Kurtosis and skewness estimation for non-life reserve risk distribution

ASTIN colloquium 2013

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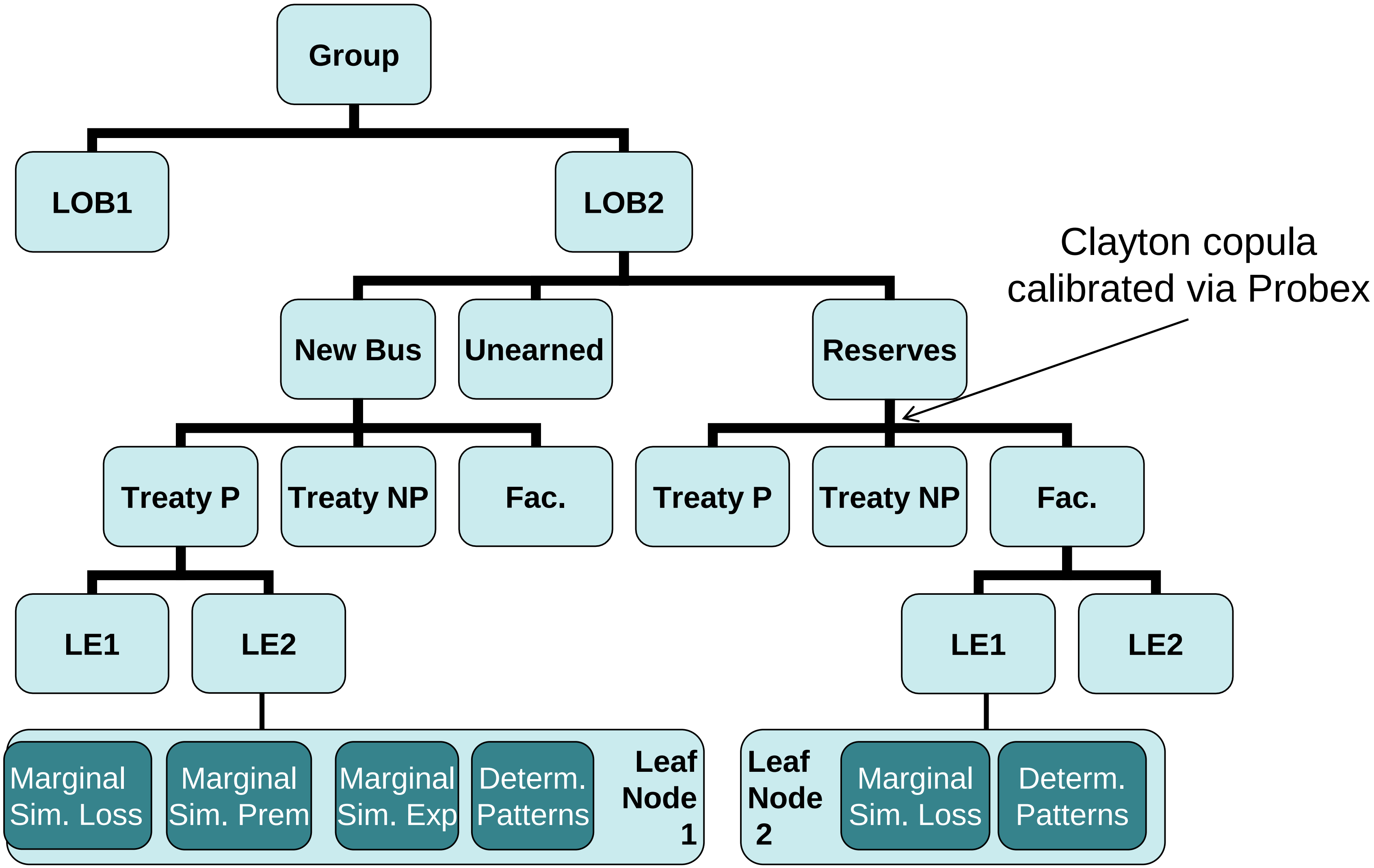
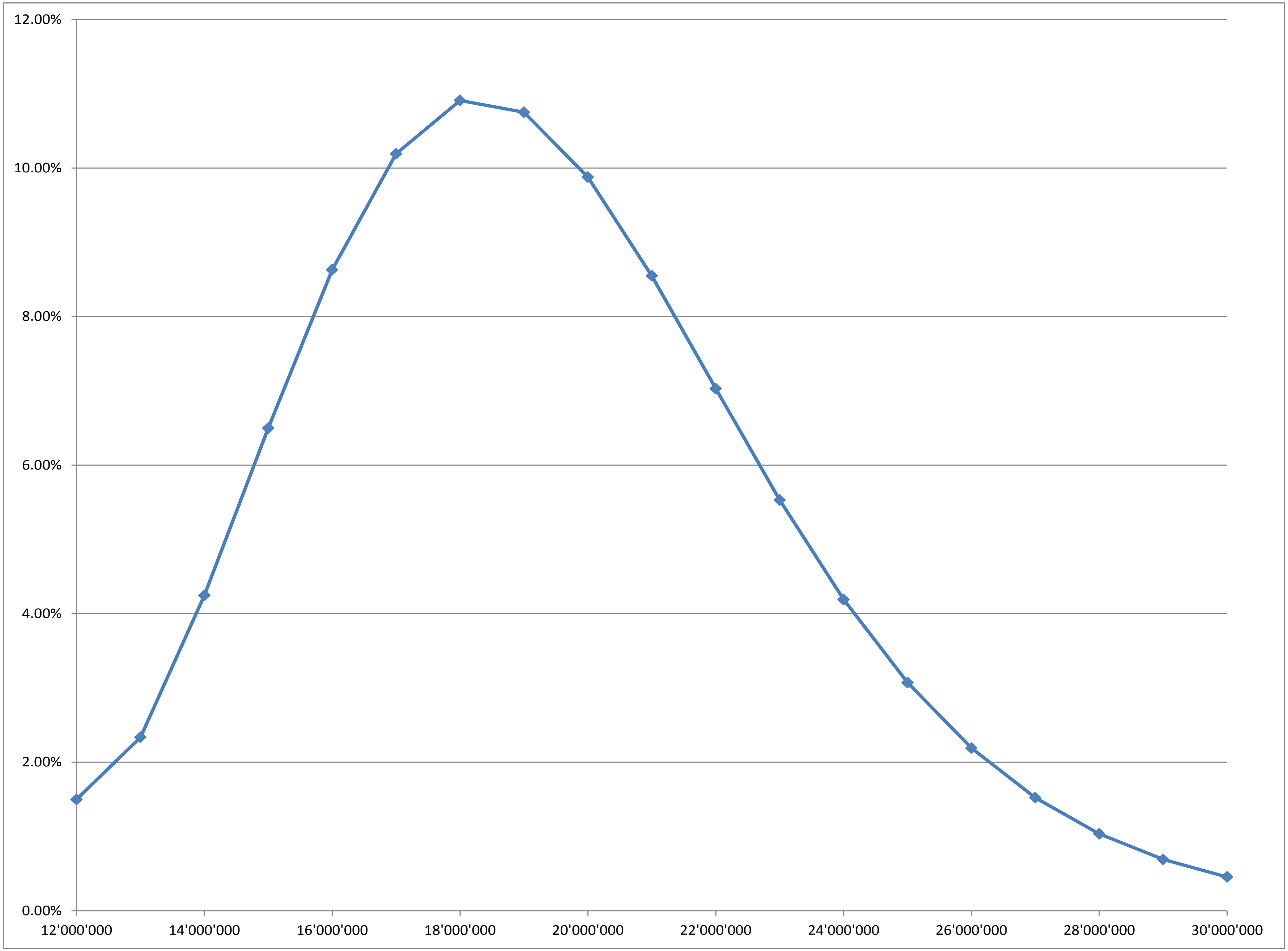
SCOR Internal Model - P&C

