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# Concentration Risk

## Where we are

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# Concentration Risk

**“Risk concentrations are arguably the single most important cause of major problems in banks” (see BCBS (2006a, §770))**

# Concentration Risk: where we are

1. *Diversification: what is it worth?*
2. *Loss Distribution- Based Risk Measures. Economic Capital*
3. *Concentration Risk. Main current Approaches*
  - I. *Basel Approach to Concentration Risk*
  - II. *Rating Agency Approach to Concentration Risk*
  - III. *Multifactor Models and Concentration Risk*
4. *Some Conclusions*

# 1. Diversification: what is it worth?

# Concentration Risk

## Risk And Return of Portfolios

$$\sum_{i=1}^N W_i E(R_i) = E(R_p)$$

The Expected Return (ER) of a portfolio is the Weighted Average of the ER of the securities.

$$Corr_{AB} = \frac{Cov(R_A, R_B)}{\sigma_A \sigma_B}$$

The Variance of Returns (VR) of a portfolio depends on how returns move together: Covariance of Returns (CR) and Correlation Coefficient .

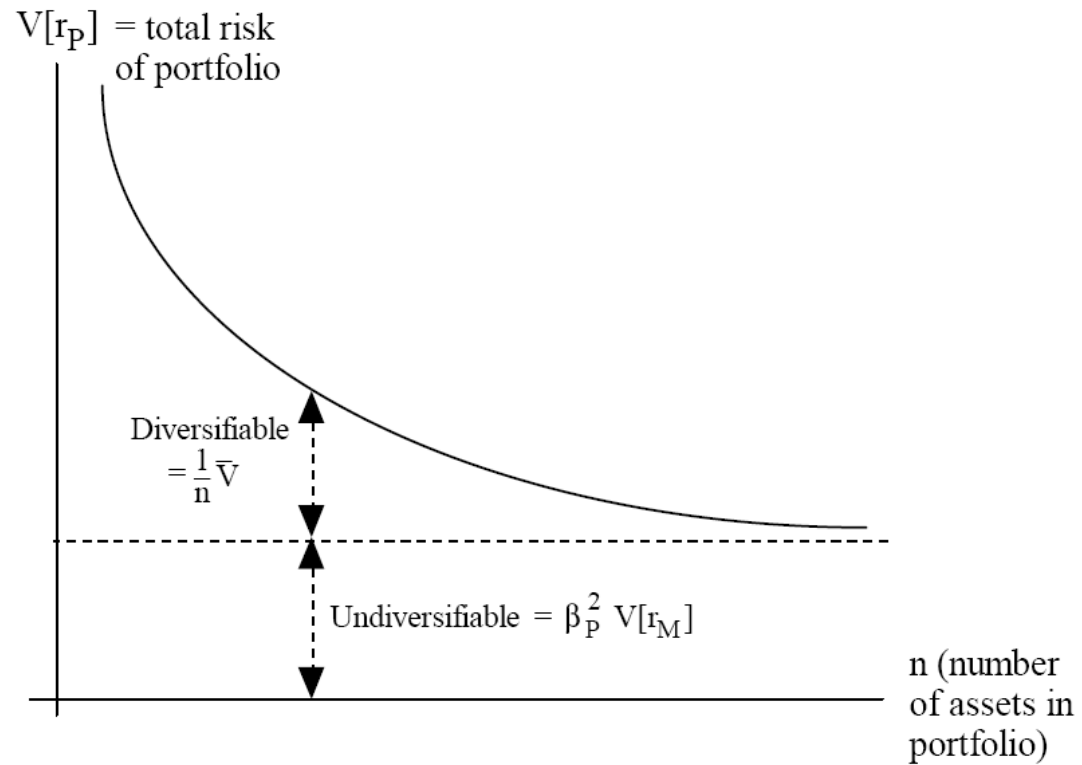
**Harry Markowitz:** the variance of a portfolio is less than a weighted average of the individual variances of the portfolio securities (Correlation is between +1 and -1). Lower Correlation results in greater diversification benefits.

$$\sigma_P^2 = W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + 2W_A W_B Cov(R_A, R_B)$$

$$\sigma_P^2 = W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + 2W_A W_B Corr_{AB} \sigma_A \sigma_B$$

Risk that disappears in a well-diversified portfolio is called Diversifiable Risk. the risk that remains is called Systematic Risk.

# Concentration Risk



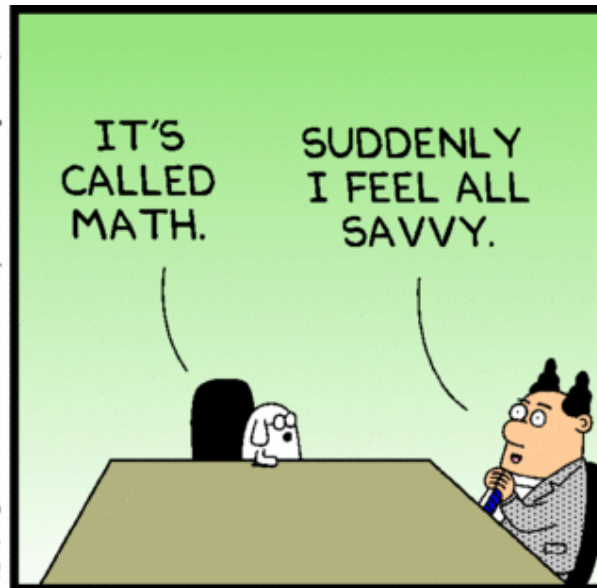
# Diversification. What is it worth?



www.dilbert.com scottadams@aol.com

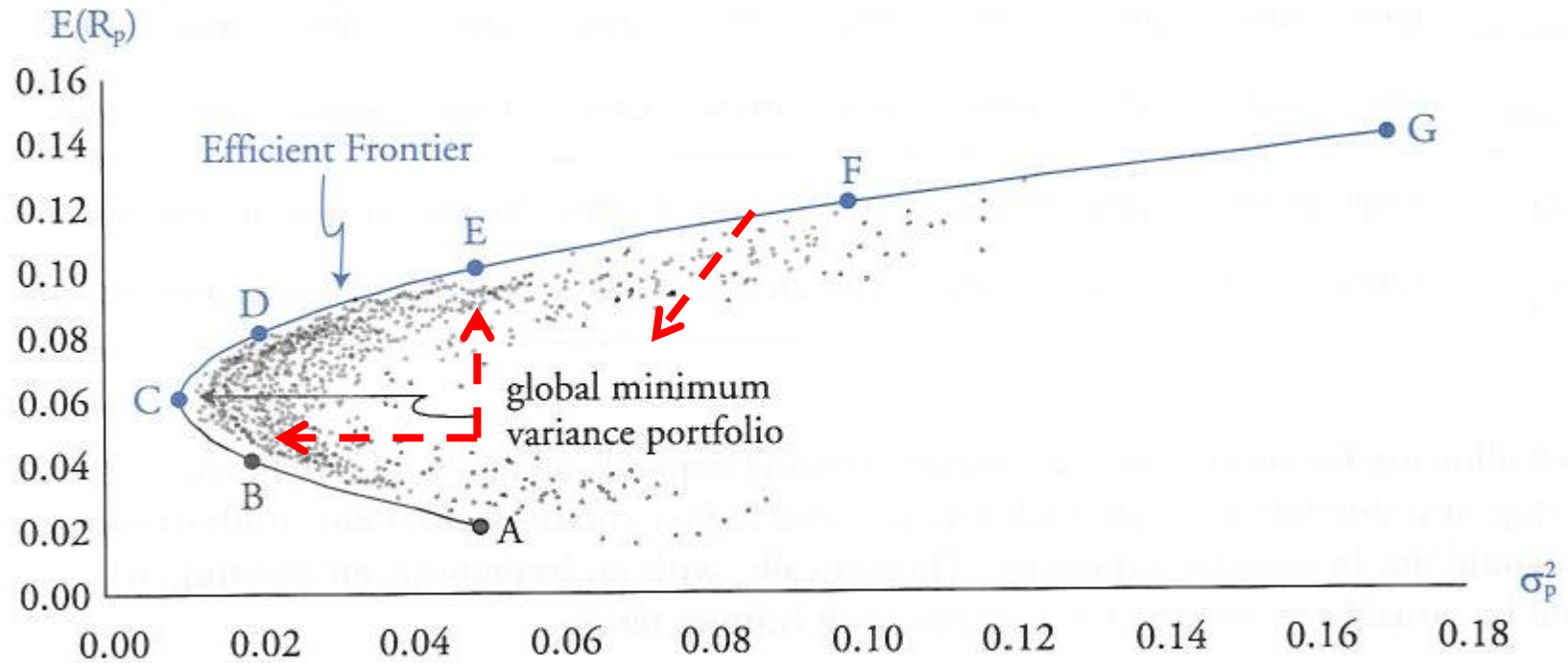


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## Diversification. What is it worth?



