Stochastic modelling of catastrophe risks in DFA models

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Abstract

Negative developments on the capital markets at the beginning of the millennium along with the increase in natural catastrophes and terrorist attacks have substantially altered the risk situation of the insurance industry. Insurance companies have reacted to the altered prevailing conditions with a paradigm shift in corporate strategy developing from classical turnover orientation to *value- and risk-based management*. Companies will only be able to assess the level of risk capital and moreover the complete distribution of results according to corporate risk structure with the help of high-quality internal models – DFA models – matched as closely as possible to the risk situation of the individual company. Measuring and evaluating catastrophe risk has come to be a very important issue, as a substantial share of the company's entire risk capital is committed to natural catastrophes. Whether or not internal models can be applied depends largely on adequate catastrophe risk modelling. The following study aims to present two actual approaches in modelling loss due to natural catastrophes taking storms as an example. Both models use results from natural risks models. The first method is based on processing complete event loss tables, while the second mathematical statistical approach uses information from certain return periods. Both methods will be compared using example data, and their advantages and disadvantages will be pointed out as applicable to value- and risk-based management. Finally, the study will calculate risk capital, and test the impact of strategies on risk capital requirement.

Keywords: Internal models, catastrophe risk, storm risk, natural risk models, event loss tables, mathematical statistical models, risk capital, value- and risk-based management

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Contents

1 Introduction

Negative developments on the capital markets at the beginning of the millennium along with the increase in natural catastrophes and terrorist attacks have substantially altered the risk situation facing the insurance industry. As an example, total corporate capital resources decreased by around between 25% from 2000 to 2002 in non-life insurance and reinsurance across the world according to SWISS RE.[1] Insurance companies have reacted to the altered prevailing conditions with a paradigm shift in corporate strategy developing from classical turnover orientation to *value- and risk-based management* in economic terms.

This involves measuring success at reaching a risk adjusted return on the risk capital provided by the investors. The risk capital should be derived from the actual risks facing the company. Companies will only be able to assess the level of risk capital and the complete distribution of results according to their individual risk structure with the help of high-quality DFA models[2] matched as closely as possible to the risk situation they aim to represent, thus addressing issues as to risk-bearing ability and profitability of the company as a whole as well as in different lines of business.

Adequately assessing the risk situation of a company involves appropriately representing the individual risks that the company is exposed to in DFA models. Modelling catastrophe events plays a major role in this matter, as natural catastrophes often involve considerable loss potential that the company's risk management must take into account sufficiently. A considerable share of risk capital is often committed to insurance divisions affected by catastrophe events, which is why risks of natural catastrophes have a major impact on selecting a suitable reinsurance policy. This should lead to intensive discussion with reinsurance departments on the adequate level of reinsurance protection with regard to risk and return. The matter is made worse by the brief experience in catastrophe events – the small number of observations in the history – amongst most insurance companies, while long return periods such as a hundred, five hundred, thousand or ten thousand-year events pose a great challenge to adequate modelling for most companies.

Catastrophe claims refer to loss caused by any single event affecting a large number of insured policies within the same time frame. The following natural catastrophes play an especially important role:

- Storms,
- Earthquakes,
- Hailstorms,
- Floods.

Apart from the losses resulting form natural catastrophes, accumulative loss can be caused by terrorist attacks. Classical examples for accumulative loss in other divisions are general accidents and fires with unknown accumulation which should also be modelled.

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[1] See [Swiss Re 2002].

[2] Interested readers will find an actual proposal for the development of a stochastic internal model useful as a basis for value- and risk-based management in [Diers 2007a]. The necessary steps from initial concept to complete preparation and implementation are presented here. The individual modelling approaches are shown with reference to data from a model company.

This study aims to provide a quantitative analysis for two different approaches in catastrophe modelling taking storms as an example, with reference to model data.[3]

2 Modelling catastrophe risk in DFA models

DFA models are aimed at covering the broadest possible range of all possible profit and loss accounts in the next year, or years in models covering several years, in order to represent the risk situation of the company adequately and determine the indicators relevant to corporate strategy, such as expected company results and risk capital. However, insurance-related risks in non-life insurance are subject to serious deviation resulting from the high level of volatility by both claim severity and frequency.[4] As an example, storms may lead to extremely high total claims due to an enormous number of more minor claims. This is why both claim and capital market development should be stochastically modelled. To this end, a DFA model should be understood as a simulation model. Analytical models are unsuitable for non-life insurance, as the total results distribution can only be determined by very restrictive assumptions.[5]

Two main aspects need to be considered modelling catastrophe claims. On the one hand, the basic assumptions of the collective model – which are usually valid in the case of non-catastrophe modelling concerning attritional and large claims – usually fail regarding the independence of claim sizes and claim number.[6] An example which can be given here is flood loss, where the number of claims and severity of each claim rise with flood water level.

Moreover, the natural catastrophe may affect different insurance divisions at the same time. In catastrophe modelling, diversification only ever applies in risks placed far apart. So modelling the adequate dependencies amongst the losses of the different divisions which often have a non-linear structure is a very difficult problem.

This means that natural catastrophes should be regarded in terms of events rather than individual claims. The assumptions placed by the collective model may be considered as satisfied, as the frequency of events and event claim sizes can be assumed to be independent of one another.[7] The loss should then be distributed amongst the different lines of business affected. This division may be based on historical experience or according to degree of exposure (number of risks affected by the event as a percentage of the number of risks insured) on the current portfolio. Beyond that, modelling theory does not always require deterministic division according to a fixed key. Rather, division factors can also be stochastic (such as depending on the level of loss arising from the event).

On the other hand, catastrophe modelling should take account of the possibility that far more serious events may occur in the future compared to those observed to date. This is why companies refer to exposure analyses for modelling types of event loss in which they

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[3] The remaining storm losses may be modelled as attritional losses that are far less volatile.
[4] Claim severity and claim frequency refer to ultimate loss.
[5] See [Diers 2007a].
[6] See [Mack 2002].
[7] See [Mack 2002].

have no previous experience. As an example, what are referred to as event sets[8] are required as outputs for natural risks models from external suppliers along with a wealth of existing data such as exact descriptions of risk locations, ideally in address form; risk type, whether private, commercial or industrial; and insurance terms such as deductibles, limits, or coinsurance policies. The insurance company is provided with information on return periods and the associated PMLs[9] or complete event loss tables.[10] As a rule such natural risks models are developed by external suppliers.

