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Aggregation of risks and Allocation of capital

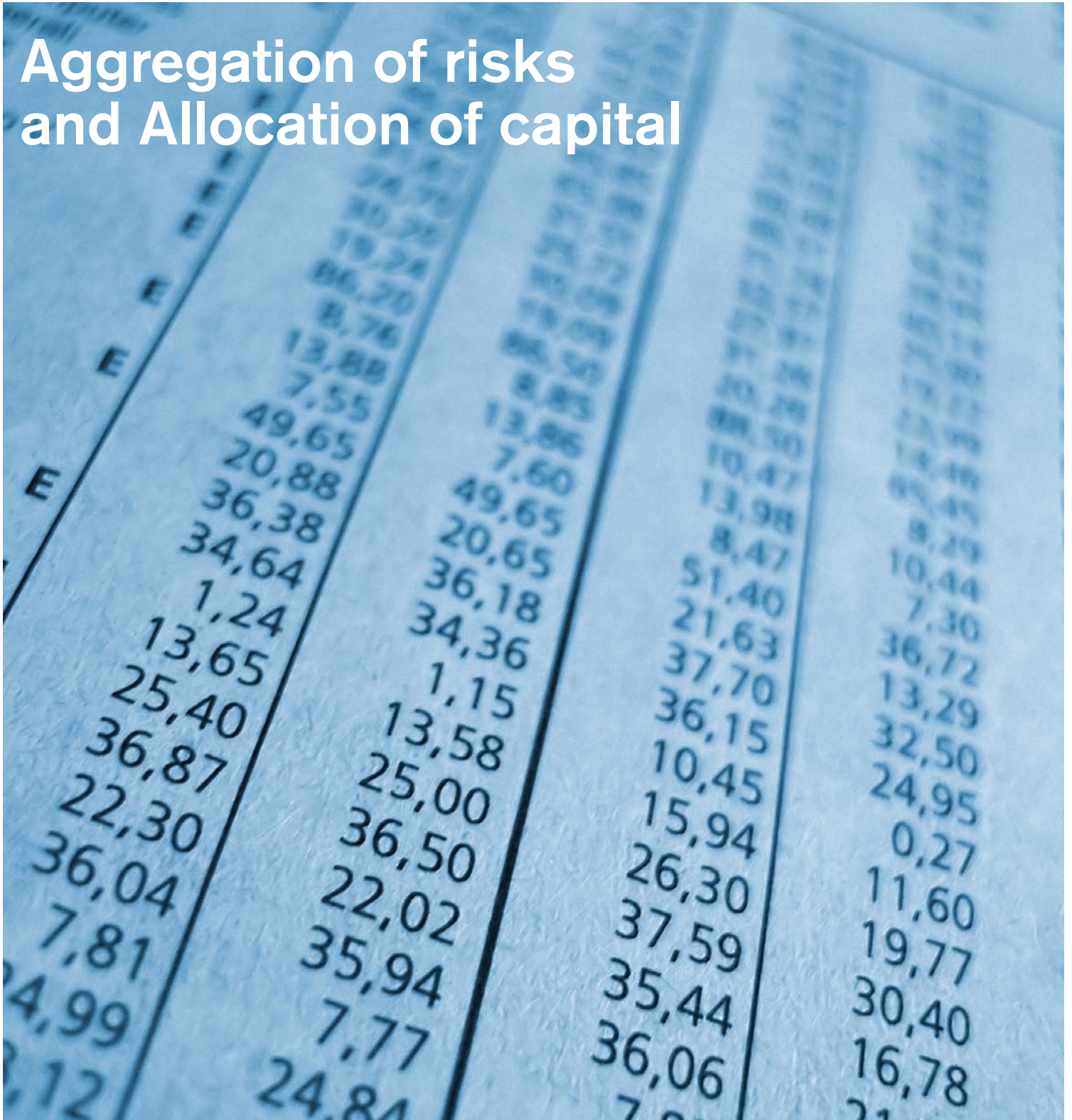




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1. INTRODUCTION AND BACKGROUND

Risk management and risk measurement are two hot topics at the moment. The financial crisis of 2008 made it clear that adequate risk management and monitoring of risks is key for survival in difficult times. However, simply employing high standards of risk management does not guarantee a secure future.

The identification and quantification of the risks that most companies face are relatively straight forward. This is in part due to the recent supervisory developments such as the Swiss Solvency Test (SST), the Individual Capital Assessment Standards (ICAS) used in the UK, the C3 Phase II standards in use in the US, and the harmonised Solvency II currently under development for the EU). The issues associated with the aggregation of risks and capital allocation are the next area of focus in the economic capital modelling practices of insurance companies.

The current focus is on calculating economic capital. However, business decisions need to be made based on risk budget and risk/return optimization. Economic capital plays a central role in prudential supervision, product pricing, risk assessment, risk management and hedging, capital allocation / project financing, performance management, and financial reporting.

Available capital is defined as financial resources available as risk-bearing funds to absorb adverse experience. This capital is held as a buffer to meet policyholder claims during adverse climates. (Required) economic capital is calculated based on a risk measure, from which there are many to choose. Different risk measures satisfy different purposes of capital determination.

Economic capital is aggregated across products, lines of business, business units, geographic and regulatory areas in order to calculate capital requirements at different levels of the organisation. Aggregation generally allows for some diversification benefits between the risks being aggregated, thus resulting in the aggregated capital being less than the sum of the parts.

Capital requirements are calculated at the lowest level first (for example, per line of business). Then the aim is to aggregate the capital requirements up to higher levels (for example, at business unit level) to arrive at capital requirements and risk measurement that take into account the interactions between the risks being aggregated (for example, the interaction between two lines of business, say annuities and mortality products). Ultimately, all the capital requirements are aggregated to arrive at holding-level total capital requirements.

Total capital requirements (for example, at group level) are therefore smaller than the sum of the capital requirements (for example, at product level.) For a range of purposes (for example, pricing or performance measurement), the total capital needs to be allocated back to the lower levels. That is, the diversification benefit achieved by aggregating the risks need to be allocated back to the individual risks. Again, there are a range approaches for doing this, depending upon the intended purpose. The allocation of capital is essential for pricing insurance products and is an important part of the planning and control cycle (risk budgeting and return measurement).

The financial crisis of 2008 made it clear that adequate risk management and monitoring of risks is key for survival in difficult times. However, simply employing high standards of risk management does not guarantee a secure future.

The allocation of capital is essential for pricing insurance products and is an important part of the planning and control cycle (risk budgeting and return measurement).

The diagram below illustrates the aggregation and allocation processes.

FIGURE 1



This white paper objectively examines the different techniques for aggregation and allocation of capital, but also includes our views on the various techniques.

The audience for this paper consists of actuaries and risk managers interested in better understanding the aggregation and allocation of economic capital. The paper will be of special interest to those who are in the process of developing economic capital models. This white paper objectively examines the different techniques for aggregation and allocation of capital, but also includes our views on the various techniques. Different methods are appropriate in different situations, and it is important the techniques presented here (and their shortcomings) are fully understood, both by the managers using the outputs, as well as the technicians creating the outputs. Furthermore, capital aggregation and allocation techniques have received much attention of late, so we encourage users of these techniques to stay up to date. The focus of the paper is on life insurance.

Chapter 2 introduces different risk measures, chapters 3 and 4 describe risks and risk distributions. Chapter 5 introduces possible capital aggregation techniques, and chapters 6 and 7 describe different methods and applications of capital allocation. Finally, chapter 8 describes some operational implementation issues given an economic capital framework.

2. RISK MEASURES

2.1. Introduction

Risk can be defined in many ways: the expected loss, the variance of a loss, the probability that a loss will exceed a defined amount, or the average amount by which it does exceed a defined amount. A risk measure needs to be selected that corresponds to how the risk in question is defined.

There exist a wide range of risk measures. Each one has its own unique characteristics, so the risk measure an insurer adopts will ultimately depend on the purpose for which it will be used. Different uses include pricing, capital allocation decisions, risk management / hedging, determining solvency requirements and capital adequacy or examining the risk appetite of the insurer.

The complexity of the risk measure can range from simply adding up notional amounts (ignoring diversification effects) to complicated option pricing approaches.

In this section we give a description of some key aspects of risk measurement from a holistic point of view. We will elaborate on a number of measures in sections to follow.

Perspective

Risk and capital can be viewed from many perspectives with different stakeholders requiring different risk assessments:

- **Shareholders** view risk from a performance measurement perspective. They are mainly interested in earning a good return on the capital they have invested in the firm. Shareholders are generally not interested in extremes beyond ruin.
- **Policyholders** and the regulator are most interested in extreme events that threaten the firm's ability to pay claims and meet obligations. Events that do not threaten ruin are of little interest.
- **Managers** require a sound basis for determining risk loadings in pricing, assessing the performance of business divisions and products, and allocating capital in a way that balances the needs of both shareholders and policyholders.