$$Sharpe = \left[ \frac{E(R_p) - R_f}{\sigma_p} \right]$$

$$Treydor = \left[ \frac{E(R_p) - R_f}{\beta_p} \right]$$

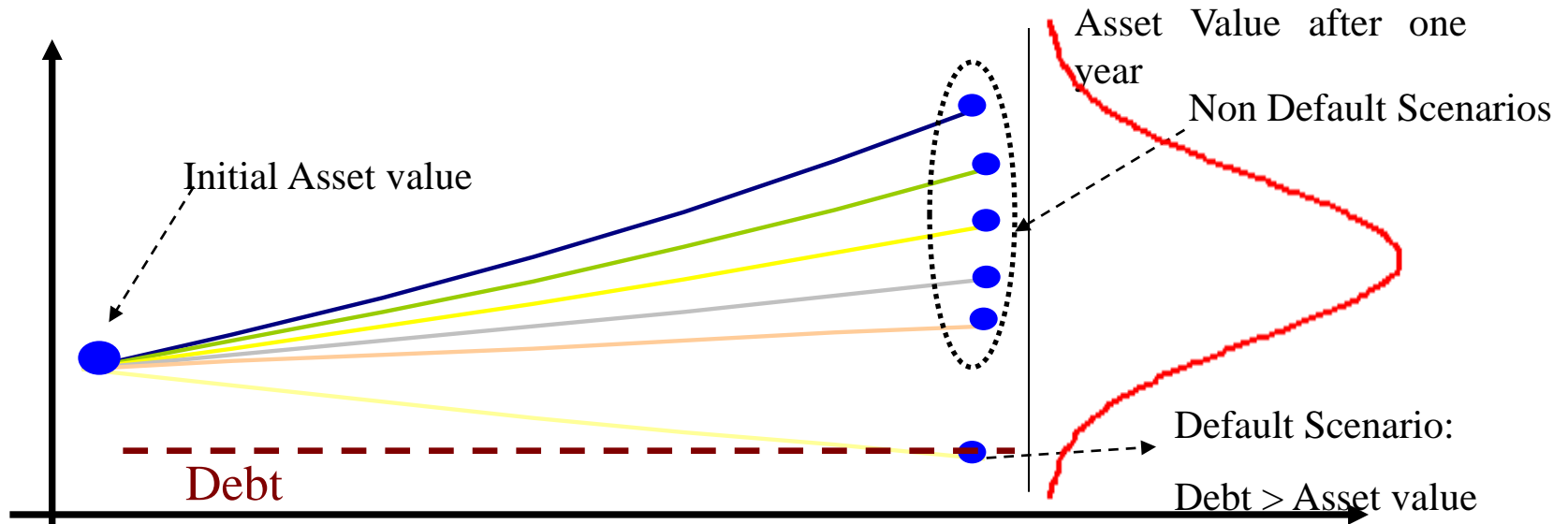
## 2. Loss Distribution- Based Risk Measures. Economic Capital

# Diversification

- Invest €1MM in one Asset Class: 100% BBVA shares.
- Invest €1MM in one Asset Class: 50% BBVA and 50% Santander shares.
- Invest €1MM in two Asset Classes:
  - 25% BBVA.
  - 25% Santander.
  - 50% Pistachos.
- Solvency: How much Capital do you need to get a Rating level?
  - Three drivers that work together: asset volatility, financial leverage and solvency rating.

## Merton Model (I)

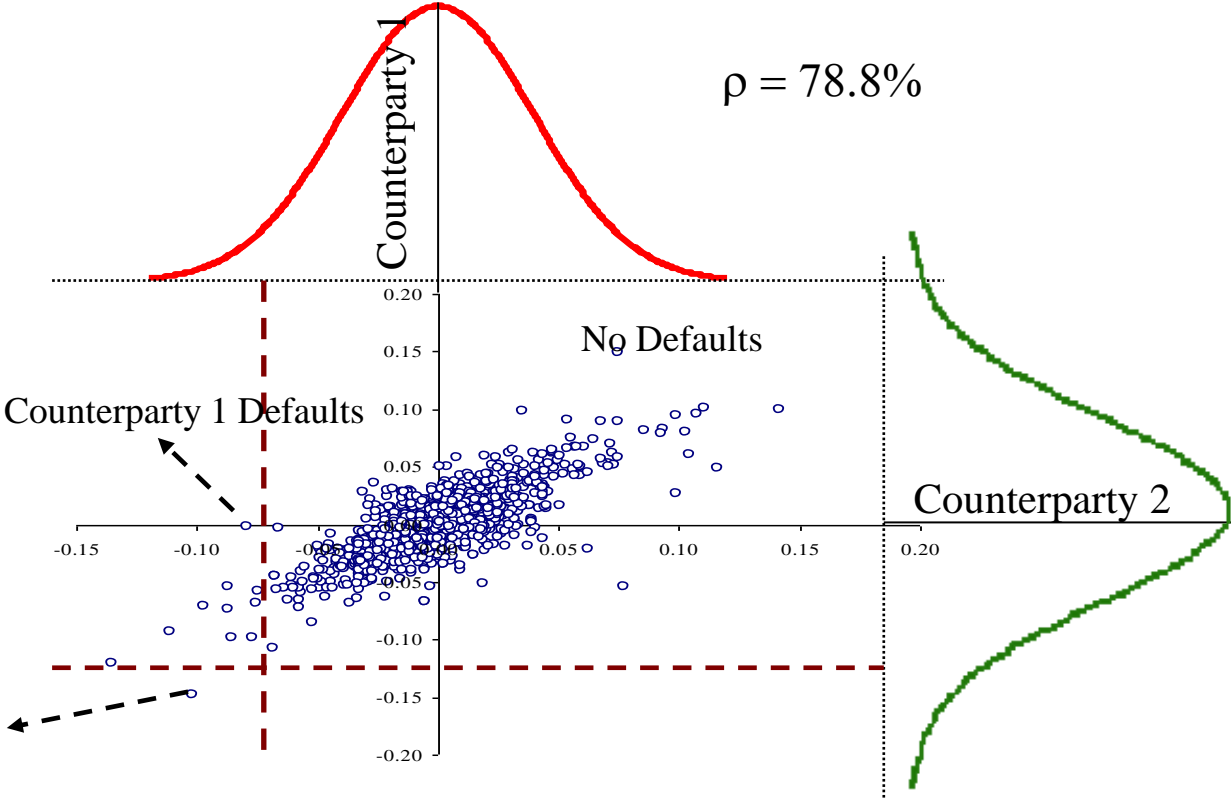
A borrower defaults when its asset value falls below a **threshold** defined by its liabilities.



# Merton Model (II)

Correlation between borrowers' defaults arise from correlation between their asset values.

**The Basel II approach is based on the Merton Model.**



Both Counterparties default

# Loss Distribution- Based Risk Measures

Quantity risk measures: the distribution of Losses over a specified time horizon. Different approaches:

Expected losses (EL).

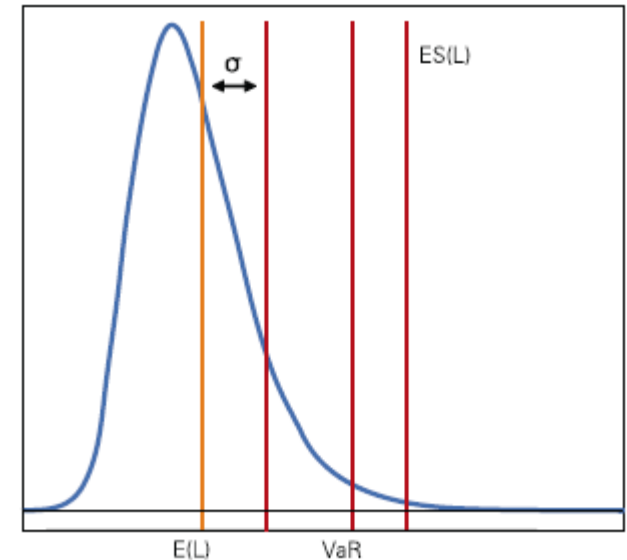
Standard Deviation.

