	–	19.30%	25.49%	–	16.68%	23.11%
20	–	16.80%	25.64%	–	15.49%	22.67%	–	12.97%	20.11%

Table 7. $rbc_{1-\epsilon}(t,T)$ for 1,000 ELA: stochastic approach with safety loading

	$rbc_{0.995}(t,T)$			$rbc_{0.99}(t,T)$			$rbc_{0.98}(t,T)$		
t	T=15	T=30	T=50	T=15	T=30	T=50	T=15	T=30	T=50
0	1.80%	4.68%	5.42%	1.59%	4.15%	5.08%	1.46%	3.71%	4.46%
5	0.67%	1.63%	4.51%	0.63%	1.51%	4.08%	0.58%	1.36%	3.59%
10	0.42%	1.54%	4.37%	0.39%	1.34%	3.80%	0.37%	1.20%	3.47%
15	–	1.87%	3.93%	–	1.67%	3.31%	–	1.45%	2.84%
20	–	2.54%	3.87%	–	2.33%	3.56%	–	2.05%	3.26%

Given the high level of risk in managing LTC insurance, safety loading and capital allocation strategies may be insufficient to manage it. To this purpose different reinsurance strategies can be realized in order to make the LTC risk insurable by its partial transfer to reinsurer.

7 Stop-loss reinsurance

Reinsurance allows the insurer to reduce the underwriting risk and leads to more effective risk management. Many reinsurance strategies can be considered to carry mortality and disability risk on a tolerable level. In insurance practice LTC risk is usually covered by quota share treaties and insurance companies are willing to ceded in reinsurance high portions of their LTC portfolios (up to 70%) due to the high riskiness of such covers. Nonetheless, it is well-known that among reinsurance treaties stop-loss gives the smallest variance of the insurer's retained risk. As we analyse risk-homogeneous portfolios and relative risk measures, the quota share treaty should not affect results of our risk analysis. So, we concentrate on stop loss treaty.
A stop-loss reinsurance protects the reinsured against an unacceptable degree of variance in the aggregate loss experience of a reinsured portfolio of business during any defined period of time. In practice, the cedant takes on the risk up to a certain amount, after which reinsurance begins. Such kind of risk is tail-end and represents the most volatile part of a LTCA. Some analyses about reinsurance in life annuity have been performed by Olivieri (see [Oli02]).
In this paper the stop-loss reinsurance (applied to a portfolio of 1,000 LTCA) is developed assuming that the reinsurer's intervention is linked to the loss sustained by the insurer expressed in terms of risk reserve. We consider stop-loss treaties covering h years, so for longer periods reinsurance arrangements are set every h years. Let $L(T)$ be the insurer loss at portfolio level for the time horizon $(0,T)$ $(T = h, 2h, ...)$:

$$L(T) = \frac{U(0)}{v(0,T)} - U(T) \tag{32}$$

Let r be a coefficient defining the reinsurance excess limit (so-called ritention) and $E[V(t)]$ the expected actuarial reserve in t $(t = h, 2h, ..., T)$ considering information available at time 0. The possible payment from the reinsurer at the end of the year t is defined as:

$$B^{SL}(t) = \max\{0; L(t) - rE[V(t)]\} \tag{33}$$

The reinsurance premium for the period $(t-h,t)$, paid by the cedant at initial period $t-h$, is computed considering the $(1-\gamma)$-th quantile of the $B^{SL}(t)$ distribution. It is denoted by $P^{SL}_{1-\gamma}(t)$. When the stop-loss treaty is in force the risk reserve at time T becomes:

$$U^{SL}(T) = U(T) + \sum_{s=h}^{T} \left(\frac{B^{SL}(s)}{v(s,T)} - \frac{P^{SL}_{1-\gamma}(s)}{v(s-h,T)} \right) \tag{34}$$

Note that as the proposed reinsurance treaty starts from year 0, its effects are completely visible on solvency requirements referring to time horizon $(0,T)$. In numerical application reinsurance effects are evaluated under stochastic approach, the ritention coefficient r is fixed to 4% consistently with the minimum solvency margin required by EU Directive Solvency I, while parameter γ is fixed to 5%. Reinsurance premia, $P^{SL}_{1-\epsilon}(t)$, for a portfolio of 1,000 LTCA are presented in Figure 7. It should be noticed that the greater is the reinsurer's participation the higher is the premium required to insurance company. As shown in Fig. 7 the most reinsurer payments concentrate in $t \in (20,45)$ with a maximum level in $t = 30$.