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Agenda

- Reserve risk distribution – What do we know ?
- Skewness and kurtosis – Some basic properties
- Reserve risk distribution – A proposal for a new approach
- Simulations to the ultimate
- Application to real triangles
- The Johnson distribution
- Conclusion
- References

Reserve risk distribution – What do we know in a chain-ladder framework ?

From Mack (1993):

UWY	Dvpt					Ultimate
	1	2	3	4	5	
1	$C_{1,1}$	$C_{1,2}$	$C_{1,3}$	$C_{1,4}$	$C_{1,5}$	$\hat{C}_{1,I}$
2	$C_{2,1}$	$C_{2,2}$	$C_{2,3}$	$C_{2,4}$		$\hat{C}_{2,I}$
3	$C_{3,1}$	$C_{3,2}$	$C_{3,3}$			$\hat{C}_{3,I}$
4	$C_{4,1}$	$C_{4,2}$				$\hat{C}_{4,I}$
5	$C_{5,1}$					$\hat{C}_{5,I}$
						$\sum_{j=1}^{I}\hat{C}_{j,I}$

Reserves

$$\hat{R}_j = \hat{C}_{j,I} - C_{j,I-j+1}$$

Best Estimate

$$\sum_{j=1}^{I}\hat{C}_{j,I} = \sum_{j=1}^{I} C_{j,I-j+1} \prod_{k=I-j+1}^{I}\hat{f}_k$$

$$\hat{f}_k = \frac{\sum_{j=1}^{I-k} C_{j,k+1}}{\sum_{j=1}^{I-k} C_{j,k}}, 1 \le k \le I\text{-}1$$

$$\hat{\sigma}_k^2 = \frac{1}{I-k-1}\sum_{i=1}^{I-k} C_{i,k}\left(\frac{C_{i,k+1}}{C_{i,k}} - \hat{f}_k\right)^2 \quad for\ 1 \le k \le I\text{-}2$$

$$mse\left(\sum_{i=2}^{I}\hat{R}_i\right) = \sum_{i=2}^{I}\left(mse\left(\hat{R}_i\right) + \hat{C}_{i,I}\left(\sum_{j=i+1}^{I}\hat{C}_{j,I}\right)\sum_{k=I+1-i}^{I-1}\frac{2\hat{\sigma}_k^2 / \hat{f}_k^2}{\sum_{n=1}^{I-k} C_{nk}}\right) \qquad (1)$$

with

$$mse\left(\hat{R}_i\right) = \hat{C}_{i,I}^2 \sum_{k=I+1-i}^{I-1}\frac{\hat{\sigma}_k^2}{\hat{f}_k^2}\left(\frac{1}{\hat{C}_{i,k}} + \frac{1}{\sum_{n=1}^{I-k} C_{nk}}\right)$$

Reserve risk distribution – What do we know in other frameworks ?

- Bornhuetter – Ferguson
  - Best estimate known
  - Estimate of the standard deviation known (see Mack 2008)
- Mix Bornhuetter – Ferguson / Chain-ladder
  - Best estimate known
  - Hybrid chain-ladder method provides an estimate of the standard deviation (see Arbenz 2010)
- GLM based on incremental triangles
  - Best estimate known
  - Different estimates of the standard deviation given (See Merz-Wüthrich 2008)
- Cape-Cod
  - Best estimate known
  - No estimate of the standard deviation

Reserve risk distribution – How do we get the distribution today ?

