

# Calibration of the Premium and Reserve Risk Factors in the Standard Formula of Solvency II

Report of the Joint Working Group on Non-Life and Health NSLT Calibration

12 December 2011

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# 1. Executive Summary

- 1. When delivering its advice for Level 2 measures, EIOPA (CEIOPS successor from January 2011) committed to carry out a comprehensive revision of the calibration of the premium and reserve risk factors in the non-life and health non-SLT underwriting risk module of the SCR standard formula.
- 2. For this purpose, EIOPA established a Joint Working Group (JWG) consisting of representatives of AMICE, Groupe Consultatif, CEA and CRO Forum as well as observers from the European Commission to discuss the most appropriate calibration methods and to derive recommendations for setting the above mentioned risk factors.
- 3. This report summarises the findings and recommendations of the JWG on the settings of the premium and reserve risk factors.

#### Data Collection

- 4. In October 2010 EIOPA launched a European wide statistical data request to carry out this calibration exercise. All the parties of the JWG were involved in the design of this data request. To ensure that the data collected was comprehensive and representative of the whole European market, the participation of as wide a range of undertakings (of all types and sizes) and Member States as possible was strongly encouraged.
- 5. Special provisions were made to ensure the confidentiality of the submitted data, and access to the centralized database was restricted to EIOPA exclusively. This implied that the work on the data could only be carried out by the EIOPA members of the JWG and could only be undertaken at the EIOPA premises, and under strict IT security arrangements. However, the intermediate results of this analysis were then shared in a timely manner within the whole JWG, which jointly took further decisions on the proposed methodological framework to be applied.

#### Methodology

- 6. The methodology used to carry out the calibrations took as its starting point the methodology which CEIOPS had previously applied in deriving its technical advice on the setting of the premium and reserve risk factors for non-life underwriting risk in the QIS5. Building on this, the discussions in the JWG led to significant further improvements and streamlining of the methodology that is documented in Annex 3.
- 7. The publication of this report includes extensive annexes with detailed results of the various methodologies considered by the JWG to derive the final recommendations.

#### Mandate

8. The JWG was tasked with deriving recommendations on the setting of the premium and reserve risk factors within the current design of the non-life and health non-SLT underwriting risk module of the SCR standard formula. This implied that for premium and reserve risk, respectively, a single market-wide factor per line of business had to be derived.

9. For reserve risk, the design foresees a factor net of reinsurance, i.e. a factor which already reflects the risk mitigation effects of the undertakings reinsurance cover. For premium risk, the design foresees a factor gross of reinsurance and the risk mitigation effects of non-proportional reinsurance are intended to be captured in a separate factor, which design was outside the scope of the JWG's work.

#### Recommendations

10. The final recommended calibration for premium and reserve risk factors are as follows<sup>1</sup>:

Segment	Premi	um risk - gross	Res	erve risk - net
	QIS5	Recommended	QIS5	Recommended
Motor vehicle liability	10%	9,6%	9.5%	8,9%
Other motor	7%	8,2%	10%	8,0%
Marine, aviation & transport	17%	14,9%	14%	11,0%
Fire / property	10%	8,2%	11%	10,2%
General liability	15%	13,9%	11%	11,0%
Credit and suretyship	21.5%	11,7%	19%	
Legal expenses	6.5%	6,5%	9%	12,3%
Assistance	5%	9,3%	11%	
Miscellaneous financial loss	13%	12,8%	15%	20,0%
Medical expenses	4%	5,0%	10%	5,3%
Income protection	8.5%	8,5%	14%	13,9%
Workers' compensation	5.5%	8,0%	11%	11,4%

11. This summary excludes factors for the Credit and suretyship reserve risk, Assistance reserve risk and the non-proportional lines of business for which too few observations were available to draw statistically founded conclusions.

#### Calibration future review

12. One of the main limitations of the exercise was related to the heterogeneity of data which was used and which is inherent to Solvency I and local accounting rules. In order to benefit from data homogeneity that will result from Solvency II guidance and take into account potential breakthroughs in actuarial development on calibration methodologies a recalibration exercise should be carried out in an appropriate number of years (in relation to the short tail/long tail characteristics of the line of business considered) for each line of business.

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Note that, mathematically speaking, the entries in the table above represent standard deviations for premium and reserve risk, respectively. Consistent with the mandate of the JWG as explained above, the recommended standard deviations for reserve risk are net of reinsurance, whereas the standard deviations for premium risk have been derived on basis of gross of reinsurance data and do not include an additional adjustment for the risk mitigation effects of non-proportional reinsurance.

#### 2. Introduction

#### Purpose of this report

- 13. When delivering its advice for Level 2 measures, CEIOPS committed to carry out a comprehensive revision of the calibration of the premium and reserve risk factors in the non-life and health non-SLT underwriting risk module of the SCR standard formula in the framework of QIS5.
- 14. For this purpose, EIOPA established a Joint Working Group (JWG) consisting of representatives of AMICE, Groupe Consultatif, CEA and CRO Forum as well as observers from the European Commission to discuss the most appropriate calibration methods and to derive recommendations for setting the above mentioned risk factors. Annex 2 contains a list of the members of the JWG.
- 15. This report summarises the findings and recommendations of the JWG on the settings of the premium and reserve risk factors.

#### Process followed in the JWG to conduct its work

- 16. To collect the statistical data needed to carry out the calibration exercise, in October 2010 EIOPA launched a European wide data request. To guarantee the feasibility of the revision exercise, all the parties of the joint working group were involved in the design of this data request. Insurers were asked to submit data to their national supervisors as part of the QIS5 exercise with an extended submission date for this specific data requirement of 31 December 2010. To ensure that the data collected was comprehensive and representative of the whole European market, the participation of as wide a range of undertakings (of all types and sizes) and Member States as possible was strongly encouraged.
- 17. Data was submitted by undertakings to national supervisors on basis of a data template issued by EIOPA. The data template and a detailed description of the data requirements were published on EIOPA's website. National supervisors then carried out an initial review of the data with the aim of ensuring that it met the requirements of the specifications in the data request, and was sufficiently credible and reliable to be used as part of the calibration exercise. During this review, particular attention was given to cases where undertakings amended or "cleaned" their raw data in line with the specifications in the data request. In January 2011, supervisors then submitted the data to EIOPA, in an anonymised way.
- 18. Special provisions were made to ensure the confidentiality of the submitted data, and access to the centralized database was restricted to EIOPA exclusively. This implied that the work on the data could only be carried out by the EIOPA members of the JWG and could only be undertaken at the EIOPA premises, and under strict IT security arrangements. However, the intermediate results of this analysis were then shared in a timely manner within the whole JWG, which jointly took further decisions on the proposed methodological framework to be applied. Where there were different views these were reflected in the report.

Template for non-life and health-non-SLT calibration data collection (28.09.2010) and explanatory note (23.09.2010). The explanatory note is also included in annex 4 of this report.

- 19. On the basis of the collected data from member states the JWG undertook an exercise which combined data validation processes (carried out by EIOPA's side of the JWG) with a comparison of different estimation methods for the risk factors applied to that data to receive robust, empirical results for the calibration of the premium and reserve risk.
- 20. The methodology used to carry out the calibrations took as its starting point the methodology which CEIOPS had previously applied in deriving its technical advice on the setting of the premium and reserve risk factors for non-life underwriting risk in the QIS5. Building on this, the discussions in the JWG led to significant further improvements of the methodology.
- 21. This included a more systematic approach to the underlying statistical framework as well as the development of a more comprehensive set of validation tools which could then be applied to assess the reliability and robustness of the calibration. For one of the triangle-type calibration methods applied for reserve risk, the JWG used an implementation tool which was provided by one of the insurance associations.
- 22. A detailed mathematical description of the estimation methods (the 'manual on methods for calibration') is provided in annex 3.
- 23. The JWG conducted seven meetings between October 2010 and May 2011. In addition it held telephone conferences to keep the members of the group updated on the continuous validation of the data received and of the methodology applied in the calibration.

#### Scope of work of the JWG

- 24. The JWG was tasked with deriving recommendations on the setting of the premium and reserve risk factors within the current design of the non-life and health non-SLT underwriting risk module of the SCR standard formula. This implied that for premium and reserve risk, respectively, a single market-wide factor per line of business had to be derived.
- 25. For reserve risk, the design foresees a factor net of reinsurance, i.e. a factor which already reflects the risk mitigation effects of the undertakings reinsurance cover. For premium risk, the design foresees a factor gross of reinsurance and the risk mitigation effects of non-proportional reinsurance are intended to be captured in a separate factor, the design of which was outside the scope of the JWG's work.

#### Structure of the report

- 26. This report is structured as follows:
  - Section 3 provides a summary description of the volume and type of data collected from member states, their limitations and necessary data cleaning processes to receive stable results, which could be endorsed by all representatives participating in the JWG.
  - The different estimation methods for premium and reserve risk factors and respective data selection processes are outlined in section 4.
  - Section 5 sets out the range of 'goodness-of-fit' validation tools and instruments applied by the JWG to support the assessment and interpretation of the results of the different estimation methods

- In carrying out its work, the JWG had to take decisions on a number of general statistical aspects such as the treatment of reinsurance or the choice of assumption on the underlying probability distribution. The JWG's findings on these aspects are presented in section 6.
- Finally, section 7 presents the main results and recommendations of the JWG on the settings of the premium and reserve risk factors.

#### 3. Data

## 3.1 General description

- 27. A detailed description of the structure of the collected data, the template that was used to submit the data and the note on data requirements for non-life and non-SLT health calibration (CEIOPS-SEC-116/10, 22 September 2010) are included in annex 4.
- 28. The following sub-sections give a description of the volume and type of data requested and received for the premium and the reserve risk analysis, respectively. The section concludes with a description of the tests which were carried out to ensure the consistency of the data used in the analysis.

#### 3.2 Premium risk data

Volume and type of data requested

- 29. Undertakings submitted the following data split by LoB<sup>3</sup> and accident year:
  - Volume of earned premium for the accident year gross of acquisition costs
  - Acquisition costs / earned commission
  - Expense information, if available, comprising relevant Unallocated Loss Adjustment Expenses (ULAE) as well as relevant other paid expenses
  - Information on the current estimate of ultimate loss<sup>4</sup> (henceforth referred to as CE ultimate loss data)<sup>5</sup>, comprising:
    - Paid claims up until 2009 for that accident year
    - QIS5 best estimate claims provisions (including IBNR) as at year end 2009
  - Information on the ultimate loss as at the end of the first development year (henceforth referred to as YE ultimate loss data), comprising:
    - Paid claims in the first development year for that accident year
    - Best estimate claims provisions (including IBNR) posted at the end of the first development year
- 30. Undertakings were asked to submit this set of data items separately for:
  - raw data gross of reinsurance;
  - adjusted data gross of reinsurance, excluding catastrophe loss; and
  - adjusted data net of reinsurance, excluding catastrophe loss.

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This comprised all Lines of Business in Non-Life and Health Non-SLT insurance relevant to the insurers' business as listed in annex 5.

Here 'ultimate loss' denotes the estimated aggregate claims expenditure that will have to be paid to finally settle the claims for the accident year considered. For the purposes of this exercise, 'current estimate' is intended to refer to the estimate of the ultimate loss as at the end of 2009.

To streamline the data request, in some cases this information was not explicitly requested, but was automatically calculated in the Excel sheet from other data provided. For example, the information on the ultimate loss as at the end of the first development year was calculated as described below from the information provided on paid claims and best estimate claims provisions posted at the end of the first development year.

31. For this purpose, catastrophe claims were regarded as claims covering all events or exceptional losses that would fall within the scope of the relevant catastrophe risk module of the standard formula SCR. Other adjustments to the data could be made where these were aimed to ensure that any set of data is made internally consistent and comparable. Undertakings were expected to explain and justify all adjustments made to their national supervisors.

#### 3.3 Reserve risk data

Volume and type of data requested

- 32. The following data split by LoB<sup>6</sup> and accident year was submitted:
  - triangles of paid claims;
  - triangles of best estimate claims provisions; and
  - reported triangles, if available
- 33. Undertakings were asked to submit this set of data items separately for:
  - raw data gross of reinsurance;
  - adjusted data gross of reinsurance, excluding catastrophe loss; and
  - adjusted data net of reinsurance, excluding catastrophe loss.
- 34. For this purpose, catastrophe claims were regarded as claims covering all events or exceptional losses that would fall within the scope of the relevant catastrophe risk module of the standard formula SCR. Other adjustments to the data could be made where these were aimed to ensure that any set of data is made internally consistent and comparable. Undertakings were expected to explain and justify all adjustments made to their national supervisors.
- 35. As to the triangles of best estimate claims provisions, in some cases Solvency II compatible provisions were not available to the undertakings. In these cases, they usually delivered provisions based on accounting data instead. This issue was taken into account in the assessment of the reserve risk calibration methods as explained in section 7.

## 3.4 Data availability

Volume and type of data requested

36. The data harvest is presented in the table below, which sets out per individual line of business the number of undertakings and countries for which data was received:<sup>78</sup>

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As for premium risk, this comprised all lines of business in Non-Life and Health Non-SLT insurance relevant to the insurers' business.

See annex 5 for a list of the lines of business considered.

For the health medical expenses (HME) line of business, this includes a data set containing 34 portfolios from the French market (with information based on the gross year end loss concept) which was submitted directly to EIOPA by the AMICE representative in the Joint Working Group. Although this data followed a different procedure than the rest of the data (i.e. was not submitted via national supervisors), it was considered in the analysis at the same level as the other submissions in order to achieve a wider market cover.

LobCode	HME	HIP	HWC I	MTPL	МО	MAT	<b>PROP</b>	LIAB	CS	LE	AS	MISC	NPH	NPC	NPM	NPP	Σ
Undertakings	269			308	300		343	333	97	149		140					
Countries	<b>25</b>	24	″ 16 <sup>'</sup>	26	26	25	<b>"</b> 26	<b>26</b>	21	19	21	22	6'	10	7	<b>" 11</b> "	26
Austria	1	4	-	9	8	6	8	8	-	8	2	6	-	-	-	-	60
Belgium	6	6	5	12	13	4	14	13	1	12	1	1	-	-	-	-	88
Bulgaria	-	-	-	1	1	1	1	1	1	-	1	1	-	-	-	-	8
Cyprus	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0
Czech Republic	1	1	-	1	1	-	1	1	2	-	-	-	-	-	-	-	8
Germany	56	201	-	68	66	2	68	75	-	31	-	-	-	-	-	-	567
Denmark	7	11	7	13	13	8	12	12	2	2	3	3	-	1	1	4	99
Estonia	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0
Spain	14	14	-	18	18	15	24	19	5	3	6	18	1	1	1	1	158
Finland	2	-	4	4	11	10	11	9	3	10	-	10	-	1	-	6	81
France	51	34	2	25	24	3	23	25	3	8	5	4	1	1	1	1	211
Greece	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0
Hungary	3	7	-	10	9	5	9	8	3	2	2	3	-	-	-	1	62
Ireland	11	8	-	13	10	16	26	24	8	4	5	16	1	2	2	3	149
Iceland	1	3	3	3	3	3	3	3	-	-	-	-	-	-	-	-	22
Italy	34	25	1	34	33	27	35	34	26	32	20	31	-	-	-	-	332
Liechtenstein	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0
Lithuania	6	3	_	5	5	5	_	5	6	-	4	4	-	1	-	-	50
Luxembourg	1	4	1	4	3	5	5	5	3	4	1	1	2	3	2	3	
Latvia	8	6	-	6	6	5	6	6	4	1	3	4	-	-	-	-	55
Malta	8	7	4	8	7	8	14	10	2	3	4	7	1	1	-	1	85
Netherlands	21	14	1	14	14	11	15	17	4	8	4	4	-	-	-	2	
Norway	1	1	7	5	5	4	5	5	-	-	1	-	-	-	-	-	34
Poland	2	2	1	1	1	2	3	3	2	-	-	2	-	-	-	-	19
Portugal	5	5	6	6	5	4	5	5	2	3	2	1	-	-	-	-	49
Romania	8	3	2	9	10	8	10	9	9	2	7	6	-	-	-	-	83
Sweden	5	5	1	6	6	6	5	4	-	-	3	1	-	-	-	-	42
Slovenia	5	9	-	6	6	4	6	7	5	5	4	6	-	1	1	1	66
Slovakia	1	1	-	2	2	2		2	1	1	2	1	-	-	-	-	17
United Kingdom	11	7	5	25	20	11	27	23	5	10	6	10	2	4	5	6	177

- 37. This represents a significant improvement compared to the previous calibration exercise undertaken by CEIOPS to recommend factors for the QIS5, in relation to both the number of undertakings which supplied data as well as to the number of Member States which could be included in the analysis.
- 38. The following tables present more granular information on the availability of data for the premium and reserve risk analysis, respectively.
- 39. For premium risk, the table below sets out:
  - The number of companies for which valid data gross of reinsurance could be used<sup>9</sup>, differentiating between "raw" (i.e. unadjusted) data and adjusted data excluding catastrophe loss;
  - The average size in gross earned premiums (in million Euros) of the undertakings' business in the respective line of business; and
  - The average length of the time series of historic loss ratios data derived from the data supplied by the undertakings.

After data cleaning as described in section 3.5

NL Calibration - Data availability											
number	number of companies that submitted valid data										
			Premiu	m Risk							
	Ray	w data (gro	ss)	Δdi	justed (gro	(22					
	Number	Average size	Mean length	Number	Average size	Mean length					
Non-life	-	-	-	-	-	-					
Motor TPL	265	160.7	10.9	153	183.6	9.0					
Motor Other	262	103.6	10.3	155	88.6	8.7					
MAT	135	67.1	9.3	102	69.5	8.6					
Fire	283	152.2	9.8	176	157.0	8.7					
General Liability	280	51.5	10.4	161	47.5	8.5					
Credit & Suretyship	68	58.6	7.8	57	31.4	8.2					
Legal Expenses	119	35.7	9.8	63	11.2	8.4					
Assistance	70	15.4	7.2	58	5.7	7.1					
Miscellaneous	104	82.1	7.7	81	18.1	7.4					
NPRI - property	19	438.3	6.5	17	476.7	6.6					
NPRI - casualty	9	240.4	7.9	9	240.3	7.9					
NPRI - MAT	10	435.6	8.0	10	394.1	8.9					
Health	-	-	-	-	-	-					
Medical Expenses	192	71.1	8.5	119	113.6	7.3					
Income Protection	217	36.9	10.2	107	36.9	8.3					
Workers Comp	43	37.1	9.2	21	50.2	6.7					
NPRI - health	5	107.3	8.0	5	103.8	8.0					

- 40. As set out in section 3.2, the premium risk data comprised information on both the current estimate of ultimate loss as well as on the year end ultimate loss. In some markets, these two kinds of ultimate loss estimates were both available. However this was not the case for all markets. For example, for the non-life lines of business the German data contained only current estimates of ultimate loss data.
- 41. For reserve risk, the following table provides similar information, with a split being made between the triangular and premium risk type methods. Here, the average size relates to the size of the undertakings' reserves gross of reinsurance.