Choice of risk appetite

To be able to calculate the economic capital or to do risk-adjusted performance management, the insurer needs to specify its risk appetite. A company's risk appetite defines the risks the company is willing and unwilling to take. The risk appetite relates directly to the amount of capital at risk or the probability of default which shareholders of the insurer are willing to accept. Often, a desired credit rating is targeted to provide an upper bound on the acceptable level of default.

New Business

The decision of whether or not to include future new business in the economic capital calculations is an important one. For regulatory capital requirements, new business will usually not be included. However, for internal management decisions, making allowance for anticipated new business in the economic capital calculations may provide valuable insights. Note, not every new business contract need be included - some insurers choose only to include larger anticipated new contracts (for example, large group pension's contracts.) This allows the insurer to more realistically monitor how their economic capital requirements are developing over the next year, or longer.

Coherence of risk measures

There are many alternative risk measures, which can be judged against a set of requirements a *good risk measure* should meet – the risk measure is then said to be coherent.

The risk appetite relates directly to the amount of capital at risk or the probability of default which shareholders of the insurer are willing to accept. Often, a desired credit rating is targeted to provide an upper bound on the acceptable level of default.

A risk measure is coherent if it obeys:

- **Translation invariance:** adding a deterministic amount to the loss distribution changes the risk by that same amount.
- **Sub-additivity:** merging two portfolios doesn't generate additional risk.
- **Positive homogeneity:** scaling a portfolio implies analogous scaling for that risk.
- **Monotonicity:** positions that lead to higher losses in any situation produce higher risk and require more capital.

Time horizon

The insurer must choose the time horizon over which a risk is to be measured. For regulatory purposes (e.g. Solvency II) a time horizon of one year is often used for determining economic capital. The objective here is to ensure that the entity has sufficient capital at the end of this time horizon to pass on its liabilities at market value after a severe adverse scenario has occurred. We can distinguish here between the following three periods:

1. **Shock application period:** the time period over which the shock is applied. Under Solvency II, the shock period is defined as instantaneous when using the standard formula method; however, a one year period is also allowed when using an internal model based upon a projection methodology (refer to section 4.5).
2. **Shock calibration period:** the time period over which the shock is calibrated. Under Solvency II, shocks are calibrated over a one year period (even though they are typically applied instantaneously).
3. **Effect period:** the time period beginning after the shock application period which runs to the end of the policy term.

Shocking key variables during the *shock period* impacts results over the *effect period*. This impact is captured in the economic capital requirements at the calculation date.

For internal purposes different time horizons may be more appropriate. Longevity risk is often seen as long-term risk, while mortality risk and lapse risks are short-term risks. That is, the impact of mortality and lapse shocks is generally felt during the *shock period*, and there is very little impact during the *effect period*.

Time horizons used in different risk measures should be unified before risks are aggregated.

In the next paragraphs the following risk measurement methods will be described:

- (Un)expected Loss
- Variance
- Value at risk
- Cost of default put option
- Transform measurement
- Probability of ruin

The insurer must choose the time horizon over which a risk is to be measured. For regulatory purposes (e.g. Solvency II) a time horizon of one year is often used for determining economic capital.

2.2. (Un)expected loss

The expected loss is the mean of a loss distribution defined statistically as follows:

$$E[X_i] = \sum_i X_i / n$$

Where $X_i, i=1, 2, \dots, n$ are the n losses from the loss distribution

This is the amount of losses that can be expected on a portfolio and can be interpreted more as an average cost of writing business than a risk measure. This measurement is often used for general reserving, market consistent guarantee pricing and dynamic hedging. For risk measurement purposes, the unexpected loss is often preferred. The unexpected loss measures the difference between actual losses and expected losses.

2.3. Variance

The variance of a loss distribution gives insight into the uncertainty of future losses or gains. It measures the amount of dispersion in the results and is defined as follows:

$$\text{Var}[X_i] = \sum_i (E[X_i] - X_i)^2 / n$$

Where losses are assumed to be normally distributed, the variance and expected losses are all that are required to fully specify the loss distribution. Variance, and the related statistic standard deviation, is the most common definition of risk used within the funds management industry to measure portfolio risk (standard deviation of returns) and active risk or tracking error (standard deviation of excess returns).

Variance is only a good measure of risk where the losses are (at least approximately) symmetrically distributed. Semi-variance or downside-variance measures can be used when the loss distribution is asymmetric. It is defined as follows:

$$\text{Semi-Var}[X_i] = \sum_{i: X_i < E[X_i]} (E[X_i] - X_i)^2 / n$$

Variance fails on the coherence requirement of monotonicity. This can be illustrated by considering two portfolios, A and B. A has a large variance and equal probability of a profit or loss of ten. Portfolio B has a certain loss of ten (with a variance of zero.) A is clearly the *better* risk, but is more volatile and would usually get a larger allocation of capital.

2.4. Value at Risk

Value at risk (VaR) is one of the most widely used risk measures by insurers and banks in quantitative risk management. The VaR is the maximum loss not exceeded with a given level of confidence over a given time horizon. A loss distribution needs to be specified first and the VaR is just a quantile of that distribution. VaR is commonly used throughout the banking industry, defined statistically as follows:

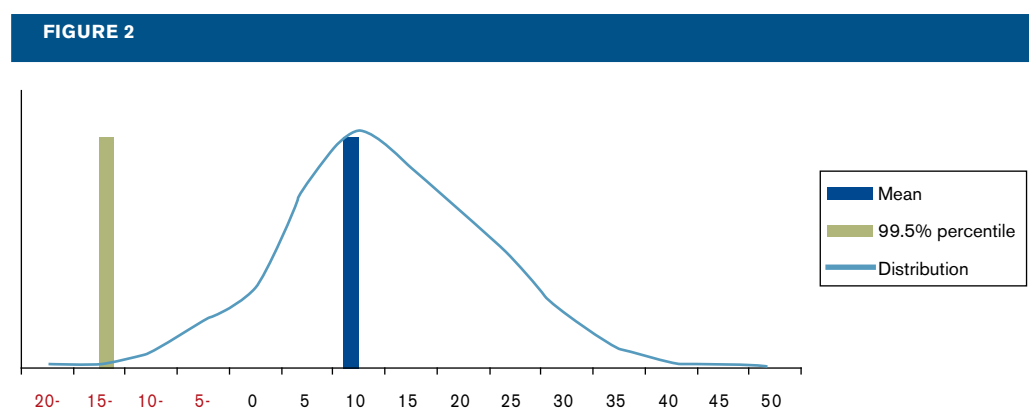
$$\text{VaR}(\epsilon) = \inf\{x \in \mathbb{R} : F_X(x) \geq \epsilon\}$$

Where $F_X(x)$ is the cumulative distribution function of the loss distribution of risk X .

Value at risk (VaR) is one of the most widely used risk measures by insurers and banks in quantitative risk management. The VaR is the maximum loss not exceeded with a given level of confidence over a given time horizon.

The time horizon is often taken as one year and the confidence level will depend on the risk appetite of the institution and the purpose of the exercise. For example, the time horizon may be specified in days when market risk is considered. VaR is the risk measure of Solvency II with a confidence level of 99.5% for the calculation of the Solvency Capital Requirement (SCR) and 85% for the Minimum Capital Requirement (MCR). The UK's Internal Capital Assessment (ICA) also uses VaR with a confidence level of 99.5%. VaR satisfies all coherence requirements except for sub-additivity.

The following graph shows the distribution of expected profits, the mean and the 99.5% percentile of distribution of profits (X) for which the gains are fixed to 60 and the losses are assumed to be normally distributed with $X \sim N(10,10)$. Based on a monte-carlo stochastic simulation using 1,000 scenarios, the VaR equals 15.9 for a given a confidence level of 99.5%. This is illustrated in the following chart:



A shortfall of VaR is that it gives no information about the severity of losses that fall above the confidence level chosen. Tail VaR overcomes this problem.

A shortfall of VaR is that it gives no information about the severity of losses that fall above the confidence level chosen. Tail VaR overcomes this problem. Tail VaR is the expected loss conditional on the loss exceeding VaR, defined as follows:

$$\text{Tail VaR}(\epsilon) = E[X_i | X_i < \text{VaR}(\epsilon)]^1$$

Note, however, that although tail VaR may overcome some of the drawbacks of VaR theoretically, in practice, it requires a suitable amount of data in the tail in order to understand the distribution of the tail. Many would argue that there is seldom access to enough suitable data to confidently use a tail measure.

If VaR is determined using a confidence level of α , then tail VaR is the expected loss over the other $1-\alpha$ part of the loss distribution. Tail VaR is the risk measure used for the Swiss Solvency Test with a confidence level of 99%. Tail VaR satisfies all coherence requirements.

1 Note that TVaR and conditional tail expectation (CTE) are equivalent for continuous distributions. In practice, both TVaR and CTE would be calculated by averaging the VaR's across simulations above the given $\text{VaR}(\epsilon)$.

In the figure 2, the $1-\alpha$ part of the loss distribution results in five scenarios (0.5% of 1,000) with the following losses:

- 22.58
- 22.57
- 19.03
- 16.56
- 16.03

Based upon these results, the Tail VaR of this example equals 19.4.

