

MEASURING CONCENTRATION RISK IN BANK CREDIT PORTFOLIOS USING GRANULARITY ADJUSTMENT: PRACTICAL ASPECTS

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The paper analyses the practical aspects of granularity adjustment for quantification of the contribution of name concentrations to portfolio risk: proposals are made for the unique choice of systemic risk variance; aggregation of credit risk parameters from exposure to counterparty level is analysed; granularity adjustment capital allocation to individual counterparties is being discussed, proposing to include single name granularity adjustment capital into performance measures and risk based pricing tools; Monte Carlo approach for estimating single name concentration risk capital is being introduced. Practical aspects of granularity adjustment estimation are illustrated by empirical calculations using real bank portfolio data and the comparison with Gordy and Lütkebohmert results is presented. Keywords: granularity adjustment; value at risk; Monte Carlo simulation; idiosyncratic risk; systemic risk.

Introduction

Starting from year 2007 many banks in different countries have adopted the new capital requirements, which are known as Basel II (BCBS 2004). The main difference between previous capital requirements (Basel I) and current Basel II is that banks were allowed to use internal credit risk models for estimating supervisory required capital. As banks are exposed to other types of risk than credit, market and operational risk, Basel II capital requirements were expanded to cover other types of risk. For this purpose Basel II requirements are divided into three parts or three pillars. Pillar I covers minimum requirements

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for estimating capital for the three main banking risks, namely credit, market and operational. Pillar II is devoted to other risk types, not captured by Pillar I: business, credit concentration, information technologies, legal and compliance, liquidity, reputation, residual, settlement, strategic. Risks under Pillar II are covered under internal capital assessment process (ICAP) and supervisory review and evaluation process (SREP). ICAP and SREP require that risks not covered under Pillar I must be assessed by banks on their own via Pillar II ICAP, and supervisors during SREP check whether ICAP process undertaken by banks is adequate to cover other risks*. In general Pillar II is about good risk management practices in banks. Pillar III concerns market discipline, i.e. disclosing of information to public about risks banks take, methods used to estimate and manage risk.

One of sub-types of credit risk is credit concentration risk, i.e. the possibility for a bank to incur relatively (compared to bank's capital, assets or total risk if it possible to estimate the latter) large loss from credit portfolio, so that this loss would endanger normal activity of a bank. Concentration risk might appear both in bank assets (banking and trading books) and liabilities, in making transactions and from other banking operations. Being more specific, concentration risk might arise from large credits to single borrower, related borrowers, borrowers having high risk ratings, borrowers from the same country, geographic region, economic sector, the same type of collateral, maturity, currency of denomination, the same type of credit product, etc. (Valvonis 2007).

Concentration of exposures in credit portfolios is an important aspect of credit risk. It may arise from two types of imperfect diversification. The first type, name concentration, relates to imperfect diversification of idiosyncratic risk in the portfolio either because of its small size or because of large exposures to specific individual obligors. The second type, sector concentration, relates to imperfect diversification across systematic components of risk, namely sector factors. The existence of concentration risk violates one or both of two key assumptions of asymptotic single risk factor (ASRF) model that underpins the capital calculations of the internal ratings based (IRB) approaches of the Basel II Framework. Name concentration implies less than perfect granularity of the portfolio, while sector concentration implies that risk may be driven by more than one systematic component (factor).

As due to specific assumptions of Basel II capital calculation requirements for credit risk it was not possible to cover credit concentration risk under Pillar I, banks themselves have to estimate capital requirements for this risk under Pillar II. One of the methods proposed by some authors (see BCBS, BIS 2001) is so called granularity adjustment (GA) and its revised version (see Gordy and Lütkebohmert 2007). Advantage of the GA is that this approach is compatible with Basel II credit risk model. Other methods, for example, proposals by Tasche are based on expected shortfall risk measure (Martin et al. 2007). Although expected shortfall risk measure is believed to be superior to value at risk (VaR)***, but as Basel II credit risk model is based on VaR risk measure as well as GA, for Basel II purposes GA is preferred. Moreover, revised GA has a closed form mathematical solution, which makes it easier to implement in practice in banks.

On the one hand banks are required to calculate capital requirements for concentration risk, on the other hand there seems to exist methods for evaluating this risk. The goal of this paper is to analyze practical aspects of applying GA in banks, namely: the aggregation of multiple exposures into a single exposure for the purpose of assessing the effect of the single name concentration risk; the calculation of the credit risk drivers, i.e. Probability of Default (PD) and Loss Given Default (LGD) to this single exposure so that the final parameters are unique and the largest exposures make the largest influence; also paper considers questions involving the choice of the systemic risk and its parameters (variance of the systemic risk for example) and small practical correction of the GA formula which allows to avoid additional new data requirement, i.e. the volatility of the inputs to IRB formula. As credit risk capital is used for measuring risk adjusted performance of credit exposures as well as in risk based pricing, capital for credit concentration risk must also be included in these calculations. In other words, in risk adjusted performance measure-

*For more details see CEBS (2006).

**See for example Artzner et al. (1999), Acerbi and Tasche (2002).

ment banks cannot use just Basel II Pillar I capital required for credit exposure, as in this way capital would be underestimated for credit concentration risk. As GA is estimated for portfolio of exposures, the natural wish is to distribute GA capital to individual exposures. This issue is also discussed in the paper. The paper also gives some empirical GA estimation results and shows how GA works on real bank credit portfolio data.

Chapter 1 of the paper reviews theoretical assumptions behind Basel II credit risk model and why it is needed to estimate capital for credit concentration risk separately. Chapter 1 also discusses in detail GA estimation methodology proposed by some authors as it will be challenged from the practical perspective in the Chapter 2 of paper. Chapter 3 gives some empirical estimation results and practical conclusions from one real bank portfolio. Chapter 4 analyses GA allocation problem and the last chapter concludes.

1. Theoretical background

This chapter briefly describes theoretical aspects of Basel II IRB model, assumptions of this model and practical implications. Analysis of IRB model directly leads to motives why, what and how IRB models need to be supplemented, for example, with GA estimate.

1.1. Basel II IRB model

The Basel II IRB model was built in order to allow banks to apply their own standards for evaluation of credit risk parameters and ultimately supervisory required capital. On the other hand calculation of the required capital must involve the following properties: universality for all banks (all banks should be able to apply required capital calculation rules, independently from geographic location, type or size of bank), capital portfolio invariance (the model is portfolio invariant if the capital required for any given loan should only depend on the risk of that loan and must not depend on the portfolio it is added to), and so on*. Thus seeking to satisfy all these assumptions the simple and also elegant decision was made – ASRF portfolio credit risk model. Single risk factor and asymptotic are the two main assumptions of Basel II IRB model.

Asymptotic means that the portfolio is infinitely fine grained and thus it consists of a nearly infinite number of credits with comparatively small exposures. Single risk factor means that only one systematic risk factor influences the default risk of all loans in the portfolio.

In ASRF model, credit risk in a portfolio is divided into two categories, systematic and idiosyncratic risk. Systematic risk is the market risk or the risk that cannot be diversified away. It refers to the movements of the whole economy. Idiosyncratic risk is described as the risk of value changes due to the unique circumstances of a specific obligor. ASRF model under IRB approach assumes that bank portfolios are perfectly fine-grained, that is, idiosyncratic risk is fully diversified away, so that economic capital depends only on systematic risk (this way portfolio invariance condition is satisfied).

Assume that the normalized asset return R_i of the i 'th obligor in the credit portfolio is driven by a systematic risk factor X and an idiosyncratic noise component ε_i . For consistency with the ASRF framework of Basel II assume that X is one-dimensional, i.e. that there is only single systematic risk factor $R_i = \sqrt{\rho_i}X + \sqrt{1 - \rho_i}\varepsilon_i$, here X and ε_i are independent identically distributed normal random variables (i.i.d. $N(0,1)$). This means that R_i has a standard Gaussian distribution. The component ε_i represents the risk specific to i 'th obligor, while X is a common risk to all. Note that ρ_i represents the asset correlations, where $E[R_i R_j] = \sqrt{\rho_i} \sqrt{\rho_j}$. It should be mentioned that asset return correlations and default correlations are not the same. Typically, the default correlation is much smaller than the asset correlation**. According to the theory of Merton (1974) the i 'th obligor defaults with probability PD_i if $R_i \leq \Phi^{-1}(\rho_i)$, denoting this indicator with Z_i . Here and throughout the paper Φ is cumulative distribution function of standard normal random variable. Thus the conditional probability of default equals to

*These properties are described in Gordy (2003).

**For more details see Frey et al. (2003).

$$P(R_i \leq \Phi^{-1}(PD_i) | X = x) = P(\sqrt{\rho_i}X + \sqrt{1-\rho_i}\varepsilon_i \leq \Phi^{-1}(PD_i) | X = x) =$$

$$= P\left(\varepsilon_i \leq \frac{\Phi^{-1}(PD_i) - \sqrt{\rho_i}X}{\sqrt{1-\rho_i}} \middle| X = x\right) = \Phi\left(\frac{\Phi^{-1}(PD_i) - \sqrt{\rho_i}x}{\sqrt{1-\rho_i}}\right).$$

Next consider credit portfolio consisting of n obligors. The portfolio has asset correlations ρ_i , exposures at default EAD_i , probabilities of default PD_i and loss given default LGD_i^* . The portfolio loss rate is given by $L = \left(\sum_{j=1}^n EAD_j\right)^{-1} \sum_{i=1}^n EAD_i \cdot LGD_i \cdot Z_i$.