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See section 4 for a description of the methods that were applied for the parameter estimation

	NL Calibration - Data availability												
			number	of compa	anies that	submitte	ed valid d	ata					
						Reserv	ve risk						
			Triangl	e type					Premium	methods	ods		
		Gross data			Net data			Gross data			Net data		
	Original sample size	Modified sample size	Mean length	Original sample size	Modified sample size	Mean length	Number	Average size	Mean length	Number	Average size	Mean length	
Non-life	-	-		-	-	-	-	-	-	-	-	-	
Motor TPL	265	255	12.4	120	114	9.8	187	1505	8.9	115	144	8.1	
Motor Other	255	249	11.5	117	115	9.4	173	70	8.7	103	27	8.0	
MAT	133	129	10.6		67	9.7	126	113	8.6	69	70	8.2	
Fire	290	282	11.0		126	9.6	202	122	8.6	-	119	7.9	
General Liability	261	259	12.1	113	111	9.5	191	339	8.4		146	7.6	
Credit & Suretyship	38	35	9.9	21	19	10.2	52	59	7.8		39	7.5	
Legal Expenses	104	97	11.5	36	33	9.9	87	97	8.5		7	8.5	
Assistance	62	60	8.1	37	32	7.6	53	2	7.2		1	6.4	
Miscellaneous	86	81	9.6	47	46	8.9	84	24	8.2	_	6	7.7	
NPRI - property	10	9	8.6		2	7.5	13	630	7.6	_	583	7.0	
NPRI - casualty	4	1	10.0		1	10.0	8	1175	8.1	5	676	8.2	
NPRI - MAT	8	7	9.7	5	2	7.0	10	881	7.2	6	797	7.2	
Health	-	-	-	-	-	-	-	-	-	-	-	-	
Medical Expenses	129	125	8.1	80	78	7.5	117	47	7.1	67	38	6.5	
Income Protection	203	202	11.7	82	80	9.6	268	55	6.7	_	47	7.7	
Workers Comp	40	36	12.5	17	14	10.2	45	86	9.1	18	99	6.4	
NPRI - health	1	0	-	1	0	-	4	367	8.3	3	313	9.0	

## 3.5 Data limitations and data cleaning

- 42. Tests were carried out on the data to ensure it was consistent. This process started with an automatic filter in order to detect anomalies in the data, concerning:
  - a) unit of values, where the unit described by the undertaking does not correspond to the underlying unit in the submitted values,
  - b) negative values, e.g. in earned premiums,
  - c) zeros e.g. in earned premiums,
  - d) double submissions of datasets,
  - e) short time series with few observations (see also section 3.6),
  - f) inconsistency between earned premium and loss estimates which leads to incomprehensible loss ratios,
  - g) inconsistency between loss estimates and triangle data, e.g. where the year end estimate value is materially lower than the sum of paid claims during the accident year and of claims provision at the end of that year,
  - h) inclusion of catastrophe events and other outliers in premium risk data.
- 43. Once the anomalies were identified, an individual analysis of each portfolio with a potential anomaly was carried out in order to decide whether the information should be included, excluded or corrected. The final aim was to have a homogeneous database with an improved reliability to be used as input in the following steps of this calibration exercise.
- 44. Regarding each single issue named above, a particular analysis was followed. Concerning the unit problems (a) national supervisors were contacted envisaging the clarification of the ambiguities and according to the answers

- received, the units were corrected. Approximately 5% of the sample collected was affected by this type of problem.
- 45. The anomaly referred in (d) was solved by excluding the effect of double counting from the data.
- 46. In respect to the anomaly described in (e) the decision was to exclude the whole portfolio if the number of observations submitted was fewer than three, once it was concluded that it is not possible to understand the statistical behavior of the portfolio with only one or two observations. Also for time series with more observations unstable and unrealistic results could occur. Therefore the JWG analysed the results regarding robustness and compared the statistical parameters of the different methods.
- 47. In relation to the anomalies referred in (b), (c), (f), (g) and (h), those observations were excluded.
- 48. Finally, some cases were detected where gross, adjusted and net data within the same portfolio contained exactly the same values. Although such a situation would be rather unusual, it could still occur in practise, for example in cases where no catastrophe adjustments were necessary (e.g. no catastrophe events occurred) and where there was no reinsurance cover in place. It was therefore decided to keep these data.
- 49. The JWG is aware that, apart from above mentioned data anomalies (missing data, incomplete data, outliers,...), there could be further cases where the data used in the analysis was not fully consistent with the given specifications which are not identifiable and therefore could not be excluded. For example, as mentioned above for the reserve risk data it seems likely that in some cases instead of triangles of best estimate claims provisions undertakings provided triangles of claims provisions based on accounting data.
- 50. Note that the table on the "data harvest" shown in section 3.4 shows the original data set received, while the more detailed tables above on the availability of data for premium and reserve risk reflect the data that in the end was used following the automatic filtering process as described above.

## 3.6 Sensitivity analysis in filtering by length of data series

- 51. As described above, as one aspect of the automatic filtering of the data to remove anomalies, the decision was taken to exclude an undertaking's portfolio from the analysis if the number of observations submitted was fewer than three.<sup>11</sup>
- 52. With the aim to assess the appropriateness of this choice of a threshold of three observations, the JWG carried out a sensitivity analysis on the impact of filtering the data base to exclude the data sets at three different levels:
  - Exclude datasets with less than three years data history
  - Exclude datasets with less than five years data history
  - Exclude datasets with less than seven years data history

See point (e) in the automatic filtering process as described above

- 53. The results showed that no consistent increase or decrease in the volatility was observed across all the line of business for the two concepts of loss, the normal model for premium risk and for the reserve risk premium type method. So the changes in the unbiased sigma obtained to apply the different filters were attributed to the fact that some undertakings were excluded of the sample, increasing the volatility of the line of business analysed almost in the same number of cases and amounts than decreasing. The results are included in annex 7.
- 54. Considering the results of this sensitivity testing, the JWG decided to keep the filter at three years of data history in order to maintain a data sample as large and representative of the EU market as possible.

## 4. Parameter estimation methods

- 55. This section provides a summary description of the methods that were applied in order to derive estimates of the premium and reserve risk factors. A more detailed description of these methods including the underlying statistical framework is contained in annex 3.<sup>12</sup> This section also describes the selection of the data that was needed to feed these methods.
- 56. Note that the results of the application of these methods are summarised in section 7.

#### 4.1 Premium risk

## 4.1.1 Methods applied

- 57. For premium risk, the mean of aggregate year loss was modeled as being proportional to the volume of gross earned premiums where the proportionality factor was an undertaking-specific loss ratio parameter subject to statistical parameter estimation. For the variance of aggregate year loss a general quadratic expression in gross earned premium was used. This formulation contains as special cases both the case where the variance is proportional to gross earned premiums and also the case where the variance is proportional to its square. In this way, two different models used in earlier calibration exercises were put together into a single more general and flexible model.
- 58. As regards the variance parameterization, eventually only the first one as described in section 5.1 of annex 3 has been used to derive the sigmas. The second parameterization as stated in section 5.2 of annex 3 was for statistical analysis purposes but in the end was found to be insufficiently aligned with the design of the Standard Formula.
- 59. After the specification for the mean and variance, this was embedded in both a normal probability model as well as into a lognormal probability model. Therefore, just two models were obtained to fit to the data and to compare them as regards their goodness of fit.
- 60. By embedding the parameter estimation into the coherent statistical framework as described in annex 3, the assumptions underlying the calibration could be made transparent. This allowed the JWG to carry out a range of different 'goodness of fit' analysis tools to assess the adequacy of the results of the estimation methods (see section 5). Transparency of the underlying assumptions will also facilitate the ORSA process where the undertaking has to compare these assumptions against its individual risk profile. Therefore such an embedding into a modeling framework was preferred against more ad-hoc approaches (as e.g. taking the mean or the median of undertaking-specific empirical coefficients of variations) where the underlying assumptions would not be explicit.

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As mentioned in the introduction, the publication of this report also includes the Excel spreadsheet developed and used in the estimation of those factors.

#### 4.1.2 Selection of data

- 61. To assess the premium risk, it is desirable to give as input data gross of reinsurance, adjusted for catastrophe events and any other adjustments that may distort the behavior of the risk being analyzed. Although adjusted gross data was received, it did not present sufficient quantity and quality comparing to non-adjusted gross data. Thus, it was decided that the data which should be used was the one with a higher level of completeness, accuracy and reliability<sup>13</sup>.
- 62. To obtain estimates of ultimate losses, the JWG considered two different concepts: the year end estimate and the current estimate of the ultimate losses. These different loss concepts are explained in section 6.6 of this report. Both approaches were investigated in the analysis.

#### 4.2 Reserve risk

## 4.2.1 Methods applied

- 63. For reserve risk, two different model approaches were considered:
  - A model approach based on financial year end data under which the premium risk methodology was applied in an analogous way to reserve risk (hereafter referred to as premium risk type methods); and
  - A model approach based on runoff triangle accident year data (hereafter referred to as triangle methods).

Premium risk type methods applied for reserve risk

- 64. Based on financial year end data, reserve risk can be modelled completely analogous to the methods described for premium risk. This is possible by using claims provisions instead of premiums as volume measure, and by considering as aggregate loss the run-off losses incurred in a financial year t for accidents years less than  $t.^{14}$  Such an approach enables the application of a single and consistent methodology across both premium and reserve risk. In particular, it allowed the use of a single Excel analysis tool for both premium risk as well as reserve risk on financial year end data.
- 65. After a preliminary analysis, the JWG decided that where the premium risk tool is used the undertaking-specific runoff ratio parameters should be assumed to be subject to parameter estimation. It was found that such an assumption would better fit the data than an assumption under which these were fixed at the value of 1.

### Triangle methods

66. Several triangle methods have been considered by the JWG for non-life calibration purposes<sup>15</sup>. The JWG subsequently focused its attention on two methods which were applied to the data:

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On the volume of adjusted and non-adjusted data received for premium risk see table in section 3.4.

See section 7 in annex 3.

Note however that the "combined approach" which is proposed by the JWG as a methodological basis for the calibration (see section 7) uses the premium risk type method for reserve risk. The recommended reserve risk factors are therefore based on this method.

- A method based on calculating the relative root mean squared error of prediction (RRMSEP)<sup>16</sup> of the undertakings' reserve risk; and
- A method based on the undertaking-specific coefficient of variation.
- 67. Since compared to paid claims data only few triangle data on incurred claims were collected, only paid triangle claim data could be analysed for these methods.
- 68. These two methods are described in more detail, below. Note that the method based on calculating RRMSEP has already been applied in CEIOPS' previous calibration exercise for determining premium and reserve risk factors for QIS5. In this previous exercise, this method was referred to as 'Method 5'. For simplicity, this denotation is also used in this report.

'Method 5'

- 69. This method involves a three stage process:
  - a. Calculate undertaking-specific Relative Root Mean Squared Error of Prediction (RRMSEP)<sup>18</sup> over a one year time horizon, after implementing the data modification.
    - The RRMSEP is defined as the ratio of undertaking specific RMSEP divided by the volume measure. The RMSEP are calculated using the approach detailed in "Modelling The Claims Development Result For Solvency Purposes" by Michael Merz and Mario V Wüthrich, Casualty Actuarial Society E-Forum, Fall 2008. The volume measure is chosen to be the chain ladder reserves.
    - The model underlying the calculation of RMSEP makes the following assumptions on the claims data triangles:
      - 1. Cumulative payments  $C_{i,j}$  in different accident years i are independent.
      - 2. For each accident year i and development year j, the cumulative payments  $C_{i,j}$  are a Markov process and, there are constants  $f_j$  and  $s_j$  such that  $E(C_{i,j} \middle| C_{i,j-1}) = f_j \cdot C_{i,j-1}$  and  $Var(C_{i,j} \middle| C_{i,j-1}) = s_j^2 \cdot C_{i,j-1}$ .
  - b. Fit a model to the undertaking specific RRMSEP, calculated in the first step. This model assumes that:

See <u>CEIOPS'</u> advice on the calibration of premium and reserve risk for QIS5 (Consultation paper CP 71, CEIOPS Doc 67 - 10)

See explanation in para. 50. below.

It should be noted that a macro provided by the German insurance association GDV has been used to calculate RMSEP and Chain ladder reserves. This macro uses runoff paid claims for each undertaking

- The variance of the claims development result is proportional to the square of the current best estimate for claims outstanding; and
- The least squares fitting approach of the undertaking specific standard deviations is appropriate.
- c. Modify the method after the first fitting by removing outlying residuals.
  - These residuals are calculated by taking the difference between the individual undertaking specific RRMSEP and the method 5 result.
  - The range of the residuals to be maintained is calculated using the sample size n. The threshold level is the quantile of the standard normal distribution with probability n/(n+1).
- 70. Specifically if the following terms are defined:

$PCO_{C,lob}$	Current best estimate for claims outstanding as derived by the chain ladder by undertaking and LoB
$V_{C,lob}$	Volume measure by undertaking and LoB
RRMSEP <sub>c,lo</sub>	Root mean squared error of prediction of the claims development result in one year's time, divided by volume measure, by undertaking and LoB
$\sigma_{(res,lob)}$	Standard deviation for reserve risk by LoB

71. Then the following relationship can be defined:

$$V_{c,lob} = PCO_{c,lob}$$

72. The least squares estimator of the standard deviation is the value of  $\sigma_{(res,lob)}$  which minimises the following function:

$$\sum_{C} (V_{C,lob} \sigma_{(res,lob)} - RMSEP_{C,lob})^{2}$$

73. By differentiating this function with respect to  $\sigma_{(res,lob)}$  and setting this to zero the following least squares estimator is obtained by:

$$\hat{\sigma}_{(res,lob)} = \frac{\sum_{C} V_{C,lob} \cdot RMSEP}{\sum_{C} V_{C,lob}^{2}} = \frac{\sum_{C} V_{C,lob}^{2} \cdot RRMSEP}{\sum_{C} V_{C,lob}^{2}}$$

Triangle method based on undertaking-specific coefficient of variation

74. This method is consistent to the compliance model for premium risk and consists of two stages. The first stage is the same as the first stage in the triangle 'Method 5'. The second stage involves determining thresholds for these calculated undertaking-specific RRMSEP. These thresholds are determined so as to ensure that certain pre-specified percentages of undertakings or claimants in the sample could be protected, i.e. are covered by a reserve risk capital charge

- which would correspond to a level of safety of at least 99.5% over a one year time horizon. Here, the number of claimants is measured by the undertakings' chain ladder reserves.
- 75. Note that this triangle method is particularly sensitive to the inconsistent data unit issue and small sample size issue. To deal with the first issue, this approach assumes a negative dependency between the undertaking's volume and its RRMSEP. This means that large undertakings are supposed to have lower RRMSEP. These undertaking volumes are then modified again according to this assumption.
- 76. However, the second issue of small sample size is difficult to overcome given the current situation. Hence this is one of the causes in some material impact on the results shown in section 7.2.
- 77. In addition, this method has been used to analyse the relation of the reserve risk gross of reinsurance to the reserve risk net of reinsurance. This analysis was performed by using two gross and net subset data samples, as described in the following sub-section.

#### 4.2.2 Selection of data

Choice of gross versus net data

78. For reserve risk, factors net of reinsurance needed to be calibrated. 19 Therefore, the JWG first considered whether the collected reserve risk data net of reinsurance could be used directly for this purpose. However, it was found that much fewer data net than gross of reinsurance was available in the individual lines of business.<sup>20</sup> To achieve a more representative analysis, it was therefore decided to base the calibration on gross of reinsurance data, and to apply a separate "gross-to-net" factor (see description in para. 76, below) to derive an estimate of the standard deviation net of reinsurance.<sup>21</sup>

Selection of data for triangle type methods

- 79. For the selection of data for the triangle type methods for reserve risk, a twostep process has been followed to modify the data for the intended purpose. In a first step, a data modification was carried out in order to produce undertaking specific Root Mean Squared Error of Prediction (RMSEP) and Chain Ladder reserves.
- 80. In a second step, an additional data modification was performed in order to better fit the calibration methods. For this purpose, the undertaking's chain ladder reserve was considered as a volume measure, and for each undertaking the relative measure RRMSEP was defined as the ratio of RMSEP divided by the undertaking's volume. This involved identifying undertakings with both high RRMSEP and large volumes. In cases where both the RRMSEP and the volumes were found to be higher than the market average RRMSEP and volume measure, both the volume and the RMSEP were divided by 1,000 to achieve a better fit. In this context, the market average RRMSEP was defined as the ratio

See mandate of the JWG as described in the executive summary. 20

See table on availability of reserve risk data in section 3.5.

See also section 6.5 for a more detailed exploration of this issue.

- of the sum of all undertaking RMSEPs divided by sum of all these undertakings' volumes.
- 81. In order to estimate the gross to net ratio, undertakings which submitted both gross and net runoff triangle paid claims were identified. These undertakings' data were then selected to form another two gross and net samples, respectively. These samples are subsets of the initial gross and net samples. As a result, these subset sample sizes are equal or smaller than the initial sample sizes. These data samples were then used to analyze the gross to net relations, using the undertaking-specific coefficient of variation triangle method. Thus the data samples used in order to estimate the gross to net ratio were slightly smaller in term of size then the main ones, however a high consistency was achieved in this way.

## 4.3 Three step procedure to eliminate outliers

- 82. In order to achieve a more robust and reliable statistical estimation of the premium and reserve risk factors, the JWG decided to apply a 'three step procedure' to further eliminate outliers in the data. This procedure as detailed hereunder was applied for the premium risk methods as well as for the premium risk type methods applied in the assessment of reserve risk.
- 83. The analysis was performed in the same manner for each line of business and data variant and proceeded as set out below:
  - a. Data was given as input into the spread sheet, and "solver" was run as necessary for each distribution /variance specification within each method in order to derive the estimates of the parameters. We noted some very large residuals and a large kurtosis. We took the general view that these outlier observations were very likely to be the result of data "errors" rather than simply extreme observations. This is because we did not detect those outliers when we carried out our own detailed accuracy checks on the data submitted.<sup>22</sup>
  - b. As a consequence we decided to eliminate observations that generated outlying standardised residuals: being outside the interval that may be expected for standard normal random variables with the given sample size. Next we repeated the analysis in step (a). This generally resulted in a significant improvement in observed fit and a reduction in observed standard deviation (sigma) for premium risk. But there were still some issues on fit.
  - c. We thus repeated step (b) one further time to try to get an improved fit. We decided not to perform any further iterations of elimination of extreme residuals as this may remove genuine volatility of results and in any case we noted that by this stage the goodness of fit had generally been significantly improved.

This residual based rejection procedure was applied for the normal and lognormal model separately.

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Note that the procedure described to eliminate outliers was carried out separately for the normal and lognormal model and also for premium and reserve risk.

84. The automatic use of three stages of elimination of outliers is likely to have removed some valid data points as well as unreasonable errors. This means that, acknowledging the limitations of the underlying model, the results from the analysis should be considered as lower bounds of the actual experience.

## 4.4 Averaging approach to address heterogeneity

- 85. Originally, the modelling focussed on the efficient estimation of a pan-European volatility parameter. However, in order to more fully address the issue of heterogeneity between different markets, this method can also be applied at the level of an individual member state. The intermediate output by member state can then be grouped by taking a weighted average also resulting in a single pan-European volatility. One could regard this as a kind of voting procedure where voting power is proportional with market share. This idea was followed in the 'combined approach' recommended by the JWG (see section 7).
- 86. Under this approach, the output of the analysis by member state consists of an unbiased estimate of the standard deviation, the average size of the insurance portfolio as a standard portfolio and a measure for the curvature, which measures the effect of the size of the underlying business on the volatility. This curvature parameter allows calculating, using the transformation documented at the end of section 5.1 in annex 3, an unbiased estimate of the standard deviation at the member state level for any portfolio size.
- 87. For the calculation of a pan-European weighted average, this presents the difficulty that standard portfolio sizes would typically differ between different member states. An approach which would then average across unbiased estimates in individual member states based on such different portfolio sizes would lead to inconsistencies. To overcome this difficulty, the following two step method was applied:
  - In a first step, unbiased estimates per member state for a common European portfolio size – selected as the average portfolio size of all the undertakings in the sample across the countries – was calculated.
  - The pan-European factor was then derived in a second step as a weighted average of these unbiased sigmas per individual member state.
- 88. For the implementation of this approach, the analysis at the level of member state was carried out for all lines of business, except for the non-proportional reinsurance lines of business due to scarcity of data. A threshold was applied to only include in the analysis countries with at least 5 portfolios<sup>23</sup> in the relevant line of business

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This threshold was lowered to 4 for the Workers' compensation line of business.