Value at Risk (VaR).

Expected Shortfall (eVaR).

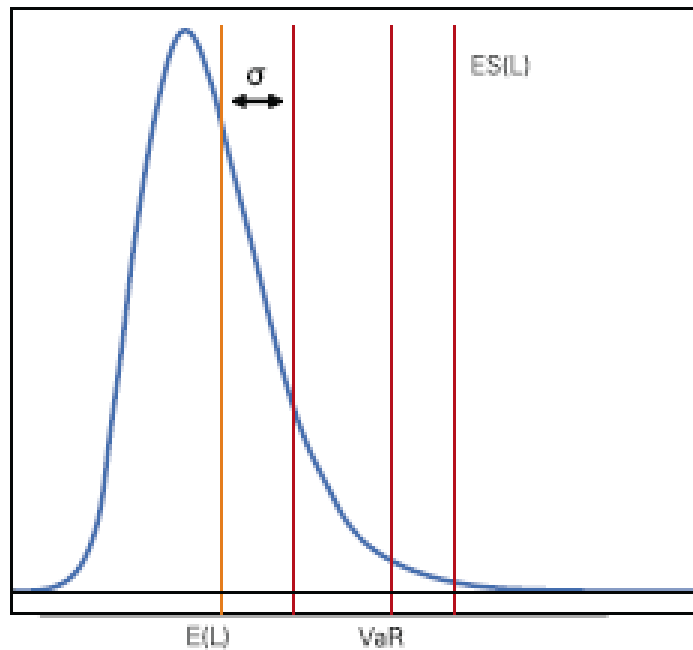
Economic Capital (EC).

- + The most flexible and accurate method.
- + It can be calculated for any level of granularity.
- + It can be aggregated along any dimension.
- + It does not reduce information to a single number.
- + **It allows for netting or diversification effects.**
- + It allows to measure and aggregate the risk of derivative instruments.
- The development of models can be challenging, and the data availability and estimation of parameters can be as well.
- Furthermore, the suitability of models and measures will have to be verified by performing back-testing exercises on a regular basis..



# Expected Shortfall

**Expected shortfall (ES):** the expected loss conditional on the real loss exceeding VaR, overcoming VaR main weakness as it is not sub additive (a merger of two portfolios does not generate additional risk).



# Economic Capital: Some Choices

## Choice of a Time Horizon

- Credit risk, a time horizon of one year is commonly used.
- Trading risk: the 1-day or 10-day VaR.

## Choice of the Confidence Level

- Link with the Risk appetite and the level of solvency (AAA-D).

## Choice of type of calculation

- Stand Alone vs Marginal EC.
- Allocation methods.

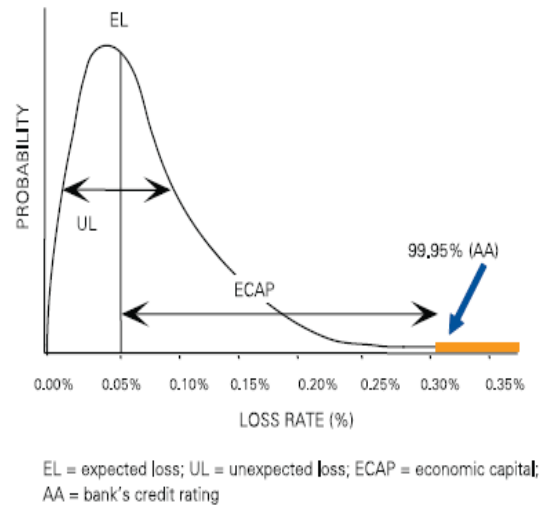


Rating (S&P)	Maximum Default Probability	Required Confidence Level
AAA	0.015%	99.985%
A	0.060%	99.940%
A-	0.070%	99.930%
BBB+	0.110%	99.890%
BBB	0.200%	99.800%



# Economic Capital Purpose

Capital fulfils the purposes of shielding the bank against unexpectedly high realizations of risks (credit losses from defaults or downgrades, a fall in the value of market instruments, or processing errors) and, potentially, bankruptcy.



Banks commonly create buffers in the form of general provisions for losses that might be reasonably expected to occur. However, actual losses are often different from expectations, and capital is held to cover unforeseen possibilities.

EC therefore explicitly links the risk appetite of the shareholders to the actual risk assumed by the bank. Economic capital is a key tool for the risk management function in understanding and quantifying the risk undertaken so as to support capital adequacy and value-based management.

Economic Capital is an indispensable concept because it allows banks to measure and manage the overall risks of a bank in a “common currency”.

**It is a Portfolio Measure: it takes into consideration correlations.**

### 3. Concentration Risk. Main current Approaches

- I. Basel Approach to Concentration Risk
- II. Rating Agency Approach to Concentration Risk. An example with S&P
- III. Multifactor Models and Concentration Risk

# Concentration Risk

- Significant improvements in understanding and measuring of Concentration Risk in credit portfolios.
- **Single-name, Industry and Country concentrations** are major risks and constraining rating factors for many financial institutions.

# Concentration Risk

The **measurement** of Concentration Risk in credit portfolios is necessary for :

- Determining regulatory capital under **Pillar II of Basel II**.
- **Rating Agencies** recognize single-name concentration in their ratings methodologies and highlight Concentration Risk as potential ratings negative.
- Concentration Risk measurement is important for **managing portfolios internally and allocating economic capital**.

### 3. Concentration Risk. Main current Approaches

#### I. Basel Approach to Concentration Risk

# Basel Approach

**The Basel II formula for measuring the VaR of credit portfolios is based on the so-called asymptotic single risk factor (ASRF) framework as explained in Gordy (2003).**

- The portfolio is infinitely fine grained and thus it consists of a nearly infinite number of credits with small exposures;
- Only one systematic risk factor influences the default risk of all loans in the portfolio.

## Basel II, Assumptions

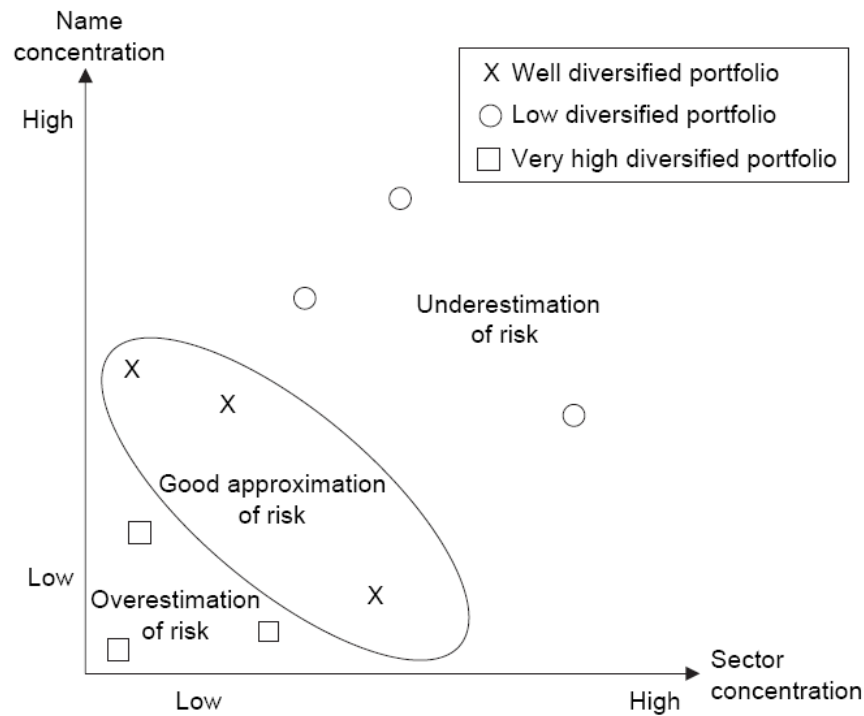
- The first assumption implies that there are **no name concentrations** within the portfolio, thus all idiosyncratic risk is diversified completely.
- The second assumption implies that there are **no sector concentrations** such as industry- or **country-specific risk concentrations**.