The xTVaR is similar to tail VaR, but instead of being the mean of all losses over VaR, it is the mean of the excess of the losses over the mean. In this example the xTVaR equals 9.4 (19.4 – 10.0).

The wxTVaR (weighted excess TVaR) is the same as xTVaR, but with adjusted probabilities. Larger losses are given greater weights – this overcomes the problem of TVaR measures that treat all losses above VaR linearly.

2.5. Cost of default put option

The shareholders of a company with limited liability do not suffer any further liability after the available capital of the company becomes exhausted; therefore, the shareholders hold an option to put the default costs to policyholders and bond holders. The value of this option can be used as a risk measure and is usually valued with Black-Scholes option pricing theory.

2.6. Transform measures

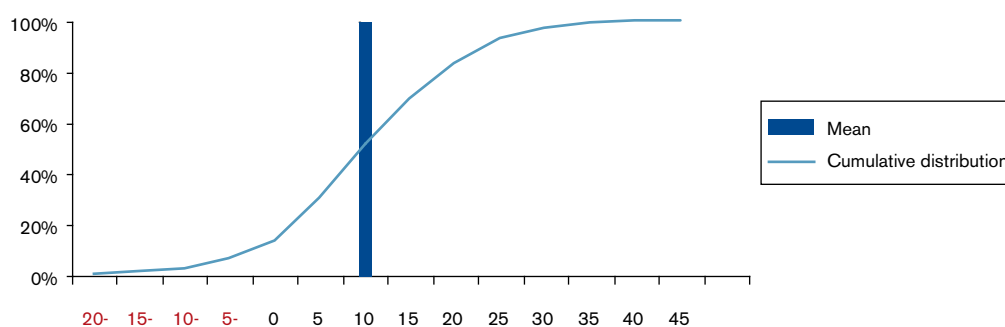
Adjustments are applied to different parts of the original distribution to form a new transformed distribution, giving more weight to upside or downside outcomes considered more important for a particular decision. The difference between the mean of the new distribution and that of the original distribution gives a measure of risk. Examples of transform measures are:

- The proportional hazards transform that raises the cumulative distribution by a selected power
- The Wang and Essher transforms
- The concentration charge that gives weights to undesirable outcomes

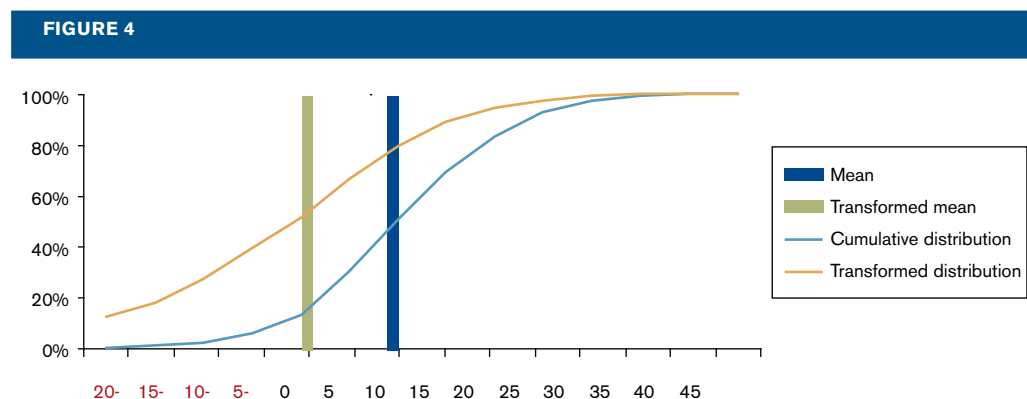
Using the example of company X, the expected profits have the following cumulative distribution:

The shareholders of a company with limited liability do not suffer any further liability after the available capital of the company becomes exhausted; therefore, the shareholders hold an option to put the default costs to policyholders and bond holders.

FIGURE 3



For example, if we raise the cumulative distribution to the power 1/3rd, the distribution will be as follows:



This transform has given greater weight to the less desired outcomes: the likelihood of outcomes in the left tail has increased, while those in the right tail have not. This is demonstrated by the mean of the transformed distribution being ten less than the mean of the original distribution. This gap between the means of the two distributions can be used as a risk measure; it is influenced by how the less desirable outcomes are weighted in the transform process.

The probability of ruin can be used as a risk measure. This is the probability that the aggregated losses exceed the available capital.

2.7. Probability of ruin

The probability of ruin can be used as a risk measure. This is the probability that the aggregated losses exceed the available capital. It is the reciprocal of VaR and is defined as follows:

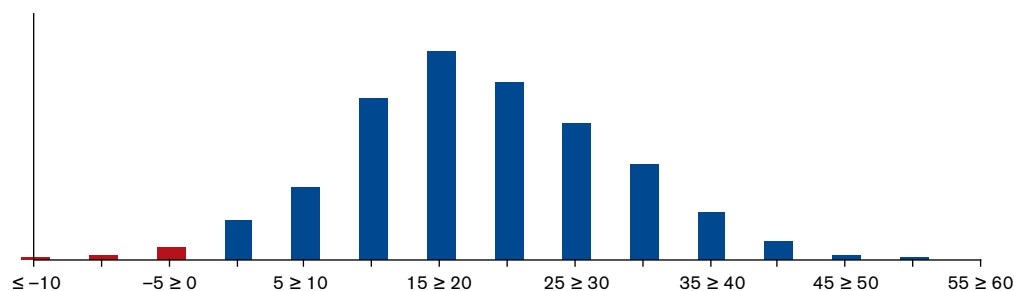
$$\text{Probability of ruin} = 1 - \epsilon$$

Where VaR(ϵ) amount of capital is available to use as a buffer against losses. Note the conceptual relationship between VaR(ϵ) and a $1 - \epsilon$ probability of ruin. If capital of VaR(ϵ) is held, then an insurer is ruined when losses exceed VaR(ϵ), and the probability of that occurring is $1 - \epsilon$.

Probability of ruin approaches are more widely used in property and casualty claims processes.

If the available capital of company X equals ten, the sum of the available capital and the aggregated profits has the following distribution in which the three red blocks on the left show the (19) negative scenarios. Negative scenarios are those for which the losses exceed the available capital (10).

FIGURE 5



The probability of ruin of company X is 1.9%. This calculated as the sum of the red parts above.

2.8. Other techniques

A number of other performance measures could be used as risk measures, depending on the type of risk to be measured:

- The Sharpe ratio of earnings could be compared to some standard or pre-defined benchmark. This would be a measure of earnings risk and would be used in performance measurement or value risk management. This is a widely used risk measure in funds management.
- The Risk Coverage ratio is calculated by dividing the expected return by the product of expected downside result and the probability of the particular downside result. This gives a measure of how many times risk is covered by the expected return.
- The Omega function is calculated as the expected upside (the expected result given is greater than a defined threshold) divided by the expected downside. This can be interpreted as the expected *winnings* divided by the expected *losses*.

2.9. Application of different measures

A survey of the CEIOPS' Internal Model Expert Group² on the use of internal models in the insurance industry shows that most companies used a Value-at-Risk (VaR) approach measured over a one-year period, and probabilities of solvency varied from 99.5% to 99.95%, with 99.93% and 99.97% also being used. Some companies used a Tail VaR measure. This again was typically over one year and probabilities of insolvency tended to be at the 99.0% percentile. Other companies expressed their economic capital requirement as a multiple of the VaR or TailVaR measure. Firms that used a longer period than one year used up to 25 years.

A survey of the CEIOPS' Internal Model Expert Group on the use of internal models in the insurance industry shows that most companies used a Value-at-Risk (VaR) approach measured over a one-year period.

² Stock-taking report on the use of Internal Models in Insurance

3. RISKS AND RISK DISTRIBUTIONS

3.1. Introduction

This section provides an overview of some of the more important risks faced by life insurers. Short explanations of each risk are provided in addition to comments about how each risk is usually assessed for calculating a capital requirement to support that risk.

3.2. Market risk

Market risk is one of the most significant risks for many insurers. Article 105 of the Solvency II Framework Directive explains which market risks should be taken into account when calculating the Basic Solvency Capital Requirement (BSCR) under the proposed Solvency II regime. While market risk could be calculated for each of the risks below and then aggregated, it may also be calculated using economic scenarios that reflect all risks simultaneously, which we discuss further in chapter 6. Market risk covers the following capital market risk factors.

Interest rates

Interest rate risk exists for all assets and liabilities for which the net asset value is sensitive to changes in the term structure of interest rates or interest rate volatility. It is typically broken down further into parallel shift risk, twist risk, key rate or duration bucket risk, and residual shape change risk.

Interest rate volatility risk exists whenever the value of an asset or liability depends upon the volatility of bonds, or where options on interest rates are held. In this case stochastic models may be used to value securities including the Vasicek, Black-Karasinski and Hull-White models.

Interest rate risk capital requirements can be assessed by shocking interest rates and interest rate volatility either up or down. Capital used to support these risks is typically determined as the change in net asset value due to these shocks. Different shocks may be applied (for example, the level and shape of the forward rates may be shocked separately) and the scenario with the greatest capital requirement can be used.