# 5. Goodness-of-fit inspection

89. To further assess the reliability and adequacy of the results of the estimation methods, a range of different 'goodness of fit' analysis tools were applied. This included the use of both scatter plots and PP-plots, as described below. The use of such tools allowed the JWG to consider to what extent the data was fitting to the underlying modeling assumptions.

### Scatter plots

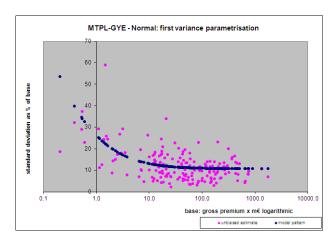
- 90. Scatter plots were produced for the estimation of each of the premium and reserve risk factors.<sup>24</sup> Annex 6 of this report contains a complete list of the statistical results and includes these scatter plots for both the normal and lognormal model for each line of business.
- 91. An example of such a scatter plot for the line of business 'Motor third party liability' and for the normal model is provided in the diagram below. Here, each of the pink points corresponds to the unbiased estimate of the volatility of the business of an individual undertaking<sup>25</sup>, so that the pink points illustrate the degree of volatility to which the individual undertakings in the markets are exposed. The ordering of these points on the x-axis is with respect to the volume of the insurance business so the larger the undertakings' business, the more to the right the pink point corresponding to the volatility of this undertaking is placed.
- 92. The blue model pattern is derived from the results of the estimation methods and represents the industry-wide common "law" of dependency between the volume of the business and the resulting volatility. For the given example, two observations can be derived which are also typical for a range of other lines of business:
  - The degree of volatility is decreasing with increasing volume of the business; and
  - Also for very large portfolios there is a positive bottom-level for the volatility. This means that even for very large portfolios the volatility does not decrease to zero.

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With the exception of the triangle type methods for reserve risk, which followed a different methodology.

For consistency these are calculated using the normal model applied as if the dataset would consist of a single



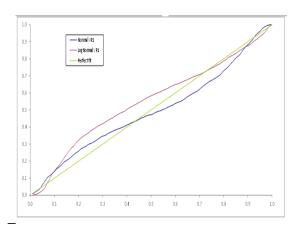
- 93. Supplementary to these scatter plots, for each line of business annex 6 sets out all relevant statistical parameters derived from the analysis. These comprise:
  - The unbiased estimate of the standard deviation for the line of business (the 'unbiased sigma');
  - The mean and the median of the sigmas corresponding to individual undertakings (the 'pink points'); and
  - The mean and the median of the points on the blue model pattern.
- 94. In the analysis it was found that the mean and the median of the 'blue curve' would often differ from the unbiased sigma. For example, using the normal model for motor third party liability and on the basis of the pan-European data set, an unbiased sigma of 10,8% was derived, whereas the mean and the median of the blue model pattern amounted to 12,9% and 11,1%, respectively.<sup>26</sup>
- 95. From a statistical point of view, such differences are not completely surprising. For a well-specified model differences will have a probability distribution with mean close to zero and a non-zero variance that might be non-negligible. For an additional lack-of-fit test this probability distribution itself would be needed.

Probability-Probability plots (PP-plots)

96. In addition to the scatter plots, Probability-Probability plots (PP-plots) were derived, as illustrated in the next picture. Such plots can be used to assess the adequacy of the assumption on the underlying probability distribution model. As a general rule, the closer the curve derived from the model fits the straight line (which corresponds to a perfect fit), the better the model fits to the data.

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See page 2 in annex 6-1.



- 97. Note that, where PP-plots are applied to standardized or studentised residuals as it is the case in this study, these display deviations resulting from a variety of different factors such as:
  - Distribution family (e.g. normal, lognormal, gamma,...)
  - Variance modeling
  - Convergence to the normal distribution of the residuals with regards to the Law-of-Large-numbers
  - Independence (or clusters) of the sample values

Hence where deviations are detected, further analysis would be needed to identify the exact source of the deviation.

# 6. Calibration aspects

98. In the context of implementing the methodologies described in section 4 to derive estimates of the calibration factors, the JWG analysed a number of general calibration issues. This section sets out descriptions of these issues and also describes how these issues were reflected in the methodologies applied.

## 6.1 Probability distribution assumptions

- 99. As described above, for the premium risk analysis as well as for the reserve risk analysis based on premium risk type methods, models based on the assumption of a normal probability distribution for the underlying data ('normal' models) and also models based on the assumption of a lognormal probability distribution for the underlying data ('lognormal' models) were used.
- 100. With an appeal to the law of large numbers the normal probability distribution forms an obvious and simple candidate for the specification of the probability distribution for year-aggregate loss for a line of business. Actuarial risk theory has modifications, such as the normal power method or the translated Gamma distribution, but these need three parameters and even then approach the normal distribution for large portfolios.
- 101. We also recall that the mixing operation for the fluctuation in the basic parameters that generates the quadratic variance in portfolio size has a behind the scene distribution such as the lognormal distribution. For indeed large portfolios this lognormality dominates the randomness that is present in the normal distribution.
- 102. It will therefore be difficult to discriminate on theoretical grounds between the normal and lognormal distribution. The empirical findings on this issue for example, with regard to the various goodness-of-fit diagnostics and PP-plots were also inconclusive. In addition, some technical issues on the technical implementation of the lognormal model arose which could not be resolved fully due to time constraints.
- 103. In view of the considerations above, from a practical point of view the normal distribution was preferred considering that its (numerical) mathematics is simpler.

## 6.2 Heterogeneity of results across different markets

- 104. It is acknowledged that heterogeneity between portfolios, also in the same line of business, will generally apply. This is the reason to use undertaking-specific parameters for loss ratios in case of premium risk and runoff ratios in case of reserve risk. But even then there will remain heterogeneity when (observable) strata such as Member State are formed. The main reason for such heterogeneity across Member States is that there are significant differences between the different markets in terms of for example types of products written, underwriting guidelines or claims management which lead to different level of volatilities.
- 105. For example, regional exposure to different Natural Perils influences the risk characteristics and reinsurance requirements of Fire and Other Damage, of the physical damage elements of Motor and MAT covers and Non proportional

- reinsurance on the basis of the geographic scope of the underlying business. By way of example, winter storm and freeze risk is inherently different for Northern, Western and Southern Europe.
- 106. Other examples are lines of business covering personal injury, health and lines covering pure financial loss as they are strongly influenced by legal and regulatory differences between member states, principally as a result of the following issues:
  - Strength of Public Health System
  - Access to health services
  - Funding of health costs
  - Strength of welfare systems
  - Access to courts
  - Basis of court awards
- 107. Also, funding of the health system is a key consideration. In some member states, insurers are obliged to cover hospitalisation and treatment costs in at fault accidents. There are frequently differences in responsibility for respect of accidents at work, road accidents and other health costs. Similarly, damages pursued through tort constitute different proportions of overall injury compensation systems depending on the particular circumstances of individual member states, with the strength of the welfare system being a key determinant of the extent of loss suffered by injured parties.
- 108. In the methodology applied, these issues were addressed by using an 'averaging approach' across different member states to derive a pan-European estimate.<sup>27</sup>

## 6.3 Recognition of size variations in recommended factors

- 109. As mentioned before and as is demonstrated in the 'scatter plots' produced as part of the goodness-of-fit analysis (see section 5), the volatility factors for premium and reserve risks are typically impacted by on the size of the portfolio (in the sense that with increasing size the volatility will typically decrease). However, the JWG was mandated to derive single factors for each of the individual lines of business (separately for premium and reserve risk), irrespective of portfolio size since this is consistent with the current design of the standard formula approach to measuring non-life premium and reserve risk..
- 110. "Size" and country are directly interlinked because small EU-countries usually have small companies but also different legal framework. Country/ regional calibration might have a much bigger impact on the results than the size of an undertaking.
- 111. Furthermore, "size" (= expected number of claims or premiums) is an exposure measure which represents a mixture of several influential factors such as "type of cover", "legal framework within a specific country".
- 112. There may be a selectivity problem if the data submitted is not sufficiently representative of the portfolio size distribution of the pan-European market. In

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See section 4.4.

the analysis a curvature parameter modelling the decrease of volatility with size was estimated and the calibration process was then pursued using the average portfolio size of undertakings in the sample. If no adjustment were introduced in order to correct a potential mismatch between the average size of portfolios in the sample and the average size of the portfolios in the market, the volatility factors produced would likely be underestimated if the average size of the sample is above the average market size, or overestimated in the opposite situation.

113. This issue was solved by applying a corrective factor to obtain a calibration appropriate for the median portfolio size at market level. This median at market level was calculated using the distribution of portfolio sizes derived from the QIS5 submissions, and adjusting for the absence of a number of small undertaking in the distribution (68% of the market participants likely to be under the Solvency II scope participated, for a premium-wise market share of 85% or a claims-wise market share of 95%) by retaining a 65% QIS5 distribution quantile as a proxy for the pan-European market-wide median. Premiums (for premium risk) and claims provisions (for reserve risk) were used to make this adjustment. The procedure followed is explained in Annex 3 section 10.2.

## 6.4 Underwriting cycle effects in premium risk

- 114. Premium is a poor proxy for exposure owing to the fact that it is itself an estimate. Indeed, the main sources of misstatement of premium are the use of unreliable or unrepresentative data, errors in estimation of key parameters and the effects of commercial pressures and the underwriting cycle. The underwriting cycle is driven by results in the overall insurance market, the market segment itself and the general business cycle. Other things being equal, the effect of the cycle becomes more pronounced in lines of business where the length of claims tail and/or the capital (and risk) intensity is increased.
- 115. The JWG recognises the possible existence of an underwriting cycle but did not find it practicable to incorporate or embed an explicit recognition of such cycles into the calibration methodology. To achieve such an implementation, knowledge on the position of the premiums on the underwriting cycle would need to be available. Then, volatilities would become dependent on the current premium-position, in the end resulting in lower or higher undertaking-specific volatilities. The current statistical approach is more pragmatic and is based on an averaging 'look-through' analysis.
- 116. However, this issue should be analysed further in future calibration exercises.

#### **6.5** Treatment of reinsurance

- 117. The calibration of the reserve risk factors undertaken by the JWG was mainly based on gross data. The main reason for this was that a significant proportion of the insurance undertakings responding to the data request have been unable to provide a history of reserve development net of reinsurance.
- 118. In addition, there is little or no information in the public domain on either the effect or structure of reinsurance arrangements with in most markets,

- information on variability of results only available in financial rather than statistical form.
- 119. However, although the calibration of standard approach is based on gross data, due recognition in the analysis of the results has to be made to the fact that volatility deriving from net data should be expected to be typically lower than volatility deriving from gross data (however in practice in some cases the opposite experience has been made).
- 120. Indeed, reinsurance is frequently purchased in respect of short tail lines either on a whole account or per risk non proportional basis. Per risk is more common where there is variation in the size of risks covered. In either of these circumstances it is clear that gross calibration results will produce an overestimate of reserve risk, and that the reinsurance risk adjustments should take account of the actual type of reinsurance in place.
- 121. For proportional reinsurance and other lines of business such as Motor Vehicle Liability, General Liability and Workers' Compensation (WC) that are more difficult to reinsure except on an individual excess of loss basis (and usually with WC subject to exclusion of the accumulation of industrial disease claims) it is likely that volatility in reserve risk gross and net will be closely aligned.

Use of gross versus net data for premium risk

122. The calibration of the premium risk factors was performed using data gross of reinsurance (excluding catastrophe events) as input. However the final capital charge for premium risk needs to be on a net basis. This gross to net adjustment is introduced a posteriori within the design of the standard formula of this capital charge. Thus, the calibration process should not take into account such adjustment.

Use of gross versus net data for reserve risk

- 123. The calibration of the reserve risk factors may be carried out by using data gross of reinsurance (excluding catastrophe events) or data net of reinsurance, as input. The final capital charge for reserve risk needs to be on a net basis, therefore if gross data is used, a gross to net adjustment needs to be introduced within the process of calibration of such factors.
- 124. Both gross and net paid claims data are modelled. However, the data sample of gross remains much larger than the sample of net claims. Although the ratios of gross to net is calculated using a common data sample, the insufficient sample size problem remains.

## 6.6 Ultimate loss: use of year end versus current estimate

125. To illustrate the differences between the concepts of Year End (YE) and Current Estimate (CE) ultimate loss<sup>28</sup>, we consider a cumulative payment triangle:

See section 3.2 for a description of these concepts.

	cumulativ	e paymen	its	at the end of accounting year				
	(1,1)					(1,6)		
	(2,1)				(2,5)			
accident	(3,1)			(3,4)				
year	(4,1)		(4,3)					
	(5,1)	(5,2)						
	(6,1)							

together with a triangle with claims provisions:

	claims pro	ovision		at the	unting year	
	(1,1)					(1,6)
	(2,1)				(2,5)	
accident	(3,1)			(3,4)		
year	(4,1)		(4,3)			
	(5,1)	(5,2)				
	(6,1)					

- 126. The aggregate loss prediction by accident year according to Year End (YE) then follows as the summation of the first column of the payment triangle and the first column of the claims provision triangle. The interpretation is ultimate loss as predicted at accident year end.
- 127. In contrast to this, the aggregate loss prediction by accident year according to Current Estimate (CE) follows as the summation of the last diagonal of the cumulative payment triangle and the last diagonal of the claims provision triangle.
  - The interpretation is ultimate loss as predicted at the end of the current accounting year. From this we see that YE and CE coincide for the most recent accident year.
- 128. To assess which of these two concepts is better suited for the purposes of the calibration exercise, we recall that in the SCR standard formula underwriting risk in non-life insurance (excluding catastrophe risk) comprises both premium risk and reserve risk.
- 129. Under the one-year horizon used for the SCR calculation, premium risk arises through the possibility that the sum of claim payments during the first settlement year and the (best estimate) claims provision at the end of this accident year may exceed the volume of earned premiums. In the subsequent calendar year, this claims provision is put at risk in the run-off reserve risk process and hence will contribute to reserve risk. These considerations could be seen as indicating that it would be adequate to use the Year End concept for the purpose of calibrating premium risk.
- 130. However, a drawback on this approach is that the current data on historic claims provisions (such as is needed under the YE approach) need not represent unbiased predictions of the future and may contain prudence margins which would not be compatible with the Solvency II valuation principles. In such cases, assessing the volatility of the underwriting risk based on Year End data may lead to distortions in the estimated premium risk factors.
- 131. Using Current Estimate data instead where such current estimates are consistent with Solvency II valuation rules could help to avoid this drawback. However, in view of the considerations above such an approach may not be fully

- consistent with the conceptual split between premium and reserve as envisaged in the standard formula.
- 132. Considering these pros and cons on the two loss concepts, the JWG agreed to follow a combined approach which maximises the use of both loss concepts. This applied the principle that the loss concept used is the one which allows for each line of business to take into account the largest number of data sets. This approach was seen in line with the observation that in most cases, the results obtained from the two different concepts were found to be comparable..

## 6.7 Adjustments to data in respect of catastrophe claims

- 133. To avoid any double-counting with catastrophe risk, the calibration of premium risk carried out by the JWG was intended to reflect premium excluding catastrophe risk. In the data inquiry for our statistical calibration of premium (excluding catastrophe) risk, we faced the problem that under normal conditions most undertakings won't have experienced catastrophes or did not perceive these as catastrophes with a dominating impact on their revenue account. As a result we did receive data on gross premium risk and relatively few data adjusted for catastrophes.
- 134. Except for the property line of business, it was found that most adjusted data was identical to the gross data. Therefore, for these other lines of business a pragmatic approach was followed in order to quantify a "potential" catastrophe effect.
- 135. The approach followed was to view the premium risk analysis results at the initial stage and examine the time series of loss ratios for each undertaking separately. When such a time series showed a smooth flat or somewhat cyclic pattern this was viewed as evidence of a catastrophe free experience for this undertaking. If on the other hand such a smooth pattern was distorted by a sudden upward outlying loss ratio (typically exceeding twice the neighbouring level), this was viewed as an observation where the occurrence of a catastrophe was a real possibility. Removing such observations created in the end a new dataset for analysis in the standard way.
- 136. Finally, by comparing the results arising from the data cleaned (which was the result of this pragmatic approach) and the risk factors arising from the original gross (raw) database, it was possible to estimate a factor to adjust the gross volatility factors for catastrophe events. These factors, obtained on the accident year end loss concept and the normal distribution assumption, are presented in the table below.

Catastrophe claims adjustment	Observed unbia	sed sigma	CAT	adjustment	Observa	ations
Based on eliminating selected observations	Gross	Ex CAT	Implied	Recommended	Original	Excluded
Motor, other classes	9.1%	9.2%	0.1%	0.0%	1677	25
Motor, third party liability	10.8%	10.7%	-0.1%	0.0%	1728	-9
Fire and other damages to property (see under)				-2.0%		
General liability	17.2%	16.5%	-0.7%	-1.0%	1670	49
Health:Medical expenses	6.3%	6.1%	-0.2%	0.0%	934	19
Health: Income protection	9.2%	9.3%	0.1%	0.0%	1106	7
Maritime, Aviation Transport	20.7%	18.7%	-2.0%	-2.0%	1130	37
Credit and Suretyship	28.6%	20.3%	-8.2%	-8.0%	465	82
Miscellaneous	16.9%	13.2%	-3.7%	-3.5%	658	106
Health: Workers' compensation	11.6%	11.5%	-0.1%	0.0%	392	1
Based on adjusted data submitted						
Fire and other damage to property	14.5%	13.0%	-1.5%	_	1700	N/A
Fire and other damage to property (common)	11.9%	9.5%	-2.4%		177	N/A

137. For reserve risk no such procedure as described for premium risk has been undertaken or viewed necessary.

## 6.8 Compliance analysis

- 138. The compliance analysis is based on the observation that in many cases the underwriting volatility will be impacted by the size of the undertakings' business. <sup>29</sup> In such cases, the compliance analysis may be used as a bridge from the statistical estimation (unbiased sigma) based on the data sample to a calibration choice which is reflective of the distribution of the size of the undertakings' portfolios to which the factor is applied.
- 139. The idea of this analysis<sup>30</sup> is to multiply the unbiased sigma from the statistical estimation with a calibration factor "kappa" and determine the implied properties of the combined factor in terms of the *compliant share* of undertakings or policyholders in the market. This means that, given a calibration factor kappa, the compliance analysis would identify:
  - Under the **company view:** the share of *portfolios* in the industry with security level of at least 99.5% when the *SCR* is calculated according to a risk factor of kappa × Unbiased sigma.
  - Under the **policyholder view:** the share of *policyholders* (or claimants in case of reserve risk) that are insured by undertakings with a security level of at least 99.5% when the *SCR* is calculated according to kappa × Unbiased sigma.
- 140. Likewise, given a "confidence level of compliance" of say 90%, the compliance analysis could be used to derive a calibration factor "kappa" such that an application of this factor would ensure that the calculated SCR would be such that at least 90% of the portfolios (or policyholders) are insured with a security level of at least 99.5%.
- 141. We note that due to the skewness of the distribution of portfolio sizes, the compliant shares in terms of policyholders usually take on higher values than in terms of compliant companies.