Under ASRF assumptions the α -percentile unexpected loss without maturity adjustment is

$$q_\alpha(L) - E(L) = \left(\sum_{j=1}^n EAD_j\right)^{-1} \sum_{i=1}^n EAD_i \cdot LGD_i \cdot \left[\Phi\left(\frac{\Phi^{-1}(PD_i) - \sqrt{\rho_i}\Phi^{-1}(\alpha)}{\sqrt{1-\rho_i}}\right) - PD_i\right], \quad (1)$$

here $E(L)$ is the expected loss of the portfolio.

From Vasicek (1991) and the law of large numbers it follows that cumulative probability of the percentage loss on a portfolio of n loans has very skewed distribution and all the formulae used in IRB model (mentioned in this chapter) hold only asymptotically**.

1.2. Granularity adjustment as a supplement to Basel II IRB model

As it was discussed previously the ASRF model assumes that bank credit portfolio consists of an infinite number of relatively small exposures to one economy or economic sector. But real bank portfolios are not infinite and exposures are not of the same size, moreover banks are not operating in single economic sector. This implies, that in practice banks are exposed to a lesser or greater concentration of single obligor risk compared to infinitely fine grained portfolio under ASRF model. On the other hand banks are more diversified with respect to economic sectors, as banks operate in more than one economic sector. Looking from a conservative point of view and from supervisory perspective, overestimation of sector concentration risk with ASRF model is not a problem***. On the other hand, underestimation of single name concentration risk is an important issue, as banks for this reason hold less capital than they should.

The GA is an extension of the ASRF model which forms the theoretical basis of the IRB model. Through this adjustment, originally omitted single-name concentration is integrated into the ASRF model. The GA *ceteris paribus* can be calculated as the difference between unexpected loss in the real portfolio and in an infinitely granular portfolio with the same risk characteristics. GA is an additional required capital to cope with unsystematic (idiosyncratic) credit risk arising from the credit portfolio. It is by means of this adjustment that one could estimate additional capital to cover single name concentration risk for portfolios which contain very large or varied exposure sizes, relative to those which are more "granular" or contain large numbers of smaller exposures.

There are at least three approaches proposed in credit risk literature how GA could be estimated: by Vasicek (2002), by Emmer and Tasche (2005), by Gordy and Lütkebohmert (2007). The intuition behind the Vasicek method is to augment systematic risk (by increasing factor loadings) in order to compensate for ignoring idiosyncratic risk. An important problem is, however, that the systematic and idiosyncratic components of the risk have very different distribution shapes. This method is known to perform poorly in practice (see Gordy and Lütkebohmert 2007). The approach proposed by Emmer and Tasche (2005) is based on the default-mode version of CreditMetrics and so shares the Merton model foundation with the IRB model. In contrast to the approach proposed by Gordy and Lütkebohmert (2007), it does not maturity-adjust the input parameters and does not account for idiosyncratic recovery risk. However, in principle it could be extended to capture both aspects. Its major drawback is that the formula itself is quite complex, especially compared to the one proposed by Gordy and Lütkebohmert.

*Here and throughout the paper LGD is not random and all expectations are empirical.

**More detailed derivation of Basel II IRB model can be found in Gordy (2003).

****Ceteris paribus* and under assumption that correlation is calibrated more precisely.

The revised GA proposed by Gordy and Lütkebohmert (2007) serves as a revision and extension of the methodology proposed in the Basel II second consultative paper (BCBS, BIS 2001). Revised GA develops a relatively simple and what is most important applicable in practice methodology for approximating the effect of undiversified idiosyncratic risk on VaR. Also in keeping with the Basel II second consultative paper (BCBS, BIS 2001), the data inputs to the revised GA are drawn from parameters already required for the calculation of IRB required capital. The other advantage of GA calculation method proposed by Gordy and Lütkebohmert is the closed form solution and consistency with Basel II IRB model. Thus further in this paper GA approach proposed by Gordy and Lütkebohmert is discussed.

An important question is how much additional capital is required if a single loan is added to the credit portfolio. In order to answer this question, the derivative of the VaR must be calculated. It can be shown mathematically that the derivative is given by the conditional mean of the marginal loan, on condition that the value of the credit portfolio and VaR are exactly identical*. If this general result is applied to a simple one-factor model, the Basel II IRB model can be obtained.

Let us write portfolio loss in the following mathematical expression: $L = u(1)$, where

$$u(\varepsilon) := E[L | X] + \varepsilon(L - E[L | X]), \quad (2)$$

and $E[L | X]$ ** representing losses driven only from the systemic risk (conditional expectation serves as a projection). The second order Taylor expansion for the q -th quantile of the portfolio loss is

$$q_\alpha(L) \approx q_\alpha(E[L | X]) + \frac{\partial}{\partial \varepsilon} q_\alpha(E[L | X] + \varepsilon(L - E[L | X])) \Big|_{\varepsilon=0} + \\ + 1/2 \frac{\partial^2}{\partial \varepsilon^2} q_\alpha(E[L | X] + \varepsilon(L - E[L | X])) \Big|_{\varepsilon=0}.$$

First derivative in Taylor expansion of quantile vanishes, since the idiosyncratic component conditional on the systematic component $E[L | X]$ vanishes. The second derivative in Taylor expansion is GA, because it represents the additional fraction to the VaR due to the undiversified idiosyncratic component. This second derivative in Taylor expansion can be expressed as

$$GA \approx - \frac{1}{2h(q_\alpha(X))} \cdot \frac{d}{dx} \left(\frac{\sigma^2(x)h(x)}{\mu'(x)} \right) \Big|_{x=q_\alpha(X)},$$

where $\mu(x) = E[L | X]$, $\sigma^2(x) = V[L | X]$ and h is the density of the systematic factor X .

Next one gets $GA = q_\alpha(L) - q_\alpha(E[L | X]) \approx - \frac{1}{2} \left(\frac{\sigma^2(x_q) \cdot h'(x_q)}{\mu'(x_q) \cdot h(x_q)} + \frac{d}{dx} \left(\frac{\sigma^2(x)}{\mu'(x)} \right) \Big|_{x=x_q} \right)$, where $x_q = q_\alpha(X)$.

From T. Wilde (2001) result for CreditRisk+ framework, the derivative can be expressed as

$$\frac{d}{dx} \left(\frac{\sigma^2(x)}{\mu'(x)} \right) \Big|_{x=x_q} = \frac{1}{\sum_{i=1}^n EAD_i} \cdot \frac{\sum_{i=1}^n EAD_i^2 \cdot \mu'_i(x_q) (C_i - 2\mu_i(x_q))}{\sum_{i=1}^n EAD_i \cdot \mu'_i(x_q)},$$

where $C_i = \frac{E[LGD_i^2]}{E[LGD_i]}$ (3)

and $\mu_i(x_q) = PD_i \cdot E[LGD_i] \cdot (\omega_i \cdot (q_\alpha(X) - 1) + 1)$, ω_i are CreditRisk+ factor loadings. The derivative is the following $\mu'_i(x_q) = E[LGD_i] \cdot PD_i$. Recall that in CreditRisk+ model the expected loss is $R_i := E[LGD_i] \cdot PD_i$ and the unexpected loss capital requirement is $K_i := E[LGD_i] \cdot PD_i \cdot \omega_i \cdot (q_\alpha(X) - 1)$ ***. The complete GA takes the following form:

*For more details see Gouiroux et al. (2000).

**Starting from formula (1) X has gamma distribution and differs from the systemic risk in the ASRF model.

***For more details see Gordy and Lütkebohmert (2007).

$$GA_n = \frac{1}{2K^* \cdot \left(\sum_{i=1}^n EAD_i\right)^2} \sum_{i=1}^n EAD_i^2 \cdot \left[\left(\delta C_i (K_i + R_i) + \delta (K_i + R_i)^2 \cdot \frac{E[LGD_i^2] - (E[LGD_i])^2}{(E[LGD_i])^2} \right) - K_i \left(C_i + 2(K_i + R_i) \cdot \frac{E[LGD_i^2] - (E[LGD_i])^2}{(E[LGD_i])^2} \right) \right], \quad (4)$$

$$\text{where } K^* = \frac{1}{\sum_{i=1}^n EAD_i} \sum_{i=1}^n (EAD_i \cdot K_i)$$

$$\text{and } \delta \equiv (q_\alpha(X) - 1) \cdot \left(\xi + \frac{1 - \xi}{q_\alpha(X)} \right). \quad (5)$$

Following Gordy and Lütkebohmert methodology it is assumed that X is gamma distributed with mean 1 and variance $1 / \xi$ for some positive ξ .

The simplified version of GA^* looks like this if one drops the second term members:

$$GA_n = \frac{1}{2K^*} \sum_{i=1}^n (s_i^2 \cdot C_i (\delta(K_i + A_i) - K_i)), \text{ where } s_i := EAD_i / \left(\sum_{j=1}^n EAD_j \right).$$

In the above formulae of GA all the parameters are available in IRB model (like PDs, EADs, LGDs) and the only unavailable parameters are δ , ξ and factor loadings ω_i .