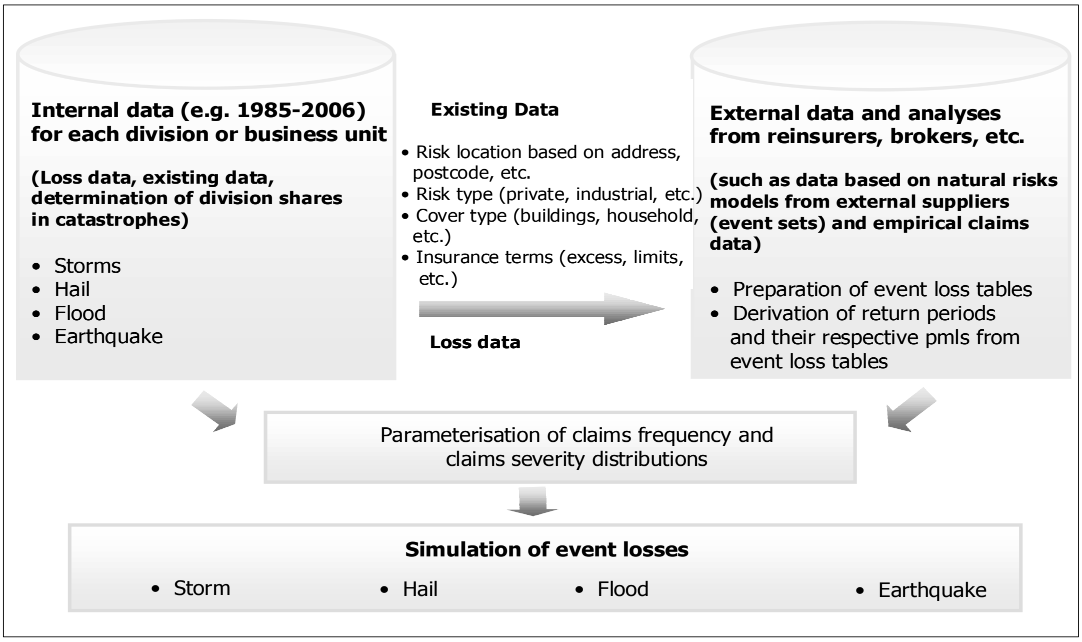

Figure 1: Catastrophe modelling

The information must be tested for plausibility with the help of empirical claims data. The next step is to fit the underlying distributions of catastrophe claim frequency and claim severity for the events.[11] This provides a basis for simulating event loss. Figure 1 presents the general approach in modelling catastrophe claims.

External data can be used in a number of ways. On the one hand, complete event loss tables – the results of natural risks models – can be used for modelling catastrophe claims. On the other hand, statistical models can be used as by only including certain outputs from natural risks models, that is, those with long return periods including their respective PMLs (in addition to adjusted empirical claims data). Both approaches will be presented in the following sections and compared using sample data. In the following we restrict to model catastrophe losses which result from catastrophe events. The attritional and large claims which also play a role in lines of business affected by storm risks are not considered here.[12]

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8 See Section 3.
9 PML: probable maximum loss
10 See Section 3.
11 See Sections 3 and 4.
12 See [Diers 2007a].

3 Catastrophe risk modelling based on natural risks models

As previously described, various approaches exist for modelling catastrophe claims in DFA models. This section will present the use of complete event loss tables generated as outputs of natural risks models. The advantages and disadvantages of this method in the context of value- and risk-based management are addressed at the end of section 4. A variety of suppliers provide models of this type.

These geophysical meteorological models are based on representing the physical forces causing the loss and their effects on insurance business.[13] The effects of climate changes can also be adequately applied. These models rest upon assessing many physical influences, such as wind speed, geographic alignment of storm zones, wind fields etc., with the aim of adequately representing all possible events. Natural risks models result in *event sets*, which serve as a numerical representation of the events. The number of event sets varies according to each individual supplier. Event sets can be used to calculate local intensity parameters, that is, numerical descriptions of local effects for any event. The next step is to calculate loss degree curves for each scenario of the event set as applicable to the portfolio of the insurance company.[14] These vary according to each risk type such as building, household, extended coverage, industrial storm insurance, etc., and are heavily dependent on factors such as building type. These calculations are mostly performed by reinsurers and brokers, but they can also be prepared by insurance companies themselves. The calculations take account of detailed existing information such as risk location (such as address[15] or postcode), risk types and insurance terms and conditions as well as insurance limits. Outputs from natural risks models are usually PML curves and return periods that the reinsurers use in calculating the possible recoveries for their premium calculations.

Event number	Event frequency	Event mean severity	Standard deviation	Exposure value
...	...	...	...	...
17.980	0,00000221	38.356.270	27.022.031	9.210.798.292
17.295	0,00001687	38.167.747	26.977.425	7.894.969.965
17.853	0,00001646	37.025.203	26.350.968	8.913.675.766
17.368	0,00000392	36.776.847	26.281.579	8.870.752.769
18.001	0,00001261	36.227.882	25.276.448	8.127.174.963
17.463	0,00001151	35.988.900	25.456.216	9.059.801.599
17.891	0,00001650	35.791.078	25.319.642	9.224.327.305
17.851	0,00000524	35.291.528	25.137.023	12.560.179.489
17.982	0,00000184	35.231.846	24.804.156	9.661.676.530
17.406	0,00003356	35.007.636	17.228.653	8.840.103.294
17.985	0,00000046	34.891.374	24.596.462	9.641.786.132
18.004	0,00001485	34.859.180	24.256.934	7.675.665.243
17.893	0,00001171	34.752.674	24.663.413	12.470.617.780
18.006	0,00001539	34.630.376	24.146.792	9.528.412.026
17.462	0,00000430	34.405.417	24.352.597	7.988.781.079
17.645	0,00004261	34.335.089	7.222.209	10.633.160.763
17.975	0,00000079	34.305.860	24.103.868	7.894.969.965
17.386	0,00000113	34.255.982	24.262.113	12.799.293.564
17.887	0,00000695	34.000.040	23.842.394	9.339.685.958
17.984	0,00000346	33.574.772	23.679.793	10.392.415.990
17.981	0,00000042	33.151.588	23.267.874	7.859.333.501
17.983	0,00000015	32.930.112	23.175.486	8.747.473.765
...	...	...	...	...