These are idealizations that can be problematic for real-world portfolios.

- Since it is difficult to incorporate credit risk concentrations into analytic approaches, in Basel II there is **no quantitative approach mentioned for how to deal with risk concentrations**.
- Instead, it is **only qualitatively demanded in Pillar II of Basel II** that “Banks should have in place effective internal policies, systems and controls to identify, measure, monitor, and control their credit risk concentrations” (see BCBS (2006a, §773)).
- **Supervisors interpret** concentration risk as “a positive or negative deviation from Pillar I minimum capital requirements derived by a framework that does not account explicitly for concentration risk” (see BCBS (2006b)).
- Pillar I capital rules were **calibrated on well-diversified portfolios** with low name and low sector concentration risk (see BCBS (2006b) and CEBS (2006, §18)).

# Basel and Concentration Risk

For well diversified portfolios the Basel II formula is a good approximation of the “true” risk.





# Basel II. RC. IRB Formula

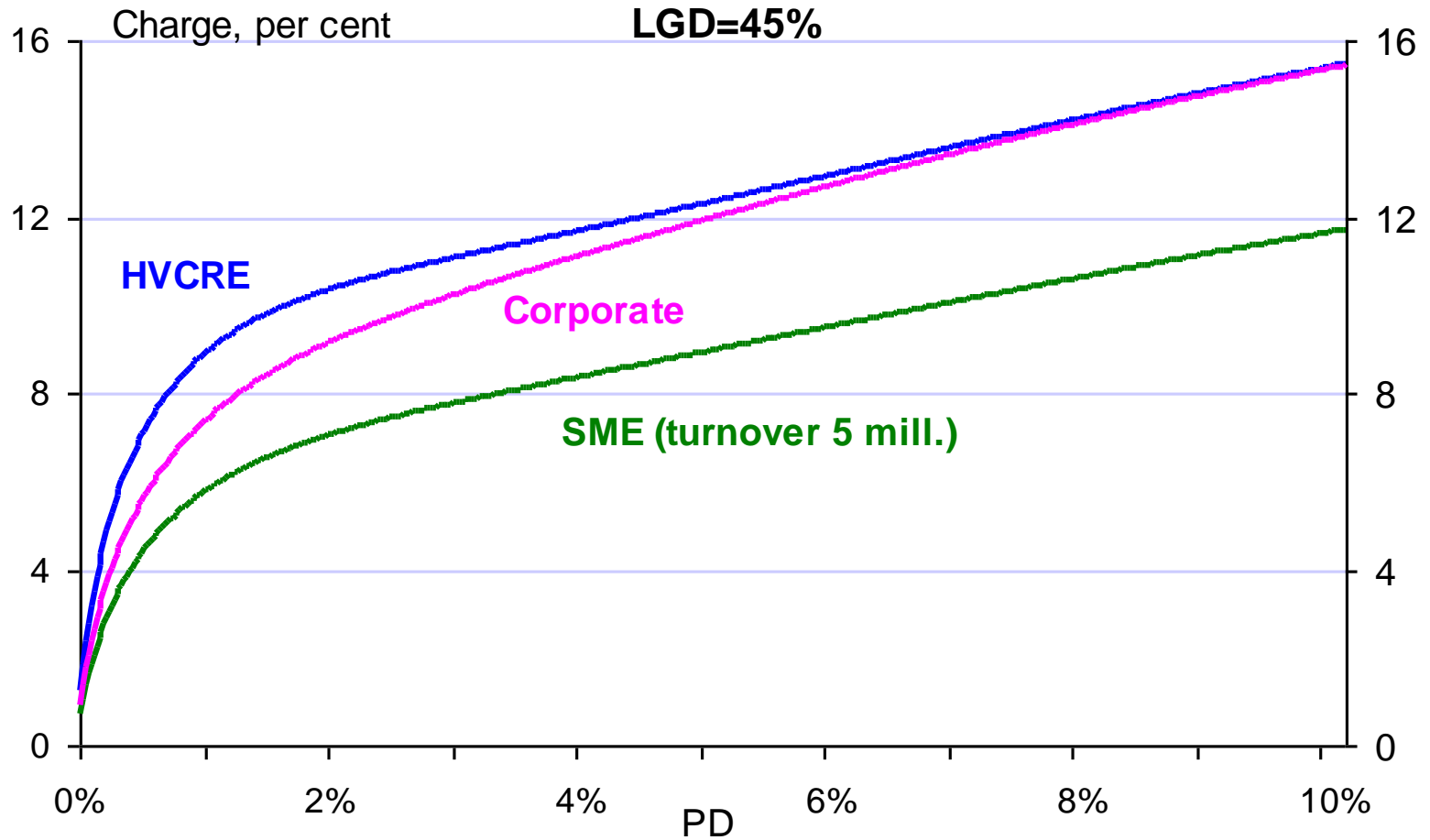
## Corporates, Sovereigns y Banks

$$K = \text{LGD} * N[(1-R)^{-0.5} * G(\text{PD}) + (R/(1-R))^{0.5} * G(0.999) - \text{PD} * \text{LGD}] * (1 - 12.5 * b(\text{PD}))^{-1} * (1 + (M - 2.5) * b(\text{PD}))$$

- $R = 0.12 * (1 - \exp(-50 * \text{PD})) / (1 - \exp(-50)) + 0.24 * [1 - (1 - \exp(-50 * \text{PD})) / (1 - \exp(-50))]$
- $b = (0.11852 - 0.05478 * \log(\text{PD}))^2$
- $\text{RWA} = K * 12.5 * \text{EAD}$
- $N(x)$  denotes the standard normal cumulative distribution function
- $G(y)$  stands for the inverse of the standard normal cumulative distribution function

**No Concentration Risk parameters**

# Basel II. RC. Enfoque IRB



## Basel II on Credit Risk Concentration

- Basel II under its Pillar II mandates that **banks should conduct an internal capital adequacy assessment** to cover all type of risks including **credit concentration risk**.
- Provided **no methodology for measuring credit concentration risk**.
- Basel created a Research Task Force to study credit risk concentration and examine the tools fit for its quantification.

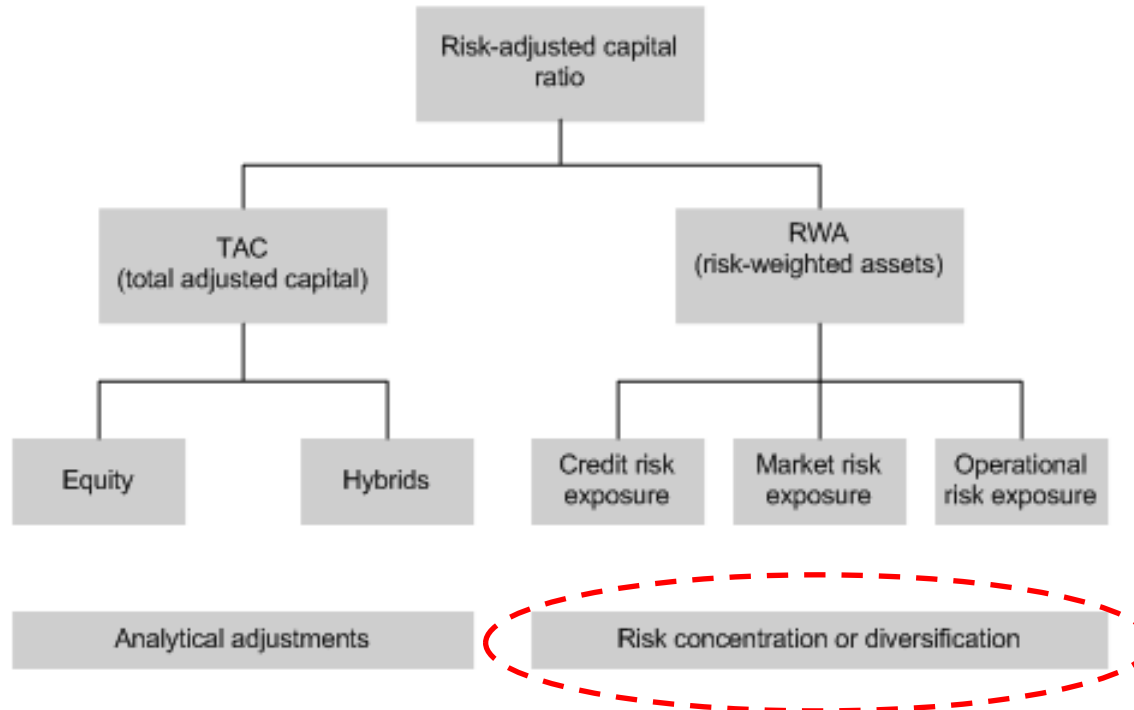
### 3. Concentration Risk. Main current Approaches