- Assume a lognormal distribution with mean given by Best Estimate and standard deviation given by Mack standard equation (see equation (1) on slide 3).
- Bootstrapping techniques based on Pearson residuals (see England and Verall 2006)
- Generalized Linear Models based on incremental triangles (see Merz and Wüthrich 2008)
  - Usual assumption: The distribution of the random element of the incremental claim $X_{i,j}$ belongs to the Exponential Dispersion Family (e.g. Poisson, Gamma …)
- Model of Salzman, Wüthrich, Merz on higher moments of the Claim Development Result in General Insurance (ASTIN Bulletin 2012)
  - Two models assumed for the distribution of the individual claims development factor $F_{i,j} = \frac{C_{i,j+1}}{C_{i,j}}$: Gamma and Lognormal models

- All of the above models use some distributional assumptions.

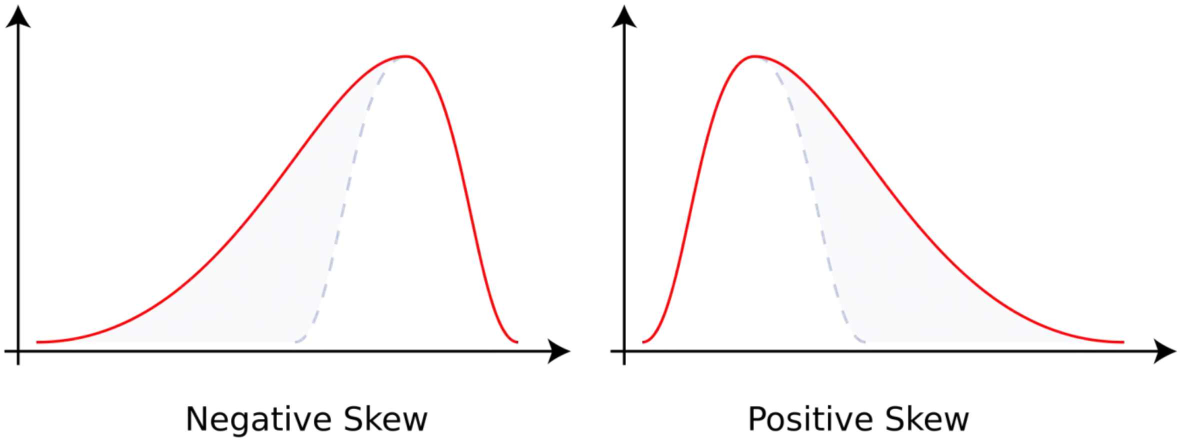
Skewness and kurtosis – Some basic properties

The following properties are taken from Wikipedia:

- The skewness of a random variable *X* is the third standardized moment, denoted γ_1 and defined as

$$\gamma_1 = E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{\mu_3}{\sigma^3}$$

where μ_3 is the third moment about the mean μ and σ is the standard deviation.



Skewness and kurtosis – Some basic properties

- Sample skewness - For a sample of *n* values the *sample skewness* is:

$$g_1 = \frac{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^3}{\left(\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2\right)^{3/2}}$$

- g1 is a biased estimator of sample skewness. H. Cramer (1946) provided an unbiased estimator of sample skewness G:

$$G = \frac{n}{(n-1)(n-2)}\sum_{i=1}^{n}\left(\frac{x_i - \overline{x}}{s}\right)^3$$

where s is the unbiased sample standard deviation.

Harald Cramer (1893 – 1985)

Swedish professor at University of Stockholm

PhD for his thesis «On a class of Dirichlet series» with the advisor Marcel Riesz

Skewness and kurtosis – Some basic properties

- The fourth standardized moment is defined as

$$\beta_2 = E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{\mu_4}{\sigma^4}$$

where μ_4 is the fourth moment about the mean μ and σ is the standard deviation.