All data in €.

Figure 2: Part of an event loss table for storm insurance divisions

[13] See [Pfeifer 2000].

[14] The loss degree curves meant here represent an analytical connection between the catastrophe event and the loss arising from it.

[15] If the portfolio data is available in address form, the portfolio can be coded – the risks can be matched to exact geographic coordinates, representing a significant advantage to the postcode approach.

Event loss tables (ELT) are another output from these geophysical meteorological models. They are synthetic catalogues of modelled event loss that refer to a specific risk, portfolio and the different lines of business with storm exposure. An event loss table consists of a variety of scenarios or events.[16] Figure 2 represents a selection of the output from an ELT for lines of business affected by storm using sample data.

The event number is set by the supplier to identify the event. The frequency parameters represent the mean frequency of storm events. The event mean severity[17] represents the average claim size of that particular insurance company's portfolio from this event. In addition the associated standard deviation is calculated. The exposure value refers to the amount by which the company is exposed to that particular risk – that is, the insurance total exposure, which therefore represents the maximum possible loss for each event. Return periods of event loss – the expected length of time between recurrences of two natural catastrophe events – and return periods of annual loss – defined using annual loss exceeding probabilities – can be derived from the ELTs. The results from various models vary widely in practice. In individual cases, companies should conduct specialised adaptation tests on their own portfolio.[18]

The ELT can be used for event modelling in DFA models to be discussed in the following with reference to storm catastrophes. Let n denote the number of ELT events.[19] In the geophysical simulation model every single scenario i, $1 \leq i \leq n$, constitutes a collective model. The individual claim sizes Z_{i_j}, $j \in IN$, of each scenario i are assumed to follow the same distribution as Z_i. All random variables (claim sizes and frequencies) are assumed to be independent. Now we want to use this information for event modelling in our internal simulation model (DFA model).

The degrees of loss X_{i_j} – individual claim severity Z_{i_j} due to the event divided by exposure value max_i – follow the same distribution as $X_i = \frac{Z_i}{\max_i}$, which in our model is assumed to be a Beta-distribution. So for each event i, $1 \leq i \leq n$, a Beta-distribution is fitted using moment fit, where event mean severity m_i and standard deviation σ_i are estimated using the corresponding entries of the ELT.[20] The expected value and standard deviation in random variables X_i for the degrees of loss is calculated according to the following equation:

$$E(X_i) = \frac{m_i}{\max_i} \quad \text{and} \quad \sqrt{Var(X_i)} = \frac{\sigma_i}{\max_i}.$$

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[16] The number of events depends on supplier and risk.
[17] Refers to the ultimate loss.
[18] An adjustment test of this type is described in Section 4 (Figure 5).
[19] The number of ELT events represents the number of scenarios or entries.
[20] Note that this causes a parameter risk. Moreover there exists a model risk. If these two kinds of risks are already taken into consideration in the geophysical models has to be clarified with the supplier. If this is not the case, they additionally have to be modelled. The modelling of theses risks exceeds the purpose of this paper.

The Beta-distribution $Beta(\alpha_i, \beta_i)$ with positive real parameters α_i und β_i has the following density:

$$f(x) = \frac{\Gamma(\alpha_i + \beta_i)}{\Gamma(\alpha_i)\Gamma(\beta_i)} (1-x)^{\beta_i - 1} x^{\alpha_i - 1}, \qquad 0 \le x \le 1,$$

where $\Gamma(y) = \int_0^\infty t^{y-1} e^{-t} dt$.

The expected value and variance of $Beta(\alpha_i, \beta_i)$-distributed random variable X_i possess the following theoretical representation:

(*)
$$E(X_i) = \frac{\alpha_i}{\alpha_i + \beta_i} \qquad \text{and} \qquad Var(X_i) = \frac{\alpha_i \beta_i}{(\alpha_i + \beta_i)^2 (\alpha_i + \beta_i + 1)}.$$

Parameters α_i and β_i in the Beta-distribution for degree of loss X_i result from the following:

$$\alpha_i = \left[\frac{E(X_i)(1 - E(X_i))}{Var(X_i)} - 1 \right] E(X_i) \qquad \text{and} \qquad \beta_i = \left[\frac{E(X_i)(1 - E(X_i))}{Var(X_i)} - 1 \right] (1 - E(X_i)).$$

So we have specified the distribution of X_i for each event *i* of the ELT.

If *MAX_Storm* refers to the possible maximum loss in the insurance portfolio caused by one single storm event, we obtain the random variable Y_i of claim severity from the following equation:

(**)
$$Y_i = MINIMUM(MAX\_Storm;\ Z_i),$$

where $Z_i = max_i \cdot X_i$ and $X_i \sim Beta(\alpha_i, \beta_i)$.

One can show that under the assumption that the frequencies N_i, $1 \le i \le n$, follow a Poisson distribution with parameter λ_i the several independent collective models of the single scenarios lead to another equivalent collective model with Poisson frequency with parameter λ.[21]

So the sum of frequency parameters λ_i, $1 \le i \le n$, can be used to calculate the mean λ of the annual event frequency:

$$\lambda = \sum_{i=1}^{n} \lambda_i .$$

[21] The independent and identically distributed claim sizes of the equivalent model follow a mixture of the given claim severity distributions. See [Pfeifer 2004a], [Straßburger 2006] and [Hipp / Michel 1990].

We have selected the Poisson-distribution as the distribution for the annual event frequency N with parameter λ. The frequency parameters λ_i can be estimated using the corresponding entries of the ELT.