#### II. Rating Agency Approach to Concentration Risk

# Standard & Poors

## Calculating the RAC Ratio

Chart 1  
Building Blocks For The Risk-Adjusted Capital Framework



# S&P- Risk Adjusted Capital

**Table 1**

Computing Risk-Adjusted Capital		
<b>Risk-adjusted capital (RAC)</b>	=	<b>Total adjusted capital (TAC)</b>
		<b>Risk-weighted assets (RWA)</b>
where		
Total adjusted capital (TAC)	=	See Table 2
Risk-weighted assets (RWA)	=	RWA credit risk + RWA market risk + RWA operational risk
RWA credit risk	=	RAC charges x 12.5 x adjusted exposure
RAC charges	=	Unexpected losses that we define as losses incurred beyond normalized losses in a given stress scenario
Adjusted exposure	=	Amount Standard & Poor's anticipates will be the bank's exposure at the point of a debtor's default. This amount may not be the same as the amount outstanding at a particular reporting date. (For Basel II* institutions, it is the same as the regulatory exposure at default with a few exceptions.)
Normalized loss	=	Average "through the cycle" annual loss rates that are expected to occur for a given class of exposure (and a given country).

\*Basel II refers to the requirements set out under the Bank for International Settlement's "Basel Committee on Banking Supervision's Basel II: International Convergence of Capital Measurement and Capital Standards: A Revised Framework - Comprehensive Version," paper of November 2005, and subsequent amendments.

# S&P. Total Adjusted Capital (TAC)

**Table 2**

## Calculation Of Total Adjusted Capital

Common shareholders' equity
Add "Minority interests: Equity"
Deduct dividends not yet distributed
Deduct revaluation reserves
Deduct goodwill and nonservicing intangibles
Deduct interest-only strips
Deduct deferred tax loss carry forwards
Add or deduct postretirement benefit adjustments
Add or deduct cumulative effect of credit-spread-related revaluation of liabilities
Add or deduct other equity adjustments
= Adjusted common equity (ACE)
Add preferred stock and hybrid capital instruments (subject to limits)
=Total adjusted capital (TAC)

## S&P. Risk-Weighted Assets (RWA)

- S&P obtains the risk weights by dividing the RAC charge by 8%, which is equivalent to multiplying the RAC charge by 12.5.
- They use the risk weights to adjust the value of a bank's assets relative to our view of their riskiness and potential for default, in a method similar to that broadly used in the industry. This helps them make comparisons between the RAC ratio and regulatory-based capital ratios.

The framework **breaks credit risk down into six categories**: governments, financial sector, corporate sector, retail and personal sector, counterparty risk, and securitizations. It then accounts for the impact of collateral and other risk mitigation on the RWA.



## S&P. RWAs. Governments

- **Governments:** They apply different risk weights according to the rating on the sovereign issuer. Those risk weights for sovereign and local authority exposures are based on S&P's foreign currency credit rating on the sovereign, except for domestic government securities in local currency that are based on the local currency rating.

Table 4

Risk Weights For Government Exposures		
Sovereign long-term foreign currency credit rating	Central government (%)	Local or regional government (%)
AA- and above	3	4
A+	5	6
A	9	11
A-	15	18
BBB+	23	28
BBB	34	41
BBB-	47	56
BB+	62	74
BB	79	95
BB-	99	119
B+	122	146
B	146	176
B- and below	173	208

## S&P. RWAs. Financial Sector

- **Financial sector:** Financial exposures fall into two categories, financial institutions and covered bonds. The framework applies risk weights according to BICRA score for the country in which the exposures are domiciled.

**Table 5**

Risk Weights For Financial Sector Exposures		
Overall BICRA score	Financial institutions (%)	Covered bonds (%)
1	15	10
2	17	11
3	23	16
4	33	22
5	48	32
6	66	44
7	88	58
8	114	76
9	144	96
10	178	118

BICRA—Banking industry country risk assessment.

## S&P. RWAs. Corporates

- **Corporate sector:** Corporate exposures fall into two categories: corporate, and construction and real estate development. It applies risk weights according to the economic risk score from BICRA analysis.

**Table 6**

<b>Risk Weights For Corporate Sector Exposures</b>		
<b>Economic risk group</b>	<b>Corporate (%)*</b>	<b>Construction and real estate development (%)</b>
1	80	180
2	88	198
3	100	225
4	116	261
5	136	306
6	161	363
7	189	426
8	223	501
9	259	582
10	300	675

\*RACF applies the risk weight to exposure at default (EAD) minus a 25% haircut, which recognizes the significant contribution to EAD from undrawn commitments.

## S&P. RWAs. Retail

- **Retail and personal:** they classify retail exposures into five categories: prime residential mortgages, auto loans, credit cards, self-certified mortgages, and other unsecured/retail lending to SMEs. S&P applies risk weights according to the economic risk score from the BICRA analysis.

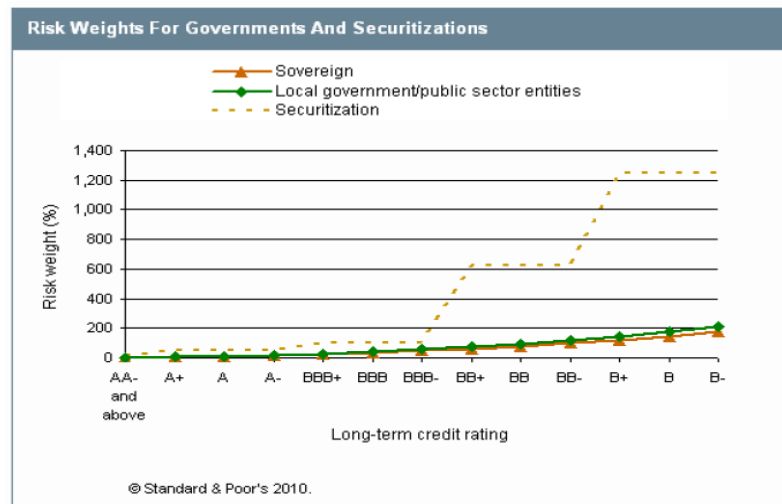
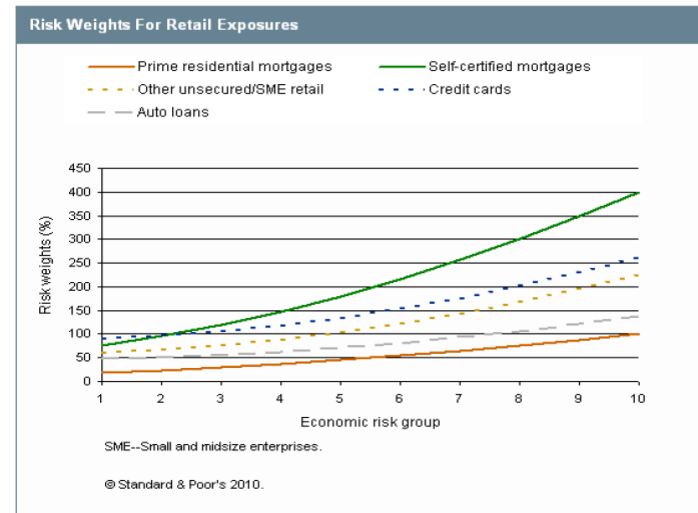
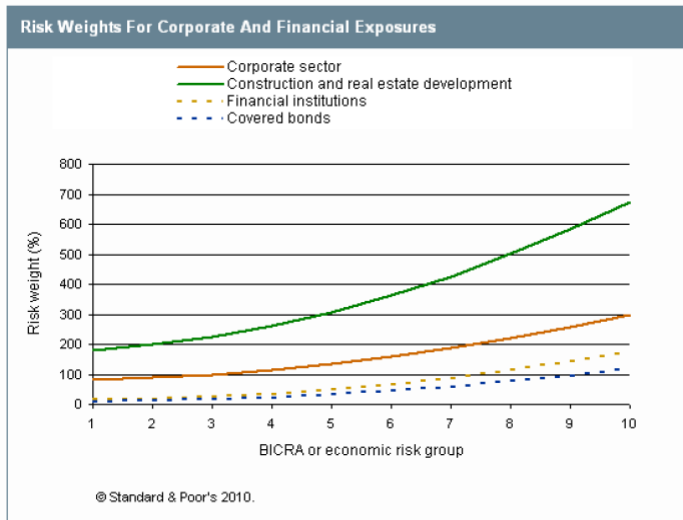
**Table 7**