Equity

Equity risk is due to the uncertainty of the level and volatility of future equity prices; it is often a significant component of the total market risk.

Equity risk is often assessed by shocking the firm's equity holdings and calculating the change in net asset value.

Equity volatility risk exists whenever the value of assets or liabilities depends upon the volatility of equities, or where options on equities are held. Stochastic models may be used to value these securities with various models being available, distinguished largely by how volatility is treated. Simpler models assume constant volatility (for example, the classical Black-Scholes log-normal models), while more complicated models, such as the Heston model, allow volatility to vary by duration as well as equity level.

Property

Property risk is due to the uncertainty of the level and volatility of property market prices. Property risk can be a significant element of the total market risk.

Property risk capital requirements are often determined by shocking the value of the property portfolio.

When property is modelled stochastically, it is usually done with an equity model that assumes constant volatility, but it could also be modelled using GARCH³ models to allow for serial autocorrelation features.

While market risk could be calculated for each of the risks below and then aggregated, it may also be calculated using economic scenarios that reflect all risks simultaneously.

Equity volatility risk exists whenever the value of assets or liabilities depends upon the volatility of equities, or where options on equities are held. Stochastic models may be used to value these securities with various models being available, distinguished largely by how volatility is treated.

3 General Autoregressive Conditional Heteroscedasticity

Spread

Spread risk relates to the risk of level or volatility of yields on risky assets that change relative to the risk free term structure.

A capital charge can be assessed by measuring the change in net asset value following a change in the level or volatility of credit spreads. Credit spread volatility risk exists whenever the value of an embedded option in an asset or liability depends upon the volatility of corporate bonds assets or corporate bond yields.

Currency

Currency risk is due to changes in the level or volatility of currency exchange rates.

Capital to support currency risk can be determined by shocking (up and down) the exchange rate against each foreign currency to which the firm is exposed. For each currency, the worst of the up and down shocks can be taken. Aggregating these capital amounts across all currencies gives the total currency risk capital.

Market risk concentrations

Concentration risks arise from large exposures to a particular market risk. Additional risk is realised due to the additional volatility in concentrated asset portfolios, as well as the fact that larger losses may be realised if an issuer defaults.

The credit quality of the counterparties can be used to determine capital amounts that should be held to guard against this risk.

3.3. Credit Risk

Credit risk is defined as the loss (or adverse change in financial standing) due to changes in the credit standing of any counterparties.

The capital required to cover credit risk should be sufficient for losses due to unexpected defaults or deterioration of the credit standing of counterparties. Credit risk is usually calculated for each counterparty and then aggregated to arrive at the total credit risk for the insurer.

Depending on the nature of the contract and counterparty, there are a number of methods for determining the capital amount for the credit risk of a given counterparty. For example, the loss-given-default (LGD) can be used for a reinsurance contract.

The LGD is the loss that the insurer would be exposed to if one of its reinsurers defaulted, taking into account the reinsurance recovery rate, the recoverables under the reinsurance contract, the risk-mitigating effect of the reinsurance on the underwriting risk and any collateral that has been posted by the reinsurer.

There is a wide range of models for the stochastic assessment of credit events. A popular model is the Jarrow-Landow-Turnbull (JLT) model. This model requires two important inputs. First, the transition/default probability matrix is usually estimated via least-squares regression on some set of historical data. The second input for the JLT model is the credit risk premium process, for which the starting values are set to fit current market data and the long term distribution is fitted to past data.

3.4. Life underwriting risks

Life underwriting risks are the traditional core risks that define insurance business. The materiality of these risks plays a central role in determining whether financial products are classified as insurance or investment business under a number of accounting standards including IFRS and US GAAP. They also form a core part of the Solvency II framework directive (article 105), which outlines specifically which risk factors need to be considered.

Capital to support currency risk can be determined by shocking (up and down) the exchange rate against each foreign currency to which the firm is exposed.

Life underwriting risks are the traditional core risks that define insurance business. The materiality of these risks plays a central role in determining whether financial products are classified as insurance or investment business under a number of accounting standards including IFRS and US GAAP.

Distinction is made between trend, level and volatility risk. Trend risk can be explained as uncertainty in the trend as observed in past years. It takes into consideration that the best estimate may change or that unknown trends may impact the best estimate assumptions negatively. The level risk is related to the misestimate of the best estimate assumption. These risks may arise due to incorrect information and/or a wrong calculation basis. The volatility risk is linked directly to the statistical fluctuation of the best estimate parameter. Each of the risks has a different statistical distribution function.

This section lists each of these risks and gives a short description of what should be considered when each risk is modelled, as well as how the capital requirement for each risk is derived.

Mortality

Mortality risk is assumed when a firm agrees to make one or more payments contingent on the death of a policyholder.

Capital supporting mortality risk should capture the uncertainty in mortality assumptions due to changes in the level, trend and volatility of mortality rates, as well as the risk that more policyholders might die than expected.

Capital supporting mortality risk should capture the uncertainty in mortality assumptions due to changes in the level, trend and volatility of mortality rates, as well as the risk that more policyholders might die than expected.

Mortality risk is often assessed by increasing the base mortality rates by a fixed amount or by a proportion. The increase should capture the level, trend and volatility referred to above. The capital requirement can be calculated as the change in net asset value due to a given permanent increase in mortality rates.

Longevity

Longevity risk is the risk that technical provisions will increase due to a decrease in the mortality rates. This is a significant risk resulting from increasing life expectancies among policyholders in most developed countries.

The capital requirement can be calculated as the change in net asset value following a permanent given decrease in mortality rates.

Disability-morbidity

Illness, accident and disability policies usually have disability risk. There are two aspects to morbidity risk: the risk that the number of claims is greater than expected, and the risk that the duration of a claim is greater than expected.

The capital requirement can be calculated as the sum of two components:

- The change in net asset value due to an increase in inception rates
- The change in net asset value due to a given permanent decrease in recovery rates

Life expense

Expense risk is the risk arising from the level of expenses of servicing life policies being greater than expected. The level, trend and volatility of expenses (including the uncertainty of future expense inflation) should be reflected in the capital requirement for life expense risk.

The capital requirement can be calculated as the change in net asset value following a given increase in future expenses and expense inflation rates.

Revision

Revision risk is the risk that annuity amounts paid out by an insurer might increase as a result of an unanticipated revision of the claims process.

The capital requirement can be determined as the change in net asset value due to an increase in the annual amounts payable for annuities that are exposed to the risk of revision.

Lapse

Lapse risk arises from higher or lower lapses, terminations, surrenders and paid-ups than expected. Due to the possible interaction of these different rates, it is difficult to accurately model the capital requirement for lapse risk.

A simple approach has been suggested to determine the lapse risk capital requirement for Solvency II. The capital requirement can be taken as the most adverse change in net asset value of:

- A permanent increase of lapse rates
- A permanent decrease of lapse rates
- A mass lapse event

Life catastrophe

Catastrophe risk comes from extreme events that are not sufficiently captured by the other risks above. For example, the outbreak of a pandemic leading to extreme mortality would be considered a life catastrophe event. Management should understand that fat tail risks pose fundamentally different issues for risk management than do risks that can be mitigated through increasing scale.

The capital requirement can be calculated as the change in net asset value due to an increase in the mortality rate of, for example, x per mille over a one-year horizon.

Other risk sources which may also be relevant include operational risk, health risk, and non-life insurance risks, each of which could further decompose into specific risk factors if required.

3.5. Examples

There are many examples illustrating the potential movements in the above risks factors. Consideration of these scenarios can be important in calibrating the size and type of stress tests used to calculate economic capital.

Market risk examples

- 1929 – black Monday equity crash (-13% in one day) plus extended bear market
- 1987 – black Monday equity crash (-23% in one day)
- 1997 Asian financial crisis – collapse of multiple currencies including the Thai Baht, Indonesian Rupiah, South Korean Won
- 2008 – equity, property bear markets (steady declines), significant falls in risk free interest rates and widening credit spreads

Interestingly, many of these examples involved both a large *instantaneous* (i.e. overnight) shock, in addition to significant and sustained falls.

Life risk examples

- Longevity risk – recognition of steadily continuing improvement in mortality rates in the 1990's and 2000's
- Mortality risk – pandemic risks including SARS, Avian flu or Swine flu outbreaks. Risk of HIV/AIDS in the mid-eighties.
- Lapse risk – lack of consideration of dynamic lapse risk on UK blocks of Guarantee Annuity Options when interest rates fell over 1990's moving the guarantees into-the-money

Lapse risk arises from higher or lower lapses, terminations, surrenders and paid-ups than expected. Due to the possible interaction of these different rates, it is difficult to accurately model the capital requirement for lapse risk.

Credit risk examples

- 2008 – collapse of Lehman brothers, Bear Sterns, Merrill Lynch
- 2008 – collapse and subsequent nationalisation of Icelandic banking system lead to a collapse in the credit rating of Icelandic banks

4. RISK ASSESSMENT METHODOLOGIES

4.1. Introduction

Once the sources of risks have been identified, various methodologies can be used to determine the economic capital required for each risk factor. These include immediate stresses and projection scenarios, based upon either single or multiple risk factor stresses. In this chapter we discuss these approaches in some detail.