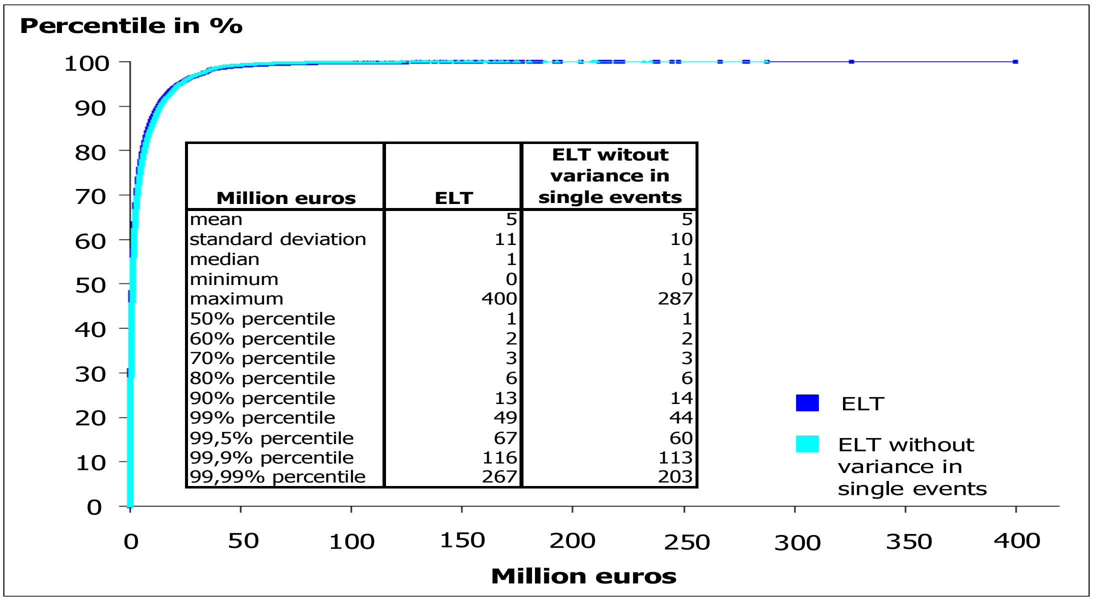
The following approach can be taken in the DFA model for each simulation:[22] first, we take a sample $\lambda^{'}$ of the Poisson(λ)-distribution, that is, $\lambda^{'}$ storm events are to be realised in the simulation in question. The next step will be to take exactly $\lambda^{'}$ event numbers with regard to the frequencies λ_i in the ELT (sampling with replacement). Let M refer to the set of these sampled event numbers i_j, $i \in \{1, ..., n\}$, $j \in IN$, with $|M| = \lambda^{'}$. We need the second index j because according to the frequency one event number can occur twice or more in the set. For each i_j we take independent samples y_{i_j} for the severity of event loss from random variable Y_i (as defined above).

The simulated annual storm loss a is calculated from $\lambda^{'}$ events in this simulation according to the following equation:

$$a = \sum_{i_j \in M} y_{ij} .$$

So using simulation techniques we create 100.000 or more random observations (the number of simulations depends on the parameter situation) from the underlying model to determine the empirical distribution of annual storm loss A.

Figure 3 shows the distribution of annual storm loss created with the help of Monte Carlo simulation.



Million euros	ELT	ELT witout variance in single events
mean	5	5
standard deviation	11	10
median	1	1
minimum	0	0
maximum	400	287
50% percentile	1	1
60% percentile	2	2
70% percentile	3	3
80% percentile	6	6
90% percentile	13	14
99% percentile	49	44
99,5% percentile	67	60
99,9% percentile	116	113
99,99% percentile	267	203

Figure 3: Annual loss distribution with and without considering variance in single events

[22] For example using Monte Carlo Simulation

Variability in event severity should still be modelled as can be seen form figure 3. No variance was assumed in the individual event severity on the ELT, eliminating the Beta-distribution described in the above model. So in (**) we replace the random variable Z_i by m_i. The table in Figure 3 shows that no large deviations can be deduced in the lower and medium percentiles, whereas remarkable deviations can be recognised in the very high percentiles (see maxima etc.), which may have a substantial impact on determining risk capital requirement.[23]

If several lines of business (divisions) are to be modelled, ELTs can be generated for the individual lines of business if there is enough data. There are two options for modelling several lines of business using ELTs. In the first option, the entries of the division ELTs can be aggregated per event number in the first step while the standard deviation can be calculated assuming suitable correlation. The second step is to simulate the total event, which can then be divided amongst the lines of business affected according to the share of the division in the total expected value of the event severity in the third step. In the second option, each line of business is modelled separately and finally aggregated by event number. Here, suitable dependencies (which have often a non-linear structure) between the random variables of event severity need to be selected. We have used both of these methods in modelling building storm insurance and industrial storm insurance. These two methods yielded almost identical results.

4 Mathematical statistical approaches

In the mathematical statistical model the event frequency is assumed to be independent from the claim sizes caused by event which are assumed to be independent and identically distributed. So the assumptions of the collective model are fulfilled. The event severity and frequency distributions are fitted with the help of the company's own historical data (observations), adding PMLs from events with long return periods with losses that the company has not yet experienced. This modelling approach uses only return periods and the associated PMLs for the insurance company itself, rather than complete ELTs, as output from natural risks models.

The empirical claims data of the company should originate from a time period reaching back as long as possible; the event loss should be extrapolated with suitable indexes – such as the building cost index – to future years. Changes in portfolio – such as the introduction of deductibles – should also be adequately taken into account. Average claim sizes and degrees of exposure should be taken into consideration in order to ensure that the scale of portfolio remains independent from the number of the insured policies.[24] Historical claims experience will usually be insufficient for the choice of an appropriate model since very long return periods (such as 100, 200, 250, 500, 1,000, 10,000-year events, etc.) also need to be included. These seldom events determine the tail part of the loss distribution, thus playing a major role in determining the risk capital requirement of the company. Only using historical data in order to fit the underlying distribution would

[23] See also Figure 6.
[24] Natural risks models may also be referred to in fitting empirical (historical) in-house data to the current portfolio. These represent storms from the past and evaluate them taking the current portfolio into account.

generally only provide an insufficient account of possible losses in the future, leading to a serious underestimation in the risk capital requirement of the company.