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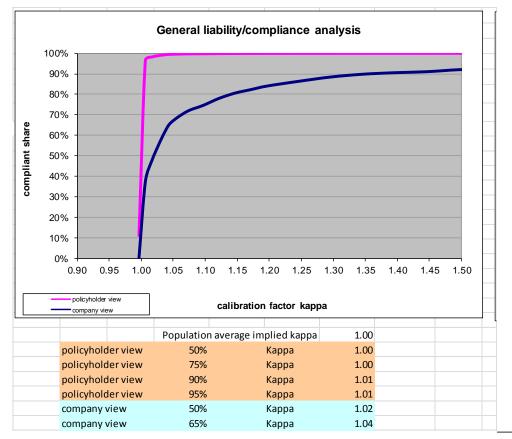
See section 6.3, above.

For a full description of this analysis we refer to section 10.2 of annex 3.

142. To illustrate this concept, consider the following example of a statistical estimation carried out for premium risk in the general liability line of business and for the normal model:

cfuesd	1.059	minimum	maximum	threshold		unbia	sed sigma
cvuesd	0.018	-4.06	4.67	3.24			0.173
mean x	31						Normal
Minimise	0.081	specific	sample		standard		
		parameters	size	mean	deviation	skewness	kurtosis
delta	0.993	179	1658	0.00	1.00	0.5	5.3
sigma	0.163						

- 143. This analysis implies an unbiased sigma of 0.173 that would be optimal for a portfolio that coincides with the average in the industry for this line of business (mean x=31). This means that if we would use just this unbiased sigma of 0.173 this would imply that portfolios that are larger than the average in the industry would comply with the 99.5% security level.
- 144. For this example, the following picture for a compliance analysis was used for assessing further the results of the statistical analysis:



- 145. This would mean that, for example, the setting of a kappa factor of 1.04 would ensure that for at least 65% of portfolios the calculated SCR would lead to a security level of at least 99.5%. At the same time, the setting of a kappa factor of 1.01 would be sufficient to ensure that for at least 95% of policyholders the calculated SCR would lead to a security level of at least 99.5%.
- 146. What has been said for premium risk before also applies to the output for runoff reserve risk of the financial year approach. Here we have as volume measure

the claims provision at the end of the financial year (instead of gross earned premium). All other interpretations of the output are similar to that of premium risk.

#### 7. Results and recommendations

## 7.1 Options considered by the JWG

- 147. In the analysis undertaken, the following two broad options emerged for the setting of the factors:
  - A pan-European approach; and an
  - Averaging approach
- 148. Under the pan-European approach, the factors are set on basis of the pooled European data set. Under the averaging approach, in a first step factors are set at a regional (country) level. The final Europe-wide factor is then determined by averaging across the regional factors.
- 149. The pan-European approach has been proposed by the EIOPA side of the JWG. The EIOPA members of the JWG consider that this approach is in line with the overall goal of the exercise to derive Europe-wide factors for premium and reserve risk in the individual lines of business. They are of the view that it would enable the use of the full data set, giving equal weight to individual observations.
- 150. However, the industry's side of JWG has raised concerns on a number of aspects of this recommendation. They pointed out that the heterogeneity of the processed data and the significant differences between Members States would not be sufficiently taken into account. Also, the increase of capital requirements that would result from such an approach might have damaging consequences for the non-life insurance markets.
- 151. Therefore, the industry's side of the JWG has proposed a simple alternative approach. Under this "averaging approach", the European factor is derived as a weighted average of the country-specific volatility factors weighted with country premiums per LoB for consistency. Such an approach allows taking into account the weight of individual regions in terms of premium and reserve volumes in the European market.
- 152. However, EIOPA members of the JWG believe that such an approach would not reflect that the data sample that was available for the analysis was not necessarily representative of the whole European market. Also, care needs to be taken to ensure that the method that is used to average across different markets is consistent with the data parameters that are derived from the statistics.

#### Reserve risk

- 153. For reserve risk, the JWG considered a further choice between a "premium risk type" method and a "triangular type" method. The premium risk type method uses the same underlying concept as the premium risk method for determining the reserve factors. The triangular type method is an alternative method which is based on paid data triangles and derives a hypothetical calculation of the value of claims reserves to measure the run-off risks.
- 154. The EIOPA members of the JWG suggested using the "premium risk type" method to ensure a consistent approach across both premium and reserve risk.

The industry side of the JWG would prefer the use of the "triangular type" method.

## Combined approach as a third option

- 155. The JWG has therefore considered a third option which is intending to combine the advantages of the two options described above. Under this combined approach:
  - As in the averaging approach, the European factor is derived as a weighted average of regional factors.
  - The regional factors are derived by a unified and consistent methodology across both premium and reserve risk. In addition to the regional factors, the methodology provides information on the nature of dependency between the portfolio size and the degree of volatility of an undertakings' business.
  - Compared to the simple averaging approach, the methodology of the averaging is improved to be fully consistent with the results of the statistical analysis.
  - The calibration is conceptually based on the median size of the portfolio in the EEA (an additional factor, described in section 6.8, is included in the calibration to ensure this). However, where the portfolios larger than the median portfolio represent more than 95% of policyholders, the calibration is reduced to ensure that at least 95% of policyholders belong to portfolios for which the risk is not underestimated.
- 156. This combined approach offers the advantage of taking into account the heterogeneity of the non-life risks in the individual markets for the setting of the European factors. At the same time, it ensures that the final factors are reflective of the average size of the portfolios of insurers in the European markets to which they are applied.
- 157. Therefore, the JWG recommends using the combined approach as a methodological basis for the calibration of the premium and reserve risk factors.
- 158. The following tables successively present the summary of results obtained following the pan-European approach with a distinction between the two loss concepts studied for the premium risk –, the pan-European reserve risk results obtained with the "premium risk type" method and the triangle type method, the averaging approach results and the combined approach results. Full sets of more detailed results are presented in annex 6.1 (pan-European approach following the premium risk type method for both premium and reserve risk) and 6.2 (Country data used in the averaging method and combined approach).
- 159. This summary of results excludes the non-proportional lines of business for which too few observations were available to draw statistically founded conclusions.

# 7.2 Premium risk following a pan-European approach

Table 1 - Pan-European approach - gross year end

premium risk	Before	Policyhold	er kappa	Company	/ kappa	Pan-european
Gross year end loss concept	kappa <sup>(*)</sup>	95%	90%	65%	50%	approach (gross)
Non-Life lines of business						
Motor, third-party liability	10,8%	1,06	1,02	1,18	1,08	11,4%
Motor, other classes	9,2%	1,07	1,03	1,20	1,08	9,8%
Marine, aviation, transport (MAT)	18,4%	1,01	1,01	1,08	1,02	18,6%
Fire and other property damage	12,8%	1,00	1,00	1,00	1,00	12,8%
General liability	16,3%	1,01	1,01	1,04	1,02	16,5%
Credit and suretyship	17,6%	1,04	1,01	1,69	1,28	18,3%
Legal expenses	7,6%	1,05	1,00	1,64	1,25	8,0%
Assistance	9,1%	1,01	1,00	1,06	1,02	9,1%
Miscellaneous	11,9%	1,22	1,03	2,50	1,56	14,5%
Health lines of business						
Medical Expenses	6,8%	1,04	1,00	1,36	1,13	7,1%
Income Protection	9,4%	1,07	1,02	1,25	1,10	10,1%
Workers compensation	11,4%	1,03	1,01	1,30	1,11	11,7%

<sup>(\*):</sup> after the adjustment for catastrophic events

Table 2 - Pan-European approach - gross current estimate

premium risk	Before	Policyhold	er kappa	Company	/ kappa
Gross current estimate loss concept	kappa	95%	90%	65%	50%
Non-Life lines of business					
Motor, third-party liability	10,2%	1,13	1,05	1,37	1,17
Motor, other classes	9,4%	1,07	1,03	1,20	1,08
Marine, aviation, transport (MAT)	20,9%	1,01	1,01	1,15	1,04
Fire and other property damage	12,9%	1,02	1,01	1,06	1,03
General liability	14,4%	1,01	1,01	1,04	1,02
Credit and suretyship	30,1%	1,01	1,01	1,01	1,01
Legal expenses	7,3%	1,10	1,05	1,63	1,26
Assistance	8,9%	1,01	1,00	1,09	1,04
Miscellaneous	17,3%	1,03	1,00	1,24	1,09
Health lines of business					
Medical Expenses	7,5%	1,02	0,99	1,26	1,09
Income Protection	7,7%	1,13	1,05	1,44	1,18
Workers compensation	21,9%	1,01	1,00	1,13	1,04

## 7.3 Reserve risk following a pan-European approach

## Premium type method

reserve risk	Before	efore Policyholder kappa		Company	/ kappa	Pan-european	Pan-e
Gross claims data	kappa	95%	90%	65%	50%	approach (gross)	appro
Non-Life lines of business						,	
Motor, third-party liability	8,6%	1,71	1,36	2,70	1,92	14,8%	
Motor, other classes	13,8%	1,10	1,04	1,29	1,11	15,1%	
Marine, aviation, transport (MAT)	17,5%	1,07	1,02	2,07	1,37	18,8%	
Fire and other property damage	13,3%	1,13	1,04	1,36	1,18	15,0%	
General liability	11,1%	1,19	1,08	2,26	1,60	13,2%	
Credit and suretyship	20,4%	1,24	1,08	3,02	2,11	25,3%	
Legal expenses	15,8%	1,01	0,99	1,26	1,08	16,0%	
Assistance	35,0%	0,99	0,98	1,03	1,00	34,6%	
Miscellaneous	36,0%	1,23	1,08	2,36	1,56	44,2%	
Health lines of business							
Medical Expenses	15,8%	1,00	0,99	1,13	1,04	15,8%	
Income Protection	11,1%	1,21	1,08	1,69	1,28	13,4%	
Workers compensation	12,1%	1,27	1,15	2,56	1,69	15,3%	l

## Triangle type method

	Using Least	·							
	Squares								
Gross	Estimation	Us	ing Undertal	king specific RM	SEP%				
						Claimants	measured		
		Undertaking	gs' RMSEP%	Claimant	s Covered	by CL R	eserves		
LoB	Method 5	Median	65%	Around 90%	Around 95%	90%	95%	QIS5	DOC67
MTPL	2.3%	7.7%	10.0%	6.0%	10.0%	91%	95%	9.5%	9.5%
МО	8.2%	16.1%	21.9%	19.0%	23.0%	88%	95%	10%	12.5%
MAT	8.9%	35.1%	55.2%	19.5%	30.0%	90%	94%	14%	17.5%
PROP	13.3%	20.4%	27.3%	16.0%	21.0%	88%	95%	11%	12%
LIAB	5.5%	13.6%	19.8%	7.0%	10.5%	89%	96%	11%	16%
CS	17.2%	38.3%	59.3%	28.5%	47.0%	89%	94%	19%	25%
LE	3.2%	8.8%	17.4%	5.0%	8.0%	91%	95%	9%	9%
AS	8.1%	39.6%	53.7%	35.0%	45.0%	89%	95%	11%	12.5%
MISC	10.2%	42.5%	72.2%	35.0%	45.0%	91%	94%	15%	20%
NPP	16.7%	35.0%	50.2%			0%	0%	20%	25.5%
NPC	8.5%	34.1%	45.0%			0%	0%	20%	25%
NPM	9.5%	25.3%	30.0%			0%	0%	20%	25%
HME	0.6%	20.1%	27.6%	11.0%	21.0%	90%	95%	10%	
HIP	3.4%	14.5%	20.5%	12.0%	16.0%	90%	95%	14%	
HWC	3.7%	13.3%	21.4%	8.0%	11.0%	87%	94%	11%	12%
NPH	#NUM!								
NPR	8.9%	32.0%	43.4%	20.0%	35.0%	90%	95%		

	Using Least								
	Squares Estimation	•••			CEDO/				
Net	Estimation	Us	ing Undertai	king specific RM	SEP%	61			
							measured		
		Undertaking			s Covered	<del> </del>	eserves		
LoB	Method 5	Median	65%	Around 90%	Around 95%	90%	95%	QIS5	DOC67
MTPL	3.6%	10.0%	12.8%	9.5%	13.0%	90%	95%	9.5%	9.5%
MO	8.9%	17.3%	22.3%	19.0%	23.0%	87%	91%	10%	12.5%
MAT	12.1%	35.4%	54.9%	13.0%	19.5%	89%	95%	14%	17.5%
PROP	11.8%	21.8%	30.7%	14.3%	21.0%	91%	95%	11%	12%
LIAB	4.4%	19.1%	23.8%	8.0%	10.5%	91%	95%	11%	16%
CS	15.9%	36.1%	51.6%	29.0%	37.0%	92%	95%	19%	25%
LE	9.2%	15.0%	23.4%	15.0%	19.5%	90%	95%	9%	9%
AS	10.9%	36.8%	56.8%	31.0%	45.0%	90%	94%	11%	12.5%
MISC	9.5%	37.6%	60.5%	22.0%	23.0%	93%	95%	15%	20%
NPP	76.7%	127.8%	274.9%			0%	0%	20%	25.5%
NPC	19.0%	21.5%	21.7%			0%	0%	20%	25%
NPM	9.3%	8.5%	8.5%			0%	0%	20%	25%
HME	9.5%	24.8%	35.2%	14.0%	28.0%	91%	95%	10%	
HIP	2.2%	14.9%	24.7%	8.0%	14.0%	90%	95%	14%	
HWC	5.2%	13.5%	22.5%	10.5%	13.5%	92%	95%	11%	12%
NPH	#NUM!								
NPR	8.7%	43.8%	69.1%	9.3%	18.0%	93%	96%		

160. Note that the results from Method 5 are significantly lower compared to results from the undertaking-specific coefficient of variation method. This is due to various reasons:

- The volume measure for method 5 uses the chain ladder reserves, calculated on paid claims. This is not always the best practice, especially for long tail lines of business, such as MTPL.
- The undertaking-specific coefficient of variation triangle method is vulnerable to the inconsistent data units and small sample size. Although the first issue has been handled by various data cleaning, the small sample size still exists for some lines of business, such as Assistance.
- Runoff triangles between undertakings may have different numbers of development years. This creates an unbalanced dataset. Due to this, the small triangles may induce a seemingly short runoff with the risk of underestimation of the claims provision and generating an unreliable estimate of the RMSEP.

#### 7.4 Premium and reserve risk following an averaging approach

	premium	
Premium and reserve risk	risk	Reserve risk
Averaging approach	(gross)	(net)
Non-Life lines of business		
Motor, third-party liability	9.3%	7.0%
Motor, other classes	7.8%	12.0%
Marine, aviation, transport (MAT)	16.9%	11.2%
Fire and other property damage	10.9%	14.2%
General liability	14.8%	10.0%
Credit and suretyship	20.6%	18.3%
Legal expenses	5.4%	12.7%
Assistance	8.7%	11.0%
Miscellaneous	9.9%	19.5%
Health lines of business		
Medical Expenses	4.8%	9.3%
Income Protection	7.9%	14.8%
Workers compensation	13.6%	9.9%

## 7.5 Premium and reserve risk following the recommended combined approach

161. Following the recommended combined methodology for all factors would result in the following set of factors<sup>31</sup>:

Segment	Premi	um risk - gross	Reserve risk - net		
	QIS5	Recommended	QIS5	Recommended	
Motor vehicle liability	10%	9,6%	9.5%	8,9%	
Other motor	7%	8,2%	10%	8,0%	
Marine, aviation & transport	17%	14,9%	14%	11,0%	
Fire / property	10%	8,2%	11%	10,2%	
General liability	15%	13,9%	11%	11,0%	
Credit and suretyship	21.5%	11,7%	19%		
Legal expenses	6.5%	6,5%	9%	12,3%	
Assistance	5%	9,3%	11%		
Miscellaneous financial loss	13%	12,8%	15%	20,0%	
Medical expenses	4%	5,0%	10%	5,3%	
Income protection	8.5%	8,5%	14%	13,9%	
Workers' compensation	5.5%	8,0%	11%	11,4%	

162. This summary excludes factors for the Credit and suretyship reserve risk<sup>32</sup>, Assistance reserve risk<sup>33</sup> and the non-proportional lines of business for which too few observations were available to draw statistically founded conclusions.

#### 7.6 Impact assessment

163. The JWG has undertaken an initial quantitative impact analysis for the combined option, before any further investigations. This analysis combines a preliminary assessment of line of business impact provided by EIOPA with an assessment of the overall impact on surplus and solvency ratio based on the methodology proposed by the industry side of the JWG.

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Motor Other premium risk is slightly lower than the detailed result presented in annex, due to different rounding precision used in calculation. Miscellaneous net reserve risk was kept at the level determined before the switch to the combined methodology approach presented in the annex.

As can be seen in annex 6.2, for credit and suretyship reserve risk, applying nonetheless the methodology despite the insufficient data available would result in a factor of 33.7%, by combining 3 country-level factors ranging from 9% to 65% (given the constraint of minimum number of undertakings per country for confidentiality issues).

As can be seen in annex 6.2, for Assistance reserve risk, applying nonetheless the methodology despite the insufficient data available would result in a factor of 19.1%,by combining country level results ranging from 7.1% to 60.7%. This set of widely dispersed results are due to the very poor coverage of the data available: data collected was insufficient to perform a reliable re-calibration as the amount of claim provision on the balance sheet is usually fairly small, needing a good coverage to provide sound results.

- 164. Supposing that all factors resulting from the combined approach as presented above would be used<sup>34</sup>, we can observe that moving from **QIS5 to the combined approach** would lead to an average decrease of 3.0% in non-life for premium and reserve risk and an average increase of 3.6% in NSLT health for premium and reserve risk.
- 165. Overall this would result in a 1 pts increase in coverage ratio and amounting to a 1.8 billion euros increase in surplus available.
- 166. The JWG suggests that further impact assessment analysis is undertaken on the finally chosen set of factors. This should include the identification of types of insurers and lines of business which would be more heavily impacted than the average market.
- 167. Furthermore, it is important to consider the combined impact of the new proposals for non-life catastrophe risk together with these proposals for non-life underwriting risk.

#### First results from EIOPA IA for non-life lines as follows:

Segment	Impact
Motor vehicle liability	-4.9%
Other motor	+9.3%
Marine, aviation and transport	-15.7%
Fire / property	-14.0%
General liability	-2.7%
Credit and suretyship	-31.6%
Legal expenses	+15.8%
Assistance	+71.4%
Miscellaneous financial loss	+10.3%
Non-Life as whole	-3.0%

#### First results from EIOPA IA for health lines as follows:

Segment	Impact
Medical expenses	+4.8%
Income protection	-0.4%
Workers' compensation	+15.2%
Health as a whole	+3,6%

Keeping the QIS5 tested factors when the data available was insufficient to derive a recommendation.

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#### Calibration future review

One of the main limitations of the exercise was related to the heterogeneity of data which was used and which is inherent to Solvency I and local accounting rules. In order to benefit from data homogeneity that will result from Solvency II guidance and take into account potential breakthroughs in actuarial development on calibration methodologies a recalibration exercise should be carried out in an appropriate number of years (in relation to the short tail/long tail characteristics of the line of business considered) for each line of business.