4.2. Immediate Stress

Under the immediate stress approach, the risk factor stress is applied at the current time ($t=0$). Assets and liabilities are re-valued at $t=0$ and the net change is calculated. Receipt or payment of cash flows such as premiums received, claims paid, and interest that are external to the calculation of assets and liabilities are excluded in this calculation. The impact of future management actions and dynamic hedging is also excluded using this methodology. If this amount represents a capital strain, then the risk capital is set equal to it. Otherwise it is set to zero (if no other relevant stress scenarios are to be considered).

This is the methodology adopted under SST, ICA and Solvency II, among others. It has the advantage of being a relatively simple approach to calculate and understand. The limitation of the approach is that it is unable to capture the risks and risk mitigation impacts arising from adverse scenarios that occur over time.

4.3. Projection Scenarios

In order to deal with these limitations, the use of a projection scenario methodology can more accurately capture a wider range of adverse risk scenarios. This can be particularly important for some risk factors such as longevity and market risks, as it is quite rare for these risk factors to move catastrophically instantaneously.

Under this method, a scenario for each risk factor is postulated to occur over the shock application period. This period may be treated as a two discrete time steps ($t=0$ and $t=n$), or it may be broken up into a number of smaller time steps. The use of a larger number of time steps is needed if either the liabilities are path dependent, or if a dynamic risk management strategy is being used.

At each time step, the value of both assets and liabilities are calculated taking into account the value of the risk factor, any dynamic interactions such as lapses and moneyness of guarantees, as well as the impact of any dynamic risk management strategy. Cash flows such as premiums, claims and interest will also be calculated in order for the P&L and Balance Sheet to be projected.

The amount of risk capital is then set equal to the present value of the net P&L results.

This methodology has numerous advantages including the ability to capture dynamic interactions and risk management strategies, the impact of cash flow effects and thus model the P&L and Balance Sheet realistically. It also provides a significantly more flexible and realistic basis for specifying specific adverse scenarios.

One of the challenges in using this methodology is that it may require the use of nested stochastic techniques, which can be relatively complex and computationally intensive. However, with the advent of grid computing, this challenge has been largely solved and a wide number of companies use this technique on a regular basis.

4.4. Multivariate Stress Tests

The above two methodologies relate to the assessment of economic risk capital at the single risk factor level. However, it is also possible to use these methodologies for scenarios involving two or more risk factors at the same time. This would generate an economic risk capital amount that relates

The use of a projection scenario methodology can more accurately capture a wider range of adverse risk scenarios. This can be particularly important for some risk factors such as longevity and market risks, as it is quite rare for these risk factors to move catastrophically instantaneously.

to whatever risk factors have been included in the scenario. In this way, it is also a technique for the aggregation of risk capital, which is discussed further in section 5.

It is important to be aware that in many situations, the capital derived by use of a multivariate stress is not equivalent to the sum of the capital derived at the individual risk factor stress. This is due to the interaction between risk factors (sometimes known as cross-Greek risk). These risk factor interactions are highly product, security, and capital market specific. Indeed, one of the limitations of the use of single risk factor stress methodologies is that they are unable to capture these interactions.

If an immediate stress methodology is used, then multiple risk factors can be specified to apply instantaneously. This can be useful if trying to capture the impact of a credit event which tends to affect multiple asset classes simultaneously, such as that which happened on the day Lehman collapsed in 2008.

If a projection scenario methodology is used, then multiple risk factors need to be modelled in the form of a projection scenario. In this instance, the use of economic scenario generation models becomes necessary. The type of model chosen, its calibration, and the inter-relationships between the models used for various risk factors become the central part of the problem, since it is practically impossible to specify in advance all the scenarios that should be used. We discuss this challenge in greater detail in the next section.

One of the important benefits of this approach is that it allows for the generation of the full distribution of P&L results under a range of assumptions and risk management bases. As the calibration of the scenarios is generally set to be realistic and is thus independent of the economic capital confidence level required, this methodology enables economic capital to be calculated from the end results, potentially for a range of different confidence levels. It also has the potential to provide significant insight into the pros and cons of alternative risk management strategies, and the business management issues involved across the entire shock application period and potentially beyond. As ESGs can be used to appropriately model both *normal* and *extreme* market conditions, a single result set can be used simultaneously for those interested in the tail for economic capital purposes, as well as for profitability analysis dependent upon the modelling of the central part of the distribution of results. The downside is that it involves significantly greater modelling complexity and computational requirements.

4.5. Calibration and ESG Models

When single risk factor methodologies are used, the calibration of each risk factor stress is typically specified directly by the external party (e.g. IFSRA for SST and CEIOPS for Solvency II). However, if the model is being developed for internal management purposes, then it may be incumbent upon the actuary to determine these calibrations. In this case, calibration typically starts with an examination of historical data. It is important here to be conscious of the difference between the shock calibration period and the shock application period. In general these periods should be consistent, however, they do not necessarily have to be (as is the case under Solvency II).

One of the key issues to be aware of in the calibration of models is the difference between market-consistent and real-world approaches. The ESG parameters used for market-consistent approaches are calibrated such that the models reproduce the market prices of the instruments in the calibration set. Thus, these are determined objectively. In contrast to this, the use of a real-world approach means that parameters can be calibrated such that the model produces a distribution of risk factor results more aligned to expected realistic experience. Risk factor stresses used to calibrate economic capital models are typically real world in nature.

The use of specific historical events such as those listed in section 3.5, can be a useful, relatively objective way of calibrating single or multiple factor immediate stress methodologies, or single factor projection methodologies. When it comes to calibrating economic scenario generators for use in

One of the key issues to be aware of in the calibration of models is the difference between market-consistent and real-world approaches.

multiple risk factor projection methodologies, history can be used to help calibrate the parameters of each of the ESG models.

The following is a non-exhaustive list of various ESG models that could be used to model various risk factors. Note that many of these models can be calibrated using both a market-consistent and real-world approach.

FIGURE 6

RISK FACTOR	TYPICAL DISTRIBUTION ASSUMED	MODEL
Interest rates	Changes normally distributed; nominal rates floored at zero	Hull-White, Libor market model, Jarro-Yildirim model for inflation and real rates. Bond returns a function of interest rates, credit spreads and duration.
Equity	Equity capital returns normally distributed	Excess returns above cash; modelled using lognormal Brownian motion. ARCH or GARCH models for dividend yields if necessary. Stable distribution or regime switching models can be used to model fat tails.
Property	Property capital returns normally distributed	Typically similar to equity models. Use of GARCH models may be appropriate to capture serial autocorrelation.
Spreads	Credit spread transition matrix, changes in credit spreads normally distributed	Jarrow-Landow-Turnbull
Currency	Currency changes normally distributed	Lognormal Brownian motion models, although potentially arbitrage free model as well.
Correlations	Typically constant	Could use regime switching model to address tail dependencies.

5. AGGREGATION TECHNIQUES

5.1. Introduction

Having chosen a risk measure and calculated the risks, the next step is to aggregate risks across different products, lines, geographic areas, etc. It is generally believed that the aggregated capital should be less than the sum of capital required for each risk being aggregated. Though this is likely true for most risks in a well-managed organization, operational risk requires special consideration, and is beyond the scope of this white paper.

However, the recent financial crisis has highlighted the fact that significant interactions can exist between risks. These interactions can have a compounding effect. For example, credit risk and market risk can, in some cases, compound each other. In such cases aggregating these risks by assuming some diversification effect between them can significantly underestimate the total risk.

The interaction of credit and market risk is currently receiving much attention. The Basel Committee on Banking Supervision published Working Paper number 16 earlier this year. This paper examines the interaction between credit and market risks and suggests that top-down approaches (that is, calculating capital requirements separately for each risk and then aggregating) can underestimate the total capital required to support credit and market risks. It is suggested that a bottom-up approach (calculating the capital requirements for each contract taking capital and market risk into account together) is a more appropriate way of determining capital requirements for these risks. For example, the interaction between interest rates and default probabilities can then be captured.

See Neil Cantle's article, *Make Proper Allowances for Risk Interactions*, at www.milliman.com/expertise for more on risk interactions.

Two top-down approaches for aggregating capital are considered in this section – correlations and copulas.

5.2. Correlation

Correlation is a measure of the strength and direction of a linear relationship between random variables. Statistically it is measured as:

$$\text{Corr}(X, Y) = E[(X - E[X])(Y - E[Y])] / \sigma_X \sigma_Y$$

Correlation measures how two variables move relative to one another. It is a scale invariant statistic that ranges from -1 to +1. For example, if two variables:

- Tend to move in the same direction, regardless of the size of movement, they will have a correlation near +1
- Tend to move in the opposite direction, regardless of the size of movement, they will have a correlation near -1
- Tend to move in the completely random ways with respect to one another, regardless of the size of movement, they will have a correlation near 0

The recent financial crisis has highlighted the fact that significant interactions can exist between risks. These interactions can have a compounding effect. For example, credit risk and market risk can, in some cases, compound each other.