Empirical claims data is thus enriched with the PMLs from long return periods from the natural risks models in order to fit the underlying severity distribution. The external data serves to adapt the tail part of the catastrophe loss distribution that cannot be fitted to a satisfactory extent by historical data, by adding claims with long return periods. This means that several distributions are fitted according to the maximum-likelihood method (for estimating the parameters).[25] Statistical goodness-of-fit tests can then be used to select the best-fitting model from these models.[26]

Statistical tests can also show whether the external dates are appropriate in relationship to historical in-house data. So it can be evaluated if the PMLs are adequate or too high or too low for the portfolio. This addresses the problems arising from the wide discrepancy between many results from various external studies for the same risk and portfolio. The validation of the model is a very important but difficult process because it requires the comparison of losses from particular storm events with the losses that the model would estimate for occurrences with the same physical characteristics, given the same geographical distributions of exposed properties. These data are often unavailable or not available in the quantity necessary for statistical testing.[27]

The loss distributions calculated as a base for current portfolio structure should then be subject to continuous review and immediately adjusted for changing conditions.

The approach to catastrophe modelling based on empirical data is often criticised due to the lack of basis in historical loss development in estimating seldom return periods. However, this objection also applies to geophysical models, as the parameters they use are also derived from historical data. As Pohlhausen commented, extrapolating the future from the past is not unproblematic. However, it is a sensible activity. There is no other possibility for addressing future uncertainty.[28]

The catastrophe-modelling approach using mathematical statistical models as described here will be presented, again taking storm events as an example, in the following. The modelling proposal presented here requires the following definitions:

- *NumR*: number of risks insured,
- *DE*: random variable for the degree of exposure per event (number of risks affected by the event as a percentage of the number of risks insured),

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[25] This causes a parameter risk which has additionally to be modelled. An example for modelling the parameter risk in internal models is given in [Diers 2007c].

[26] Examples of mainly quantitative test methods include the χ^2-test, Kolmogorov-Smirnov test and Anderson-Darling test. Apart from the quantitative test methods, more qualitative or intuitive methods such as the mean-excess plot, Hill plot, P-P plot and Q-Q plot may be used.

[27] See [Clark 2002]. Clark states further that "The nature of statistics is such that one can never prove that the sample is a true representation of the population. Statistical tests of significance merely provide confidence intervals for parameter estimates which are based on certain assumptions. These tests are used to choose between alternatives or competing hypotheses."

[28] See [Pohlhausen 1999].

- *MAX_DE:* maximum degree of exposure = 100%,[29]
- *AC*: random variable for the average claim severity from one event, that is, the claim severity from one event divided by the number of risks affected by this event,
- λ: expected value for the number of events in the year to be modelled,
- *MAX_Storm*: maximum loss that can be caused by a storm event.

The random variable of claim severity *CS* is calculated as follows:

$$CS = Accept\,(NumR \cdot DE \cdot AC;\, NumR \cdot DE \cdot AC \leq MAX\_Storm),$$

with distribution function *F* for random variable *DE*:

$$F_{DE}(x) = \frac{1}{F_X(MAX\_DE)} F_X(\mathrm{x}), \text{ for } x < MAX\_DE, \qquad F_{DE}(x) = 1 \text{ otherwise.}$$

The *Accept*-function causes each case where the claim severity simulated exceeds *MAX_Storm* to be simulated again until all of the results fall below *MAX_Storm*.[30] We can fit the underlying distributions for random variables *X*, which represents the degree of exposure per event before maximum,[31] and *AC* by suitable matching between the in-house and external data, which represent the observations, according to the statistical approach described above.

The next step is to model annual loss due to storm events. Following the collective model the random variable of annual storm loss *A* can be represented as the sum of the independent and identically distributed claim severities CS_i that follow the same distribution as CS and are assumed to be independent of random variable of event frequency *N*:[32]

$$A = \sum_{i=1}^{N} CS_i \ .$$

We assume that *N* follows a Poisson-distribution with parameter λ, which can be estimated from historical data. Using Monte Carlo simulations a large number of random observations (e.g. 100.000) can be simulated from the model in order to create the empirical distribution of the annual storm loss.

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[29] The maximum degree of loss can be less than 100% depending on the portfolio.
[30] Using the *Accept*-function represents the possibility of capping; however, this only applies in cases where only a few simulations are lying above the condition. If this is not the case, the selected model should be reviewed for validity. An alternative approach in capping is the minimum function *CS = MINIMUM*(*NumR* $\cdot$ *DE* $\cdot$ *AC*; *MAX_Storm*). The two approaches lead to different results, however.
[31] Before maximum means that the degree of exposure per event is limited by *MAX_DE*, which is omitted until this point.
[32] According to the assumptions of the collective model degrees of exposure and average claim sizes are assumed to be independent and identically distributed as *DE* and *AC* respectively. If these assumptions hold in practice has to be verified. We use this modelling approach in order to be able to use this model for strategic decisions.

This modelling approach using random variables for exposure degree and average claim severity is suitable for strategic value- and risk-based management, such as in quantifying the effects of portfolio expansion[33] or reduction on the risk and return situation of a company where *NumR* and *MAX_Storm* can be adjusted as appropriate. Here, it is absolutely necessary to review how this model fits historically recorded annual loss due to storms as well as external PMLs provided for annual loss. Additionally one can directly fit the underlying event severity distribution *CS* using the statistical methods described above and compare the results for validation.

The final decision as to whether the distribution assumptions are adequate choices of loss distributions, along with a final judgement as to whether the Poisson or the Negative Binomial distribution – as example – is the adequate choice of the distribution of claim frequencies, should not be made until this point.[34]

Figure 4b shows the annual storm loss distribution calculated according to this method, and compares the results with the empirical distribution (original data: empirical and external) for the storm insurance divisions modelled here (Figures 4a, 4b).