Conditional on $X = x$, the probability of default in CreditRisk+ model is $PD_i(x) = PD_i \cdot (1 - \omega_i + \omega_i \cdot x)$, where ω_i is the factor loading which controls the sensitivity of i 'th obligor to the systematic risk factor and PD_i is the non-conditional probability of default. In CreditMetrics the variance of the conditional probability of default is

$$\text{Var}[PD_i(X)] = \Phi_2(\Phi^{-1}(PD_i), \Phi^{-1}(PD_i), \rho_i) - PD_i^2, \quad (6)$$

where Φ_2 denotes the bivariate normal cumulative distribution function. The corresponding variance in CreditRisk+ is

$$\text{Var}[PD_i(X)] = (PD_i \cdot \omega_i)^2 / \xi. \quad (7)$$

Now equating the two variance expressions ((6) and (7)) one obtains ξ :

$$1/\xi = \frac{\Phi_2(\Phi^{-1}(PD_i), \Phi^{-1}(PD_i), \rho_i) - PD_i^2}{(PD_i \cdot \omega_i)^2}. \quad (8)$$

To obtain factor loadings ω_i , one needs to compare the asymptotic unexpected loss capital charges across the two models:

$$K_i^{CR+} = E[LGD_i] \cdot PD_i \cdot \omega_i \cdot (q_\alpha(X) - 1), \quad (9)$$

$$K_i^{CM} = \left(\Phi \left(\sqrt{\frac{1}{1 - \rho_i}} \Phi^{-1}(PD_i) + \Phi^{-1}(q) \sqrt{\frac{\rho_i}{1 - \rho_i}} \right) - PD_i \right) \cdot E[LGD_i]. \quad (10)$$

Equating (9) and (10) one gets

$$\omega_i = \frac{\Phi \left(\sqrt{\frac{1}{1 - \rho_i}} \Phi^{-1}(PD_i) + \Phi^{-1}(q) \sqrt{\frac{\rho_i}{1 - \rho_i}} \right) - PD_i}{PD_i \cdot (q_\alpha(X) - 1)}. \quad (11)$$

Although the above GA estimation formulae seem to be complicated, with many variables, but they are possible to implement in practice. On the other hand, while implementing GA in practice, several challenges are faced:

- as GA measures single name concentration risk, all credit risk parameters (PD, LGD, EAD, ρ) in GA formulae are on counterparty level. As some counterparties might have several exposures one needs to aggregate PD, LGD, EAD and ρ from single exposures to counterparty level;

*The accuracy of this simplified version is described in Gordy and Lütkebohmert (2007).

- banks are required to estimate volatility of LGD ((3) formula). The proposal in Gordy and Lütkebohmert (2007) is to estimate volatility of LGD using this formula $VLGD_k^2 = \gamma ELGD_k(1 - ELGD_k)$ with supervisory set constant γ . But as supervisors have not set any value for γ , banks themselves have to overcome this problem and estimate volatility of LGD;

- estimation of ξ which is a $PD(X)$ parameter and portfolio dependent. This is a drawback, because this parameter describes systemic risk and must be unique and satisfy portfolio invariance. Thus ξ should not be dependent on any credit risk parameters;

- estimation of δ is complicated, because to obtain δ one needs to solve non-linear equation. If GA is estimated each time capital adequacy is being calculated (can be every day), each time non-linear equation for δ must be solved. Thus some quick solution to this non-linear equation is needed.

2. Challenges in implementing GA in practice

This chapter discusses practical aspects of implementing GA methodology in banks, i.e. how to estimate various parameters in GA model.

2.1. Obtaining PD, LGD and EAD on counterparty level

To estimate GA for single name concentration risk, GA estimation is done on counterparty level, not on exposure level. In other words it is natural to assume that the counterparty is defaulting, not individual exposures of this counterparty. This implies that credit risk parameters (PD, LGD, EAD, ρ) used in GA formulae must be estimated on counterparty level.

Lets assume that k'th obligor ($1 \leq k \leq n$) has n_k exposures with the following characteristics $(EAD_i, PD_i, LGD_i, \rho_i)$, ($1 \leq i \leq n_k$):

- aggregation of EAD to counterparty level if counterparty has several exposures is the most simple, as one just have to sum up EADs: $EEAD_k = \sum_{i=1}^{n_k} EAD_i$;

- as in Basel II IRB framework non retail borrowers are rated on counterparty level, so PD is also estimated on counterparty level. Thus no aggregation of PDs for non-retail borrowers' PDs is required. As under Basel II banks are allowed for retail counterparties to fix default on exposure level this way different exposures of the same counterparty might have different PDs. Thus aggregation of PDs for retail counterparties is needed. The proposal* here would be to set retail counterparty PD to the maximum PD of his exposures' PDs: $EPD_k = \max(PD_1, PD_2, \dots, PD_{n_k})$;

- if counterparty defaults, one would estimate how much bank is able to recover from all exposures of this counterparty. In other words, if total exposure of counterparty is $EEAD_k = \sum_{i=1}^{n_k} EAD_i$ and total loss from counterparty is $ELOSS_k = \sum_{i=1}^{n_k} EAD_i \cdot LGD_i$, then LGD on counterparty level would be $ELOSS_k / EEAD_k = \sum_{i=1}^{n_k} \left(LGD_i \cdot \frac{EAD_i}{EEAD_k} \right) := ELGD_k$ or in other words exposure weighted LGD;

- as in Basel II IRB framework correlation is PD dependent, then correlation should be estimated on a basis of $(PD_1, PD_2, \dots, PD_{n_k})$. Since correlation takes values dependent on the product and assessment types (see Directive 2006/48/EC) the value should be taken over the product segments (the maximum PD's from the same product type exposures (if there is such)). For the correlation expression for different product types (retail case) the priority is given for the largest exposures (exposure weighted approach).

2.2. Variance of LGD

Note that in the paper of Gordy and Lütkebohmert formula (1), C_k is expressed through the volatility of LGD, i.e. $E[LG D_k^2] = ELGD_k^2 + VLGD_k^2$. Since the volatility of LGD is not an input to the IRB formula, banks in principle could be permitted to supply this parameter for each loan. This procedure would require additional amount of new data and often it

*The alternative could be to estimate exposure weighted PD. But taking into consideration that counterparty is considered to be defaulted when counterpart defaults of any one of his exposures, thus to be consistent with this definition of default maximum on PDs should be taken. Taking exposure weighted PD would underestimate PD on counterpart level.

seems preferable to impose a regulatory assumption to avoid this burden. For example the relationship in the CP2 version of the GA uses $VLGD_k^2 = \gamma ELGD_k(1 - ELGD_k)$ with the regulatory parameter γ between 0 and 1. Let's call this LGD volatility as Basel II volatility. When this specification is used in industry models such as CreditMetrics or KMV Portfolio manager, a typical setting is $\gamma = 0.25$. This value is also used in Gordy and Lütkebohmert methodology.

This paper considers empirical approach for evaluating volatility of LGD. Let us consider the exposure weighted deviation of LGD:

$$(\text{Dev}(LGD_k))^2 := E[LGD_k^2] - ELGD_k^2 := \sum_{i=1}^{n_k} \left((LGD_i - [ELGD_k])^2 \cdot \frac{EAD_i}{EEAD_k} \right),$$

where $E[LGD_k^2] := \sum_{i=1}^{n_k} \left(LGD_i^2 \cdot \frac{EAD_i}{EEAD_k} \right)$. The only difference of this derivation from the standard ones is that instead of $1/n_k$ it uses weights $\frac{EAD_i}{EEAD_k}$.

Recall that n_k is a number of exposures for the k 'th obligor or equivalently k 'th position. Thus we have that by applying exposure weighted LGD in GA formulae parameter C_k equals:

$$C_k = \frac{\sum_{i=1}^{n_k} LGD_i^2 \cdot EAD_i}{\sum_{i=1}^{n_k} LGD_i \cdot EAD_i}. \tag{12}$$

Having empirical LGDs for each counterparty, one can compare the Basel II volatility of LGD with the exposure weighted one. For the comparison to be more clear we involve the GA component C_i . With the Basel II volatility of LGD it is

$$C_i = \frac{0.25 \cdot (ELGD_i) + 0.75 \cdot (ELGD_i)^2}{ELGD_i}. \tag{13}$$

Comparing the latter two expressions one obtains that neither Basel II C_i nor exposure weighted C_k dominates. In our portfolio we obtained that 0.685 per cent of all counterparts had the (13) expression strictly less than the (12), and these counterparts contained 1.263 per cent of all portfolio EAD.

Next let us assume the following scenario: if counterparty has an exposure equal to EUR 1,000 and $LGD = 1$, after some time this counterparty is granted new exposure of EUR 100,000 and $LGD = 0.001$. From the local (single counterparty) perspective the concentration increases since the exposure increases to 101,000 (also observe that expected loss with only first exposure included is 1,000 and with both exposures is 1,100). However the C_i from expression (13) to GA formulae doesn't reflect the local increase in the concentration, i.e. with only first exposure $C_i = 1$ and after second C_i drops to 0.2582 (about four times), while in this scenario the (12) expression drops from 1 to 0.9092. Thus in this paper we suggest the conservative approach by implementing the LGD volatility related input C_k to be the maximum of (12) and (13) expressions.