Risks are aggregated using the following formula:

$$\text{Total risk} = \{ \sum_i \sum_j \rho_{ij} X_i X_j \}^{1/2}$$

Where $i, j = 1, 2, \dots, n$ (n is the number of risks being aggregated)

ρ_{ij} is the correlation between risks i and j

X_i is the risk measure output (for example, the VaR) of risk i

The table below illustrates how capital requirements may be aggregated across lines of business (life, non-life and bank) and risks (market, credit and insurance) using the formula above. The capital amounts are first summed for each line and risk, resulting in a total capital requirement of 189. This would be the capital requirement if one did not allow for the diversification effect.

FIGURE 7

	MARKET	CREDIT	INSURANCE	TOTAL	TOTAL WITH CORRELATION
LIFE	34	14	12	60	47
NON-LIFE	16	20	11	47	36
BANK	31	42	9	82	67
TOTAL	81	76	32	189	150
TOTAL WITH CORRELATION					132

A correlation matrix is specified for the correlations between risks, and this is used to calculate new totals for each row using the formula above. Note that the capital requirements for each line are lower after the diversification between the risks of that line is taken into account. For example, the total capital requirement for life business reduces from 60 to 47. The capital requirements across the lines are then summed to arrive at a total capital requirement of 150. This has reduced from the 189 calculated by summing the capital requirements due to the diversification effect between the risks.

Finally, the capital requirements are aggregated across the lines using a different correlation matrix. This second correlation matrix specifies the correlations between the lines. The total capital required reduces further from 150 to 132.

The correlation approach assumes that the risks are normally distributed and that the dependence structure can be specified via the margins of a Gaussian distribution. The combined risk distribution is therefore multivariate normal. This assumption may well introduce unacceptable distortion where the risks are not normally distributed.

Correlations tend to behave differently in extreme situations. For example, during normal times one would expect not to see any correlation between mortality and asset market values, while a terrorism event could lead to a spike in mortality as well as a drop in asset market values. Therefore, having a normally distributed correlation (which assumes the tails are uncorrelated) between mortality and asset market values is not acceptable during extreme events. Note, capital is often calculated to provide protection during extreme events – it is this part of the loss distribution that is of greatest interest. It is under stress conditions when the correlation approach of aggregating risks fails.

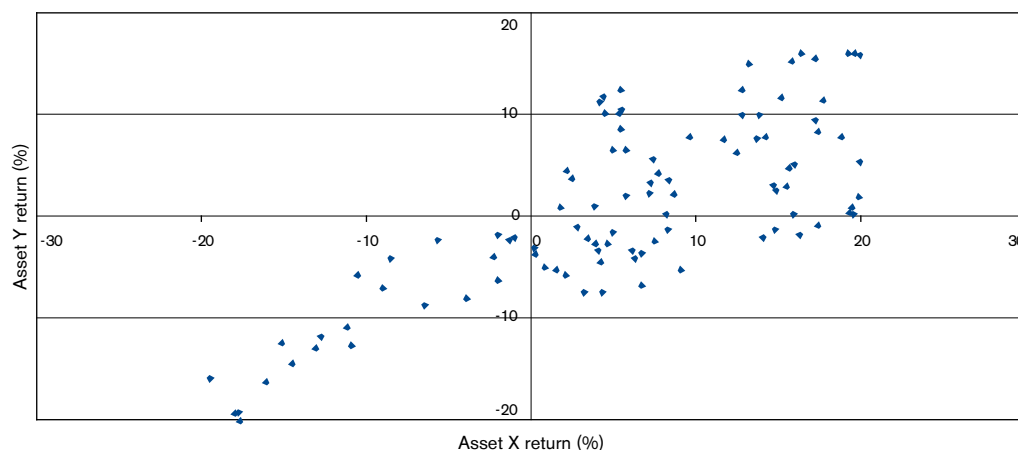
The graph below shows a scatter plot of the annual returns on two assets, X and Y. There is clearly a strong positive correlation between assets – the correlation coefficient is 0.79. However, the returns seem to have a stronger correlation as they become more negative and a weaker correlation

A correlation matrix is specified for the correlations between risks, and this is used to calculate new totals for each row using the formula above.

When things are going well, there may be less correlation between assets; it is when negative events affect an asset that one would expect greater correlation between assets.

when returns are greater than zero. The correlation between the returns where the returns on X are negative is 0.91, while the correlation for the set of returns where X is positive is only 0.46. Note also that there are many more observations where X is positive. This is not unrealistic; one would expect an asset to make positive returns more often than negative returns. Also, when things are going well, there may be less correlation between assets; it is when negative events affect an asset that one would expect greater correlation between assets (for example, a poor economic outlook, or, more extremely, a terrorist event.)

FIGURE 8: CORRELATION BETWEEN ASSET RETURNS



A company observing past data of these asset returns is likely to infer a correlation somewhere in the interval [0.46, 0.79]. For economic capital purposes this would significantly understate the correlation, since the correlation in the tail (where both X and Y produces negative returns) is understated. The diversification effect of the two assets in a combined portfolio would be overstated, which, in turn, would lead to establishing inadequate capital requirements. One way to address this limitation would be to use a multiple risk factor projection methodology with an ESG model that allows for regime switching correlations.

5.3. Copulas

The idea of the copula comes from Sklar's theorem. Sklar's theorem can be summarized as:

- Suppose $M(x)$ and $N(y)$ are the two marginal distributions of the bivariate distribution $Z(x,y)$
- Then there exists a function, C , such that $Z(x,y) = C(M(x), N(y))$
- This function, C , is called a copula.
- All continuous multivariate functions contain a unique copula.

When applied to economic capital, M and N can be seen as (a function of) two risk distributions. Under the correlation approach, a risk measure (for example, VaR) would be applied to each of these distributions. The resulting risk measure amounts, say M' and N' would be aggregated using a correlation assumption between risks M and N (for example, a correlation coefficient of 0.4) in order to calculate the total capital requirement. This is where the correlation approach fails: it assumes the correlation between M and N is constant for all realizations $M(x)$ and $N(y)$.

For example, if M is equity risk and N is lapse risk, there may be a 0.4 correlation in normal times. However, if a realization of $M(x)$ gave an extremely negative return, the correlation may well be

somewhat higher (for example, 0.75.) For economic capital purposes, one is more interested in these more extreme realizations (and the corresponding correlations.)

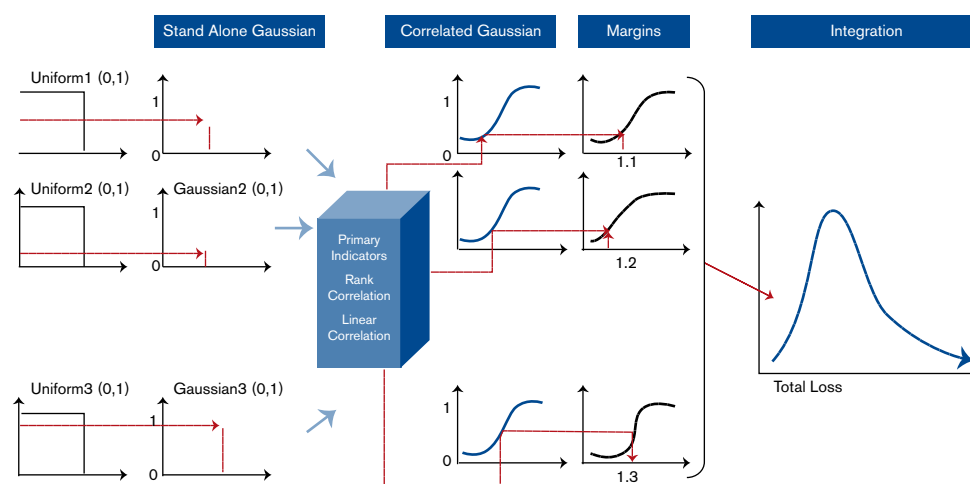
Copulas solve this problem. The function, C , is a plane (in this case in three dimensions, but can be extended to an n -dimensional plane if additional risks are added) and can be specified such that the interaction between M and N differs at different parts of each of the distributions of M and N .

In practice, M and N are usually transformed versions of their original risk distributions. This allows the copula to be easier defined. Each risk is transformed to a uniform distribution on the interval $[0, 1]$.

One way of doing this is to use the cumulative distribution function of each risk.