## **Appendices**

## Annex 1:Issues in calibrating premium and reserve risk for direct business

#### Issues to consider in calibrating Premium and Reserve risk for Direct Business

Category	Duration to Settleme nt	Claims Frequenc y	Claims Severity	Policy Form	Policy Limits and exclusion s	Reinsura nce / Retro Availabili ty	Principal Risks
Medical Expense	Short, usually < 6 months	Reasonabl y stable (ex pandemic, out of scope)	Not heavily skewed	Regulated in some markets, e.g. Holland, Ireland). individual / group may differ	Indemnity cover for selected elective and most acute services, Excludes long term care	Generally good	Medical Inflation, Changes in Public Healthcar e provision, Anti selection
Assistance	Short	Reasonabl y stable	Not skewed			Generally good	
Motor Vehicle Other	Short Tail	Reasonabl y Stable	Not heavily skewed	Competiti vely set		Whole account catastroph e and clash covers	Natural Perils, some accumulat ion potential
Fire and Other Damage	Short tail, except complex commerci al cases/ business interruptio n	Varies by insured peril	Household is not skewed, Commerci al is quite skewed.	Commerci ally set with fairly standard conditions	Policies usually contain risk managem ent conditions	Well developed market offering high capacity consistent with exposures	Narural Perils, Accumulat ions, Man made catastrop he risk
Income Protection	Claims are admitted quickly, but may take a long time to settle	Reasonabl y Stable	Not heavily skewed		Usually a fixed time limit of payment or income protection	Generally good	Economic risks

#### Issues to consider in calibrating Premium and Reserve risk for Direct Business

Category	Duration to Settleme nt	Claims Frequenc y	Claims Severity	Policy Form	Policy Limits and exclusion s	Reinsura nce / Retro Availabili ty	Principal Risks
Legal Expense	Medium tail length	Reasonabl y stable	Moderatel y skewed			Generally good	Litigation propensity & the funding legal proceedin gs
Marine, Aviation and Transport	Hull and Cargo are short tailed, liability is long tail	Hull and Liability are volatile low frequency ; Cargo less so	Very skewed except cargo which is medium	Regulated for commerci al passenger carriers		Well developed market offering high capacity consistent with exposures	Natural Perils, Collision, accumulat ion Storage, Catastrop he risk
Motor Vehicle Liability	Small numbers of litigated large claims extend the tail	Reasonabl y Stable	Medium skew	Regulated under EU Directives	No real opportunit y given regulation	Individual XOL is usual for smaller companies	Economic risks, inflation, structured settlemen ts
Workers Compensa tion	Small numbers of litigated large claims extend the tail.	Slightly volatile, particularl y for Industrial Disease	Medium skew	Regulated in many markets	Conditions prohibited in markets where insurance is compulsor y	Limitation s on industrial disease cover	Economic risks, inflation, structured settlemen ts
General Liability	Long Tailed; claims can be discovered many years after occurrenc e	Some volatility, particularl y in financial and specialty lines	Quite highly skewed	A lot of risks being written on a claims made basis	Policies are usually heavily conditione d	Market is selective; cover is usually individual excess of loss	Personal injury, Pure economic losses through negligenc e
Credit and Suretyshi p	Short claims tail, some extended exposure periods	Varies in line with economic cycle	Moderate skew, strong effects from recoveries	Commerci ally set		Selective, a lot of reinsuranc e is proportion al;	Economic risks

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#### Issues to consider in calibrating Premium and Reserve risk for Direct Business

Category	Duration to Settleme nt	Claims Frequenc y	Claims Severity	Policy Form	Policy Limits and exclusion s	Reinsura nce / Retro Availabili ty	Principal Risks
Miscellane ous Non Life	Varied, some longer term exposures are present	Varied by sum line	Varied by sum line			Varied, tends to include some "difficult" lines, e.g. Mortgage Indemnity	Economic / employme nt / Accumulat ion
Non Proportio nal Reinsuran ce ("NPR")	Generally longer then the original subject business	Varies by line, reinsuranc e type, attachmen t point and limit	Varied	Commerci al terms apply throughou t, little regulation in a profession al market	Reinsuran ce conditions limit the original business that can be accepted	Retrocessi on is only available in well defined circumsta nces	Vary by contract type and by reference to underlyin g business

# Annex 2:Composition of Joint Working Group and calendar of meetings

Chair		Peter ter Berg (DNB, Netherlands)
Members	EIOPA	Laurent Voignac (ACP, France) Aurélien Cosma (ACP, France) David Theaker (FSA, United Kingdom) Huijuan Liu (FSA, United Kingdom) Romain Labaune (FSA, United Kingdom) Matthias Heinze (BaFin, Germany) Ana Rita Ramos (ISP, Portugal) Giulia Avola (ISVAP, Italy) Marc Baran (ACP, France) John Byrne (IECB, Ireland)
	AMICE	Sílvia Herms
	CRO Forum	Francis Berthoix (AXA) Mathilde Sauvé (AXA) Paolo Loi (Generalli) Stefano Ferri (Generalli) Henry Medlam (Zurich)
	Groupe Consultatif	David Paul
	CEA	André-Philippe Sende Ulrich Stienen (GDV) Ingelore Döring (GDV)
	EIOPA staff	Pierre-Jean Vouette Daniel Perez
Observers	European Commission	Ramon Carrasco Lars Dieckhoff

#### **Physical Meetings**

- o 2010-10-26 JWG Paris
- o 2010-12-09 JWG Frankfurt
- o 2011-01-13 JWG Frankfurt
- o 2011-03-11 JWG Frankfurt
- o 2011-02-25 JWG Frankfurt
- o 2011-03-23 JWG Frankfurt
- o 2011-04-13 JWG Frankfurt

#### & Teleconferences

- 2010-09-29 JWG Telco
- 2010-11-25 JWG Telco
- 2011-01-26 JWG Telco
- 2011-02-04 JWG Telco
- 2011-03-28 JWG Telco
- 2011-05-04 JWG Telco
- 2011-05-20 JWG Telco
- 2011-05-24 JWG Telco
- 2011-06-01 JWG Telco
- 2011-06-07 JWG Telco
- 2011-06-09 JWG Telco
- 2011-06-20 JWG Telco

#### **Annex 3: Manual on Methods for Calibration**

- 1 Introduction
- 2 Aspects and Notation
- 3 Mean and Variance of Premium Risk
- 4 (Concentrated) Criterion Function and Newton-Raphson
- 5 Normal Models for Premium Risk
  - 5.1 First Variance Parametrisation
  - 5.2 Second Variance Parametrisation
- 6 Lognormal Models for Premium Risk
  - 6.1 First Variance Parametrisation
  - 6.2 Second Variance Parametrisation
- 7 Reserve Risk Models of Premium Risk Type
- 8 Reserve Risk Models of Runoff Triangle Type
  - 8.1 Over-dispersed Bilinear Poisson Models
  - 8.2 Merz-Wüthrich (2008) Inspired Models
- 9 Goodness-of Fit, Residuals and All That
  - 9.1 Detecting Outliers
  - 9.2 PP-Plot
  - 9.3 Jarque-Bera Test
- 10 Bridging Statistical Estimation and Calibration
  - 10.1 Calibration by Estimation of a Size-Invariant Model
  - 10.2 Compliance Analysis by Portfolio-Size

#### **Appendices**

- A1 Deriving the Quadratic Variance Specification
- A2 Unbiased Estimator Standard Deviation
- A3 Understanding the Earlier Premium Risk Method 1
- A4 Rephrasing the Earlier Premium Risk Methods 2-3-4

#### Literature

#### 1 Introduction

This manual evolved from rephrasing earlier existing premium risk methods. These methods are now in the appendix as more general and flexible formulations appeared to be possible. Besides formulation of estimation criterion functions based on maximum likelihood, there is also a section that bridges between parameter estimation and parameter calibration.

#### 2 Aspects and Notation

For the parameter estimation and calibration the following aspects play a role:

- Lines of busines, indexed as  $\ell = 1, \dots, L = 16$ However, as the various procedures are strictly separable for lines of business, we may get rid of any reference to line of business in the various formulae.
- Member states, indexed as  $s=1,\cdots,S=27$ Also in the harmonised world of Solvency 2 there is no direct reason to refer to member states. For data handling purposes a member state index is useful however and it facilitates parameter estimation by member state, which might be useful for testing purposes.
- Accident years, indexed as  $t = 1, \dots, T \le 20$ Discrete time, indicating accident year, will be there permanently. For some lines of business and undertakings *underwriting* year may occur instead of accident year. Care in the statistical analysis is in order here.
- (Re)insurance portfolios (undertakings), indexed as  $i = 1, \dots, I \rightarrow \text{large}$

After adding an index i for insurance portfolio, the combined set of formulae defines the framework of the pan-European calibration for a single specific line of business. In case we specialise this framework to I=1, it gets an *undertaking specific parameter* interpretation.

In what follows, summation over t will simply be denoted by  $\sum_{t=1}^{T}$ .

Likewise, for insurance portfolios we will use  $\sum_i$  and double summation over i and t will be denoted by  $\sum_i$  instead of  $\sum_i \sum_i$ . When the summation index is clear we also use  $\sum_i$ .

The total number of observations will be indicated simply by n instead of the longer  $\sum_i T_i$ .

#### 3 Mean and Variance of Premium Risk

The typical non-life insurance underwriting risk (including health) is often modeled using a compound Poisson distribution. This implies that mean and variance of aggregate loss y are proportional with the expected number of claims. Measuring exposure by (gross) earned premium x, we can rephrase this as:

$$E(y) = \beta x$$

$$V(y) = \eta x$$

where  $\beta$  has the interpretation of a loss ratio parameter and  $\eta$  is a dispersion parameter with the same monetary dimension as x and y.

This specification of the variance requires fixed basic parameters that do not change randomly through time. As soon as we have random change of basic parameters over time, this will change the variance and induce correlations between undertakings as well as over time. An analysis of these non-zero correlations is out of our scope and mandate, so we will confine to just displaying the modified variance, which becomes quadratic in exposure:

$$V(y) = \eta x + (\sigma x)^2$$

A more detailed derivation of this quadratic variance specification is in appendix A1. A drawback of this *parametrisation* is the monetary dimension of  $\eta$ . We can get rid of this dimension by writing:

$$\eta = \sigma_1^2 \overline{x}$$
 where  $\overline{x} = \frac{1}{n} \sum x_{ii}$ 

Now, the variance specification can be reparametrised as:

$$V(y_{ti}) = \sigma_1^2 \bar{x} x_{ti} + (\sigma_2 x_{ti})^2$$

where the subscript 1 and 2 in  $\sigma$  occur naturally for indicating the linear and squared nature. We may rewrite this further as:

(3.1) 
$$V(y_{ti}) = \sigma^2 \left( (1 - \delta) \overline{x} x_{ti} + \delta x_{ti}^2 \right) \qquad \text{with} \quad \begin{cases} \sigma^2 = \sigma_1^2 + \sigma_2^2 \\ \delta = (\sigma_2 / \sigma)^2 \end{cases}$$

where we have reused the earlier symbol  $\sigma$ .

In terms of loss ratios q = y/x we have:

$$E(q) = \beta$$

$$V(q) = \sigma^{2} \left( \delta + (1 - \delta) \bar{x} x^{-1} \right)$$

For large portfolios this will imply a time series of loss ratios with constant level parameter  $\beta$  and limiting standard deviation  $\,\sigma\sqrt{\delta}$  . Here portfolio size does not matter anymore.

It is good to remember that with  $\delta$ =0 the probability distribution of y will become bell-shaped approaching a normal distribution under the forces of the central limiting law. When  $\delta$ >0 a mixing operation enters the scene that at best results in a *mixed normal* distribution, which in general will have more heavy tails, such as these of the *Student* distribution.

As a slight mathematical variation we consider also a variance that is proportional in  $\beta^2$ :

(3.2) 
$$V(y_{ii}) = \beta^2 \sigma^2 ((1 - \delta) \bar{x} x_{ii} + \delta x_{ii}^2)$$

which gives rise to a coefficient of variation that does not depend on  $\beta$ . In case that undertaking-specific modeling of  $\beta$  shows much similarity (for a certain line of business) there will not be much difference between the two variance parametrisations.

The two probability distributions that we will consider are Normal (least squares) and Lognormal.

#### 4 (Concentrated) Criterion Function and Newton-Raphson

Typically maximum likelihood parameter estimation proceeds by taking the natural logarithm of the likelihood function. Here, we additionally will add the minus-operation to get a estimation criterion function, additive in the information contributions per observation, that should be minimised. This also will resemble a (nonlinear) least squares approach. This criterion function is generically denoted by  $\ell(\alpha,\beta \mid data)$  or  $\ell(\alpha,\beta)$  for short. Here  $\alpha$  denotes two (shape) parameters, such as  $\delta$  and  $\sigma$ , and  $\beta$  denotes the parameter vector of I undertaking-specific mean loss ratio parameters.

Direct minimisation of  $\ell(\alpha,\beta)$  can be done using the general purpose optimiser available in Excel and known as *Solver*. When I is large this computation needs much time.

In addition to that the maximum feasible parameter dimension appears to be 200.

When it is possible to have an analytical expression for the optimal  $\beta$  in terms of  $\alpha$  and the data, we denote this by  $\hat{\beta}(\alpha \mid data)$  or just  $\hat{\beta}$  for short. This gives rise to a concentrated criterion function  $\ell(\alpha, \hat{\beta}) \to \ell(\alpha)$  that is just a function of the few elements of  $\alpha$  and can be minimised using the *Solver* in a fast way.

Direct minimisation of  $\ell(\alpha,\beta)$  can also be done along the lines of iterative Newton-Raphson. In the current type of models it appears that the  $(2+I)\times(2+I)$  Hessian matrix of the criterion function has an  $I\times I$  diagonal sub-matrix that makes (partitioned) matrix inversion easy.

We need the following (analytical) partial derivatives:

Gradient: 
$$\mathbf{g} = \begin{bmatrix} \mathbf{g}_{\alpha} \\ \mathbf{g}_{\beta} \end{bmatrix}$$
 where  $\mathbf{g}_{\alpha} = \frac{\partial \ell(\alpha, \beta)}{\partial \alpha}$  and  $\mathbf{g}_{\beta} = \frac{\partial \ell(\alpha, \beta)}{\partial \beta}$ 

Hessian: 
$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{\alpha\alpha} & \mathbf{H}_{\alpha\beta} \\ \mathbf{H}_{\beta\alpha} & \mathbf{H}_{\beta\beta} \end{bmatrix} = \begin{bmatrix} \mathbf{C} & \mathbf{B}' \\ \mathbf{B} & \mathbf{D} \end{bmatrix}$$
 for short

$$\mathbf{C} = \mathbf{H}_{\alpha\alpha} = \frac{\partial^2 \ell(\alpha, \beta)}{\partial \alpha \partial \alpha'} \qquad 2 \times 2$$

$$\mathbf{B} = \mathbf{H}_{\beta\alpha} = \frac{\partial^2 \ell(\alpha, \beta)}{\partial \beta \partial \alpha'} \qquad I \times 2$$

$$\mathbf{D} = \mathbf{H}_{\beta\beta} = \frac{\partial^2 \ell(\alpha, \beta)}{\partial \beta \partial \beta'} \qquad I \times I \quad \text{diagonal}$$

When **D** is invertible, the Hessian can be decomposed as:

$$\mathbf{H} = \begin{bmatrix} \mathbf{C} & \mathbf{B}' \\ \mathbf{B} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{D}^{-1}\mathbf{B} & \mathbf{I} \end{bmatrix}' \begin{bmatrix} \mathbf{A} & \mathbf{O} \\ \mathbf{O} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{D}^{-1}\mathbf{B} & \mathbf{I} \end{bmatrix} \text{ where } \mathbf{A} = \mathbf{C} - \mathbf{B}'\mathbf{D}^{-1}\mathbf{B}$$

For Newton-Raphson minimisation the Hessian should be positive-definite. This will be the case if the  $2\times 2$  matrix **A** and the diagonal matrix **D** are positive-definite. That is easily tested.

In case that  $\mathbf{H}$  fails to be positive-definite we replace  $\mathbf{H}$  by its expectation  $E(\mathbf{H})$  which is known as the *Information* matrix in maximum likelihood theory and that is guaranteed positive-definite. The inverse of  $\mathbf{H}$  follows as:

$$\mathbf{H}^{-1} = \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ -\mathbf{D}^{-1}\mathbf{B} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{A}^{-1} & \mathbf{O} \\ \mathbf{O} & \mathbf{D}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ -\mathbf{D}^{-1}\mathbf{B} & \mathbf{I} \end{bmatrix}' = \begin{bmatrix} \mathbf{A}^{-1} & -\mathbf{A}^{-1}\mathbf{B}'\mathbf{D}^{-1} \\ -\mathbf{D}^{-1}\mathbf{B}\mathbf{A}^{-1} & \mathbf{D}^{-1} + \mathbf{D}^{-1}\mathbf{B}\mathbf{A}^{-1}\mathbf{B}'\mathbf{D}^{-1} \end{bmatrix}$$

and may serve as an estimate of the covariance matrix of the parameter estimators. The square root of the diagonal elements will give the standard deviations. The Newton-Raphson search direction  $\mathbf{H}^{-1}\mathbf{g}$  can be computed in the following

stepwise way:

$$\mathbf{v} = \mathbf{D}^{-1}\mathbf{g}_{\beta} \rightarrow \mathbf{w} = \mathbf{A}^{-1}(\mathbf{g}_{\alpha} - \mathbf{B}'\mathbf{v}) \rightarrow \mathbf{H}^{-1}\mathbf{g} = \begin{bmatrix} \mathbf{w} \\ \mathbf{v} - \mathbf{D}^{-1}\mathbf{B}\mathbf{w} \end{bmatrix}$$

This iterative routine is known to converge quickly. It needs a few iterations, irrespective of the size of I. If programmed as a string of iterations it can be viewed as a finite step algorithm that generates results upon data entry instantaneously. In the current implementation Newton-Raphson was not used, as analytical expressions for gradient and Hessian were not available.

In what follows we will use the reciprocal of the variance, known as *precision* and generically denoted as a parametric function  $\pi$ . This simplifies analytical manipulation as well as Excel-programming using SUM, SUMPRODUCT and SUMSQUARES over rectangular arrays with numerical cells. This allows a multi-linear oriented programming style, using (0,1)-Boolean indicator functions to account for empty observations and taking care that 0/0=0.

#### **5 Normal Models for Premium Risk**

#### **5.1 First Variance Parametrisation**

The data are generated by:

$$E(y_{ii}) = \beta_i x_{ii}$$

$$\widetilde{\pi}_{ii}^{-1} = (1 - \delta) \overline{x} x_{ii} + \delta x_{ii}^2$$

$$V(y_{ii}) = \sigma^2 \widetilde{\pi}_{ii}^{-1}$$

$$u_{ii} = y_{ii} - \beta_i x_{ii}$$

We have closed form expressions for  $\beta$  and  $\sigma$ , conditionally on  $\delta$ :

$$\hat{\beta}_{i}(\delta) = \frac{\sum_{t} x_{ti} \widetilde{\pi}_{ti} y_{ti}}{\sum_{t} x_{ti} \widetilde{\pi}_{ti} x_{ti}} \quad \Rightarrow \quad \hat{u}_{ti} = y_{ti} - \hat{\beta}_{i}(\delta) x_{ti} \quad i = 1, \dots, I$$

$$\hat{\sigma}(\delta) = \left(\frac{1}{n} \sum \widetilde{\pi}_{ii} \hat{u}_{ii}^2\right)^{1/2}$$

Finally we get a one-dimensional estimation criterion function:

$$\ell(\delta \mid data) = n \log \hat{\sigma} - \frac{1}{2} \sum_{i} \log \tilde{\pi}_{ii} \qquad 0 \le \delta \le 1$$

When I=1 all parameters are undertaking-specific and the two variance parametrisations boil down to the same mathematical model.