### Risk Weights For Retail And Personal Exposures

Economic risk group	Prime residential mortgages (%)	Self-certified mortgages (%)	Credit cards (%)	Auto loans (%)	Other unsecured/SME retail (%)
1	19	76	89	48	60
2	24	96	96	51	66
3	30	120	105	56	75
4	37	148	118	63	87
5	45	180	134	71	102
6	54	216	153	81	121
7	64	256	176	93	142
8	75	300	201	107	167
9	87	348	230	122	194
10	100	400	263	139	225

SME--Small and midsize enterprises.

# S&P. RWAs



## S&P- RWAs.

### Collateral and other credit risk mitigation

- It is accounted through a combination of different risk weights, reduction of exposure amounts, recognition of credit substitution, and by making standard adjustments.
- We may lower our risk weights to reflect our view of the effects of credit risk mitigation, which may take the form of:
  - Financial collateral;
  - Guarantees from a financial institution or a sovereign; or
  - Credit default swaps.

## S&P. Market Risk

- **Trading activities:** S&P applies a risk weight for market risk from trading activities, which is a multiple of the regulatory risk weight, derived either from a value-at-risk (VAR) calculation validated by regulators, the Basel standardized approach, or a combination of the two.
- **Equity investments:** The S&P applies risk weights to three different types of equity investments: listed securities, unlisted securities, and investments in unconsolidated subsidiaries. It classifies listed equity investments into four equity market groups by country, based on the volatility we have observed in that country's main stock market index over the past 30 years.

# S&P. Market Risk

## Risk Weights For Equity Investment Exposures

Equity market group	Listed securities (%)	Unlisted securities (%)	Minority holdings in unconsolidated financial institutions (%)
1	563	688	1,250
2	688	813	1,250
3	813	938	1,250
4	938	1,063	1,250

- For **unlisted equities**, they add 10% (equivalent to a 125% risk weight add-on) to the charge we apply for listed equity investments.
- The RAC charges apply to the fair value of equity holdings.



# Operational risk and associated Risk Weights

- S&P applies risk weights to all business lines according to either their **revenue contribution** or the **size of assets** under management or custody.

## Risk Weights For Business Lines By Revenue

<b>Business line</b>	<b>Risk weight to be applied to revenue (%)</b>
Asset management, retail banking, retail brokerage	150
Commercial banking and custody	188
Payment and settlement	225
Corporate finance, trading and sales	313
Other or no details to allocate in the first four buckets	188

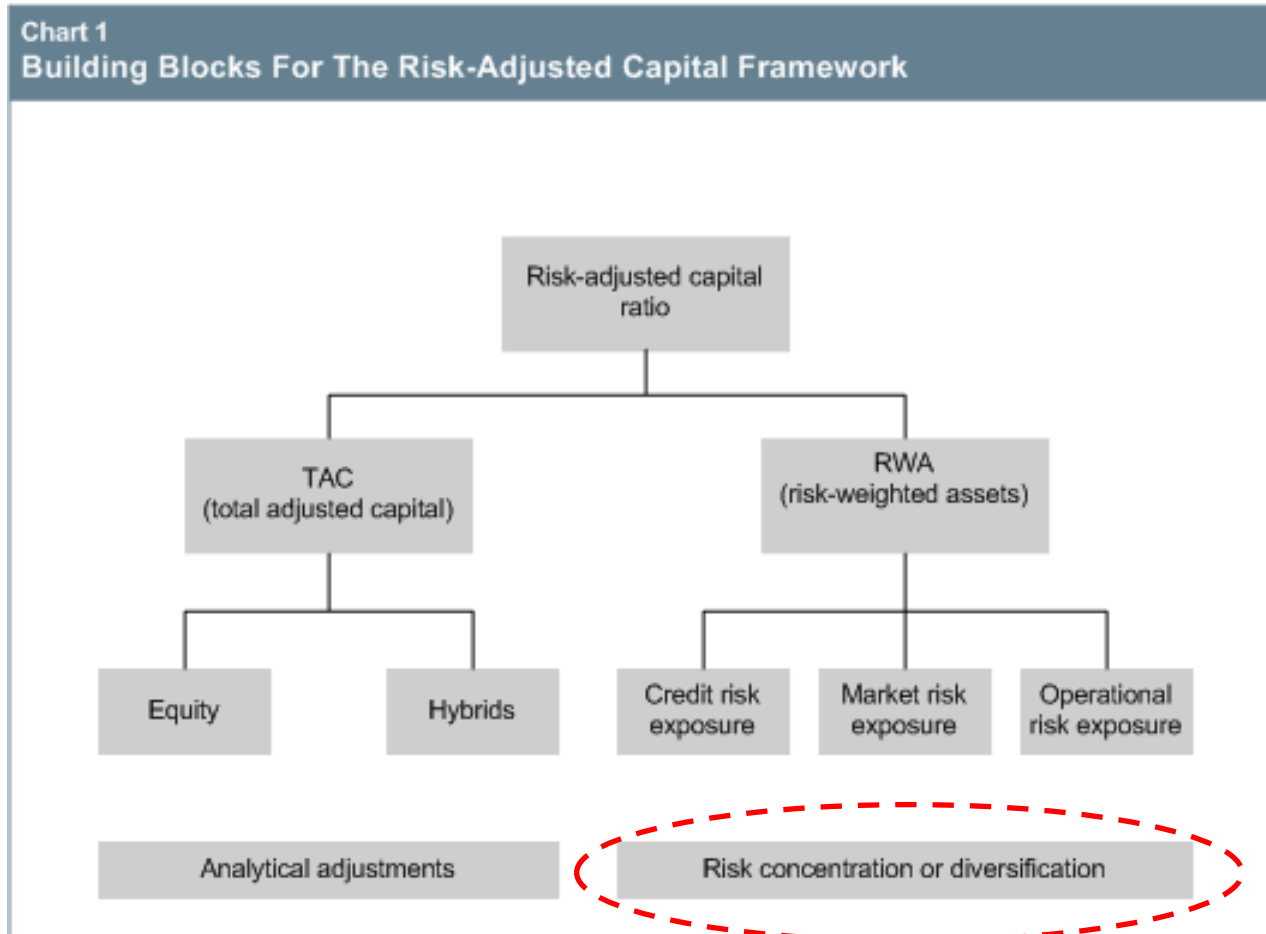
- If a breakdown of revenues by business line is not available, S&P applies a 188% risk weight to the highest annual revenue of the past three year.

## S&P. Other Risks Not Covered

- S&P framework is not intended to capture risks such as:
  - Interest rate and currency risk in the banking book;
  - Volatility of pension funding;
  - Funding risk;
  - Reputation risk; or
  - Strategic risk.
  
- Such risks are covered **qualitatively** in other areas of the methodology.

# Standard & Poors

## Calculating The RAC Ratio



## S&P. Risk Concentration/ Diversification

- **S&P also quantifies the potential impact of risk concentration or diversification on RWA.**
- S&P's framework takes into account **single-name concentration** (the aggregate of large exposures to a single borrower or counterparty), as well as the correlation of risk by **geography**, **sector** type, and **business line**.

## S&P. Risk Concentration

S&P calculates **an adjustment to RWA** to reflect the impact of concentration or diversification of risks.

- Adjustment to RWA in corporate exposures for correlations among different industries;
- Adjustment to total RWA for correlations among country or regional exposures;
- Adjustment to total RWA for correlations among different business lines;
- Add-on to total corporate RWA to capture single-name concentrations in the corporate book using the largest 20 named corporate exposures.

S&P sets a **cap** on the overall benefit of concentration and diversification adjustments to 30% for the most diversified global financial institutions.