2.3. Solving non-linear equation to estimate δ

Expression of δ in equation (5) has a non-linear form. If bank wishes to estimate GA every time when required capital is estimated (this can range from daily to quarterly estimation, and most probably shall be done monthly), each time non-linear equation for ξ must be solved.

If factor loadings from formula (11) are put into the (8) formula, non-linear equation for ξ is obtained:

$$\frac{1}{\xi(\alpha_q(X) - 1)^2} = \frac{\Phi_2(\Phi^{-1}(PD_k), \Phi^{-1}(PD_k), \rho_k) - PD_k^2}{\left[\Phi \left(\sqrt{\frac{1}{1-\rho_k}} \Phi^{-1}(PD_k) + \Phi^{-1}(q) \sqrt{\frac{\rho_k}{1-\rho_k}} \right) - PD_k \right]^2}.$$

This expression has an obvious drawback since the estimated value of ξ depends on the chosen PDs and thus on portfolio characteristics. To be consistent with the exposure weighted approach and at the same time avoid the above mentioned dependencies, the following aggregation is proposed:

$$\frac{1}{\xi(\alpha_q(X)-1)^2} = \sum_{k=1}^n \left(\frac{\Phi_2(\Phi^{-1}(PD_k), \Phi^{-1}(PD_k), \rho_k) - PD_k^2}{\left[\Phi\left(\sqrt{\frac{1}{1-\rho_k}}\Phi^{-1}(PD_k) + \Phi^{-1}(q)\sqrt{\frac{\rho_k}{1-\rho_k}}\right) - PD_k \right]^2} \cdot \frac{EEAD_k}{\sum_{k=1}^n EEAD_k} \right). \quad (14)$$

To solve the non-linear equation for ξ^* and make it computationally not burdensome, as this equation needs to be solved each time GA is estimated, so called secant method could be used. In numerical analysis, the secant method is a root finding algorithm that uses a successions of roots of secant lines to better approximate a root of a function. The secant method is defined by the recurrence relation: $x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n)$.

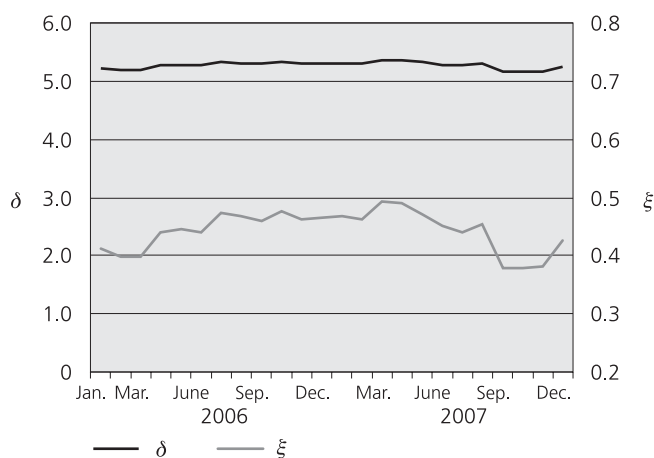
In GA case the function is $f(\xi) = \frac{1}{\xi(q_\alpha(X)-1)^2} - const$ and 'const' is the right hand side of the equation (14). The first two steps in the recurrent relation we choose as $x_0 = 0.1$ and $x_1 = 0.2$ and the tolerance parameter is $eps = 0.00000001 = 10^{-8}$. The number of iterations of the secant method in our case is 100. The method is computationally simple since it doesn't involve any derivatives.

In Gordy and Lütkebohmert GA baseline parameterization ($\xi = 0.25$) the relation between ξ and δ is the following (quantile $q = 0.999$):

ξ	0.2	0.25	0.35	0.5	0.75	1.00	1.50
δ	4.66	4.83	5.09	5.37	5.68	5.91	6.23

For empirical portfolio using two years of historical data delta (δ) is depicted below.

Figure 1. Evolution of δ



Source: data from one Eastern Europe bank.

The average δ is $\delta^{av} = 5.212206$ with $\xi^{av} = 0.412447$. Note that the lower values of ξ imply greater systematic risk which leads to higher unexpected loss, but in turn it minimizes the GA. Also observe that in the Gordy and Lütkebohmert (2007) appendix it is mentioned that the range $4.5 < \delta < 6.5$ is in line with common practice.

Having analysed mathematical aspects of estimating GA, next step is to see how the proposals work in practice and what are the empirical results.

3. Empirical estimation results of single name concentration risk

This chapter gives an overview of empirical GA estimation results using real data of one bank**. The goal of this chapter is to show how GA works in practice, how it

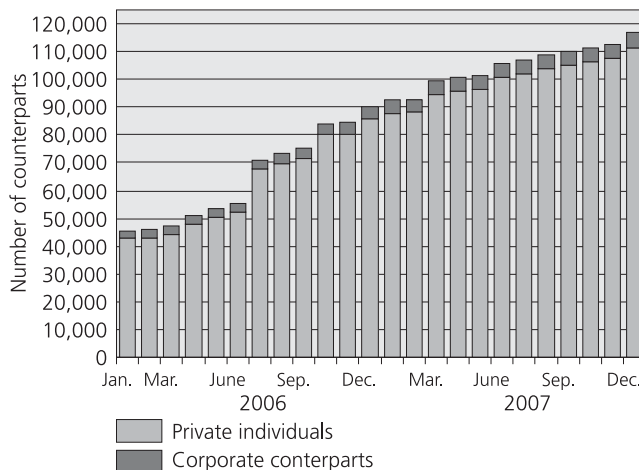
*This equation has a unique solution in the interval [0; 2].

**This Eastern Europe bank operates only in one country, but is among the largest three banks in the country. Bank is undertaking universal operations, it has strong market positions both in non-retail (corporate) and retail credit segments. As bank is a leader in crediting corporate (majority of largest corporate have exposures in this bank) so naturally this has effect on GA. The portfolio analyzed below encompasses all exposures, of all types and to all types of counterparts, excluding exposures to central government and subsidiaries of bank.

changes with time, how empirical estimation results compare with those given in Gordy and Lütkebohmert and how proposals in Chapter 2 for GA estimation work on real bank data and affect GA. As for smaller and less sophisticated banks, especially those not using IRB model for calculating the required credit risk capital, it might be impossible to implement GA in practice, empirical calculation results could be a good benchmark how much capital might be needed to cover single name concentration risk.

Figure 2 below shows the evolvement of number of counterparts in the reference portfolio.

Figure 2. Number of counterparts in reference portfolio

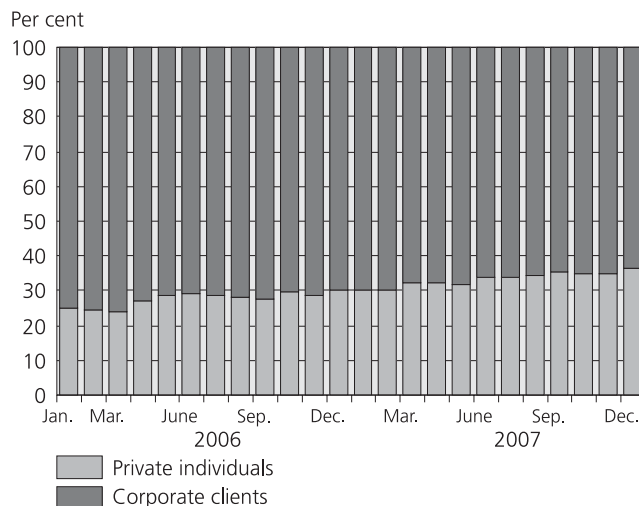


Source: data from one Eastern Europe bank.

The Figure 2 above shows that during two years of observation period the reference portfolio was rapidly growing in the number of counterparts. This is explained by the fact that the reference bank operates in one of the new EU member states, where financial sector was not developed for many years. Only during the observation period banking and the whole financial sector experienced credit boom. To the analysis of GA this adds additional interest as this fact would enable to investigating how GA behaves in changing economic environment and radically changing credit portfolio.

Figures 3 and 4 below show that during the observation period credit portfolio of private individual retail counterparts has increased considerably. Increase in funding of private individuals was mainly driven by increased number of mortgage and consumer loans.

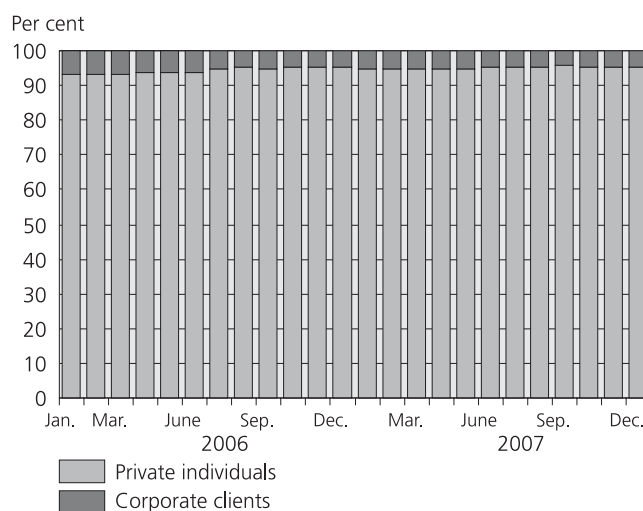
Figure 3. Distribution of portfolio between borrower types (measured on exposure size level)



Source: data from one Eastern Europe bank.