In the parametrisation defined by the triple  $(\bar{x}, \delta, \sigma)$  the sample mean  $\bar{x}$  occurs as a kind of standardisation. This would justify an indication of this dependence to the parameters. This can be done by adding a bar. If we had used a different "average", denoted as  $\bar{x}$  this would give different maximum likelihood estimates. The relation between both triples is given by:

$$\overline{x} \to \overline{x} \qquad \overline{\delta} \to \overline{\delta} = \frac{\overline{\delta} \overline{x}}{\overline{\delta} \overline{x} + (1 - \overline{\delta}) \overline{x}} \qquad \overline{\sigma} \to \overline{\sigma} = \overline{\sigma} \sqrt{\overline{\delta} + (1 - \overline{\delta}) \overline{x} \overline{x}^{-1}}$$

This is of importance when comparing parameter estimates based on a data sample that not necessarily coincides with the population.

#### **5.2 Second Variance Parametrisation**

The data are generated by:

$$E(y_{ii}) = \beta_{i} x_{ii}$$

$$\tilde{\pi}_{ii}^{-1} = (1 - \delta) \bar{x} x_{ii} + \delta x_{ii}^{2}$$

$$V(y_{ii}) = \pi_{ii}^{-1} = (\sigma \beta_{i})^{2} \tilde{\pi}_{ii}^{-1}$$

$$u_{ii} = y_{ii} - \beta_{i} x_{ii}$$

For a single undertaking we display the estimation criterion function:

$$\ell(\alpha, \beta \mid data) = \frac{1}{2} \sum_{t} \pi_{t} u_{t}^{2} - \frac{1}{2} \sum_{t} \log \pi_{t}$$

$$= T \log \sigma + T \log \beta + \frac{1}{2} \sigma^{-2} \sum_{t} \widetilde{\pi}_{t} (y_{t} \beta^{-1} - x_{t})^{2} - \frac{1}{2} \sum_{t} \log \widetilde{\pi}_{t}$$

This function allows analytical minimisation with respect to  $\beta$ , conditional on  $\alpha$ :

$$\hat{\beta}_{i} = \frac{2\sum_{t} \tilde{\pi}_{ti} y_{ti}^{2}}{\sum_{t} \tilde{\pi}_{ti} x_{ti} y_{ti} + \sqrt{\left(\sum_{t} \tilde{\pi}_{ti} x_{ti} y_{ti}\right)^{2} + 4T_{i} \sigma^{4} \sum_{t} \tilde{\pi}_{ti} y_{ti}^{2}}} \qquad i = 1, \dots, I$$

The concentrated criterion function over all undertakings follows as:

$$\ell(\alpha \mid data) = n\log\sigma + \sum_{i} T_{i}\log\hat{\beta}_{i} + \frac{1}{2}\sigma^{-2}\sum\tilde{\pi}_{ii}(y_{ii}\hat{\beta}_{i}^{-1} - x_{ii})^{2} - \frac{1}{2}\sum\log\tilde{\pi}_{ii}$$

The two-dimensional numerical procedure should take place under the restrictions on  $\alpha$ .

#### **6 Lognormal Models for Premium Risk**

An aggregate loss y with parametric functions for mean and variance can be approximated by a lognormal distribution with mean and variance  $\mu$  and  $\omega$  for  $\log(y)$  as follows:

$$E(y) = \exp(\mu + \frac{1}{2}\omega) = \beta x$$

$$V(y) = \exp(2\mu + 2\omega) - \exp(2\mu + \omega) = (\beta x)^2 (e^{\omega} - 1)$$

From this we can express the mean  $\mu$  and the variance  $\omega$  as:

$$\mu = \log(\beta x) - \frac{1}{2}\omega$$
 and  $\omega = \log\left(1 + \frac{V(y)}{(\beta x)^2}\right) = \pi^{-1}$ 

So, the nature of the variance specification becomes crucial. In case that the variance is proportional with the square of  $\beta$ , the specification for  $\omega$  does not depend on  $\beta$ . This is the second variance parametrisation that is mathematically most tractable. The first variance parametrisation is awkward from a mathematical and computational point of view.

After a logarithmic transformation  $z_t = \log q_t$  we may write:

$$u_{ti} = z_{ti} + (2\pi_{ti})^{-1} - \log \beta_i \sim N(0, \omega_{ti})$$

which shows that the parameters in  $\omega = \pi^{-1}$  enter both the mean and the variance.

#### **6.1 First Variance Parametrisation**

From (3.1) it follows that:

$$\omega = \log \left( 1 + \frac{\sigma^2 \left( (1 - \delta) \bar{x} x + \delta x^2 \right)}{\left( \beta x \right)^2} \right) = \pi^{-1}$$

We formulate the estimation criterion function for all undertakings as:

$$\ell(\alpha, \beta \mid data) = \frac{1}{2} \sum_{i} \pi_{ii} \left( z_{ii} + (2\pi_{ii})^{-1} - \log \beta_{i} \right)^{2} - \frac{1}{2} \sum_{i} \log \pi_{ii}$$

This function is additive separable in the components of  $\beta$  but does not allow convenient reduction for optimisation. A convenient simplification is to use the  $\beta$ -estimates from the normal model and optimise with respect to  $\alpha$ .

#### **6.2 Second Variance Parametrisation**

From (3.2) it follows that:

$$\omega = \log\left(1 + \frac{\beta^2 \sigma^2 \left((1 - \delta)\overline{x}x + \delta x^2\right)}{(\beta x)^2}\right) = \log\left(1 + \sigma^2 \left((1 - \delta)\overline{x}x^{-1} + \delta\right)\right) = \pi^{-1}$$

We formulate the estimation criterion function for all undertakings as:

$$\ell(\alpha, \beta \mid data) = \frac{1}{2} \sum_{i} \pi_{ii} \left( z_{ti} + (2\pi_{ti})^{-1} - \log \beta_{i} \right)^{2} - \frac{1}{2} \sum_{i} \log \pi_{ti}$$

We derive: 
$$\log \hat{\beta}_i = \frac{\sum_t z_{ti} \pi_{ti} + \frac{1}{2} T_i}{\sum_t \pi_{ti}}$$
  $i = 1, \dots, I$ 

and the concentrated criterion function follows as:

$$\ell(\alpha \mid data) = \frac{1}{2} \sum_{i} \pi_{ii} \left( z_{ii} + (2\pi_{ii})^{-1} - \log \hat{\beta}_{i} \right)^{2} - \frac{1}{2} \sum_{i} \log \pi_{ii}$$

which should be minimized under the restrictions on  $\alpha$ .

#### 7 Reserve Risk Methods of Premium Risk Type

For reserve risk, using premium risk type methods, *financial* (accounting) year will occur.

Reusing the earlier symbols with a new interpretation:

x = total claims provision at the start of financial year t

y = aggregate loss incurred in financial year t for accidents years < t

we can analyse runoff reserve risk with the earlier discussed premium risk methods. If that is done this would imply an undertaking *specific* runoff ratio  $\beta$ . Whatever our appreciation of these methods, it is convenient that these can be used from the premium risk methods without any further investment.

#### 8 Reserve Risk Methods of Runoff Triangle Type

#### 8.1 Overdispersed Bilinear Poisson Models

An overdispersed Poisson model with accident-year effects and development duration effects, as described in Renshaw & Verrall (1998). Due to the chain-ladder algorithm this procedure can be programmed such that it only needs manipulation of some matrices as well as some additional matrix algebra, such as the inverse. Chain-ladder mathematics in Matitschka (2010) shows that a chain-ladder oriented reparametrisation exists that makes the Information matrix diagonal, avoiding even matrix inversion! The results of this chain-ladder method seem to be similar to the ones explained in the next paragraph, which is the triangle method that eventually was analysed.

#### 8.2 Merz-Wüthrich (2008) Inspired Models

These procedures have also a chain-ladder as basis.

For the variance there is a specification proportional in x as well as squared in x. Using least squares a coefficient of variation or a standard deviation is determined.

#### 9 Goodness-of Fit, Residuals and All That

#### 9.1 Detecting Outliers

After data entry and optimisation we should have a look at the standardised residuals and focus on potential outlying values. If these are there, we should verify whether these are

- Caused by data entry errors
- Intrinsic by odd observations, which need further explanation before adoption
- To be adjusted manually (always difficult to explain)

#### 9.2 PP-Plot

After the detection of and protection against outliers the standardised residuals are believed to be a sincere expression of the probability law that is thought to generate the data. Transforming these standardised residuals with the inverse (log)normal cumulative distribution function maps these residuals on the interval (0,1) in a uniform pattern. The graphical display is known as the PP-plot.

#### 9.3 Jarque-Bera Test

This (asymptotic) test for (log)normality follows a  $\chi^2$ -distribution with 2 degrees of freedom (= Exponential distribution with mean and standard deviation equal to 2) and is based on the sample skewness S and kurtosis K:

$$JB = \frac{n}{24} \left( 4S^2 + (K - 3)^2 \right) \qquad P(JB > v) = \exp(-\frac{1}{2}v)$$

Care should be exercised with this test statistic as the asymptotic distribution only holds for fairly large (>>100) numbers of observations n.

#### 10 Bridging Statistical Estimation and Calibration

#### 10.1 Calibration by Estimation of a Size-Invariant Model

The quadratic variance specification will only be size-invariant when  $\delta=1$ . In case that the estimate for  $\delta$  is close to 1, re-estimation of the model under the restriction that  $\delta=1$  may make sense.

#### 10.2 Compliance Analysis by Portfolio-Size

After parameter estimation for a line of business with average portfolio size  $\bar{x}$  and estimates for  $\delta$  and  $\sigma$  we replace  $\sigma$  by an unbiased estimate  $\bar{\sigma} = cfuesd \cdot \sigma$ . The appropriate standard deviation for a portfolio of size x results as:

$$\overline{\sigma}\sqrt{\delta + (1-\delta)\overline{x}x^{-1}} > \overline{\sigma}\sqrt{\delta}$$

A common calibrated level of the standard deviation can be expressed as a multiple of the unbiased estimate for the appropriate standard deviation of an average sized portfolio  $\kappa \overline{\sigma}$ . Whatever the choice of  $\kappa$  it will imply that the *SCR* will be too large for the larger portfolios and too small for the smaller ones. The question arises when and how often this occurs and to what degree. An undertaking with portfolio size  $x_i$  will be compliant when:

$$\overline{\sigma}\sqrt{\delta + (1-\delta)\overline{x}x_i^{-1}} \leq \kappa\overline{\sigma} \qquad \text{or} \quad \kappa \geq \kappa_i \quad \text{where} \quad q = \sqrt{\delta + (1-\delta)\overline{x}x^{-1}} > \kappa_0 = \sqrt{\delta}$$

In the industry there are (observed) portfolio sizes, denoted and ordered as:

$$x_1 > x_2 > \dots > x_L > 0$$
 implying  $\kappa_0 < \kappa_1 < \kappa_2 < \dots < \kappa_L$ 

We define the Boolean indicator as a function of  $\kappa$ :

$$1(\kappa \ge \kappa_i) = \begin{cases} 1 & \text{if } \kappa \ge \kappa_i \\ 0 & \text{if } \kappa < \kappa_i \end{cases}$$

and define a family of compliant shares depending on a further control parameter  $\rho$ :

$$C_{\rho}(\kappa) = \frac{\sum_{i} x_{i}^{\rho} 1(\kappa \ge \kappa_{i})}{\sum_{i} x_{i}^{\rho}} \qquad 0 \le \rho \le 1$$

This ratio can be interpreted as:

- $\rho$ =0 compliant share of *portfolios* in the industry with security level ≥0.995 when the *SCR* is calculated according to  $\kappa \overline{\sigma}$
- $\rho$ =1 compliant share of *policyholders* that are insured by undertakings with security level  $\geq$ 0.995 when the *SCR* is calculated according to  $\kappa \overline{\sigma}$

This compliant share is a right-continuous step-function of  $\kappa$  that increases from 0 to 1:

$$\begin{split} C(\kappa) &= 0 & \kappa < \kappa_1 \\ &= C(\kappa_i) & \kappa_i \leq \kappa < \kappa_{i+1} & i = 1, \cdots, (I-1) \\ &= 1 & \kappa_I \leq \kappa \end{split}$$

When the statistical estimate for  $\delta$  equals 1 each portfolio is compliant as soon as  $\kappa \ge 1$  and the step function boils down to the simple form:

$$C(\kappa) = 0 \qquad \qquad \kappa < 1$$
$$= 1 \qquad \qquad \kappa \ge 1$$

This compliant share is inspired from voting theory where THEIL & SCHRAGE (1977) show that the case  $\rho$ =0.5 has optimal relevance too.

When calibrating one may single out a representative portfolio size, such as population mean, median or other, calculate the corresponding standard deviation that implies a value for  $\kappa$  that defines the level of the compliant share  $C(\kappa)$ .

The choice of  $\kappa$  could also be made by having it satisfy an acceptable level of  $C(\kappa)$  and solving for  $\kappa$ . Unfortunately, as  $C(\kappa)$  is a step-function it does not have a straight-forward inverse. If we replace  $C(\kappa)$  by a piece-wise linear function by linking the points of increase this numerical problem can be settled:

$$C^{*}(\kappa) = 0$$

$$= C(\kappa_{i}) + \frac{C(\kappa_{i+1}) - C(\kappa_{i})}{\kappa_{i+1} - \kappa_{i}} (\kappa - \kappa_{i})$$

$$= 1$$

$$\kappa < \kappa_{0}$$

$$\kappa_{i} \le \kappa < \kappa_{i+1}$$

$$\kappa_{i} \le \kappa < \kappa_{i+1}$$

$$\kappa_{i} \le \kappa$$

Solving  $C^*(\kappa) = p$  gives:

$$\kappa = \kappa_0 \qquad p = 0$$

$$= \kappa_i + \frac{p - C(\kappa_i)}{C(\kappa_{i+1}) - C(\kappa_i)} (\kappa_{i+1} - \kappa_i) \qquad C(\kappa_i) \le p < C(\kappa_{i+1}) \qquad i = 0, \dots, (I - 1)$$

$$= \kappa_I \qquad p = 1$$

#### **Appendices**

#### A1 Deriving the Quadratic Variance Specification

We consider an insurance portfolio consisting of m similar risks generating n claims according to a Poisson law with mean  $m\lambda$ . An individual severity of a claim follows a probability distribution with mean  $\mu$  and a variance that is proportional with the square of the mean:  $(\kappa\mu)^2$  where  $\kappa$  is the coefficient of variation of the positive nonlife insurance risk. The probability distribution of the aggregate loss  $\gamma$  conditional on the number of claims  $\gamma$  is just the  $\gamma$ -fold convolution with mean  $\gamma$  and variance  $\gamma$  is However, for prediction purposes we cannot condition on the number of claims. Taking the  $\gamma$  ratio distribution with respect to the number of claims generates the  $\gamma$  compound Poisson distribution for the aggregate loss and has mean, second moment and variance given by:

$$E(y) = m\lambda\mu$$

$$E(y^{2}) = m\lambda\mu^{2}(1+\kappa^{2}) + (m\lambda\mu)^{2}$$

$$\Rightarrow V(y) = m\lambda\mu^{2}(1+\kappa^{2})$$

which is a well-known result.

We envisage now that the basic parameters  $\lambda$ ,  $\mu$  and  $\kappa$  are not fixed through time but are subject to a (stationary) stochastic process themselves. To denote this potential randomness we equip the time-fluctuating basic parameter symbols with a  $\sim$  and consider an additional expectation (and variance) operation on (functions of) these parameters:

$$\begin{split} E(y) &= mE(\widetilde{\lambda}\widetilde{\mu}) \\ E(y^2) &= mE\left(\widetilde{\lambda}\widetilde{\mu}^2(1+\widetilde{\kappa}^2)\right) + m^2E\left((\widetilde{\lambda}\widetilde{\mu})^2\right) \end{split} \Rightarrow V(y) &= mE\left(\widetilde{\lambda}\widetilde{\mu}^2(1+\widetilde{\kappa}^2)\right) + m^2V(\widetilde{\lambda}\widetilde{\mu}) \end{split}$$

implying a variance that is quadratic (not linear) in the portfolio size m. We rewrite matters in terms of a per capita premium p, a premium volume x=mp and a random loss ratio parameter  $\widetilde{\beta}=\widetilde{\lambda}\widetilde{\mu}/p$ :

$$E(y) = xE(\widetilde{\beta})$$

$$E(y^{2}) = xE(\widetilde{\beta}\widetilde{\mu}(1+\widetilde{\kappa}^{2})) + x^{2}E(\widetilde{\beta}^{2}) \Rightarrow V(y) = xE(\widetilde{\beta}\widetilde{\mu}(1+\widetilde{\kappa}^{2})) + x^{2}V(\widetilde{\beta})$$

If we write:

$$E(\tilde{\beta}) = \beta$$

$$E(\tilde{\beta}\tilde{\mu}(1 + \tilde{\kappa}^2)) = \sigma_1^2 \bar{x}$$

$$V(\tilde{\beta}) = \sigma_2^2$$

we arrive at the quadratic variance specification:

$$E(y) = \beta x$$
$$V(y) = \sigma_1^2 \bar{x} x + \sigma_2^2 x^2$$

#### **A2 Unbiased Estimator Standard Deviation**

In the framework of the standard linear model with normal disturbances (see any econometric text), the maximum likelihood estimator for the variance is biased, but asymptotically consistent. This consistency disappears however in case of incidental parameters, that is parameters that occur only a finite time in the observational process. Well-known examples are panel data where the size of the panel increases but the number of observations for a member of the panel remains limited, as explained in LANCASTER (2000, 2002). In the current observational plan we too have a panel with a lot of portfolios for a line of business but a limited unbalanced length of the time series.

Even more, for solvency purposes we do not need an unbiased estimator for the variance but an unbiased estimator for the standard deviation. The strategy is to use a correction factor that is applied to the (biased) maximum likelihood estimator. This correction factor can be used for the various standard deviation functions that form part of the *SCR*.

In case of a total sample size of n and k unknown parameters in the linear expected value, we have n/(n-k) as the traditional correction factor for the variance. For the standard deviation this implies a correction factor:

$$c_1 = \sqrt{\frac{n}{n-k}}$$

To achieve unbiasedness for the standard deviation we need a second additional correction factor given by:

$$c_{2} = \frac{\Gamma(\frac{1}{2}(n-k))}{\Gamma(\frac{1}{2}(n-k+1))} \sqrt{\frac{1}{2}(n-k)}$$

Here  $\Gamma$  denotes the Gamma-function that most easily is evaluated in Excel through its logarithm. Combining these two correction factors we get:

$$c = c_1 c_2 = c(n, k) = \frac{\Gamma(\frac{1}{2}(n - k))}{\Gamma(\frac{1}{2}(n - k + 1))} \sqrt{\frac{1}{2}n}$$
 (cfuesd)

The coefficient of variation of the resulting unbiased estimator follows as:

$$c(n-k) = \sqrt{\frac{n-k}{n}c^2 - 1} = \sqrt{c_2^2 - 1}$$
 (cvuesd)

In the spreadsheet for models with undertaking-specific means these constants are indicated with the mnemonic abbreviations *cfuesd* and *cvuesd*. For the full undertaking-specific case, we have k = I = 1 and n = T.

#### A3 Understanding the Earlier Premium Risk Method 1

All parameters of this method are undertaking-specific.

