INTEREST RATE RISK FACTORS IN THE AUSTRALIAN BOND MARKET

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Abstract

This paper reports the results of an empirical analysis of the nature of the factors that explain changes in Australian bond market yields. Consistent with overseas studies it is found that three factors are sufficient to explain most of the historical yield curve variation in Australia. These factors can be interpreted as a "parallel" shift factor, a "slope" factor and a "curvature" factor. Although one factor interest rate models are often used to price and hedge interest rate dependent claims it is clear that, at least for hedging purposes, a two or three factor model will be essential.

Keywords:
Interest rate risk, duration, convexity, factor analysis

Introduction

Modeling interest rates for the purposes of pricing and hedging of interest rate dependent cash flows has been a topic of much interest to actuaries over recent years. Developments in financial economics have included a theory of arbitrage pricing with the most general case of a multi-factor model given by Heath, Jarrow and Morton (1992).

For computational reasons, a single factor interest rate model is often used for pricing and hedging although an increasing number of practitioners are developing multi-factor models. Such single factor models are the stochastic equivalent of the deterministic parallel shift model for insurance liability immunisation developed by Redington (1952) since they imply that all yield changes are perfectly correlated.

It is therefore of interest to determine empirically how many factors drive interest rates from historical data in order to formulate a parsimonious model for pricing and hedging interest rate dependent cash flows. Although for pricing purposes a one factor interest rate model might be sufficient to determine an adequate arbitrage free pricing model this will not be adequate for hedging purposes if the yield curve is in fact driven by more than one factor. Once the number of factors is determined techniques as covered by Tilley (1992) can be used to generate interest rate models that 'fit' current yield curve and volatility parameters.

Studies of the interest rate risk factors in the various bond markets including the U.S. bond market, the Danish market and the Italian bond market have been carried out (see D'Ecclesia and Zenios (1994) for details). These studies demonstrate both similarities and differences in international bond markets. It appears that three factors explain the major portion of yield curve changes but the relative importance of each of these factors differs in different bond markets.

The purpose of this study is to apply factor analysis to determine the number of factors that explain the changes in the Australian bond market yield curve. The structure of the paper is as follows: the first section provides some background on the Australian bond market and describes the yield data, the second section sets out the results of the factor analysis of yield changes and the final section summarises the implications for pricing and hedging.

The Australian Bond Market

The Australian Commonwealth government issues by tender both short term Treasury Notes with maturities of 13 and 26 weeks at issue and coupon paying Treasury Bonds with various maturities. Treasury Notes have been issued by tender since 1979 and Treasury Bonds since August 1982. Prior to the tender system these securities were issued by a variety of methods including a 'tap' system. There is an active secondary market in these Treasury securities. Graph 1 provides a plot of secondary market yields to maturity for 13 week Treasury Notes, 2 year, 5 year and 10 year Treasury Bonds for the period January 1972 to October 1994. The Bond yields are determined by the Reserve Bank of Australia based on reported secondary market transactions.

Yields to maturity are used in this study. It would be theoretically more correct to use spot or forward yields determined from these yields to maturity. This has been left for later study. One of the reasons that the results for yields to maturity have been reported is that the results of this study were found to be consistent with studies of other bond markets based on spot yields.

Table 1 provides summary statistics for this yield data. Note that the data is monthly in the form of yields as per cent per annum. From Graph 1 and Table 1 it can be seen that on average over this period yield curves were upward sloping although there were periods when the yield curve was inverse. Short term yields were more volatile than long term yields. From mid 1982 the yield curve appears to be more volatile. This is assumed to be related to the introduction of the tender system for Treasury bonds and the development of a more active secondary market. Because this period was different from the total period the study is also carried out for the period August 1982 to October 1994 separately. Statistics for this time period are provided in Table 2.