Although consumer loans are very small in amount and mortgage exposures are somewhat larger, but on average exposures to private individuals are far smaller than to corporate borrowers (compare Figures 3 and 4). On the other hand, as private individuals' credit portfolio has increased considerably during the observation period as well as the number of private individuals counterparts (see Figure 2), this caused to increase portfolio diversification (which is evident from changes in Herfindahl-Hirschmann Index (HHI)).

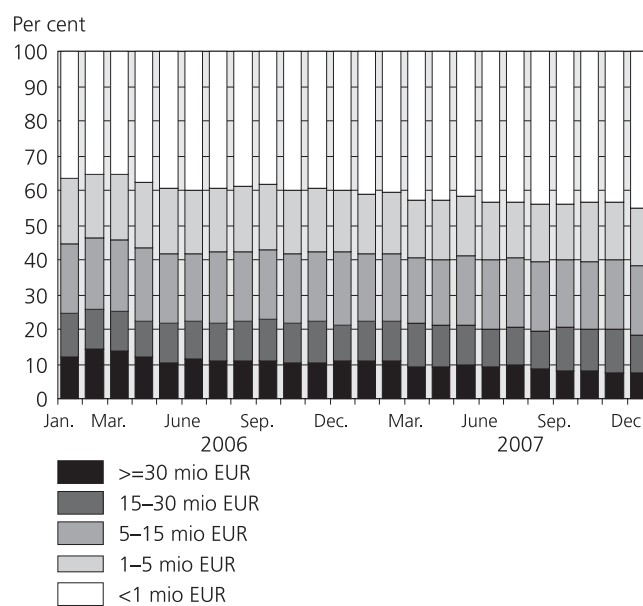
Figure 4. Distribution of portfolio between borrower types (measured on number of clients level)



Source: data from one Eastern Europe bank.

Portfolio structure in Figure 5 below also shows that the share of counterparts with small total exposure (that would mainly be consumer, mortgage exposures to private individuals and exposures to small corporations) has increased during the observation period. Again, this means that during the observation period portfolio diversification increased, implying that one would expect to observe a decreasing trend in GA although the main impact on portfolio concentration and GA comes from largest counterparts and their share in total credit portfolio.

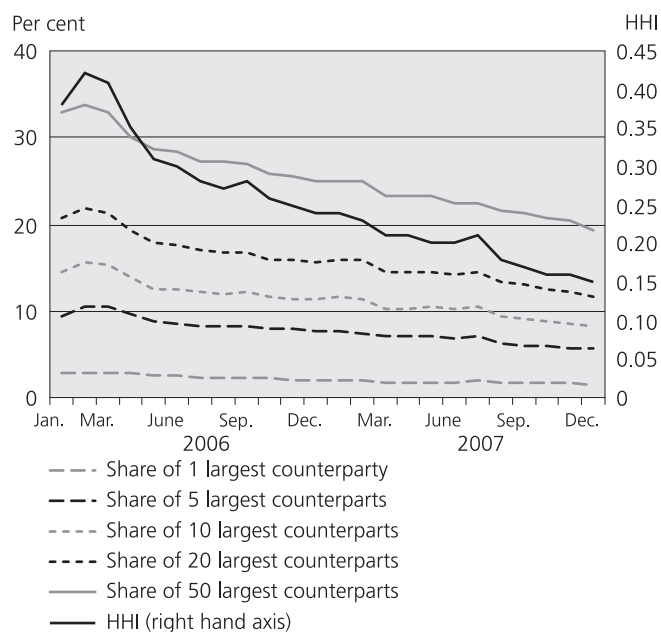
Figure 5. Distribution of portfolio with respect to exposure size (measured on counterparty level)



Source: data from one Eastern Europe bank.

Figure 6 below shows how HHI was evolving during the observation period. In parallel information is presented on how the share of largest counterparts was evolving. As it could have been expected from changes in portfolio structure, shown in previous figures, HHI was decreasing during observation period, i.e. single name concentration risk was decreasing. All the indicators on share of largest counterparts were decreasing, again showing decreasing trend in single name concentration risk.

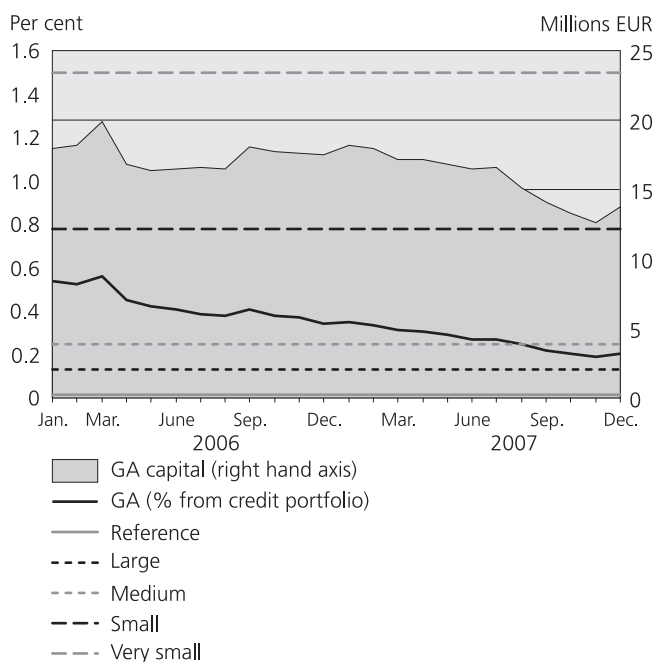
Figure 6. Herfindahl-Hirschmann index and share of largest counterparts



Source: authors' calculations.

Finally Figure 7 below depicts GA calculation results (the detailed numbers are provided in Table 1 in the Appendix).

Figure 7. Empirical GA⁺



Note: *aggregated GA estimation data is provided in Table 1 in the Appendix.

Source: authors' calculations.

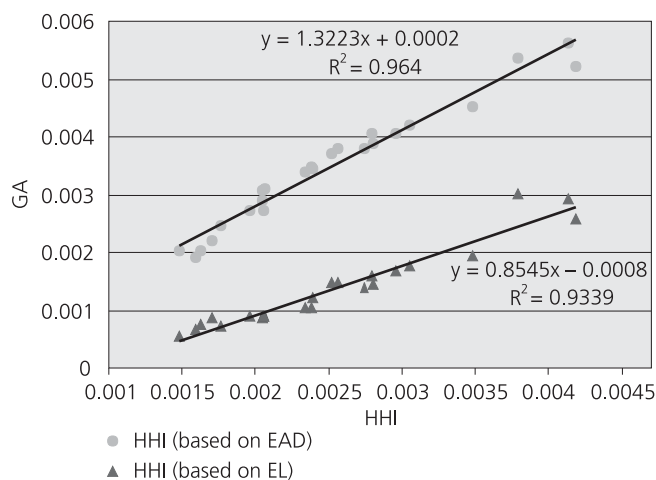
For the reasons described earlier, portfolio GA was decreasing over the observation period. At the beginning of the observation period, when number of counterpart and credit portfolio was small, GA was high (required capital to cover single name concentration risk was 0.55 per cent from total credit portfolio under consideration). Comparing this result with that in Gordy and Lütkebohmert (2007), GA of 0.55 per cent corresponds to small bank credit portfolio. By the end of observation period, as credit portfolio and number of counterpart increased considerably, GA dropped to 0.2 per cent and this corresponds to medium credit portfolio GA.

It should be also noted, that throughout the whole observation period GA required capital in absolute terms did not change significantly: at the beginning of observation period capital to cover single name concentration risk was close to EUR 18 millions and at the end of the period when portfolio increased in size and number of counterpart and concentration reduced (HHI dropped from 0.004 to 0.0015) GA required capital decreased to EUR 14 millions. From the figures above and statistical data in the appendix of the paper, it can be concluded that during observation period capital for single name concentration risk did not change drastically, although portfolio increased in size several times. This implies, that single name concentration risk capital did not change in absolute terms, but its share in total capital was changing dramatically. This would mean that for small banks with relatively concentrated portfolios single name concentration risk capital might take up relatively large proportion of the total capital. Increased capital requirements for such banks, having no parent banks abroad, might cause serious reductions in capital ratios.

Empirical estimation results provided above, could be a good benchmark for banks, which do not estimate PD, LGD and other credit risk parameters, needed to calculate GA required capital. Banks not opting for Basel II IRB, could set capital for single name concentration risk based on their credit portfolio HHI. In other words from Figure 6 and Figure 7 above banks could find out what capital should be set aside for single name concentration risk, corresponding to their portfolio HHI. This would yield a rough estimate of the required capital.

Figure 8 below shows that there is a strong linear relationship between HHI and GA. From this figure banks having no PDs and LGDs and not opting for IRB model, might find out what GA per cent corresponds to their portfolio HHI. It should be noted however, that empirical estimation results on GA and HHI presented below are for a bank that is local market leader in large corporations credit segment. This implies that this bank might have relatively larger share of large credits in its portfolio compared to its peers. Thus presented relationship between HHI and GA should be considered as with high conservative margin.