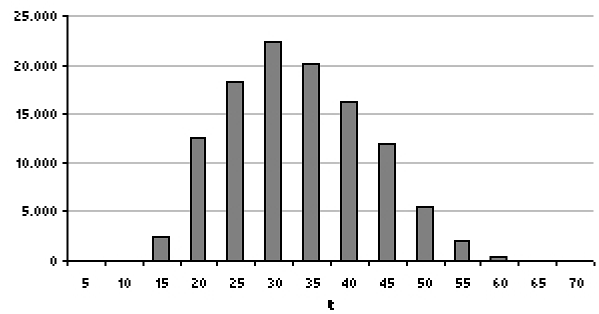

Fig. 7. Reinsurance premia, $P^{SL}_{95\%}(t)$, for a portfolio of 1,000 LTCA

Table 8 shows the solvency requirements after stop-loss reinsurance computed according to VaR and TVaR at time T with a $(1-\epsilon)$ confidence level. As in the previous section simulation outcomes are expressed as a percentage of the actuarial value at time 0 of future benefits under central scenario.

Table 8. $rbc^{VaR(SL)}_{1-\epsilon}(0,T)$ and $rbc^{TVaR(SL)}_{1-\epsilon}(0,T)$ for 1,000 LTCA: stochastic approach

$rbc^{VaR(SL)}_{1-\epsilon}(0,T)$	T=15	T=30	T=50
$\epsilon = 0.005$	9.39%	31.63%	42.77%
$\epsilon = 0.01$	8.97%	31.23%	42.35%
$\epsilon = 0.02$	8.59%	30.35%	40.98%
$rbc^{TVaR(SL)}_{1-\epsilon}(0,T)$	T=15	T=30	T=50
$\epsilon = 0.005$	9.59%	32.16%	43.41%
$\epsilon = 0.01$	9.40%	31.77%	42.96%
$\epsilon = 0.02$	9.10%	31.27%	42.22%

Positive effects of reinsurance on risk are recognizable comparing Table 8 with Table 4. Greater reductions in RBC requirements (either VaR or TVaR) are obtained for lower values of ϵ. Anyhow, reinsurance effect is stronger on $rbc_{1-\epsilon}^{TVaR}(0,T)$ rather than $rbc_{1-\epsilon}^{VaR}(0,T)$ as the stop-loss treaty cuts U-distribution tail, where TVaR is calculated. As previously stated, riskiness of ELA in relative terms is strongly lower than LTCA making stop-loss reinsurance less necessary.

8 Concluding remarks

The risk analysis gives evidence of high biometric risk in insurance products with LTC cover. Deterministic and stochastic approaches highlight risk of random fluctuations and systematic deviations, respectively. The former risk can be reduced by increasing the portfolio size; on the contrary the latter one is more difficult to control, since it depends from future evolution of biometric functions. Such an evolution is hard to represent, causing difficulties to define projection model for mortality and disability and to choose biometric parameters.

We focused our analysis on parameter risk, demonstrating that it could be partially reduced by:

- combining LTC with other insurance covers (in particular a life annuity);
- introducing a safety loading, whose principal drawback is to produce a premium increase (such a choice is limited by competitiveness on insurance market);
- including an appropriate capital flows policy, whose disadventage is to reduce profitability of stockholders;
- involving reinsurance strategies with a consequent reduction on the net volume of business.

Future works could focus on finding an optimal reinsurance strategy under different premium principles, as well as including stochastic models for financial risk.

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