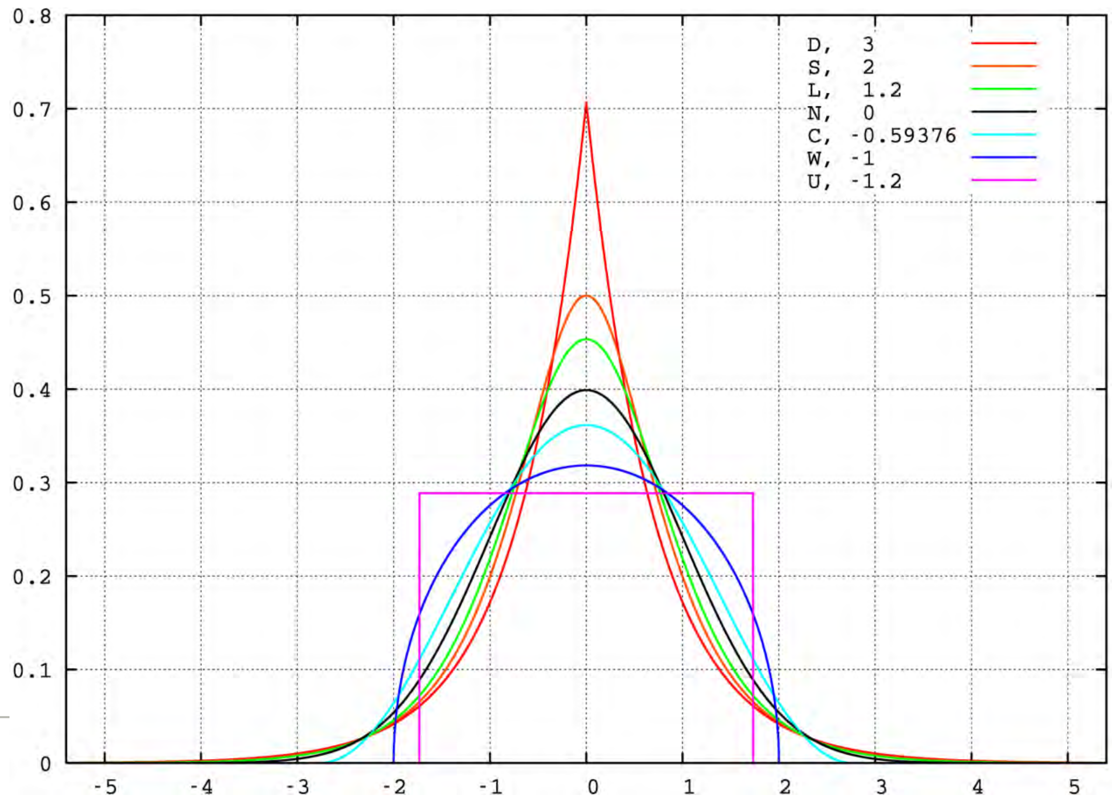
- Excess kurtosis is defined as: $\gamma_2 = \frac{\mu_4}{\sigma^4} - 3$

Excess kurtosis

Leptokurtic:
D: Laplace distribution
S: Hyperbolic secant distribution
L: Logistic distribution

N: Normal distribution

Platykurtic:
C: Raised cosine distribution
W: Wigned semicircle distribution
U: Uniform distribution



Skewness and kurtosis – Some basic properties

- For a sample of *n* values the sample excess kurtosis is

$$g_2 = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^4}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2\right)^2}-3$$

- g2 is a biased estimator of the sample excess kurtosis. H. Cramer (1946) provided an “unbiased” estimator of sample excess kurtosis as follows. We denote :

$$m_4=\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^4 \qquad m_2=\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

Then an unbiased estimator of the fourth centered moment is:

$$M_4=\frac{n(n^2-2n+3)}{(n-1)(n-2)(n-3)}m_4-3m_2^2\frac{n(2n-3)}{(n-1)(n-2)(n-3)}$$

SCOR

Reserve risk distribution – A proposal for a new approach

UWY	Dvpt					Ultimate
	1	2	3	4	5	
1	$C_{1,1}$	$C_{1,2}$	$C_{1,3}$	$C_{1,4}$	$C_{1,5}$	$\hat{C}_{1,I}$
2	$C_{2,1}$	$C_{2,2}$	$C_{2,3}$	$C_{2,4}$		$\hat{C}_{2,I}$
3	$C_{3,1}$	$C_{3,2}$	$C_{3,3}$			$\hat{C}_{3,I}$
4	$C_{4,1}$	$C_{4,2}$				$\hat{C}_{4,I}$
5	$C_{5,1}$					$\hat{C}_{5,I}$
						$\sum_{j=1}^{I} \hat{C}_{j,I}$

Assumption on skewness : $\dfrac{SK\left(C_{i,k+1} \mid C_{i,1},\dots,C_{i,k}\right)}{\left[Var\left(C_{i,k+1} \mid C_{i,1},\dots,C_{i,k}\right)\right]^{3/2}}$ depends on k but not on i

where: $SK\left(C_{i,k+1} \mid C_{i,1},\dots,C_{i,k}\right) = E\left[\left(C_{i,k+1} - E\left(C_{i,k+1} \mid C_{i,1},\dots,C_{i,k}\right)\right)^3 \mid C_{i,1},\dots,C_{i,k}\right]$

Assumption on kurtosis**:** $\dfrac{KT\left(C_{i,k+1} \mid C_{i,1},\dots,C_{i,k}\right)}{\left[Var\left(C_{i,k+1} \mid C_{i,1},\dots,C_{i,k}\right)\right]^{2}}$ depends on k but not on i

where: $KT\left(C_{i,k+1} \mid C_{i,1},\dots,C_{i,k}\right) = E\left[\left(C_{i,k+1} - E\left(C_{i,k+1} \mid C_{i,1},\dots,C_{i,k}\right)\right)^4 \mid C_{i,1},\dots,C_{i,k}\right]$

For one development year the skewness/kurtosis is the same for any UWY.
Context : Reserving portfolio which risks are similar for every UWY.

Skewness : Use of the new approach

- With the above assumption, under a Mack model for the volatility, we have:

$$\exists \gamma_k \text{ such that } \gamma_{\mathrm{k}} = \frac{SK\left(C_{i,k+1} \mid C_{i,1},\ldots,C_{i,k}\right)}{\left[Var\left(C_{i,k+1} \mid C_{i,1},\ldots,C_{i,k}\right)\right]^{3/2}} = \frac{SK\left(C_{i,k+1} \mid C_{i,1},\ldots,C_{i,k}\right)}{\left[\sigma_k^2 C_{i,k}\right]^{3/2}}$$

$$\Rightarrow SK\left(C_{i,k+1} \mid C_{i,1},\ldots,C_{i,k}\right) = \gamma_{\mathrm{k}}\left[\sigma_k^2 C_{i,k}\right]^{3/2} = Sk_k^3 C_{i,k}^{3/2}$$