Figure 8. Relationship between HHI and GA

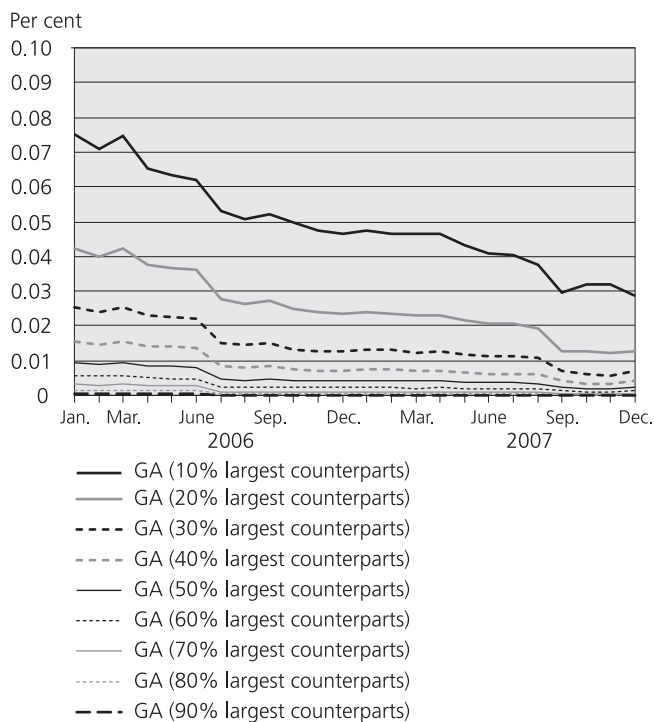


Source: authors' calculations.

Gordy and Lütkebohmert (2007) also provide simplified GA estimation method, when not all, but some predefined number of largest counterpart are included into GA

estimations. This way GA approximation is obtained. On one hand this makes all the calculations easier, as not all of the exposures should be included for estimating GA. On the other hand, approximation leads to higher GA compared to that if all exposures would be included into calculations. So the natural and interesting question is how many exposures should be included into GA estimations not to increase GA itself too much?

Figure 9. Upper bound GA (differences from full portfolio GA in percentage points)



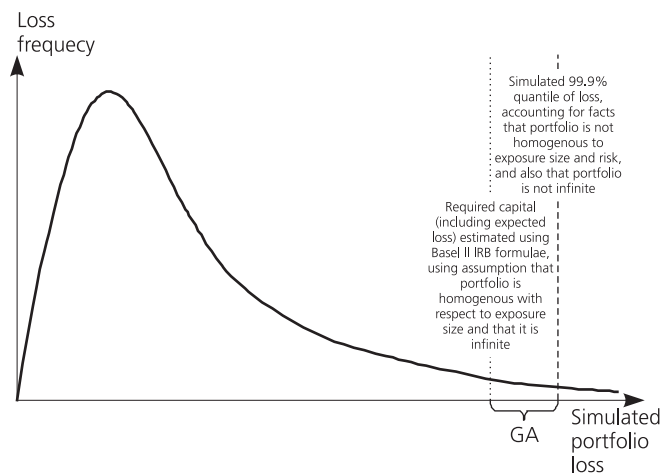
Source: authors' calculations.

The figure above shows the differences between two GAs, namely GA including x per cent of largest counterparts and GA estimated using all of the counterparts. The results in the figure above reveal that upper bounds works properly and as depicted in the figure above, one observes very fast approximation of the true portfolio GA with increasing share of counterparts used for estimating GA. The apparent difference is only observed in cases when 10 per cent, 20 per cent, 30 per cent and 40 per cent of largest exposures are included into portfolio for estimating GA. Although one should also observe that with time and increasing portfolio diversification (as evident from figures earlier) difference between upper bound approximations and total portfolio GAs are stable. For example at the beginning of the observation period, when portfolio concentration was the highest, difference between 10 per cent portfolio GA and GA of total portfolio was slightly beneath 1 percentage point. At the end of the observation period this difference between approximated GA and total portfolio GA decreased 2.6 times. If one includes at least 40 or 50 per cent of total portfolio counterparts in GA estimation, increase in capital compared to total portfolio GA shall be negligible.

It is also possible to estimate or back-test the above GA calculations using "straight-forward and dirty" method. In Basel II IRB framework ASRF credit risk model is used. As in IRB model asymptotic assumption is not met, in GA calculations above two quantiles of loss, namely of true portfolio risk and infinitely granular portfolio risk are compared. Using Monte Carlo simulations and single risk factor model it is possible to obtain loss distribution of bank credit portfolio (not infinitely granular portfolio) and by comparing the unexpected loss of this simulated loss distribution with actual Basel II IRB required capital, one would obtain required capital to cover single name concentration risk or GA. In other words, usage of single risk factor credit risk model and Monte Carlo simu-

lations shall account for differences in exposure size, credit portfolio concentrations and other factors, that have impact on single name concentration risk (see Figure 10 below)*.

Figure 10. Monte Carlo simulation of GA



Source: formed by the authors.

Simulation is dependent on assumption that both systematic and idiosyncratic risk components are normally distributed and default is modelled using Merton type model, i.e. simulation is done using the same model as that in Basel II IRB, but as simulations are done for many times this way asymptotic assumptions are met and loss distribution obtained. In reality, for example systematic risk component might be distributed not normally, for example take gamma distribution. In this case simulation would lead to other loss distribution, other capital for unexpected loss and comparison results between Basel II IRB required capital and simulated capital would be different.

Monte Carlo simulation on empirical data (end of December 2007) yielded, that after 6 millions of iterations GA is close to EUR 9.2 millions. Whereas closed form estimation results for the same date indicate that GA is equal to EUR 13.7 millions (see Figure 7). Simulation results differs from closed form estimation results (they cannot match exactly for some methodological differences, like treatment of systemic risk (in closed form estimation systemic risk is modelled using gamma distribution, whereas in Monte Carlo simulation systemic risk factor follows normal distribution)).

Although Monte Carlo simulation might seem as a straightforward way for obtaining GA required capital, it has some disadvantages compared to GA estimation. First of all, as for GA estimation purposes one needs 99.9 per cent quantile, this implies that actually a quantile is simulated. This implies that very large number of simulations needs to be run in order to get stable quantile estimate. As the tail of simulations shall contain only 0.1 per cent of all observations this means that after 1 million of iterations the tail shall contain only 1,000 observations of loss. Alternatively one could say that this is equivalent to 1,000 iterations of quantile. Clearly 1,000 observations in tail shall not give stable quantile value. In order to get stable quantile estimate, possibly 10 millions of iterations in total (10,000 observations in tail) might be needed. It arises from Glivenko-Cantelli's law which states that the empirical loss distribution could be approximated with the \sqrt{n} rate (n is the number of simulations). Thus from 10 million simulations one would expect the $3.163e^{-4}$ accuracy, and since portfolio total amount reaches about 6.7 billions euros we get that the precision could not be better than 1–2 million euros. So many iterations require huge IT resources**. The second disadvantage of simulation is the fact that if GA required capital suddenly changes because of structural changes in credit portfolio, it might not be clear where this change comes from as simulation gives only the final result. Closed form solution to GA estimates GA_i to each counterparty and one can infer how GA_i changed and which counterpart contributed the most to the final GA figure. Contribution to final GA figure by individual borrowers is another topic of interest that is discussed in the following chapter of the paper.

*Monte Carlo simulation process is depicted in Figure 2 in the Appendix.

**Moreover, if portfolio contains 100,000 positions, then 10 million iterations should be multiplied by 100,000 positions.

The final step in analysing practical aspects of GA is its application in everyday operations of bank. For this reason next chapter considers problem of GA allocation.

4. Allocation of credit concentration risk capital to individual exposures

If calculated GA capital is used in ICAP, meaning that bank actually allocates part of capital to cover single name concentration risk, GA capital has to be allocated to individual counterparts. In risk based pricing or risk adjusted performance measurement economic capital allocated to exposure is being used. This implies that banks using Basel II IRB capital for risk based pricing or risk adjusted performance measurement have to add to this additional capital required to cover single name concentration risk. For example risk adjusted return on capital formulae for i^{th} exposure has to be adjusted this way:

$$\text{RaRoC}_i = \frac{\text{Risk_adjusted_return}}{\text{IRB_capital}_i + \text{GA_capital}_i}$$

In formula (4) in the previous chapter total portfolio GA is a sum of GA_i , i.e. total GA is equal to sum of absolute GA_i contributions of all counterparts in the portfolio. It should be noted that GA_i (absolute contributions to GA by single counterparty) does not represent additional capital needed to be included in risk based pricing or risk adjusted performance measurement. In fact so called marginal contributions to GA must be used*.

Marginal GA contribution for i^{th} counterparty is defined in the following way:

$$\text{GA}_i^{\text{marginal}} = \text{GA}_{\text{all_portfolio}} - \text{GA}_{\text{all_portfolio_excluding_i_exposure}}$$

The practical problem with marginal GA is that it is computationally demanding to recalculate portfolio GA excluding one by one each and every exposure. A practical solution here might be the fact that for very small exposures in portfolio marginal GA can be well approximated by absolute GA_i . For the largest exposures, which are the major source of concentration in credit portfolio and contribute the most to final portfolio GA, marginal GA would need to be calculated. For example using the data from the previous chapter, absolute contribution GA_i for largest counterparty in portfolio was EUR 1.495 million when for the same counterparty marginal GA was only EUR 1.328 million. For small exposures, that help to diversify credit portfolio with respect to exposure concentration, marginal GAs are negative. This is of no surprise, as small exposures help to diversify portfolio.