Simulating from the marginal and aggregate risk distributions becomes a relatively easy task after the copula structure is specified. First, a uniform random variable is simulated for each risk distribution. For example, uniform random variables p and q . The cumulative distribution of each of the risk distributions is then calculated at the point's p and q respectively. In the example, the distributions $M(x)$ and $N(y)$ are the cumulative distributions of the underlying risk distributions, so $M(p)$ and $N(q)$ are calculated. Thereafter, the interaction between $M(x)$ and $N(y)$ is captured by calculating $r = C(M(p), N(q))$. Finally, *correlated* risk realizations are calculated for each of the marginal risk distributions as the inverse of the M and N distributions. That is, the simulated losses are $M^{-1}(r)$ and $N^{-1}(r)$ and the total simulated losses for that simulation is the sum of these two amounts. Repeating this procedure across a suitable number of simulations yields a simulated total loss distribution. The simulation of total losses is shown in the figure below. There the marginal distributions are $\text{Normal}(0,1)$. Note that the yellow box represents different ways of taking account of the interaction between the marginal distributions. One of these approaches is the copula.

FIGURE 9: SIMULATION OF TOTAL LOSSES USING COPULAS.



Source: Morone, M., Cornaglia, A. & Mignola, G., Economic Capital Assessment via Copulas: Aggregation and Allocation of Different Risk Types, Intesa-Sanpaolo, 2007.

Note that the *standalone Gaussian* marginals in the figure above could be replaced with risk distributions for market risk, credit risk, insurance risk, etc. in an economic capital framework. Furthermore, these marginals need not be normally distributed.

5.4. MultiVariate Methods

Section 4.4 discussed multivariate methods to calculate economic capital. When using multiple risk factor stresses, the resulting economic capital includes the impact of risk aggregation directly and all risk factors are included. This approach is quite common when dealing with market risk factors and potentially dynamic lapses as well, as these factors can be highly interdependent. Risk aggregation is implicit in the ESG models and the models are inter-related: either directly or indirectly through risk factor correlations.

The benefit of this approach is that the risk factor relationships or correlations are defined explicitly with respect to the risk factors themselves. This is different to the Correlation approach outlined in section 5.2, in which the correlations are defined with respect to economic risk capital (i.e. the tail scenarios).

In the same way that multiple risk factors are assessed at the same time, multiple product groups, geographic locations and entities can also be simultaneously assessed to calculate an aggregate risk capital amount.

In the same way that multiple risk factors are assessed at the same time, multiple product groups, geographic locations and entities can also be simultaneously assessed to calculate an aggregate risk capital amount. Diversification effects are modelled implicitly in this type of methodology, since any interactions between various product groups, geographic locations or entities are captured directly.

6. APPLICATION OF CAPITAL ALLOCATION

Company level capital and risk are allocated down to lower levels (such as business units, lines and products) for a number of purposes. The initial reason for calculating total capital is often for regulatory reporting; however, insurers are becoming increasingly risk aware and are allocating the capital and risk more actively in order to improve areas such as pricing and performance measurement. This section considers the application of capital allocation.

6.1. Pricing & Technical provisions

One aim of allocating capital to business units, lines and products is to correctly allow for the cost of the capital in pricing exercises. The cost of capital is usually calculated as a product of the amount of capital allocated to a product and the return-on-capital requirement. Thus, the target price is generally greater where the risk is more concentrated (or less diversified), as more capital is allocated to such a risk.

Similarly, risks that are well-diversified are allocated less capital, and hence, they have a lower capital charge in the pricing exercise. A drawback to this is that there is no unique way to allocate capital. Consequently, a line written by two different insurers attracting the same amount of risk may well be allocated different amounts of capital by each insurer. This would result in different premiums being charged by the two insurers for underwriting the same risk.

One can argue that pricing of product risks should reflect the market consistent risk premium, rather than becoming a function of the risk capital of the entity being driven by diversification effects. Also, shareholder rewards should not change simply due to management's method of allocating capital to individual product lines. However, such pricing and rewards depend on how the capital requirements are set.

Capital requirements set using a market value margin approach should reflect the market price of the risk, since the market value margin is the amount needed to pay a third party to take on a portfolio of liabilities. When internal solvency capital requirements are used to define the solvency capital required to support a line of business, then these requirements would naturally vary according to the internal mechanics of each company; diversification effects and internal risk appetite differ by company, and therefore, lead to different risk capital requirements for the same risk in different companies.

Since the solvency capital requirements for similar business in different companies can differ, these companies will also place different *market-consistent* values on similar liabilities. This approach allowing for the diversification between all non-hedgeable risks when calculating solvency capital requirements in order to calculate cost of capital loadings can have some drawbacks. The technical provisions of an insurer are smaller than the technical provisions required to break the insurer up into business units that can be transferred to third parties. During the recent financial crisis it became clear that when financial institutions get into trouble, they need to have the option of unloading parts of the institution to third parties in order to stay in business.

See Neil Cante's article, *The Big Picture: Enterprise risk management can reveal key pricing concerns to insurers that may help prevent losses*, at www.milliman.com/expertise for more on ERM and pricing concerns.

6.2. Risk budgetting and Capital Allocation

Risk budgetting is a process whereby managers decide in which areas (lines, products, geographical areas, etc.) to accept risk. A total risk budget for the entire organisation is the starting point. Risk targets (or budgets) are then set for each area. This is usually done from a top-down level, starting with organisation level, then with business units, lines, etc. Capital allocation techniques are used to allocate the total risk budget down through the different layers, resulting in a budget for each level.

The initial reason for calculating total capital is often for regulatory reporting; however, insurers are becoming increasingly risk aware and are allocating the capital and risk more actively in order to improve areas such as pricing and performance measurement.

Capital allocation techniques are used to allocate the total risk budget down through the different layers, resulting in a budget for each level.

Having arrived at a budget for each level, managers can compare this to the risk amount actually taken on that level. This comparison may serve as a decision tool to determine which levels require additional capital or where capital may be released. The budget can be used to allocate responsibility to managers. For example, fund managers may be given a mandate that includes a range on the risk they are permitted to take. Furthermore, managers are made more risk aware by having explicit risk budgets available at business unit, line and product levels. This can lead to better risk management throughout the insurer.

An insurer can develop risk mitigation strategies such as re-insurance and hedging, and this can be done via its business units. These risk management strategies, if done at business unit level, will impact the risk position of that unit, as well as of the entire insurer. Note, there may be restrictions on the movement of capital around a group which could limit the impact on the rest of the group of reinsurance or hedging program in one part of the group. In this regard, there are two important concepts. First, the fungibility of capital, which refers to the ability of capital to absorb losses, no matter where the capital is held or where the losses occur within the group. Second, the transferability of capital, which refers to the actual ability of a business unit to transfer assets or liabilities to the rest of the group, taking into account the time and costs of the transfer.

Allocating capital to different business units, lines or products also helps to manage the insurers risk appetite.

Allocating capital to different business units, lines or products also helps to manage the insurers risk appetite. After capital is allocated, for example, to a business unit, then that capital amount becomes a limit for managers in that unit on the risk they are allowed to accept. For example, the investment department may be allocated a risk budget. There can be disincentives to discourage exceeding the risk budget, such as performance linked to the risk taken (and less reward for exceeding the budget.) The allocation of capital to business units also becomes an important decision-making tool. Risk decisions can be taken with a clearer view on how they impact the risk appetite of the total organisation.

Another approach to risk budgeting is to allocate capital to risk types, no matter which business unit or geographical area they arise. This is useful in managing the types of risk a company accepts across all its subsidiaries.

Another approach to risk budgeting is to allocate capital to risk types, no matter which business unit or geographical area they arise. This is useful in managing the types of risk a company accepts across all its subsidiaries. However, this is less useful for management decision-making, since it is unclear how to make use of the available budget for a specific risk.

In practice, there is often a mixture of decentralised and centralized risk management. Line-managers may do well to manage risks within a budget, but often do not apply integrated company-level risk principles. This can result in *silo* risk management.

A sufficient buffer should be maintained between economic capital and risk-taking capacity. This buffer can be split into two parts:

1. A strategic buffer, used for writing new business and for taking advantage of new business opportunities, as well as for possible future additional regulatory requirements. The strategic ambitions of management should be used to determine the size of this buffer.
2. A technical buffer, used for business cycle impacts outside the one-year horizon of economic capital, as well as used against the volatility of the capital base. An analysis of the company's financial condition will be an important part of determining this buffer.

6.3. Risk adjusted performance measurement

Return on assets (ROA), return on equity (ROE), Risk adjusted return on capital (RAROC) and risk adjusted return on risk adjusted capital (RARORAC) are all common performance measures, although only the latter two are risk adjusted performance measures.

RAROC and RARORAC are often used as a performance measures. These are a measure of the profitability of a portfolio, taking account of the risk assumed in order to generate the profits. These

risk-adjusted measures considered more accurate and comparable (between portfolios and insurers) than the more traditional return measures such as ROE and ROA.

The traditional measures focus on performance relative to accounting balance sheet items. There are two drawbacks to using these:

1. Focusing on assets ignores leverage effects. ROA fails here, whereas ROE does capture leverage impacts.
2. There is no distinction between classes or riskiness of assets. Neither ROE or ROA allow for risk being accepted to achieve the return generated.