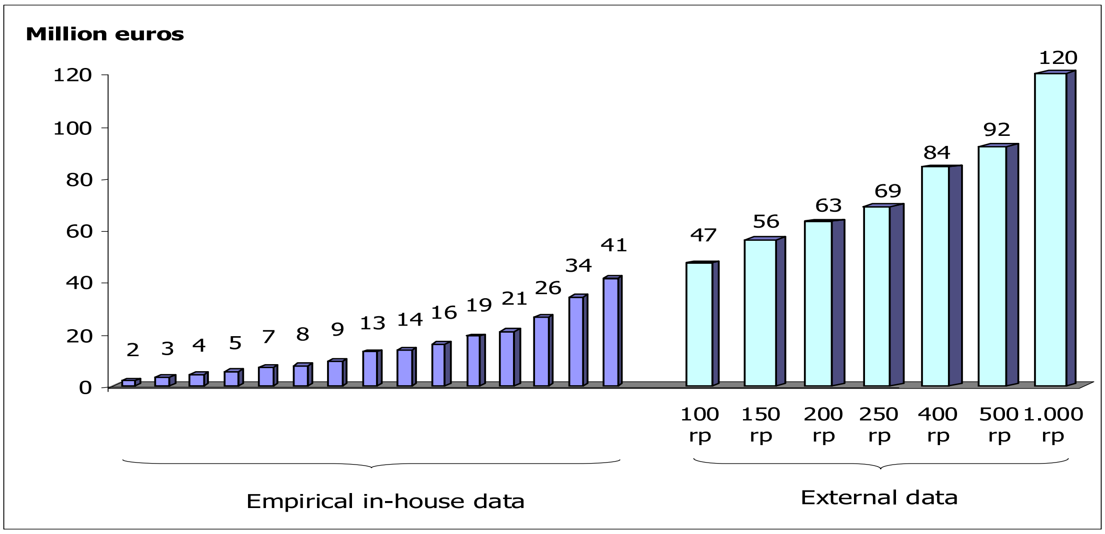

Figure 4a: Empirical in-house data for annual storm claims and external data for long return periods (rp)

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[33] This is conditional on the expansion in portfolio with similar average loss severity and degrees of exposure as in the existing portfolio.
[34] Refer to [Rosemeyer / Klawa 2006], who studied the number of storms in Germany from 1970 to 1997 and identified the Negative Binomial distribution as the more valid distribution.

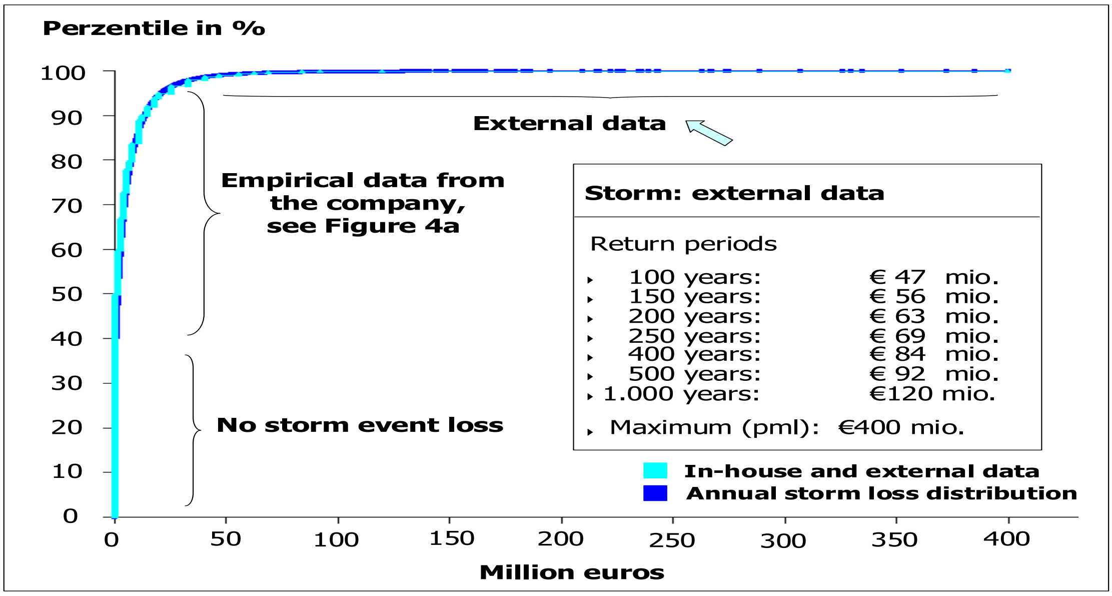
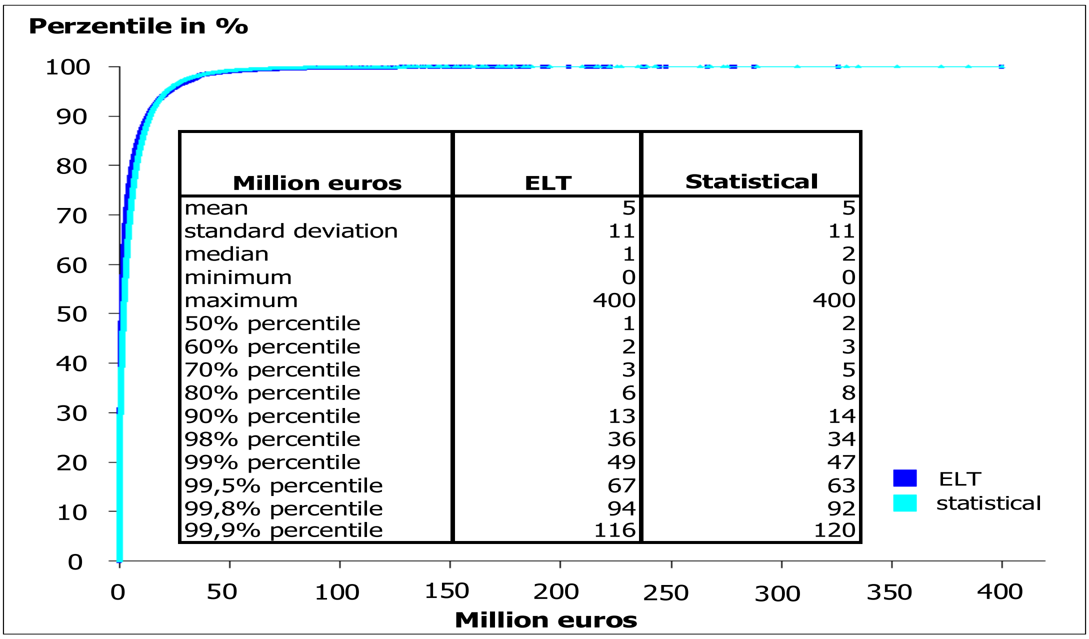


Figure 4b: Annual storm loss distribution due to the underlying model vs. empirical distribution (in-house and external data)

Figure 5 shows the distribution of annual storm loss according to the mathematical statistical model (grey) and according to natural-risk models (black) from Section 3 for comparison. Both graphs have a very similar curve, which means that the ELTs used fit well to observations for the lower percentile ranges. The agreement in the upper percentile range (99% percentile and above) results from our use of long return periods derived from the ELTs (as observations) in the mathematical statistical distribution fit. It also shows that the long return periods match our historical experience. In practice as described above, various sources are available to insurance companies for long return periods whose results often widely fluctuate. Comparison with in-house data usually helps solve this fluctuation.