# S&P. Risk Concentration

## Industry sector, geographic, and business line methodology:

A concentration multiplier to RWA, then they determine the aggregate RWA for the various portfolios using a **correlation matrix** (based on the **Markowitz** covariance/variance formula):

$$\text{Adjusted Capital Charge} = \sqrt{\begin{pmatrix} K_{1*}C_1 \\ \dots \\ K_{n*}C_n \end{pmatrix}^T \begin{pmatrix} 1 & \dots & R_{1,n} \\ \dots & \ddots & \dots \\ R_{n,1} & \dots & 1 \end{pmatrix} \begin{pmatrix} K_{1*}C_1 \\ \dots \\ K_{n*}C_n \end{pmatrix}}$$

Where:

- $K_i$  is the RAC charge for either the industry sector, geographic region, or business line (i) in order to compute the total risk weight adjusted for industry sector, geographic region, business line concentration, or diversification;
- $C_i$  is the Concentration factor for the industry sector, geographic region, or business line (i); and
- $R_{i,j}$  is the Correlation coefficient between the industry sectors, geographic regions, or business lines) i and j.

# Industry Sector Concentration Factors

- S&P calculates the concentration factors using the volatility of the respective **MSCI** sector stock market index.
- The volatility is calculated as the standard deviation of the monthly log returns over the past 20 years.

Sector Concentration Factors	
Industry sector	Concentration factor (%)
Consumer discretionary	103
Consumer staples	97
Energy	104
Financials	106
Health care	98
Telecom services	104

Sector Concentration Factors (cont.)	
Utilities	98
Information technology	113
Industrials	103
Materials	106
Capital goods	105
Commercial and professional services	106
Transportation	102
Automobiles and components	105
Consumer durables	106
Consumer services	106
Media	110
Retailing	107
Food and staples retailing	108
Food, beverages, and tobacco	98
Household and personal products	101
Health care equipment and services	106
Pharmaceutical and biotechnology	99
Banks	107
Diversified financials	110
Insurance	115
Real estate	109
Software and services	115
Semiconductors	112
Technology hardware and equipment	115

# Geographic Region Concentration Factors

- S&P uses a multiplier based on the logarithm of the **GDP** of the country in which the bank is located.
- In practice, the concentration multiplier diminishes by a constant factor each time the GDP doubles.
- This concentration factor reflects the view that, in general, **the smaller an economy is, the less diversified it is.**

<b>Region or group of countries</b>	
Africa	113
Asia Pacific	103
Baltic	123
Caribbean	124
Eastern Europe	114
European Union	102
Gulf Cooperation Council	114
Latin America	107
North Africa	116
North America	100
Southeast Asia	112



# Correlation Matrices

- For correlations by geographic regions and industry sectors, S&P uses the **MSCI** stock indexes.
  - MSCI stock indexes: monthly returns of the index as a compromise between stability and the number of data points from 1987 to 2010.
  - S&P first computed Pearson correlations of these MSCI index returns, then they stressed the results to capture more fat-tail risks. To do so, They use a Fisher transformation and stress the resulting value to a confidence interval of 99.5%.
- Business line correlations are based on analytical judgment.

# Correlation Matrices

## Sector Correlation Matrix (Selected Sample)

Industry sector	--Correlation factors (%)--										
	CD	CS	EN	FN	HC	TC	UT	IT	IN	MT	
Consumer discretionary (CD)	<b>100</b>	68	66	88	62	77	65	86	93	84	
Consumer staples (CS)	68	<b>100</b>	62	79	79	55	76	47	74	70	
Energy (EN)	66	62	<b>100</b>	68	55	53	73	56	75	81	
Financials (FN)	88	79	68	<b>100</b>	72	69	74	71	91	82	
Health care (HC)	62	79	55	72	<b>100</b>	58	69	53	67	59	
Telecommunication services (TC)	77	55	53	69	58	<b>100</b>	60	79	72	63	
Utilities (UT)	65	76	73	74	69	60	<b>100</b>	49	74	71	
Information technology (IT)	86	47	56	71	53	79	49	<b>100</b>	80	67	
Industrials (IN)	93	74	75	91	67	72	74	80	<b>100</b>	90	
Materials (MT)	84	70	81	82	59	63	71	67	90	<b>100</b>	

## Geographic Correlation Matrix (Selected Sample)

Country	--Correlation factors (%)--										
	United States	Japan	Europe	United Kingdom	France	Germany	Italy	Spain	China	World	
United States	<b>100</b>	55	83	79	77	76	64	73	60	91	
Japan	55	<b>100</b>	65	64	59	53	55	62	41	80	
Europe	83	65	<b>100</b>	94	93	93	81	87	55	94	
United Kingdom	79	64	94	<b>100</b>	81	79	68	78	57	89	
France	77	59	93	81	<b>100</b>	90	76	81	52	86	
Germany	76	53	93	79	90	<b>100</b>	77	80	54	84	
Italy	64	55	81	68	76	77	<b>100</b>	77	37	76	
Spain	73	62	87	78	81	80	77	<b>100</b>	53	84	
China	60	41	55	57	52	54	37	53	<b>100</b>	59	
World	91	80	94	89	86	84	76	84	59	<b>100</b>	

## Business Line Diversification Matrix

Business line	--Correlation factors (%)--							
	Sovereign	Financial institutions	Corporate	Real estate	Other retail	Trading and equity	Asset management	Insurance
Sovereign	<b>95*</b>	85	85	85	85	85	85	50
Financial institutions	85	<b>95*</b>	50	50	25	85	85	50
Corporate	85	50	<b>95*</b>	50	25	85	85	50
Real estate	85	50	50	<b>95*</b>	50	85	25	50
Other retail	85	25	25	50	<b>95*</b>	85	25	50
Trading and equity	85	85	85	85	85	<b>95*</b>	85	50
Asset management	85	85	85	25	25	85	<b>95*</b>	50
Insurance	50	50	50	50	50	50	50	<b>95*</b>

# Single-Name Concentration Adjustment

- **S&P** calculates the concentration charge for exposures to single names in the corporate exposures using a model based on the granularity adjustment described and tested by **Gordy** and **Lütkebohmert** (2007). They apply the model to a bank's total corporate exposures and largest 20 corporate exposures.
- The methodology is derived as a first-order asymptotic approximation for the effect of diversification in large portfolios within the **CreditRisk+ methodology** for calculating the distribution of possible credit losses from a portfolio, developed by **Credit Suisse**. The theoretical tools for this analysis were proposed first by **Gordy** (2004) and refined significantly by **Martin** and **Wilde** (2003).

## Single-Name Concentration Adjustment

$$\begin{aligned}
 \text{Add-on} &= 11.7 \left[ \frac{1}{2K^*} \sum_{i=1}^m s_i^2 Q_i C_i + \bar{s}((\delta - 1)(K^* - K_m^*) + \delta(R^* - R_m^*)) \right]^2 \\
 &+ 0.19 \frac{1}{2K^*} \left[ \sum_{i=1}^m s_i^2 Q_i C_i + \bar{s}((\delta - 1)(K^* - K_m^*) + \delta(R^* - R_m^*)) \right]
 \end{aligned}$$

In practice, S&P derives an add-on from the breakdown of the top 20 corporate exposures, according to this formula, which is a quadratic scaled version of the formula proposed as upper-bound by **Gordy** and **Lütkebohmert**:

- parameter  $\delta$  equals 4.83;
- $K^*$  is the RAC charge for the entire corporate portfolio (as a percentage of EAD);
- $R^*$  is Standard & Poor's normalized loss for the entire corporate portfolio (as a percentage of EAD);
- $s_i = \text{EAD}(i)/\text{total corporate EAD}$  is the share of the corporate portfolio corresponding to exposure  $i$ ;
- $K_i$  is the Basel II unexpected loss for exposure  $i$  (as a percentage of EAD) computed using the Basel II foundation IRB formula, where the probability of default ( $PD_i$ ) is set as Standard & Poor's long-term average global corporate default rate for the rating class if the exposure is rated. If the exposure is not rated we use the 'BB-' default rate;
- $R_i = PD_i * 45\%$  is the Basel II foundation IRB expected loss for exposure  $i$  (as a percentage of EAD);
- $K_m^*$  is the cumulative unexpected loss for the  $m$  largest exposures (as a percentage of EAD);
- $R_m^*$  is the cumulative expected loss for the  $m$  largest exposures (as a percentage of EAD);
- $C_i = (45\%^2 + \text{VLGD}_i^2)/45\%$  where VLGD is the volatility of LGD (loss-given default).  $C_i$  can be viewed as a stressed LGD using its normalized variance:
- $\text{VLGD} = \sqrt{0.25 * 45\% * (1 - 45\%)}$

$Q_i = \delta * (K_i + R_i) - K_i$  is used for notational convenience.