- Then, it is possible to show that the estimator below is unbiased.:

$$\hat{S}k_k^3 = \frac{1}{\left(I - k - \frac{\left(\sum_{i=1}^{I-k} C_{i,k}^{3/2}\right)^2}{\left(\sum_{i=1}^{I-k} C_{i,k}\right)^3}\right)} \sum_{i=1}^{I-k} C_{i,k}^{3/2}\left(\frac{C_{i,k+1}}{C_{i,k}} - \hat{f}_k\right)^3 \quad \text{for } 1 \le k \le I\text{-}3$$

Comments:
1 – The formula for $\hat{S}k_k^3$ has a similar “shape” as the formula for $\hat{\sigma}_k^2$
2 – The formula for $\hat{S}k_k^3$ uses the usual weighted average (power 1.5) of cubic differences.
3 – As for the formula of $\hat{\sigma}_k^2$, outliers can play a major role in the estimation of $\hat{S}k_k^3$
4 – The homogeneity formulas in terms of power of $C_{i,k}$ is kept in the above formulas.

Kurtosis : Use of the new approach

- With the above assumption, under a Mack model for the volatility, we have:

$$\exists \gamma_k \text{ such that } \gamma_k = \frac{KT\left(C_{i,k+1} \mid C_{i,1},\ldots,C_{i,k}\right)}{\left[Var\left(C_{i,k+1} \mid C_{i,1},\ldots,C_{i,k}\right)\right]^2} = \frac{KT\left(C_{i,k+1} \mid C_{i,1},\ldots,C_{i,k}\right)}{\left[\sigma_k^2 C_{i,k}\right]^2}$$

$$\Rightarrow KT\left(C_{i,k+1} \mid C_{i,1},\ldots,C_{i,k}\right) = \gamma_k \left[\sigma_k^2 C_{i,k}\right]^2 = Kt_k^4 C_{i,k}^2$$

- Then, it is possible to show that the estimator below is unbiased.:

$$\hat{K}t_k^4 = \frac{\left[\sum_{i=1}^{I-k} C_{i,k}^2 \left(\frac{C_{i,k+1}}{C_{i,k}} - \hat{f}_k\right)^4 - 3\left(\hat{\sigma}_k^2\right)^2 \left(2 - 6\frac{\sum_{i=1}^{I-k} C_{i,k}^2}{\left(\sum_{i=1}^{I-k} C_{i,k}\right)^2} + 4\frac{\sum_{i=1}^{I-k} C_{i,k}^3}{\left(\sum_{i=1}^{I-k} C_{i,k}\right)^3}\right)\right]}{\left(\sum_{i=1}^{I-k}\left(1 - \frac{C_{i,k}}{\sum_{i=1}^{I-k} C_{i,k}}\right)^4 + \frac{\left(\sum_{i=1}^{I-k} C_{i,k}^2\right)^2 - \sum_{i=1}^{I-k} C_{i,k}^4}{\left(\sum_{i=1}^{I-k} C_{i,k}\right)^4}\right)} \quad \text{for } 1 \le k \le I\text{-}4$$

Notes:

1 – The formula is “as expected”.

2 – There is the “usual correction” equal to 3 times the square of the variance estimator.

3 – The homogeneity formulas in terms of power of $C_{i,k}$ is kept in the above formulas.

Skewness/Kurtosis : Simulation to the ultimate

UWY	Dvpt					Ultimate
	1	2	3	4	5	
1	$C_{1,1}$	$C_{1,2}$	$C_{1,3}$	$C_{1,4}$	$C_{1,5}$	$\hat{C}_{1,I}$
2	$C_{2,1}$	$C_{2,2}$	$C_{2,3}$	$C_{2,4}$		$\hat{C}_{2,I}$
3	$C_{3,1}$	$C_{3,2}$	$C_{3,3}$			$\hat{C}_{3,I}$
4	$C_{4,1}$	$C_{4,2}$				$\hat{C}_{4,I}$
5	$C_{5,1}$					$\hat{C}_{5,I}$
						$\sum_{j=1}^{I} \hat{C}_{j,I}$

$$\hat{f}_k = \frac{\sum_{j=1}^{I-k} C_{j,k+1}}{\sum_{j=1}^{I-k} C_{j,k}}, 1 \le k \le I\text{-}1$$

$$\hat{\sigma}_k^2 = \frac{1}{I-k-1} \sum_{i=1}^{I-k} C_{i,k} \left(\frac{C_{i,k+1}}{C_{i,k}} - \hat{f}_k \right)^2 \quad for\ 1 \le k \le I\text{-}2$$

$$\hat{Sk}_k^3 = \frac{1}{\left(I-k-\frac{\left(\sum_{i=1}^{I-k} C_{i,k}^{3/2}\right)^2}{\left(\sum_{i=1}^{I-k} C_{i,k}\right)^3} \right)} \sum_{i=1}^{I-k} C_{i,k}^{3/2} \left(\frac{C_{i,k+1}}{C_{i,k}} - \hat{f}_k \right)^3 \quad \text{for } 1 \le k \le I\text{-}3$$

$$\hat{Kt}_k^4$$

Supporting distribution to estimate the distributional properties of $C_{i,I}$.

$\Rightarrow$ Recursive calculation (see Mack 1999)

Assumption: The chosen supporting distribution should not be influencing the overall simulated skewness.