Graph 1

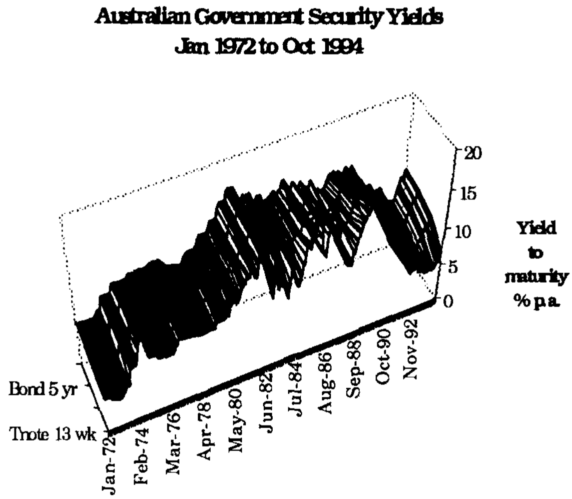

Table 1

Australian Government Security Yields Jan 1972 to Oct 1994[a]

	Mean	Median	Standard deviation	Maximum	Minimum
Treasury note 13 week	9.694	9.020	3.940	19.400	3.850
2 year Bond	10.157	9.910	3.305	16.500	4.600
5 year Bond	10.906	10.750	2.842	16.400	5.190
10 year Bond	10.630	10.240	2.874	16.400	5.690

[a] Note that the 13 week Treasury Note, 2 and 10 year Bond data include observations from July 1969 to October 1994

Table 2
Australian Government Security Yields Aug 1982 to Oct 1994

	Mean	Median	Standard deviation	Maximum	Minimum
Treasury note 13 week	11.398	11.620	4.043	19.400	4.650
2 year Bond	11.564	12.500	3.112	16.100	5.100
5 year Bond	11.862	12.800	2.576	16.100	6.000
10 year Bond	12.030	12.950	2.307	16.100	6.350

The aim of this study is to examine the number of factors that explain the changes in the yield curve. The correlation between changes in yields of different maturities provides important information about these factors. For example, if the changes in yields were to be perfectly correlated then only one factor would be required to explain yield curve changes.

If there were no correlation between yields of different maturities then a separate factor would be required for each maturity. Using a separate factor for each maturity is the underlying assumption that corresponds to the use of multivariate or key rate durations as in Reitano (1991a, 1991b) and the use of yields at different maturities as factors in interest rate models (Tilley, 1992). A more parsimonious model of yield curve changes can be developed by determining the smallest number of factors required to explain yield curve changes. This will allow for more efficient computation of prices and hedge statistics for multiple factor models.

Tables 3 and 4 summarise the correlations between the bond yields to maturity for the two data periods. It is interesting to note that the correlation structure of the yields to maturity does not vary much between the full period and the period since the start of Treasury Bond tenders. The correlation structure is closer to that reported for the U.S. Bond market than it is for other bond markets, such as Italy, where similar studies have been carried out (D'Ecclesia and Zenios (1994)).

Table 3
Correlations of changes in yields to maturity January 1972- October 1994

Maturity	13 weeks	2 years	5 years	10 years
13 weeks	1.000			
2 years	0.649	1.000		
5 years	0.560	0.936	1.000	
10 years	0.503	0.853	0.929	1.000

Table 4
Correlations of changes in yields to maturity August 1982- October 1994

Maturity	13 weeks	2 years	5 years	10 years
13 weeks	1.000			
2 years	0.712	1.000		
5 years	0.618	0.943	1.000	
10 years	0.567	0.864	0.927	1.000

The lowest correlation occurs between the short term interest yield and the medium to longer term yields. The medium term yields have much higher correlations with the longer term yields. Based on these correlations, it would be a surprising result if a single factor interest rate model, which assumes all yields are perfectly correlated, were found to be an adequate basis for explaining the future evolution of interest rates.

Factor Analysis of Yields

In order to determine the number of factors required to explain yield curve changes a multivariate factor analysis technique is used. All multivariate statistics text provide a coverage of the technique and most statistical packages provide routines for carrying out such an analysis. For example Kleinbaum, Kupper and Muller (1988) is one such text and MINITAB Version 9 includes factor analysis routines.