Conclusions

Credit concentration risk, being one of the main Basel II requirements Pillar II issues, receives big attention among practitioners and supervisors. The Basel II IRB models were built to fit all banks (involving the properties of universality for all banks, capital portfolio invariance and other), thus they contain some simplifying assumptions.

To make models simple and fit all banks elegant decision was introduced – ASRF portfolio credit risk model. Single risk factor and asymptotic are the two main assumptions of Basel II IRB model. On the other hand simplifications have led to some shortcomings in model and supervisors require banks to overcome these shortcomings. For example, asymptotic assumption in the model implies that the portfolio is infinitely fine grained and thus it consists of a nearly infinite number of credits with comparatively small exposures. As this is not true in practice, banks are required to measure additional required capital to cover single name concentration risk or alternatively account for not perfect granularity of their portfolios.

Performed survey of literature suggests that there are at least three approaches how granularity adjustment could be estimated: by Vasicek (2002), by Emmer and Tasche (2005) and by Gordy and Lütkebohmert (2007). Analysis of these three approaches leads to the conclusion that the third approach proposed by Gordy and Lütkebohmert is applicable in practice and has less drawbacks than the other two approaches.

*For more details see, for example, Bessis (2002).

Although Gordy and Lütkebohmert approach could be implemented in banks, trying to implement this approach in practice inevitable leads to the following challenges for which the paper suggests practical solutions:

1) as GA measures single name concentration risk, all credit risk parameters (PD, LGD, EAD, ρ) in GA formulae are on counterparty level. As some counterparties might have several exposures one needs to aggregate PD, LGD, EAD and ρ from single exposures to counterparty level;

2) banks are required to estimate volatility of LGD;

3) estimation of model parameter ξ which is a function of $PD(X)$ and portfolio dependent. This is a drawback, because this parameter describes systemic risk and must be unique and satisfy portfolio invariance. Thus ξ should not be dependent on any credit risk parameters;

4) estimation of δ is complicated, because to obtain δ one needs to solve non-linear equation. If GA is estimated each time capital adequacy is being calculated, each time non-linear equation for δ must be solved.

The paper also proposes an alternative approach for measuring concentration risk capital using Monte Carlo simulations. As IRB model does not meet asymptotic assumption, in GA calculations true portfolio risk and infinitely granular portfolio risk are compared. Using Monte Carlo simulations and single risk factor model it is possible to obtain actual loss distribution of bank credit portfolio and compare unexpected loss of this simulated loss distribution with empirical (IRB or infinity granular portfolio) unexpected loss. Although Monte Carlo simulation is a straightforward way to estimate GA, it has some disadvantages compared to GA estimation using closed form solution. First of all, large number of simulations needs to be run in order to get stable quantile estimate. Second, if GA required capital suddenly changes because of structural changes in credit portfolio, it might not be clear where this change comes from, as simulation gives only the final result. Third, contribution to final GA figure by individual borrowers is not known if Monte Carlo simulations are used.

Empirical analysis of GA estimation results also suggests that there is a strong linear relationship between HHI and GA. This relationship enables banks not having own PD and LGD parameters to infer what could be approximate GA required capital based on concentration of their portfolio. The latter approach serves only as an approximation, as GA, unlike HHI, considers not only exposure size, but also exposure risk. In other words, GA and HHI relationship presented in the paper holds only for similar portfolio analyzed in the paper.

Finally, the paper suggests that GA capital should be allocated to individual counterparties, and the latter figures should be used in risk adjusted performance and risk based pricing models, along with IRB capital.