Skewness/Kurtosis : Simulation to the ultimate – Generalized Pareto Distribution

i	C_{i1}	C_{i2}	C_{i3}	C_{i4}	C_{i5}	C_{i6}	C_{i7}	C_{i8}	C_{i9}	C_{i10}
2										
3										
4										
5										
6										
7										
8										
9										
10										
Asymmetry		ξ_2	ξ_3	ξ_4	ξ_5	ξ_6	ξ_7	ξ_8	ξ_9	ξ_{10}

$$\hat{C}_{i,k} \propto GPD(\mu_{i,k}; s_{i,k}; \zeta_k)$$

Parameters of all distributions at step k are known

Estimation of parameters of all distributions at step k+1

$$\hat{C}_{i,k} = \mu_{i,k} + \frac{s_{i,k}\left(U_{i,k}^{-\zeta_k} - 1\right)}{\zeta_k} \propto GPD(\mu_{i,k}; s_{i,k}; \zeta_k)$$

$$\left\{\begin{aligned}
&\frac{\hat{Kt}^4_{k+1}}{\left(\hat{\sigma}^2_{k+1}\right)^2} = \frac{3\left(3 - 5\zeta_{k+1} - 4\zeta^3_{k+1}\right)}{\left(1 - 7\zeta_{k+1} + 12\zeta^2_{k+1}\right)} \\
&\xi_{k+1} = \frac{\hat{Sk}^3_{k+1}}{\hat{\sigma}^3_{k+1}} = \frac{2\left(1+\zeta_{k+1}\right)\sqrt{1 - 2\zeta_{k+1}}}{1 - 3\zeta_{k+1}} \\
&\hat{Variance}_{i,k+1} = \hat{C}_{i,k}\hat{\sigma}^2_k\left(1 + \frac{\hat{C}_{i,k}}{\sum_{j=1}^{I-k} C_{j,k}}\right) = \frac{s^2_{i,k+1}}{\left(1-\zeta_{k+1}\right)^2\left(1 - 2\zeta_{k+1}\right)} \\
&\hat{C}_{i,k+1} = \hat{f}_k\hat{C}_{i,k} = \mu_{i,k+1} + \frac{s_{i,k+1}}{1 - \zeta_{k+1}}
\end{aligned}\right.$$

Application to real triangles

The calculations of Skewness/Kurtosis per development year as well as the simulations to ultimate on the triangle using the GPD distribution were performed on the following triangle:

- Schedule P triangles provided by G Meyers on the CAS website – Accident year 1988 to 1997 (10 x 10 triangles - http://www.casact.org/research/index.cfm?fa=loss_reserves_data):
  - Farmers Alliance – Private Motor
  - NC Farm Bureau – Private Motor
  - New Jersey Manufacturers – Private Motor
  - Pennsylvania – Product Liability
  - West Bend – Product Liability
- First example triangle in Mack 1993 (10 x 10 triangle)
- SCOR Global P&C 2011 reserve triangles – Excel files (15x15 triangle - http://www.scor.com/en/investors/financial-reporting/presentations.html)
  - Casualty proportional worldwide
  - Motor non-proportional worldwide

Application to real triangles – Skewness and Kurtosis per development year – 10x10 triangles

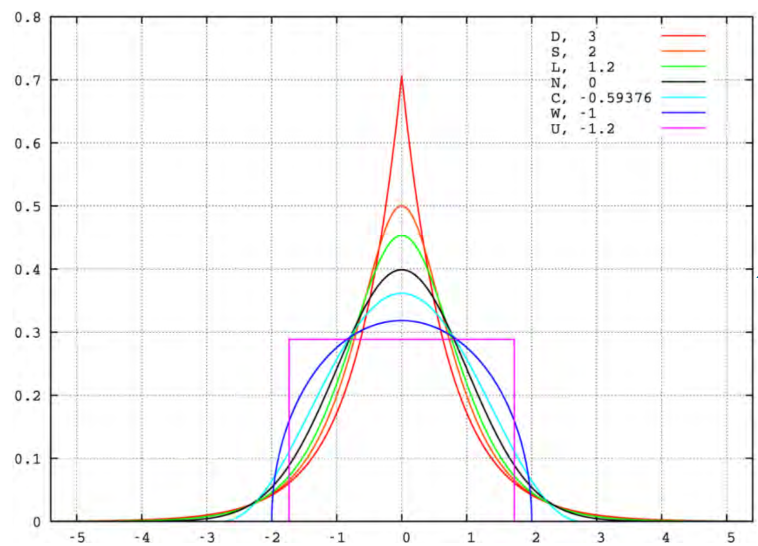
k			1	2	3	4	5	6	7
Private Motor	Farmers Alliance	$\hat{S}k_k^3/(\hat{\sigma}_k^2)^{3/2}$	0.611	-0.256	-0.349	-0.090	1.049	0.477	0.273
		$\hat{K}t_k^4/(\hat{\sigma}_k^2)^2$	317.02%	177.30%	166.23%	146.75%	340.69%	165.38%	NA
Private Motor	NC Farm Bureau	$\hat{S}k_k^3/(\hat{\sigma}_k^2)^{3/2}$	0.703	0.412	0.727	-0.047	-0.058	-0.769	0.500
		$\hat{K}t_k^4/(\hat{\sigma}_k^2)^2$	223.54%	201.65%	182.26%	78.01%	144.84%	237.28%	NA
Private Motor	New Jersey Manufacturers	$\hat{S}k_k^3/(\hat{\sigma}_k^2)^{3/2}$	0.583	0.187	0.414	-0.565	-0.141	0.230	-0.008
		$\hat{K}t_k^4/(\hat{\sigma}_k^2)^2$	204.32%	229.15%	207.06%	192.21%	103.56%	104.25%	NA
Product Liability	Pennsylvania	$\hat{S}k_k^3/(\hat{\sigma}_k^2)^{3/2}$	0.774	1.716	-0.540	-1.059	-0.620	0.164	0.111
		$\hat{K}t_k^4/(\hat{\sigma}_k^2)^2$	324.61%	620.07%	298.72%	369.19%	164.25%	119.36%	NA
Product Liability	West Bend	$\hat{S}k_k^3/(\hat{\sigma}_k^2)^{3/2}$	-0.008	1.060	0.525	-0.507	-0.030	-0.484	-0.113
		$\hat{K}t_k^4/(\hat{\sigma}_k^2)^2$	205.13%	411.28%	302.64%	200.90%	125.54%	121.50%	NA
Mack 1993		$\hat{S}k_k^3/(\hat{\sigma}_k^2)^{3/2}$	0.137	0.215	0.638	-0.433	0.402	-0.026	-0.497
		$\hat{K}t_k^4/(\hat{\sigma}_k^2)^2$	184.92%	170.29%	265.62%	162.92%	185.65%	123.00%	NA