A factor analysis of yield curve changes estimates the following relationship for each maturity for n factors:

$$\Delta y_{mt} = \sum_{j=1}^{n} \beta_{jm} F_{jt} + \varepsilon_{mt}$$

where

Δy_{mt} is the change in the yield to maturity for maturity m at time t;

F_{jt} is the value of the jth independent (random) factor at time t;

β_{jm} is the factor loading for the jth factor for maturity m;

ε_{mt} is an error term representing the variability unique to maturity m not explained by the n factors.

The factor loadings were estimated using the correlation matrix and principal component factor analysis. Three factors were assumed since three factors have been found to explain almost all of the variability in yield curve changes in overseas studies. The factor loadings based on the full period of data are given in Table 5. Table 6 gives the factor loadings based on the period from August 1982 to October 1994

Table 5
Factor loadings for Yield Changes Jan 1972 to Oct 1994

Maturity	Factor 1	Factor 2	Factor 3
13 weeks	-0.728	-0.683	0.068
2 years	-0.964	0.047	-0.244
5 years	-0.965	0.026	-0.046
10 years	-0.927	0.273	0.248
Percentage of variance explained by factor	81.2	14.6	3.2

Table 6
Factor loadings for Yield Changes Aug 1982 to Oct 1994

Maturity	Factor 1	Factor 2	Factor 3
13 weeks	-0.776	-0.627	0.069
2 years	-0.970	0.037	-0.217
5 years	-0.965	0.025	-0.072
10 years	-0.929	0.272	0.243
Percentage of variance explained by factor	83.4	12.8	2.9

These factors can be interpreted as explaining different types of change in the shape of the yield curve. The first factor affects yields at all maturities by a similar amount and in the same direction. This factor can be interpreted as a parallel shift factor for this reason. The changes are not exactly parallel since the effect at the short maturity is less than at the medium to long maturities. This factor explains as much as 83% of yield curve changes over the period of study.

The second factor has an opposite affect on the short and long yields. This factor can be interpreted as a "slope" factor since it changes the slope of the yield curve. This second factor explains about 13% of yield curve changes.

The third factor has a negative affect on medium yields and a positive affect on short and long term yields. For this reason this factor can be interpreted as a "curvature" factor. The third factor explains about 3% of yield curve changes. In total these three factors explain over 99% of yield curve changes.

Compared with overseas studies the slope and curvature factor appear to be more important in explaining the variance of yield curve changes in the Australian bond market. A single factor model would only be expected to replicate around 83% of yield curve changes.

Implications for Pricing and Hedging

For pricing purposes a single factor model is commonly used. Such an approach allows efficient computation of the value of interest dependent cash flows. Multi-factor models are often implemented using simulation and computation time is an important issue in these cases. These models are fitted to the current yield curve and poor performance from using too few factors is often compensated for by increasing the number of parameters that are fitted for pricing purposes. Although this can be satisfactory for pricing, the above results indicate that it is not going to be satisfactory for hedging or immunisation of interest dependent cash flows.

A factor model is used for immunisation of liability cash flows by matching the sensitivities of the value of the liability cash flows to each of the factors with the sensitivities of the value of the asset cash flows to the same factors. This can be formulated as a linear programming problem. Shiu (1988) demonstrates how the conventional one factor immunisation portfolio can be constructed for a set of liability cash flows using linear programming. The single factor case is identical to duration matching. The idea underlying this approach can be extended to immunisation in a multi-factor model.

Since the second and third factors appear to explain as much as 15-18% of yield curve changes in the Australian Bond market, portfolios constructed for immunising a set of liability cash flows using a one factor parallel shift model will fail to immunise against a significant proportion of yield curve shifts. For this reason it is considered essential that multi-factor models be used for immunisation of liability cash flows using Australian Government bonds. Two or three factor models will be adequate and there is not need to have a separate factor for each key rate yield to maturity because of the inherent correlation between changes in these yields. A similar result holds for other bond markets although the importance of the second and third factor appears to vary from one market to another.

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