# Single-Name Concentration Adjustment

- A number of **academic studies** provide either direct or indirect estimates of the importance of granularity risk for bank portfolios. The effect is clearly more pronounced for smaller portfolios.
- An indicative calculation of the upper boundary of the contribution of idiosyncratic risk to economic capital can be performed by reference to a portfolio having the maximum permissible concentration under the **EU's large-exposure rules**. Such calculations give estimates of 13% to 21% higher portfolio value-at-risk for this highly concentrated portfolio versus a perfectly granular one that is comparable in all other dimensions.
- For portfolios that are more typical for an "actual" bank (as opposed to a theoretical portfolio with the maximum concentration that EU large-exposure rules would allow), the impact of name concentration is substantially lower.
- **Gordy and Lütkebohmert** (2007) use characteristics of loans from the German credit register to compare the effect of name concentration on loan portfolios of the size that can be found in actual banks. For large credit portfolios of more than 4,000 exposures, they estimate that name concentration can contribute about 1.5% to 4% of portfolio value at risk. For smaller portfolios (with 1,000 to 4,000 loans), they estimate that a range between 4% and 8% is more likely.

## Gordy's vs S&P Approach

**Michael Gordy's** Upper bound estimate formula for the heterogeneous case:

$$GA_m^{upper} = \frac{1}{2K^*} \left( \sum_{i=1}^m s_i^2 Q_i C_i + \bar{s} \left( (\delta - 1)(K^* - K_m^*) + \delta(R^* - R_m^*) \right) \right)$$

**S&P's** formula for concentration on non-sovereign operations (Bank Capital Methodology, December 2010):

$$Add - on = 11.7 \left[ \frac{1}{2K^*} \left( \sum_{i=1}^m s_i^2 Q_i C_i + \bar{s} \left( (\delta - 1)(K^* - K_m^*) + \delta(R^* - R_m^*) \right) \right) \right] + 0.19 \frac{1}{2K^*} \left[ \sum_{i=1}^m s_i^2 Q_i C_i + \bar{s} \left( (\delta - 1)(K^* - K_m^*) + \delta(R^* - R_m^*) \right) \right]$$

Michael Gordy's formula alone is **complicated enough**. The revision introduced by S&P made the formula much more opaque.

For single name concentration, the approaches developed by (i) Gordy and Lütkebohmert and (ii) Emmer and Tasche are reasonable.

## 3. Concentration Risk. Main current Approaches

### III. Multifactor Models and Concentration Risk

## Multifactor Models

***The measurement and management of risk concentrations are not only important for the determination of regulatory but also for the measurement of the “true” portfolio risk: Economic Capital.***

- Name concentrations, as well as sector and Country concentrations, have already been analyzed in the literature.
- The theoretical derivation of the so-called granularity adjustment that accounts for name concentrations was done by **Wilde** (2001) and improved by **Pykhtin** and **Dev** (2002) and **Gordy** (2003).
- The adjustment formulas are derived in a more straightforward approach by **Martin** and **Wilde** (2002), **Rau-Bredow** (2002) and **Gordy** (2004).
- Furthermore, the adjustment is extended and numerically analyzed in detail by **Gürtler** et al (2008).
- An approach related to **Wilde** (2001) is the granularity adjustment from **Gordy** and **Lütkebohmert** (2007).
- In contrast, the semi-asymptotic approach from **Emmer** and **Tasche** (2005) refers to name concentrations due to a single name, while the rest of the portfolio remains infinitely granular. Thus, this type can be called “individual name concentration”.



# Multifactor Models

- There also exist analytic and semi-analytic approaches that account for sector concentrations.
- One rigorous analytical approach is **Pykhtin** (2004) that is based on a similar principle as in **Martin** and **Wilde** (2002).
- An alternative is the semianalytical model from **Cespedes** et al (2006) that derives an approximation formula through a complex numerical mapping procedure.
- Another approach from **Düllmann** (2006) extends the binomial extension technique (BET) model from **Moody's**.
- **Tasche** (2006) suggests an ASRF-extension in an asymptotic multi-factor setting.
- Some numerical work on the performance of the **Pykhtin** model has been done by **Düllmann** and **Masschelein** (2007).
- Furthermore, **Düllmann** (2007) presented a first comparison of different approaches on sector concentration risk.

# Multifactor Models

**The problem is that the derivation and the application of the approaches are often inconsistent with the Basel II framework, which is critical.**

- **Gürtler, Hibbeln, Vöhringer (2010)** suggest that Pykhtin model provides a methodology to perform multi-factor models that are able to measure concentration risk in credit portfolios in terms of economic capital and still deliver results that are consistent with Basel II.
- **Gürtler, Hibbeln, Vöhringer (2010)** proposed a methodology to perform multi-factor models that are able to measure concentration risk in credit portfolios in terms of economic capital and still deliver results that are consistent with Basel II.
- They applied that methodology to different multi-factor approaches (**Montecarlo Siumulation, Pykhtin (2004)** and **Cespedes et al (2006)**) and compared their performance showing that it is possible to achieve good approximations in a reasonable time when the approaches are adjusted in the proposed way.

## Multifactor Models

- They chose input parameters, especially the inter- and intra-sector correlations, in a way that the results are comparable with the regulatory Pillar I capital.
- Hence, it is possible to directly consider the extent of credit risk concentrations in the assessment of capital adequacy under Pillar II.
- Using these modifications, They performed an extensive numerical study similar to Cespedes et al (2006) to obtain a closed-form approximation formula. This allows to compute the Pykhtin formula much faster than Monte Carlo simulations even for a high number of credits.
- Following this methodology they detected that the **Pykhtin model** leads to very good results for homogeneous as well as heterogeneous PDs when EADs are homogeneous. The performance is slightly lower for heterogeneous EADs.
- The results of the **Cespedes model** have a high accuracy throughout. Interestingly, the approach works better for heterogeneous portfolios.
- In general, both models can be used for approximating the economic capital in a multi-factor setting when adjusted in the proposed way. **The main advantage of the Pykhtin model is that it can be directly applied to an arbitrary portfolio type**, whereas the approach of Cespedes et al (2006) should not be used without initially performing the demonstrated extensive numerical work.

## 4. Some Conclusions

## Conclusions

- **Existing approaches** for measuring Concentration Risk are mostly **not fully consistent** with the **new capital adequacy framework** (Basel II, BCBS (2006a)).
- *Basel II* under its Pillar II mandates that banks should conduct **an internal capital adequacy assessment** to cover all type of risks including credit concentration risk . It provides no methodology for measuring credit concentration risk.
- **S&P's approach** for concentration seems theoretically and practically **too punitive**.
- **Basel does not recommend an approach** since their IRB approach assumes a perfectly granular portfolio which fully diversifies away idiosyncratic risk.
- **Neither Moody's nor Fitch provide alternatives**. Some alternatives from the academic literature, ex. Emmer and Tasche, may be difficult to implement.
- For single name concentration, the approaches developed by **(i) Gordy and Lütkebohmert and (ii) Emmer and Tasche are reasonable**.
- Gürtler, Hibbeln, Vöhringer (2010) suggest that **Pykhtin model provides a methodology to perform multi-factor models that are able to measure concentration risk in credit portfolios in terms of economic capital and still deliver results that are consistent with Basel II**.
- A good approach would be **intuitive, fair and transparent**.

# Some Partial Solutions

- **Limits**

- The case for limits makes intuitive sense, but
- The choice of any particular limit may be arbitrary.
- No account taken of correlations among borrowers.
- Need to balance financial with business objectives.

- **Stress Testing**

- Also an intuitive approach, but
- The choice of stress tests is arbitrary. The same test may yield very different results based on implementation technique.
- The most stressful tests are also the most implausible. So difficult to serve as a basis for policy.

Thanks!

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