Application to real triangles – Skewness and Kurtosis per development year – 15x15 triangles SCOR

k		1	2	3	4	5	6	7	8	9	10	11	12
Casualty Prop	$\hat{S}k_k^3 / \left(\hat{\sigma}_k^2\right)^{3/2}$	1.031	0.070	-0.372	-0.119	-0.390	0.537	-0.334	0.863	-0.842	0.868	-0.843	0.518
	$\hat{K}t_k^4 / \left(\hat{\sigma}_k^2\right)^2$	233.59%	291.73%	339.24%	193.39%	194.71%	260.08%	226.84%	314.55%	294.64%	266.31%	257.91%	NA
Motor NonProp	$\hat{S}k_k^3 / \left(\hat{\sigma}_k^2\right)^{3/2}$	1.491	0.286	0.348	-0.155	0.621	0.177	0.920	0.411	0.658	0.713	0.865	-0.212
	$\hat{K}t_k^4 / \left(\hat{\sigma}_k^2\right)^2$	631.06%	201.17%	403.49%	234.55%	294.37%	167.42%	292.94%	194.28%	291.68%	245.21%	265.76%	NA

Application to real triangles – Simulation to ultimate

LoB	Company	Chain-ladder reserves	Chain-ladder stdev	CoV	Overall simulated skewness	Overall simulated kurtosis	LogN			
							Mu	Sigma2	Resulting skewness	Resulting kurtosis
Private Motor	Farmers Alliance	-374	1493	-400%	-0.01	297%	NA	NA	NA	NA
Private Motor	NC Farm	19'415	9'528	49%	0.32	298%	9.77	0.216	1.59	781%
Private Motor	New Jersey Manuf.	109'719	11'961	11%	0.07	295%	11.60	0.012	0.33	319%
Product Liab.	Pennsylvania	1'474	1'784	121%	0.06	350%	6.84	0.903	5.41	8222%
Product Liab.	West Bend	2150	1899	88%	0.35	384%	7.38	0.577	3.34	2784%
Mack 1993 triangle		18'680'856	2'447'095	13%	0.13	292%	16.73	0.017	0.40	328%
WW Casualty Prop	SCOR	219'461'925	79'722'452	36%	0.14	300%	19.14	0.124	1.14	539%
WW Motor NP	SCOR	402'645'321	53'078'447	13%	0.17	289%	19.80	0.017	0.40	328%



The Johnson distribution

We recall that the family of Johnson distribution has the following properties (see also Johnson 1949):

$$z = \gamma + \delta \ f\left(\frac{x-\xi}{\lambda}\right)$$

where f is a function of simple form and z is a unit normal variable.

Depending on f, the Johnson distribution is noted as follows:

$f = \log$: Distribution SL

$f = \sinh^{-1}$: Distribution SU

$z = \gamma + \delta \log\left(\frac{x-\xi}{\xi+\lambda-x}\right)$: Distribution SB

$z = \gamma + \delta \left(\frac{x-\xi}{\lambda}\right)$: Distribution SN

Norman Lloyd Johnson (FIA)

PhD 1948 for his thesis «A family of Frequency Curves» done under the advisor Egon Sharpe Pearson

The Johnson distribution

The Johnson distribution is available in the software R:

- Package “SuppDists”

- Fitting of a Johnson distribution on the first 4 moments can be done with the function:

  JohnsonFit

- Getting the main statistics of a known Johnson distribution (with its 4 parameters and its type can be done with the function:

  sJohnson

The Johnson distribution – Fitting to simulated data

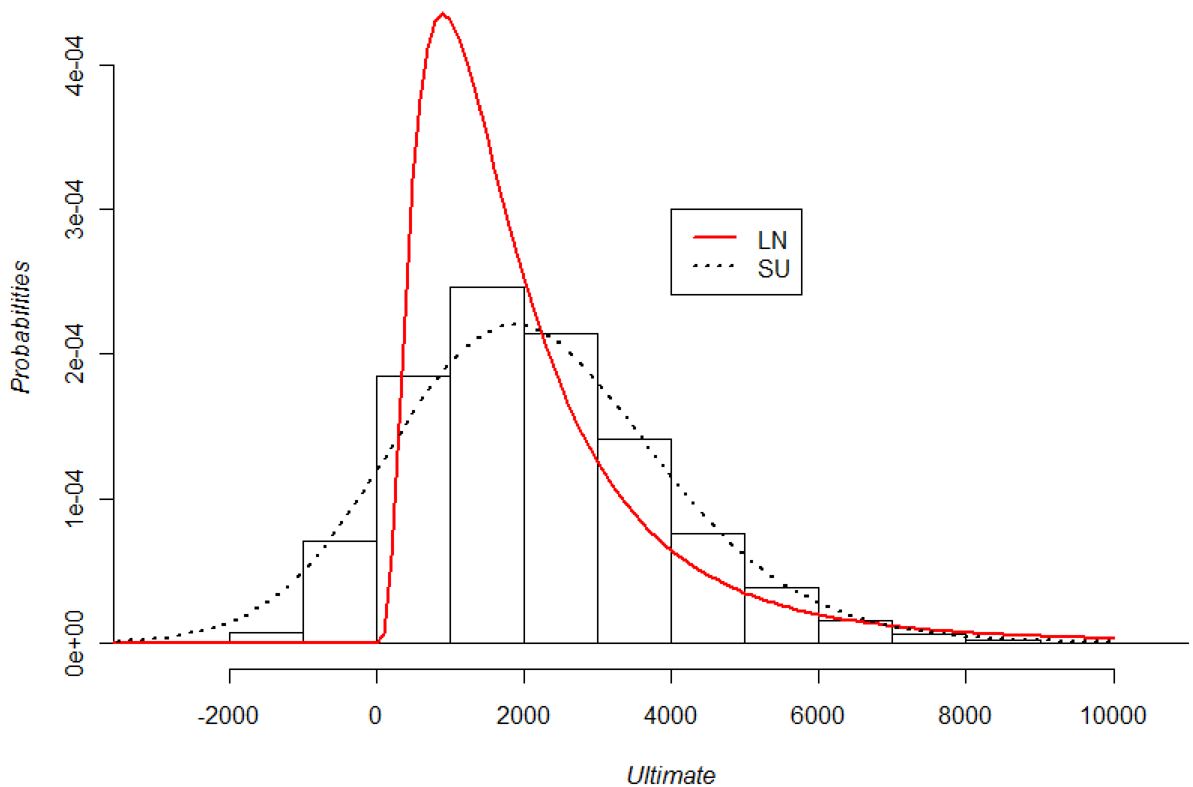
LoB	Company	Chain-ladder reserves	Chain-ladder stdev	CoV	Overall simulated skewness	Overall simulated kurtosis	Johnson fitting				
							Type	Fitted Mean	Fitted Stdev	Fitted Skewness	Fitted Kurtosis
Private Motor	Farmers Alliance	-374	1493	-400%	-0.01	297%	SN	-374	1'493	-	300%
Private Motor	NC Farm	19'415	9'528	49%	0.32	298%	SB	19'355	9'406	0.19	287%
Private Motor	New Jersey Manuf.	109'719	11'961	11%	0.07	295%	SN	109'719	11'961	-	300%
Product Liab.	Pennsylvania	1'474	1'784	121%	0.06	350%	SU	1'474	1'784	0.06	350%
Product Liab.	West Bend	2150	1899	88%	0.35	384%	SU	2'150	1'899	0.35	348%
Mack 1993 triangle		18'680'856	2'447'095	13%	0.13	292%	SB	18'645'236	2'428'748	0.18	278%
WW Casualty Prop	SCOR	219'461'925	79'722'452	36%	0.14	300%	SL	219'461'925	79'722'452	0.14	303%
WW Motor NP	SCOR	402'645'321	53'078'447	13%	0.17	289%	SB	401'885'733	52'707'797	0.21	278%

The Johnson distribution – Comparison of VaR 99%

LoB	Company	VaR 99%		Difference LogN Johnson VaR 99%
		Johnson	Lognormal	
Private Motor	Farmers Alliance	3'099	NA	NA
Private Motor	NC Farm	43'432	51'358	18%
Private Motor	New Jersey Manuf.	137'544	140'453	2%
Product Liab.	Pennsylvania	5'864	8'556	46%
Product Liab.	West Bend	7'214	9'430	31%
Mack 1993 triangle		24'555'541	25'089'172	2%
WW Casualty Prop	SCOR	411'994'159	467'889'645	14%
WW Motor NP	SCOR	531'340'556	541'742'729	2%

The Johnson distribution – Case of West Bend / Product Liability

k			1	2	3	4	5	6	7	8	9
Product Liability	West Bend	$\hat{f}_k$	1.692	1.487	1.269	1.016	1.150	1.130	0.862	1.007	1.000
		$\hat{\sigma}_k^2$	31.078	66.326	70.197	33.319	23.011	3.421	14.919	0.015	0.000
		$\hat{S}k_k^3 / \left(\hat{\sigma}_k^2\right)^{3/2}$	-0.008	1.060	0.525	-0.507	-0.030	-0.484	-0.113	NA	NA
		$\hat{K}t_k^4 / \left(\hat{\sigma}_k^2\right)^2$	205.13%	411.28%	302.64%	200.90%	125.54%	121.50%	NA	NA	NA



Reserve risk distribution – Conclusions and next steps

- The usual feelings on the reserving distribution seem to be confirmed by the study
  - The distribution is slightly positively skewed
  - The distribution is not sharp
- The use of the Lognormal distribution can fit with the above feelings in the case where the coefficient of variation is small.
- When the coefficient of variation is high (e.g. more than 36%), the lognormal distribution may not be adequate anymore. Use of alternatives should be sought.

- Next steps
  - Find formulae for overall skewness and kurtosis
  - Find distributions that can fit specific lines of business

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