Adding for a single undertaking the index for accident years  $t = 1, \dots, T$  the following two sets of estimators arise naturally within a normal weighted least squares setting:

linear in exposure: 
$$\delta = 0$$
  $\tilde{\beta} = \frac{\overline{y}}{\overline{x}}$   $\tilde{\sigma}^2 = \frac{\sum_t x_t (q_t - \tilde{\beta})^2}{(T - 1)\overline{x}}$  squared in exposure:  $\delta = 1$   $\hat{\beta} = \overline{q}$   $\hat{\sigma}^2 = \frac{\sum_t (q_t - \tilde{\beta})^2}{T - 1}$ 

squared in exposure: 
$$\delta = 1$$
  $\hat{\beta} = \overline{q}$   $\hat{\sigma}^2 = \frac{\sum_t (q_t - \hat{\beta})^2}{T - 1}$ 

It is also possible to have a hybrid approach. That is optimally estimating  $\beta$  using a variance specification linear in exposure and estimating the variance parameter using a specification squared in exposure (and reversed):

$$\mathsf{linear} \Rightarrow \widetilde{\beta} \to \mathsf{squared} \Rightarrow \widehat{\sigma}^2 = \frac{\sum_{t} (q_t - \widetilde{\beta})^2}{T - 1}$$

squared 
$$\Rightarrow \hat{\beta} \rightarrow \text{linear} \Rightarrow \vec{\sigma}^2 = \frac{\sum_t x_t (q_t - \hat{\beta})^2}{(T - 1)\bar{x}}$$

The earlier premium risk method 1 corresponds with the latter hybrid approach. Whatever the method, for the population of I undertakings this implies a series of I estimates for the dispersion parameter that must be combined into a single representative. Applied to the latter hybrid approach we have:

$$\widetilde{\sigma}_{i}^{2} = \frac{\sum_{t=1}^{T_{i}} x_{ti} (q_{ti} - \widehat{\beta}_{i})^{2}}{(T_{i} - 1)\overline{x}_{i}} \qquad i = 1, \dots, I$$

where the notation stresses allowance for different length of time series by undertaking.

The current calibration for premium risk method 1 proceeds as:

$$\sigma^* = \sum_i w_i \overline{\sigma}_i$$
 where  $w_i = \overline{x}_i / \sum_i \overline{x}_i$ 

Some reflection might be in order here. In the various estimators for the variance parameter we see division by (T-1). This makes such estimators unbiased. However, these are used by their square roots to derive estimators for the standard deviation. Jensen's inequality implies that such estimators for the standard deviation will be biased downwards.

#### A4 Rephrasing the Earlier Premium Risk Methods 2-3-4

These methods use a lognormal approximation. The specification of the variance is linear or squared. With the exception of the earlier method 3 numerical procedures are *I*-dimensional which may prove prohibitive for direct optimisation procedures.

#### Premium risk method 2

This lognormal model has specific scale parameters for the loss ratio. The variance is linear in exposure with a common dispersion parameter  $\eta$ . We reparametrise:

$$\gamma_i = \eta \beta_i^{-2} \Rightarrow \omega_{ti} = \log(1 + \gamma_i x_{ti}^{-1}) = \pi_{ti}^{-1}$$

and the estimation criterion function can be reduced to:

$$\ell(\gamma_1, \dots, \gamma_I, \eta \mid data) = \frac{1}{8} \sum_{i} \pi_{ii} \left( 2z_{ii} + \pi_{ii}^{-1} + \log \gamma_i - \log \eta \right)^2 - \frac{1}{2} \sum_{i} \log \pi_{ii}$$

This allows a closed form expression for the optimal value of  $\log \eta$ :

$$\log \hat{\eta} = \frac{2\sum z_{ti}\pi_{ti} + \sum \pi_{ti}\log \gamma_i + n}{\sum \pi_{ti}}$$

resulting in an *I*-dimensional concentrated criterion function:

$$\ell(\gamma_1, \dots, \gamma_I \mid data) = \frac{1}{8} \sum_{i} \pi_{ii} \left( 2z_{ii} + \pi_{ii}^{-1} + \log \gamma_i - \log \hat{\delta} \right)^2 - \frac{1}{2} \sum_{i} \log \pi_{ii}$$

The earlier implementation of method 2 required an (I+1)-dimensional numerical procedure.

#### Premium risk method 3

This lognormal model has a *common* scale parameter  $\beta$  and a common dispersion parameter  $\eta$  and  $\sigma$ =0. We reparameterise  $\eta = \gamma \beta^2$  leading to:

$$\omega_{ti} = \log(1 + \gamma x_{ti}^{-1}) = \pi_{ti}^{-1}$$

We formulate the estimation criterion function for all undertakings as:

$$\ell(\gamma, \beta \mid data) = \frac{1}{2} \sum_{i} \pi_{ii} \left( z_{ii} + (2\pi_{ii})^{-1} - \log \beta \right)^{2} - \frac{1}{2} \sum_{i} \log \pi_{ii}$$

Conditionally on  $\gamma$  we can estimate  $\log \beta$ :

$$\log \hat{\beta} = \frac{\sum z_{ti} \pi_{ti} + \frac{1}{2} n}{\sum \pi_{ti}}$$

After inserting this in the criterion function, this results in a one-dimensional numerical procedure to determine the optimal  $\gamma$ :

$$\ell(\gamma \mid data) = \frac{1}{2} \sum_{i} \pi_{ii} \left( z_{ii} + (2\pi_{ii})^{-1} - \log \hat{\beta} \right)^{2} - \frac{1}{2} \sum_{i} \log \pi_{ii}$$

The earlier implementation of method 3 required a two-dimensional numerical procedure.

This method is generally believed to be of little interest for premium risk calibration.

Premium risk method 4

This lognormal model has specific scale parameters for the loss ratio. The variance is squared in exposure with a common dispersion parameter  $\sigma$ . We have:

$$\omega_i = \log(1 + (\sigma\beta_i^{-1})^2) = \pi_i^{-1} \iff \log\beta_i = \log\sigma - \frac{1}{2}\log(e^{\omega_i} - 1)$$
 does not depend on  $t!$ 

and the estimation criterion function follows as:

$$\ell(\pi_1, \dots, \pi_I, \sigma \mid data) = \frac{1}{2} \sum_{i} \pi_i \left( z_{ii} + (2\pi_i)^{-1} + \frac{1}{2} \log(e^{\omega_i} - 1) - \log \sigma \right)^2 - \frac{1}{2} \sum_{i} T_i \log \pi_i$$

From this we derive:

$$\log \hat{\sigma} = \frac{2\sum z_{ii}\pi_i + \sum_i T_i\pi_i \log(e^{\omega_i} - 1) + n}{2\sum_i T_i\pi_i}$$

resulting in an *I*-dimensional concentrated criterion function:

$$\ell(\pi_1, \dots, \pi_I \mid data) = \frac{1}{2} \sum_{i} \pi_i \left( z_{ii} + (2\pi_i)^{-1} + \frac{1}{2} \log(e^{\omega_i} - 1) - \log \hat{\sigma} \right)^2 - \frac{1}{2} \sum_{i} T_i \log \pi_i$$

The earlier implementation of method 4 required an (I+1)-dimensional numerical procedure.

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## Annex 4: Data Request<sup>35</sup>

#### Introduction

When delivering its advice for Level 2 measures, CEIOPS committed to carry out a comprehensive revision of the calibration of the premium and reserve risk factors in the non-life and health non-SLT underwriting risk module of the SCR standard formula in the framework of QIS5.

For this purpose, CEIOPS is requesting data at EU level covering as wide a range of undertakings (of all types and sizes) and Member States as possible. This document summarises the data requirements. As much data as possible from the list below is needed with a clear reference to what the data includes.<sup>36</sup> As CEIOPS will be applying a number of methods, the information specified below will be useful also for testing the appropriateness of the methods.

The quality of the calibration of a capital charge depends on the quality of the data available. Therefore particular attention should be paid to ensuring that the data supplied is as accurate and complete as possible, and that where adjustments have been made to the raw data, that these are justified and fully explained. Examples of situations where adjustments to raw data may be considered necessary or desirable are included in the annex to this document.

Data should be submitted to national supervisors as part of the QIS5 exercise – with an extended submission date for this specific data requirement **of 30 of November of 2010**. A data collection template will be provided for this purpose. National supervisors will review the submissions to ensure that the data meet the overall requirements before submitting the data to CEIOPS for analysis. Special provisions have been made to ensure the confidentiality of the submitted data.<sup>37</sup>

The European Commission and selected stakeholders have been involved in the design of the data requirement from the beginning of this revision exercise. They will continue their participation in this joint work stream together with CEIOPS in discussing the most appropriate calibration methods and deriving suggestions for setting premium and reserve risk factors for non-life and health non-SLT underwriting risk in the standard formula. To this end, CEIOPS will invite the European Commission and industry representatives for meeting on a regular basis.

The results of the analysis (but not the data supplied by undertakings) will be made public, including information on the calibration methodologies which have been applied.

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This annex contains the contents of the note on data requirements for non-life and non-SLT health calibration (CEIOPS-SEC-116/10) as published on CEIOPS' website on 23 September 2010.

For a detailed description of the data required for the analyses we refer to sections 2 and 3, below See section 5 on confidentiality

#### Premium risk analyses

We will use various models and parameterisation techniques to quantify appropriate levels of premium risk by line of business (LoB), by comparing undertaking and accident year information. In particular, for each accident year, we will be looking at:

- Estimate of ultimate loss
- premium earned

Per LoB this information will be used to estimate the volatility of earned loss ratios on the level of individual undertakings (across different accident years) as well as on a market level (across different undertakings).

The methodologies to derive these estimates are intended to be in the range of the calibration methods applied by CEIOPS in its advice on the calibration of the non-life underwriting risk in the standard formula<sup>38</sup>, with further refinements of these methods as appropriate.

#### **Data Requirements**

The following data - split by  $LoB^{39}$  and accident year - shall be submitted:

#### Raw data gross of reinsurance

- <u>Volume of earned premium</u> for the accident year gross of acquisition costs. (see column (a) on the template)
- <u>Acquisition costs / earned commissions</u> (see column (b) on the template) if available
- Expense information, if available, comprising:
  - Relevant Unallocated Loss Adjustment Expenses (ULAE) (see column (c) on the template)
  - Relevant other paid expenses (all other expenses excluding Allocated Loss Adjustment Expenses (ALAE) and ULAE) (see column (d) on the template).
- <u>Current estimate of ultimate loss</u><sup>40</sup> (see column (e) on the template), comprising:
  - Paid claims up until 2009 for that accident year

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Cf. CEIOPS' Advice for Level 2 Implementing Measures on Solvency II: SCR Standard Formula: Calibration of Non-life Underwriting Risk (CEIOPS-DOC-67/10); 8 April 2010
This comprises all Lines of Business (LoB) in Non-Life and Healt Non-SLT insurance relevant to the insurers' business. For a list we refer to para. TP.1.15 (for Non-Life insurance) and para. TP.1.17 (for Non-SLT Health obligations). (See the readme in the Excel template)

Here 'ultimate loss' denotes the estimated aggregate claims expenditure that will have to be paid to finally settle the claims for the accident year considered. For the purposes of this exercise, 'current estimate' is intended to refer to the estimate of the ultimate loss as at the end of 2009.

- QIS5 best estimate claims provisions (including IBNR) as at year end 2009
- Estimate of the <u>ultimate loss as at the end of the first development year</u> (see column (f) on the template), comprising:
  - Paid claims in the first development year for that accident year
  - Best estimate claims provisions (including IBNR) posted at the end of the first development year
- The estimate of ultimate loss should:
  - Include ALAE
  - Where practicable, allow for receivables for salvage and subrogation

Adjusted data gross of reinsurance, excluding catastrophe loss

Same as gross (point 2.1.1. above) but excluding catastrophe loss and including other justifiable adjustments as those included in the annex, where appropriate – see 2.2 and 2.3 below.

Adjusted data net of reinsurance, excluding catastrophe loss

Same as point 2.1.2 above but now net of reinsurance instead of gross – see 2.2 and 2.3 below.

Information on the data provided

When unforeseen and un-modelled external factors – such as changes of regulation – had a material effect on the subsequent development of claims in an individual accident year, please provide any qualitative comment needed for the supervisors to have the relevant background information when analysing your data. However, those effects should not be removed or smoothed in the data submitted.

Exclusion of catastrophe claims and related expenses

The premium risk data referred to above shall generally be adjusted to exclude catastrophe claims and related expensed.

For this purpose, catastrophe claims cover all events or exceptional losses that would fall within the scope of the relevant catastrophe risk module of the standard formula SCR. This includes natural catastrophe claims, man-made catastrophe claims and health catastrophe claims as explained in the relevant sections of the QIS5 Technical Specifications. Because there are no simple and unambiguous definitions of catastrophe claims in the Technical Specifications, undertakings will need to form a judgement as to where they draw the line between for example catastrophe windstorm claims, and "normal" claims resulting from high wind – but a general rule of thumb may be to exclude events for which they put relevant catastrophe reinsurers on notice for a potential catastrophe recovery claim.

Undertakings should carefully explain what has been excluded, and what has been left in the data, and all adjustments made to raw data.

No adjustments should be made to earned premiums in the individual accident years.

Other data adjustments

Other adjustments to the data may be made where these are aimed to ensure that any set of data is made internally consistent and comparable. The annex contains a non-exhaustive list of situations where it may be acceptable to introduce adjustments to historical data presented by runoff triangles.

Undertakings should explain and justify all adjustments made. All adjustments shall be appropriate for the purpose of calibrating the volatility of premium risk. No adjustments should be made to earned premiums in the individual accident years.

*Underwriting year data* 

Where data is not available on an accident year basis, it may be acceptable to submit equivalent data on an underwriting year basis. In this case, undertakings should carefully explain what approach they have taken in producing the data.

CEIOPS will then make a judgement as to whether the data is able to be used in full or in part of the subsequent analysis.

Reserve risk analyses

We will use various models and parameterisation techniques to quantify appropriate levels of reserving risk by LoB, by performing analyses in two separate ways.

- Analysing by company how opening reserves compare against the amounts paid in the subsequent financial year along with the associated closing reserves.
- Implementing one year reserving risk approaches directly from the triangles of either paid and/or reported data.

The applied methodologies are intended to be in the range of the calibration methods used by CEIOPS in its advice on the calibration of the non-life underwriting risk in the standard formula<sup>41</sup>, with further refinements of these methods as appropriate.

#### Data Requirements

The following data - split by LOB and accident year - shall be submitted:

Raw data gross of reinsurance

• <u>Triangles of paid claims</u>:

Cf. CEIOPS' Advice for Level 2 Implementing Measures on Solvency II: SCR Standard Formula: Calibration of Non-life Underwriting Risk (CEIOPS-DOC-67/10); 8 April 2010

- These figures should include ALAE, but exclude ULAE
- This should allow for receivables for salvage and subrogation, if available
- All claims should be included, including catastrophe claims
- Triangles of best estimate claims provisions
  - Ideally these should be compatible with Solvency II valuation principles
  - If not available, posted reserves as shown in undertaking's published accounts and including IBNR should be submitted
- Reported triangles, if available
  - The triangles should exclude IBNR and be the sum of paid claims and case estimates
  - All reported claims should be included
  - The data should include ALAE, but exclude ULAE
  - The data should allow for receivables for salvage and subrogation, if available

#### Adjusted data gross of reinsurance, excluding catastrophe loss

Same as gross (point 3.1.1. above) but where practicable excluding catastrophe loss and including other justifiable adjustments as those included in the annex, where appropriate – see 3.2 and 3.3 below.

#### Adjusted data net of reinsurance, excluding catastrophe loss

Same as point 3.1.2 above but now net of reinsurance instead of gross – see 3.2 and 3.3 below.

#### Information on the data provided

When unforeseen and un-modelled external factors – such as changes of regulation – had a material effect on the subsequent development of claims in an individual accident year, please provide any qualitative comment needed for the supervisors to have the relevant background information when analysing your data. However, those effects should not be removed or smoothed in the data submitted.

#### Exclusion of catastrophe claims and related expenses

If practicable, the reserve risk data referred to above shall be adjusted to exclude catastrophe claims and related expensed.

For this purpose, catastrophe claims cover all events or exceptional losses that would fall within the scope of the relevant catastrophe risk module of the standard formula SCR. This includes natural catastrophe claims, man-made catastrophe claims and health catastrophe claims as explained in the relevant sections of the QIS5 Technical Specifications. Because there are no simple and unambiguous definitions of catastrophe claims in the Technical Specifications, undertakings will need to form a judgement as to where they draw the line between for example catastrophe windstorm claims, and "normal" claims resulting from high wind – but a general rule of thumb may be to exclude events for which they put relevant catastrophe reinsurers on notice for a potential catastrophe recovery claim.

Undertakings should carefully explain what has been excluded, and what has been left in the data, and all adjustments made to raw data.

#### Other data adjustments

Other adjustments to the data may be made where these are aimed to ensure that any set of data is made internally consistent and comparable. The annex contains a non-exhaustive list of situations where it may be acceptable to introduce adjustments to historical data presented by runoff triangles..

Undertakings should explain and justify all adjustments made. All adjustments shall be appropriate for the purpose of calibrating the volatility of reserve risk.

#### Underwriting year data

Where data is not available on an accident year basis, it may be acceptable to submit equivalent data on an underwriting year basis. In this case, undertakings should carefully explain what approach they have taken in producing the data.

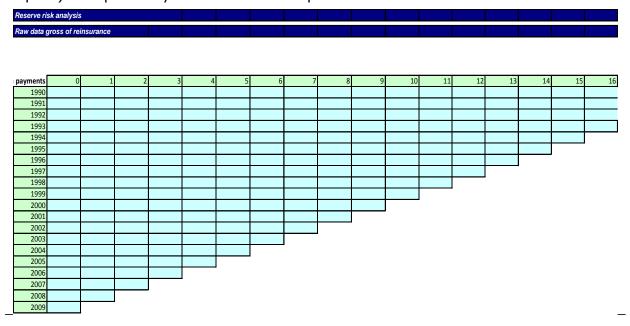
CEIOPS will then make a judgement as to whether the data is able to be used in full or in part of the subsequent analysis.

#### **Data Template**

An example of data requested above is shown below (for premium risk data gross of reinsurance). The accompanying Excel file has templates separately for premium and runoff triangle reserve risk data, and for gross and net data, respectively. It allows (but does not require) for up to 20 years of data to be provided.

2.	Premium risk an	alysis							
2. 1.1	Raw data gross	of reinsuranc	e						
								Ultima	ite loss
		Earned		Acquisition	ULAE	Other paid		Current	End of 1rst
		premiums		costs	OLAE	expenses		estimate	year
		(a)		(b)	(c)	(d)		( e )	(f)
	1990								0.00
	1991								0.00
	1992								0.00
	1993								0.00
	1994								0.00
	1995								0.00
	1996								0.00
	1997								0.00
	1998								0.00
	1999								0.00
	2000								0.00
	2001								0.00
	2002								0.00
	2003								0.00
	2004								0.00
	2005								0.00
	2006								0.00
	2007								0.00
	2008								0.00
	2009						•		0.00

An example of data requested above is shown below (for reserve risk paid claims data gross of reinsurance). The accompanying Excel file allows (but does not require) for up to 20 years of data to be provided.



#### General considerations on confidentiality

#### Data to be submitted

Individual data regarding the non-life and non-SLT health lines of business will be submitted to CEIOPS by national supervisors in order to improve the calibration of those two underwriting sub-modules. Therefore, undertakings should submit their data to their national supervisor and not direct to CEIOPS.

In some Member States, local law may not allow the supervisory authority to submit any data to CEIOPS that is not anonymised. In those cases, the supervisory authority should send the data anonymised but with an indication of the company size – undertakings should provide actual data to their national supervisor.

#### Access

The database will be located in CEIOPS premises and only accessible from there to ensure the unity and integrity of the database.

In an initial stage, 2 members of the Secretariat at least will have access to the database. If a task force is put in place to carry out the calibration, members (limited number) of that task force will be granted access to the database.