Million euros	ELT	Statistical
mean	5	5
standard deviation	11	11
median	1	2
minimum	0	0
maximum	400	400
50% percentile	1	2
60% percentile	2	3
70% percentile	3	5
80% percentile	6	8
90% percentile	13	14
98% percentile	36	34
99% percentile	49	47
99,5% percentile	67	63
99,8% percentile	94	92
99,9% percentile	116	120

Figure 5: Annual storm loss distribution according to the mathematical statistical model (grey) vs. modelling based on natural-risk models (black)

Modelling certain reinsurance contracts such as *frequency cover* requires a probability distribution for the number of claims arising from storm events.

The random variable *number of claims per event – NE_i* – can be calculated as follows:

$$NE_i = NumR \cdot DE_i.$$

The modelling approach presented here is not based on different models for the different lines of business, as is usual in non-catastrophe claims modelling.[35] So the event losses have to be distributed amongst the lines of business affected. Distribution may be applied taking fixed keys, which can be derived from the company's records. Individual portfolio structure should be taken into account. If, for example, the company's records or the use of natural risks models reveals that major storms have a greater impact on a specific division (such as industrial storm insurance due to the higher PMLs) than on other divisions (such as building or household) in comparison to minor events, this effect should be given consideration.[36] In these cases, the percentage key should not be fixed but should be applied dynamically, depending on the claim severity of the event. Modelling catastrophe claims results in functional dependencies between the divisions affected.[37]

Another aspect that needs to be considered when modelling event loss is the extension of simulation data with information on the time that the event takes place. This information is necessary for adequately calculating cash-flows. An event that occurs early in the year will mostly be settled in the same year, leading to a different cash-flow situation that would arise for events occurring at the end of the year where the most part of the settlement will be paid in the following year. Therefore, it is wise to simulate an indicator as to whether the event occurs for example in the first or second six months of the year.

So the probability p of a storm event taking place in the first six months is to be calculated on the basis of in-house data to be enriched with external information. This yields the following weighting vector:

$$Weight\ (first\ six\ months;\ second\ six\ months) = (p;\ 1\text{-}p)$$

This will facilitate simulating the occurrence of an event in the first or second six months for each event.

Since catastrophe claims have a major impact on corporate strategy due to the high level of risk involved, they must be adequately modelled in DFA models. On the one hand modelling based on ELTs from natural risks models can be validated using statistical modelling to review PMLs for long return periods. On the other hand the validity of ELTs for the shorter return periods can be checked with reference to empirical in-house data.

[35] See for example [Diers 2007a] for attritional and large claims modelling in internal models.
[36] This assumption must be justified (for example, by internal records).
[37] Additionally, the dependencies between the individual catastrophe risks (storms, earthquakes, hailstorms, floods, etc.) must be represented. Modelling dependencies (which have often non-linear structures) in internal company models plays an important role in modelling loss. See for example [Diers 2007a]. For the copula approach see [Pfeifer / Neslehova 2004].

The following presents the advantages and disadvantages of methods described in Sections 3 and 4.

- Modelling based on mathematical statistical methods encompass all of the historical events, which are used to fit the underlying distributions. This aids plausibility testing on high PMLs, as these have to fit the historical losses. This results in a good basis for management acceptance of the model, as the results are highly intuitive (see Figure 4b).
- Therefore, mathematical statistical models can and should be used to test the plausibility of ELTs as well.
- Statistical models present another advantage in that the model is based on modelling the random variables degree of loss and average claim severity, enabling quantification of the exposure of corporate strategy such as expansion in individual divisions, withdrawal from various agreements or universal introduction of deductibles, on the risk capital requirement of the entire company. This is not directly possible using ELTs as a basis, as ELTs only reflect the exposure of individual catastrophe events on the current portfolio, meaning that the corporate strategies and general strategies affecting gross business (before reinsurance) cannot be directly represented.
- However, modelling based on ELTs presents the distinct advantage of including actual events from the natural risks model into the model. Although either approach will lead to a very similar curve in the annual loss distribution in our example (see Figure 5), the number of events and loss severity may vary substantially. This is unimportant at gross level for corporate strategy and risk capital determination, as only the annual loss is important here. However, there may be major differences in annual loss distributions after reinsurance if excess-of-loss agreements are considered by event (event XLs), whose impact depends on the nature of the individual event. Mathematical statistical modelling may be not appropriate for reinsurance calculation or optimisation. So both models should be applied as necessary depending on the actual issue concerned in corporate strategy. This is possible without reservation if the results of the different models are as similar as in this example (see Figure 5).

5 Risk capital calculation and outlook

The loss resulting from catastrophe events can take on a very large scale, and therefore tie a significant share of the entire risk capital of a company; this is why the loss should be modelled to a great degree of accuracy.

The following will explain the effect of the different modelling approaches as described here on risk capital requirement. We have taken value-at-risk *VaR* and tail-value-at-risk *TVaR* as risk measure at a confidence level of $1-\alpha = 0.998$. Both risk measures are often used in practice. The tail-value-at-risk for a real random variable L is defined as follows:

$$TVaR_{\alpha}(L) = \mathrm{E}[L \mid L \geq VaR_{\alpha}(L)].$$

TVaR is defined as the expected loss of $\alpha \cdot 100\%$ worst cases, $\alpha \in (0.1)$.[38] The value-at-risk is defined as

$$VaR_\alpha(L) := \inf\left[x \in IR : F_L(x) \geq 1-\alpha\right],$$

where F_L denotes the distribution function of the loss *L*.

In the following let *A* denote the random variable of the annual storm loss. We define risk capital using the *TVaR* and *VaR* as risk measures with confidence level 99,8% and the following random variable *L* (loss):

$$L = A - E(A),$$

where *E*(*A*) denotes the expected value.