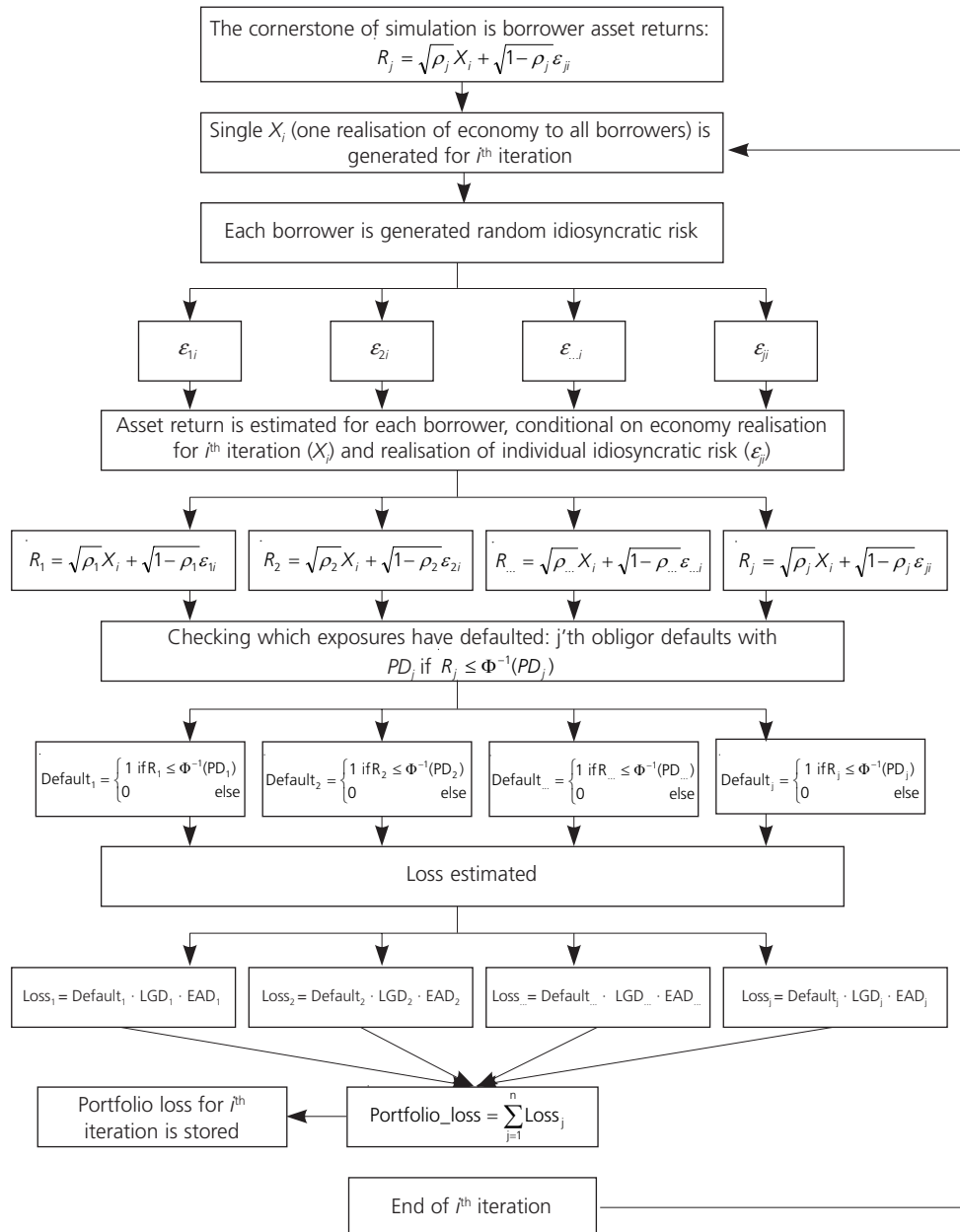
Table 1

GA estimation summary data

Period	Number of counterpart		Portfolio EAD (millions EUR)		Portfolio capital (millions EUR)	HHI (in percentage points)	Share of largest party, %	Share of largest 5 counter-parts, %	Share of largest 10 counter-parts, %	Share of largest 20 counter-parts, %	Share of largest 50 counter-parts, %	δ	X_i	γ	GA (% portfolio EAD)	GA (millions EUR)
	Corporate	Private individuals	Corporate	Private individuals												
1	3,031	42,650	2,505.4	842.1	3,347.54	0.37897	2.91	9.50	14.47	20.67	32.80	5.216562	0.41132	0.22266	0.53546	17.9
2	3,072	43,176	2,645.5	849.6	3,495.18	0.41853	2.92	10.41	15.69	21.84	33.80	5.193842	0.39939	0.22146	0.52191	18.2
3	3,114	44,032	2,688.5	854.2	3,542.73	0.41330	2.85	10.44	15.44	21.33	33.01	5.192103	0.39849	0.22463	0.56153	19.9
4	3,225	47,779	2,714.7	996.3	3,711.00	0.34799	2.70	9.60	13.96	19.38	30.13	5.268134	0.43971	0.21929	0.45316	16.8
5	3,334	50,457	2,787.8	1,109.8	3,897.61	0.30543	2.57	8.71	12.60	17.77	28.53	5.278635	0.44572	0.21858	0.42109	16.4
6	3,394	52,200	2,887.0	1,175.7	4,062.68	0.29554	2.47	8.54	12.55	17.69	28.26	5.270777	0.44121	0.21665	0.40598	16.5
7	3,596	67,487	3,040.1	1,216.3	4,256.44	0.28049	2.38	8.32	12.16	17.06	27.33	5.323445	0.47229	0.21846	0.38897	16.6
8	3,622	69,560	3,099.1	1,225.8	4,324.94	0.27436	2.34	8.15	11.96	16.79	27.18	5.315181	0.46728	0.21940	0.38069	16.5
9	3,762	71,407	3,218.7	1,229.2	4,447.94	0.27909	2.26	8.36	12.06	16.64	26.94	5.302534	0.45970	0.22099	0.40697	18.1
10	3,887	80,172	3,281.5	1,383.8	4,665.25	0.25644	2.14	7.96	11.50	15.99	25.84	5.331527	0.47725	0.21711	0.37896	17.7
11	3,933	80,403	3,385.9	1,373.4	4,759.30	0.25201	2.10	7.88	11.36	15.81	25.55	5.308362	0.46318	0.21874	0.37079	17.6
12	4,043	86,011	3,532.7	1,527.6	5,060.23	0.23921	1.99	7.57	11.27	15.68	25.05	5.313533	0.46628	0.21281	0.34533	17.5
13	4,733	87,663	3,653.9	1,585.6	5,239.54	0.23859	1.95	7.64	11.52	15.83	24.88	5.314203	0.46669	0.21092	0.34728	18.2
14	4,755	88,215	3,689.2	1,598.3	5,287.55	0.23427	1.91	7.51	11.34	15.81	24.87	5.304960	0.46115	0.21012	0.33782	17.9
15	4,977	94,832	3,749.6	1,798.2	5,547.81	0.20647	1.81	7.12	10.32	14.41	23.22	5.358903	0.49442	0.20551	0.31085	17.2
16	5,088	95,492	3,762.6	1,810.1	5,572.62	0.20510	1.79	7.03	10.35	14.48	23.35	5.351327	0.48961	0.20617	0.30776	17.2
17	5,145	96,358	3,938.0	1,832.2	5,770.22	0.20489	1.73	6.97	10.37	14.52	23.18	5.322183	0.47152	0.20592	0.29042	16.8
18	5,228	100,566	4,014.7	2,046.5	6,061.21	0.19636	1.64	6.91	10.17	14.16	22.40	5.288203	0.45127	0.19949	0.27144	16.5
19	4,987	101,724	4,053.9	2,077.8	6,131.68	0.20569	1.98	7.16	10.44	14.38	22.49	5.270891	0.44128	0.19847	0.27111	16.6
20	5,023	103,554	4,032.5	2,119.8	6,152.27	0.17654	1.61	6.17	9.46	13.36	21.61	5.291726	0.45333	0.19694	0.24588	15.1
21	4,797	105,308	4,148.3	2,285.3	6,433.57	0.17027	1.64	5.99	9.16	13.03	21.23	5.154190	0.37939	0.21850	0.21924	14.1
22	5,070	106,416	4,229.0	2,287.2	6,516.23	0.16240	1.63	5.90	8.68	12.48	20.67	5.153083	0.37884	0.23796	0.20414	13.3
23	5,237	107,444	4,301.3	2,299.6	6,600.93	0.15933	1.60	5.81	8.53	12.19	20.49	5.160466	0.38249	0.24683	0.19052	12.6
24	5,628	111,549	4,331.1	2,474.5	6,805.63	0.14766	1.55	5.66	8.26	11.71	19.41	5.243997	0.42619	0.21768	0.20145	13.7

Source: data from one Eastern Europe bank.

Figure 1. GA simulation process



Source: formed by the authors.

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Summary

PASKOLŲ PORTFELIO KONCENTRACIJOS RIZIKOS VERTINIMAS TAIKANT GRANULIARUMO MATAŲ: PRAKTINIAI ASPEKTAI

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Pastaruoju metu nemažai pasaulio bankų įdiegė naujus kapitalo pakankamumo reikalavimus ir vidaus reitingais pagrįstus kapitalo poreikio skaičiavimo modelius. Šie modeliai buvo kuriami skirtingose šalyse veikiantiems ir skirtinga bankine veikla užsiimantiems bankams, bet bankų priežiūros institucijos siekė, kad jie tiktų visiems. Todėl šie modeliai yra pagrįsti tam tikromis supaprastintomis prielaidomis. Viena iš tokių prielaidų – apie paskolų portfelių sudarančių paskolų begalinį skaičių. Bankų paskolų portfelius sudaro baigtinis paskolų skaičius, todėl taikant vidaus reitingais pagrįstą modelį ši prielaida netenkinama ir apskaičiuojamas mažesnis kapitalo poreikis, nei jis iš tikrųjų yra.

Siekiant išvengti tokių trūkumų, buvo kuriami metodai, kaip vidaus reitingais pagrįstą modelį patikslinti, didelių skolininkų koncentracijos rizikai padengti skaičiuojant papildomo kapitalo poreikį. Apžvelgtos literatūros analizė parodė, kad tokių metodų yra trys. Iš jų mažiausiai kritikuotinas ir praktiškai pritaikomas bankuose yra M. B. Gordy ir E. Lütkebohmert pasiūlymas. Šio straipsnio tikslas – išanalizuoti M. B. Gordy ir E. Lütkebohmert pasiūlyto metodo praktinius ypatumus, aptarti empirinius šio metodo taikymo rezultatus

ir pasiūlyti alternatyvų metodą, leidžiantį įvertinti kapitalo poreikį stambių skolininkų koncentracijos rizikai padengti.

Straipsnio pirmoje dalyje analizuojami vidaus reitingais pagrįsto modelio teoriniai aspektai, taip atskleidžiant šio modelio prielaidas ir priežastis, kodėl jį būtina tikslinti. Toje pačioje dalyje taip pat aptariamas M. B. Gordy ir E. Lütkebohmert pasiūlytasis metodas, skirtas didelių skolininkų paskolų koncentracijos rizikai įvertinti. Nors šis metodas ir tinkamas įgyvendinti bankuose, pradedant tai daryti susiduriama su keletu praktinio pobūdžio sunkumų. Pirma, granuliarumo rodiklis taikytinas skolininkams, o ne jų paskoloms, todėl būtina sustambinti to paties skolininko turimų kelių paskolų įsipareigojimų neįvykdymo tikimybės (PD), nuostolio įsipareigojimų neįvykdymo atveju (LGD), paskolos dydžio įsipareigojimų neįvykdymo atveju (EAD) ir turto verčių koreliacijos rodiklius. Antra, norint apskaičiuoti granuliarumo rodiklį, būtina įvertinti LGD rodiklio kintamumą, taip pat rodiklius ξ ir δ . Rodiklis ξ priklauso nuo paskolų portfelio sudėties ir skolininko PD rodiklio, tačiau turėtų būti egzogeninis. Rodiklis δ nustatomas iš netiesinės lygties, kuri turi būti išsprędžiama kiekvieną kartą skaičiuojant granuliarumo rodiklį, todėl būtinas greitas ir tikslus algoritmas, leidžiantis rasti δ rodiklį. Straipsnio autoriai pateikia siūlymų, kaip šias praktines problemas išspręsti, banke diegiant granuliarumo matą.

Straipsnyje siūloma alternatyva M. B. Gordy ir E. Lütkebohmert nurodytam metodui – kapitalo poreikį didelių skolininkų paskolų koncentracijos rizikai vertinti taikant *Monte Carlo* metodą. Žinant paskolų portfelio PD, LGD ir EAD rodiklius, galima modeliuoti paskolų portfelio nuostolį, rasti šio nuostolio skirstinį, o iš skirstinio – kapitalo poreikį atitinkantį 99,9 procento kvantilį. Palyginus taip apskaičiuotą kapitalo poreikį su kapitalo poreikiu, apskaičiuotu vidaus reitingais pagrįstu metodu, nustatomas kapitalo poreikis didelių skolininkų paskolų koncentracijos rizikai padengti. Nors *Monte Carlo* skaičiavimai nesudėtingi, tačiau, palyginti su M. B. Gordy ir E. Lütkebohmert pateiktu metodu, jie pasižymi keletu trūkumu. Pirma, tokie skaičiavimai ilgai trunka, nes modeliuojamas skirstinio 99,9 procento kvantilis. Antra, jeigu galutinis rezultatas pakinta, nežinoma, kodėl šis pokytis atsirado, nes *Monte Carlo* skaičiavimais gaunamas tik galutinis rezultatas, be tarpinių skaičiavimų. Trečia, *Monte Carlo* skaičiavimais negalima įvertinti, kiek reikšmės galutiniam rezultatui turi vienas ar kitas skolininkas.

Atlikta empirinė skaičiavimo rezultatų analizė parodė, kad egzistuoja stiprus tiesinis ryšys tarp granuliarumo mato ir *Herfindahl-Hirschmann* indekso (HHI). Šis faktas svarbus dėl to, kad ne visi bankai skaičiuoja PD, LGD ir EAD rodiklius, o jų neturėdami jie negali apskaičiuoti granuliarumo rodiklio. Tačiau visi bankai gali įvertinti paskolų portfelio HHI rodiklį. Taigi visi bankai kapitalo poreikį didelių skolininkų koncentracijos rizikai padengti gali įvertinti, pasinaudodami nustatyta priklausomybe tarp granuliarumo rodiklio ir HHI.

Paskutinėje straipsnio dalyje atskleidžiama, kad apskaičiuotas papildomas kapitalo poreikis didelių skolininkų rizikai padengti turi būti paskirstomas atskiriems skolininkams ir jį turi būti atsižvelgiama taikant kainodarą, pagrįstą rizika, taip pat paskolų pelningumo analizę.