The persons who will be granted access to the database will sign a dedicated confidentiality agreement.

IT solution to ensure a safe exchange and storage of data

Central databases will be stored in encrypted containers and accessed using On-The-Fly-Encryption technology<sup>42</sup> to ensure that the sensitive content will be never stored in a non-encrypted form<sup>43</sup>.

Passwords allowing access to these encrypted containers will be disclosed to the defined list of people through physical meetings.

The list of passwords used will be stored in the CEIOPS physical safe.

Encrypted data will be received and sent from a dedicated CEIOPS e-mail address (<u>qis5db@ceiops.eu</u>), as normal e-mail with the sensitive data included in encrypted attachments created using the same technology<sup>44</sup>.

Passwords allowing access to these encrypted attachments will be disclosed using a different communication channel than the one used to transfer the encrypted data<sup>45</sup>.

Using the FreeOTFE (www.freeotfe.org) implementation of this technology.

This goes a step further than the technology used to centrally store and analyse QIS3 and QIS4 data.

Either the full version, or the "FreeOTFE Explorer" version that doesn't require being granted special IT rights to be used. Both are mutually compatible.

Physical meeting, phone call or SMS for passwords related to encrypted e-mail attachments.

Annex: Circumstances where adjustments to historical data may be acceptable

The following is a non-exhaustive list of situations where it may be acceptable to introduce adjustments to historical data presented on the form of runoff triangles. These adjustments are aimed to ensure, as far as reasonably possible, that any set of data is made internally consistent and comparable. Undertakings are asked to thoroughly justify any adjustments made, having due regard to the objectives of the calibration exercise.

#### a) Reflection of changes in cover (e.g. undertakings may decide to introduce/change/remove an excess in its policies, and the past claims data reflects a different reality in policy covers)

One of the reasons for experiencing heavier or lighter claims in some periods may be due to structural changes in the level or scope of coverage of risks. These may be entity specific (e.g. changes in policy design, excesses, perils covered, etc.) or industry-wide (e.g. changes in the minimum capital insured for certain type of policies, such as motor third party liability or in the minimum set of perils covered). Past data may need to be adjusted, to the extent possible, to make it consistent with the current and future policy and coverage characteristics.

In practice, these features are likely to be difficult to identify on an aggregate basis, especially when they are entity-specific. In some cases, it may also be impossible to adjust past data on a sufficiently reliable basis because of the unavailability of additional data (e.g. increase in the perils covered by the contracts, reduction of the level of excesses).

#### b) Reflection of changes in the reinsurance policies

Where the aim is to estimate figures net of reinsurance, it will be necessary to assess the stability of the reinsurance policies in force during the observation period. Where the retention levels remained broadly equivalent no adjustments are needed. However, where this is not the case, it may be necessary to adjust the past data and make it consistent with the reinsurance policies that will be in force during the projection period. This will require reconstructing the past data as if the current and future reinsurance policies were in force.

For non-proportional reinsurance, this may be a difficult exercise to perform, but the impact of not adjusting data may be significant if there is evidence of occurrence of a material number of past claims that triggered outwards reinsurance contracts that are no longer expected to be covered in future, or would be expected to be captured in future.

#### c) Occurrence of large or exceptional claims

Where claims experience seems to be unusually large for one or a few cohorts, it needs to be investigated if this is caused by a small number of particularly large claims.

If the underlying events fall within the definition of catastrophic loss (as included within CAT risk in the QIS5 Technical Specifications), then the losses would need to be stripped out from the premium risk data (see above – note that no adjustments for catastrophe losses shall be made for the reserve risk data).

Where the claims are large but not catastrophic, data should be left unmodified, but undertakings are asked to identify separately the amount and characteristics of these claims.

It should also be noted that a large claim with a distorting impact on the undertaking's own loss ratio still will be a proper representative observation for the industry-wide calibration of the volatility. Indeed, adjustment of such observations would lead to a smoothed *unjustified* lower volatility at the industry-wide level.

## **Annex 5:Lines of business**

The following table contains the lines of business for non-life and health NLST business as defined in the current draft Level 2 text and as has been sprecified in QIS5:

Abbreviation	Line of Business
HME	Health medical expenses
HIP	Health income protection
HWC	Health workers' compensation
MTPL	Motor vehicle liability
МО	Motor, other classes
MAT	Marine, aviation and transport
PROP	Fire and other damage
LIAB	General liability
CS	Credit and suretyship
LE	Legal expenses
AS	Assistance
MISC	Miscellaneous non-life insurance
NPH	Non-proportional reinsurance – Health
NPC	Non-proportional reinsurance – Casualty (other than health)
NPM	Non-proportional reinsurance – Marine, aviation and transport
NPP	Non-proportional reinsurance – Property

### **Annex 6: Results in details**

- Details of the Pan-european approach: See separated file "EIOPA 11-163-B-Annex 6\_1 Report JWG on NL and Health non-SLT Calibration.pdf"
- Details of the Averaging and Combined approach: See separate file " EIOPA-11-163-C-Annex 6\_2 Report JWG on NL and Health non-SLT Calibration.pdf"

### **Annex 7:Length of data series sensitivity analysis:**

This annex shows the results of the sensitivity analysis that was carried out on the impact of filtering the data base to exclude the data sets at three different levels:

- a. Exclude dataset with less than three years data history
- b. Exclude dataset with less than five years data history
- c. Exclude dataset with less than seven years data history

The table below shows for the Premium risk gross year end concept the impact either in the unbiased sigma and the number of portfolios and undertakings, of excluding of the analysis those data series with less than 5 years of history, in comparison with the same exercise but excluding data series with less than 3 years of history:

			Filte	ring on le	ngth: 3			variation	Filtering on length: 5							
Premium risk	Sigmas(3) Policyholder kappa		Company kappa		Numbers			Sigmas(5)	Sigmas(5) Policyholder kappa		Company	kappa	Numbers			
Gross year end loss concept	Normal	95%	90%	65%	60%	Companies	Sample		Normal	95%	90%	65%	60%	Companies	Sample	
Assistance	9.1%	1.01	1.00	1.06	1.02	53	434	0.1%	9.1%	1.01	1.00	1.06	1.02	41	380	
Credit and suretyship	25.6%	1.04	1.01	1.69	1.28	52	461	-1.2%	24.4%	1.05	1.01	1.77	1.31	44	425	
Income protection	9.4%	1.07	1.02	1.25	1.10	127	1155	-0.1%	9.3%	1.07	1.02	1.26	1.10	100	1029	
Medical expense	6.8%	1.04	1.00	1.36	1.13	194	1755	0.1%	6.8%	1.02	0.98	1.33	1.11	157	1590	
Workers' compensation	11.4%	1.03	1.01	1.30	1.11	39	383	0.1%	11.5%	1.04	1.01	1.41	1.15	29	335	
Legal expenses	7.6%	1.05	1.00	1.64	1.25	75	685		7.6%	1.06	1.00	1.61	1.25	62	626	
General liability	17.3%	1.01	1.01	1.04	1.02	179	1658	-0.1%	17.2%	1.01	1.01	1.06	1.02	145	1499	
Marine, aviation and transport	20.4%	1.01	1.01	1.08	1.02	120	1136	0.1%	20.4%	1.01	1.01	1.09	1.03	99	1040	
Miscellaneous financial loss	15.4%	1.22	1.03	2.50	1.56	80	679	-2.1%	13.3%	1.31	1.06	2.77	1.93	63	596	
Motor other	9.2%	1.07	1.03	1.20	1.08	175	1655	0.1%	9.3%	1.07	1.03	1.19	1.07	146	1521	
Motor third party liability	10.8%	1.06	1.02	1.18	1.08	176	1721	0.1%	10.9%	1.07	1.03	1.20	1.09	145	1583	
Fire and other damage to property	14.8%	1.00	1.00	1.00	1.00	188	1767	-0.3%	14.6%	1.01	1.01	1.02	1.01	156	1626	
Non-proportional casualty reins.	13.8%	1.01	1.01	1.04	1.02	8	68	-0.4%	13.5%	1.01	1.00	1.01	1.01	7	62	
Non-proportional health reins.	12.7%	1.17	1.07	8.59	3.91	4	38		12.7%	1.17	1.07	8.59	3.91	4	38	
Non-prop. marine, aviation and train	25.2%	1.03	1.01	1.83	1.27	9	79		25.2%	1.03	1.01	1.83	1.27	9	79	
Non-proportional property reins.	31.0%	1.00	1.00	1.00	1.00	18	125	3.1%	34.1%	1.00	1.00	1.00	1.00	13	103	
Non-proportional health & casualty	16.4%	1.01	1.00	1.01	1.01	12	111	0.1%	16.5%	1.01	1.01	1.02	1.01	11	106	
Non-proportional Prop. & MAT	32.9%	1.00	1.00	1.00	1.00	27	206	1.2%	34.1%	1.00	1.00	1.00	1.00	22	183	

The column labelled "variation" contains the variation in percentage of the unbiased sigma from one filtering criteria at least 3 years of history to the at least 5 years criteria. Only miscellaneous line of business and non-proportional reinsurance property showed clear sensitivity, although in the opposite sense.

The columns labelled "numbers" represent the number of portfolios (companies) and the number of observations (sample) considered each time which results naturally reduced when the filtering criteria is fixed at a higher level.

For example, for Assistance line of business it can be seen that the number of companies considered in the first run decreases from 53 to 41 (decrease in 12) when replacing the filter to exclude companies with less than 3 years of history by the filter at less than 5 years. It can also be seen that the number of observations (sample) decrease in 54 (from 434 to 380))

The exercise was done also for the normal model, for the same concept of loss comparing the filter with a criteria of less than 7 years with the criteria of less than 5 years reaching similar conclusions. The table below contains the result of this comparison:

			Filte	ring on len	gth: 5			variation	Filtering on length: 7							
Premium risk	Sigmas(5) Policyholder kappa Company kappa			kappa	Numbers			Sigmas(7) Policyholder kappa			Company	kappa	Numbers			
Gross year end loss concept	Normal	95%	90%	65%	60%	Companies	Sample		Normal	95%	90%	65%	60%	Companies	Sample	
Assistance	9.1%	1.01	1.00	1.06	1.02	41	380		9.2%	1.01	1.00	1.07	1.03	28	295	
Credit and suretyship	24.4%	1.05	1.01	1.77	1.31	44	425	-3.5%	20.9%	1.07	1.02	2.00	1.42	35	367	
Income protection	9.3%	1.07	1.02	1.26	1.10	100	1029		9.3%	1.07	1.02	1.26	1.10	83	920	
Medical expense	6.8%	1.02	0.98	1.33	1.11	157	1590	1.3%	8.1%	1.00	0.98	1.18	1.05	99	1194	
Workers' compensation	11.5%	1.04	1.01	1.41	1.15	29	335	-0.2%	11.3%	1.04	1.01	1.43	1.15	27	322	
Legal expenses	7.6%	1.06	1.00	1.61	1.25	62	626	-0.2%	7.4%	1.07	1.00	1.70	1.29	49	542	
General liability	17.2%	1.01	1.01	1.06	1.02	145	1499	-1.1%	16.1%	1.01	1.00	1.12	1.05	113	1290	
Marine, aviation and transport	20.4%	1.01	1.01	1.09	1.03	99	1040	-0.5%	19.9%	1.01	1.01	1.12	1.03	76	889	
Miscellaneous financial loss	13.3%	1.31	1.06	2.77	1.93	63	596	-0.9%	12.3%	1.00	1.09	2.98	2.14	53	530	
Motor other	9.3%	1.07	1.03	1.19	1.07	146	1521		9.3%	1.09	1.04	1.26	1.11	119	1351	
Motor third party liability	10.9%	1.07	1.03	1.20	1.09	145	1583	-0.2%	10.7%	1.07	1.03	1.21	1.09	124	1447	
Fire and other damage to property	14.6%	1.01	1.01	1.02	1.01	156	1626	-0.6%	13.9%	1.02	1.01	1.05	1.02	122	1404	
Non-proportional casualty reins.	13.5%	1.01	1.00	1.01	1.01	7	62	-0.1%	13.4%	1.01	1.00	1.01	1.01	5	50	
Non-proportional health reins.	12.7%	1.17	1.07	8.59	3.91	4	38	1.1%	13.8%	1.44	1.16	5.15	3.58	2	25	
Non-prop. marine, aviation and tra	25.2%	1.03	1.01	1.83	1.27	9	79	0.4%	25.5%	1.02	1.01	1.48	1.16	5	53	
Non-proportional property reins.	34.1%	1.00	1.00	1.00	1.00	13	103	-7.1%	27.0%	1.16	1.04	2.86	2.00	5	52	
Non-proportional health & casualty	16.5%	1.01	1.01	1.02	1.01	11	106	0.5%	17.0%	1.01	1.00	1.01	1.01	9	93	
Non-proportional Prop. & MAT	34.1%	1.00	1.00	1.00	1.00	22	183	-0.1%	34.0%	1.02	1.01	1.33	1.13	11	115	

The tables below show the results for the Gross current estimate loss concept under normal model:

Comparison filtering at 3 years against filtering at 5 years:

Filtering (	vai	riation Filtering on length: 5													
Premium risk	Sigmas	Policyholder kappa Company kappa		Numbers variation			on Sigmas Policyholder kappa			Company	kappa	Numbers			
Gross current estimate loss concept	Normal	95%	90%	65%	60%	Companies	Sample		Normal	95%	90%	65%	60%	Companies	Sample
Assistance	8.9%	1.01	1.00	1.09	1.04	59	480	0.3%	9.2%	1.02	1.00	1.11	1.05	43	410
Credit and suretyship	30.1%	1.01	1.01	1.01	1.01	57	535	-0.3%	29.8%	1.01	1.01	1.01	1.01	50	501
Income protection	7.7%	1.13	1.05	1.44	1.18	192	2144		7.7%	1.13	1.05	1.44	1.18	169	2039
Medical expense	7.5%	1.02	0.99	1.26	1.09	159	1533	0.4%	7.8%	1.00	0.98	1.20	1.06	127	1393
Workers' compensation	21.9%	1.01	1.00	1.13	1.04	41	414	0.2%	22.2%	1.02	1.00	1.24	1.09	28	350
Legal expenses	7.3%	1.10	1.05	1.63	1.26	107	1128		7.3%	1.11	1.05	1.67	1.29	97	1080
General liability	14.4%	1.01	1.01	1.04	1.02	263	2911	-0.2%	14.2%	1.01	1.01	1.06	1.03	223	2725
Marine, aviation and transport	20.9%	1.01	1.01	1.15	1.04	129	1269	-0.3%	20.6%	1.01	1.01	1.18	1.05	105	1157
Miscellaneous financial loss	17.3%	1.03	1.00	1.24	1.09	90	776	-4.0%	13.2%	1.10	1.01	1.69	1.30	70	675
Motor other	9.4%	1.07	1.03	1.20	1.08	245	2668		9.4%	1.07	1.03	1.21	1.08	216	2533
Motor third party liability	10.2%	1.13	1.05	1.37	1.17	251	2927	-0.1%	10.1%	1.14	1.06	1.40	1.18	218	2773
Fire and other damage to property	13.5%	1.00	1.00	1.00	1.00	261	2741	-0.7%	12.8%	1.01	1.01	1.04	1.02	226	2584
Non-proportional casualty reins.	27.7%	1.00	1.00	1.00	1.00	9	108		27.7%	1.00	1.00	1.00	1.00	9	108
Non-proportional health reins.	27.5%	1.00	1.00	1.00	1.00	5	60		27.5%	1.00	1.00	1.00	1.00	5	60
Non-prop. marine, aviation and trai	45.0%	1.00	1.00	1.00	1.00	10	103		45.0%	1.00	1.00	1.00	1.00	10	103
Non-proportional property reins.	38.4%	1.00	1.00	1.00	1.00	19	171	1.4%	39.8%	1.00	1.00	1.00	1.00	15	153
Non-proportional health & casualty	29.5%	1.00	1.00	1.00	1.00	14	172		29.5%	1.00	1.00	1.00	1.00	14	172
Non-proportional Prop. & MAT	43.2%	1.00	1.00	1.00	1.00	29	277	1.9%	45.1%	1.00	1.00	1.00	1.00	25	260

Comparison filtering at 5 years against filtering at 7 years:

Filtering o	vari	ation			Filtering o	on length:	7								
Premium risk	Sigmas	Policyholder kappa		Company kappa		Numbers		variation	Sigmas	Policyholder kappa		Company kappa		Numbers	
Gross current estimate loss concept	Normal	95%	90%	65%	60%	Companies	Sample		Normal	95%	90%	65%	60%	Companies	Sample
Assistance	9.2%	1.02	1.00	1.11	1.05	43	410	-1.0%	8.3%	1.02	1.00	1.12	1.05	34	349
Credit and suretyship	29.8%	1.01	1.01	1.01	1.01	50	501	-3.5%	26.3%	1.01	1.01	1.04	1.02	41	442
Income protection	7.7%	1.13	1.05	1.44	1.18	169	2039	-0.5%	7.2%	1.17	1.06	1.57	1.23	143	1871
Medical expense	7.8%	1.00	0.98	1.20	1.06	127	1393	-0.2%	7.6%	0.99	0.96	1.24	1.06	101	1222
Workers' compensation	22.2%	1.02	1.00	1.24	1.09	28	350	-0.1%	22.0%	1.03	1.00	1.27	1.09	26	337
Legal expenses	7.3%	1.11	1.05	1.67	1.29	97	1080	-0.5%	6.8%	1.16	1.08	2.00	1.43	82	984
General liability	14.2%	1.01	1.01	1.06	1.03	223	2725	-0.3%	13.8%	1.01	1.01	1.09	1.04	194	2535
Marine, aviation and transport	20.6%	1.01	1.01	1.18	1.05	105	1157	-1.3%	19.2%	1.02	1.01	1.39	1.12	85	1024
Miscellaneous financial loss	13.2%	1.10	1.01	1.69	1.30	70	675	-1.6%	11.6%	1.15	1.02	1.95	1.41	57	588
Motor other	9.4%	1.07	1.03	1.21	1.08	216	2533		9.4%	1.09	1.04	1.26	1.10	188	2357
Motor third party liability	10.1%	1.14	1.06	1.40	1.18	218	2773	-0.1%	10.0%	1.15	1.06	1.41	1.18	197	2639
Fire and other damage to property	12.8%	1.01	1.01	1.04	1.02	226	2584	-0.1%	12.7%	1.02	1.01	1.06	1.03	195	2386
Non-proportional casualty reins.	27.7%	1.00	1.00	1.00	1.00	9	108		27.7%	1.00	1.00	1.00	1.00	9	108
Non-proportional health reins.	27.5%	1.00	1.00	1.00	1.00	5	60	5.2%	32.7%	1.00	1.00	1.00	1.00	4	56
Non-prop. marine, aviation and trai	45.0%	1.00	1.00	1.00	1.00	10	103	4.7%	49.7%	1.00	1.00	1.00	1.00	6	80
Non-proportional property reins.	39.8%	1.00	1.00	1.00	1.00	15	153	-8.3%	31.5%	1.11	1.02	2.64	1.83	10	121
Non-proportional health & casualty	29.5%	1.00	1.00	1.00	1.00	14	172	1.2%	30.8%	1.00	1.00	1.00	1.00	13	166
Non-proportional Prop. & MAT	45.1%	1.00	1.00	1.00	1.00	25	260	0.8%	45.9%	1.01	1.01	1.06	1.02	16	204

It should also be noted that the kappa parameters are affected in both directions (increase – decrease) as a consequence of the different filtering processes. The conclusion of this analysis is contained in subsection 4.4 of the report.