Figure 6 shows that the risk capital requirements based on *TVaR* lie much beyond those based on *VaR* because of the extremely high losses with very small probabilities. The risk capital calculated using the confidence level of 99,8% is €135 respectively €139 million for each of the modelling alternatives shown in Sections 3 and 4 (using *TVaR*). Note that the similar risk capital requirements result from the PMLs for extreme events, which were used to fit the underlying distributions in the mathematical statistical model and were taken from the ELT, as were applied to the ELT-based model.

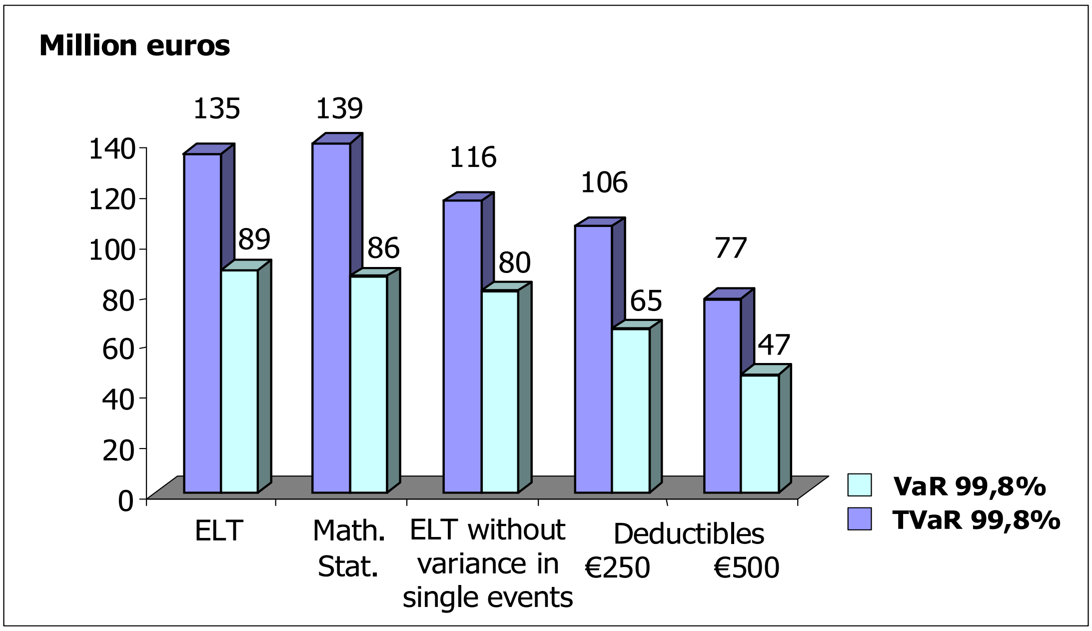


Figure 6: Value-at-risk and Tail-value-at-risk at a confidence level of 99.8% for annual storm loss

Ignoring the variance of the severity of individual events in the ELT, the result is a substantially lower risk capital requirement of €116 million (using *TVaR*). This emphasises the need to include this variability in the model.

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[38] There exist many publications where the tail-value-at-risk, which is recommended by IAA [IAA 2004], is criticised, see for example [Pfeifer 2004b], [Rootzén / Klüppelberg 1999], [McNeil / Embrechts / Frey 2005]. A further discussion about the use of risk measures is absolutely necessary but exceeds the purpose of this paper.

Strategic value- and risk-based management often involves discussing alternatives for risk reduction in divisions with very high risk capital requirement, such as where the capital available is exceeded. In those cases adequate reinsurance protection is an important factor. Other concepts are possible in storm risk, such as the introduction of deductibles. Since large numbers of claims with low average losses are typical of storm events, even low deductibles will have a great effect. Figure 6 shows the risk capital requirement with a universal introduction of deductibles in our sample portfolio. Somewhat lower deductibles for €250 and €500 have been assumed, as higher may otherwise lead to the undesirable effect of losing customers due to the general unpopularity of deductibles.

Modelling based on mathematical statistical methods as shown in Section 4 has been adjusted to these altered conditions by exactly calculating the deductibles to historical loss data according to as-if calculations. The relief from loss in the longer return periods has also been approximated from as-if calculations. Modelling using ELTs requires the individual events to be fitted to conditions. Here, we have omitted this step since this is only possible with direct access to natural risks models. At €250 deductibles, the risk capital requirement decreases by around 24%, and a clear reduction of 45% results from €500 deductibles. This confirms the positive affect of deductibles on the risk situation of the company with regard to storm loss. However, successfully introducing deductibles heavily depends on customer acceptance as described earlier, and the marketing aspects should not be ignored in such strategic decisions.

To summarise, we conclude that adequate catastrophe modelling is especially important in internal modelling in order to create a solid decision base for companies in deciding on corporate strategy. So for decisions in the areas of underwriting policy, changing insurance terms (as introduction of deductibles, limits, etc.), expansion, withdrawal from special segments, reinsurance buying, pricing, marketing, etc., these models will be an essential benefit for the management in future.

Modelling with mathematical statistical methods can be used to review the suitability of PMLs in long return periods with reference to historical data. If modelling based on natural risk models and mathematical statistical models yield similar results, the former can be judged as matching well. The latter present the distinct advantage of enabling direct modelling for corporate strategy in that a variety of gross strategies such as portfolio expansion, withdrawal from various agreements or the universal introduction of deductibles can be directly modelled. The effects of reinsurance contracts and alternative reinsurance strategies such as event XLs can be tested with both modelling types. The approach based on natural risk models usually leads to results that are more beneficial to the company since all of the events in the ELT are explicitly included in the model. As a result, in future insurance companies should use own natural risks models, which allow them to change the respective parameters for calculating the effects on management strategies.

Individual company modelling in DFA models create distributions of results of all different lines of business, reinsurance contracts, assets-classes, etc., as a basis for defining important strategic indicators such as return on risk adjusted capital, economic value added, economical profit and loss accounts, balance sheets, etc., by using simulation methods. The models described here represent an important step in supporting management in a thorough value- and risk-based corporate strategy that will lead to a lasting increase in corporate value while providing solid support for risk management.

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