RAROC is calculated as the risk adjusted return (the long-term expected return across the insurance cycle) divided by available capital.

$$\text{RAROC} = \text{risk adjusted return} / \text{capital}$$

RARORAC is calculated as risk adjusted return divided by required capital. Required capital captures the risks assumed by the insurer in generating the numerator (the economic return), after allowing for the diversification effect between the risks at company level.

$$\begin{aligned} \text{RARORAC} &= \text{risk adjusted return} / \text{required capital} \\ &= (\text{revenues} - \text{costs} - \text{expected losses}) / \text{required capital} \end{aligned}$$

RAROC is more interesting for shareholders. It is a measure of risk adjusted return on the *total capital* supplied by the shareholders. RARORAC measures this return against the capital *required* to generate it, thus making it more suitable for internal management purposes. Since RARORAC does not take the additional capital available (between required and available capital), it is often insightful to reconcile RARORAC with RAROC.

The difference between the total capital for RAROC and required capital for RARORAC makes up the two buffers discussed in the previous section: the strategic and technical buffers. A part of this difference may be the result of business units not fully using up their allocated capital budget or capital not being fully allocated.

RARORAC can be calculated at any level where comparison of performance is desired, for example, at the company level, business unit level or product level. The key step in this calculation is to correctly allocate the risk bearing capital. For example, life business capital can be allocated to each life product in order to calculate the RARORAC for each and get insight into the relative performance of the different lines of life business written. This may well feed into further management decisions concerning which product lines to expand or which places to free up some capital.

A simple formula approach for economic capital can be used to better understand risk adjusted performance measurement. If we define L as the largest liability portfolio that assets A can support over the next year (while remaining a going concern), for an expected extreme loss corresponding to a 1 in 100 year event, l (expressed as a percentage of the assets), then

$$L = A (1 + r_a) (1 - l) / (1 + r)$$

Where r_a is the return on the assets during the year and r_l is the increase in liabilities over the year. The economic capital, EC, required to support this loss is then the difference between the assets and liabilities,

$$EC = A \{ 1 - [(1+r_a)(1-l) / (1+r_l)] \}$$

Suppose company ABC has 100 assets, split between two classes: 60% in asset X (with an expected return of 7.8%) and 40% in asset Y (with an expected return of 7.5%). The weighted average return is 7.68%. The liabilities are expected to grow by 6% over the next year. Suppose also that the 1 in 100 year loss for asset X is a 20% fall in value (that is, $l = 20\%$ for X) and a 10% fall in asset Y, and that asset X and Y have a correlation of 0.5. The combined fall is then 16% (this assumes that there is no diversification between X and Y). The economic capital required for such a scenario would be

$$EC = 100 \{ 1 - [(1 + 7.68\%) (1 - 16\%) / (1 + 6\%)] \} = 14.67 \text{ (ignoring the diversification)}$$

$$EC = 100 \{ 1 - [(1 + 7.68\%) (1 - 14.4\%) / (1 + 6\%)] \} = 13.07 \text{ (taking account of diversification)}$$

Considering the correlation between X and Y, the combined fall becomes 14.4% and the economic capital required reduces to 13.07. Assume that this 13.07 is exactly how much economic capital company ABC has. ABC's liabilities are therefore 86.93.

Suppose now that we want to compare the returns on assets X and Y. To do this on a risk-adjusted basis, the capital must be allocated to each asset. The capital can be allocated using the same formula above – the following are the capital requirements for each asset ignoring diversification:

$$EC(X) = 60 \{ 1 - [(1 + 7.8\%) (1 - 20\%) / (1 + 6\%)] \} = 11.18 \quad (76\%)$$

$$EC(Y) = 40 \{ 1 - [(1 + 7.5\%) (1 - 10\%) / (1 + 6\%)] \} = 3.49 \quad (24\%)$$

Using a pro-rata allocation, 76% of the total capital is allocated to asset X ($76\% \times 13.07 = 9.96$) and 24% to asset Y (3.11). The following table summarizes the balance sheet items at the start of the year, by asset class.

FIGURE 10

	CLASS X	CLASS Y	TOTAL
ASSETS	60.00	40.00	100.00
LIABILITIES	50.04	36.89	86.93
EC	9.96	3.11	13.07

The next step is to determine the RARORAC for each asset. Note that in this example the RARORAC formula collapses to:

$$\begin{aligned} \text{RARORAC} &= (\text{revenues} - \text{costs} - \text{expected losses}) / \text{required capital} \\ &= (A \times r_a - L \times r_l - \text{expected losses}) / EC_0 \\ &= EC_1 / EC_0 \end{aligned}$$

Assume that the expected losses (taking into account the diversification effect) over the next year are 1.2% for asset X and 1% for asset Y. This results in the balance sheet items at the end of the year.

FIGURE 11

	CLASS X	CLASS Y	TOTAL
ASSETS	63.90	42.57	106.47
LIABILITIES	53.04	39.10	92.14
EC	10.86	3.47	14.33

The assets are increased by the growth rate and decreased by expected losses. For example, $63.90 = 60 \times (1 + 7.8\%) \times (1 - 1.2\%)$. The liabilities are increased by the liability growth amounts and the new EC values are the difference between year end assets and liabilities. RARORAC can now be calculated for each asset:

$$\text{RARORAC}(X) = 10.86 / 9.96 - 1 = 9.0\%$$

$$\text{RARORAC}(Y) = 3.47 / 3.11 - 1 = 11.5\%$$

Therefore, although asset X has the greater expected return, asset Y is a more attractive investment based on risk-adjusted returns. This is because asset X requires a great portion of the capital due to its higher risk (in terms of the extreme risks and expected losses over the year.) Note, when comparing sub-portfolios based on a risk-adjusted performance measure, it is important the denominator represents that sub-portfolio's capital contribution to the total portfolio, not its stand-alone capital. So, for example, the denominator for RARORAC(X) should be asset X's capital contribution to the portfolio (9.96), not its stand alone capital (11.18).

7. ALLOCATION OF CAPITAL

7.1. Introduction

After aggregating risks in order to take into account the effect of diversification, companies want to allocate the capital back to the lower levels for a range of purposes. In other words, the diversification benefit achieved by aggregating the risks needs to be allocated back to the individual risks. The allocation of capital is an important measure for profitability in relation to risk and is essential for pricing insurance products as well as for the planning and control cycle (risk budgeting and return measurement).

There are a range of approaches for allocating capital with different ones being appropriate for different purposes. Capital may be allocated by a number of factors, including:

- Risks
- Products / Product groups
- Geographical locations
- Entities / organizational units

Allocation methods can be judged against a set of requirements that a *good allocation method* should meet – the method is then said to be coherent.

Allocation methods can be judged against a set of requirements that a *good allocation method* should meet – the method is then said to be coherent. An allocation method can be considered coherent if it meets the following properties:

- **No undercut:** a sub-portfolio's allocation should be no more than its standalone capital requirement
- **Symmetry:** if the risk of two sub-portfolios (as measured by the risk measure) is the same, then the allocation should be the same for each
- **Risk-free allocation:** capital allocated to a risk-free line of business should be zero

This section presents different methods for allocation of capital and illustrates each with the following practical example.

There are three portfolios of company X, Y and Z. Losses from each portfolio are assumed to be normally distributed, with $X \sim N(50,10)$, $Y \sim N(40, 7)$ and $Z \sim N(70,12)$. Gains of the portfolios are fixed to 60 for X, 50 for Y and 80 for Z. 1,000 simulations are taken from each portfolio and the 99.5 percentile of the simulated losses is taken as the VaR for each portfolio. VaR is used as a risk measure to determine the capital requirement for each of X, Y and Z in this example; in practice, any risk measure can be used. Note, the VaR can be calculated directly via formula and the results below are an approximation due to the limited number of simulations used.

The following correlations are specified between the 3 portfolios:

FIGURE 12

	X	Y	Z
X	1.0		
Y	0.8	1.0	
Z	0.3	0.2	1.0

The following table summarizes the capital requirement results used throughout this section. The capital requirement for combined portfolio XYZ is calculated by combining the capital requirements of each portfolio and taking diversification into account.

FIGURE 13

PORTFOLIO	CAPITAL REQUIRED
X	15.9
Y	8.8
Z	22.7
TOTAL XYZ SUMMED	47.4
TOTAL XYZ WITH DIVERSIFICATION	37.0

Note, the sum of the parts (47.4) is greater than the total portfolio capital requirement (37). The aim of this section is to allocate the 37 back to portfolios X, Y and Z.

The following approaches will be discussed:

- Marginal approaches
- Game theory

7.2. Marginal Approaches

There are a number of marginal approaches to allocating capital. The following are examined in this section:

- Pro-rata or linear marginal contributions
- Discrete marginal contributions
- Continuous marginal contributions
- Myers-Read allocation method

7.2.1. Pro-rata or linear marginal contributions

Pro-rata (or linear) allocation is the simplest approach. The total capital requirement (including diversification effect) of the combined portfolio XYZ is allocated pro-rata to each of X, Y and Z. For example, portfolio X receives an allocation of $37 \times (15.9/47.4)$.

This approach allocates the diversification benefits across the portfolios in proportion to each portfolio's individual capital requirement. Therefore, it does not penalize highly correlated portfolios. Similarly, it does not reward those portfolios that increase the overall diversification effect.

The following table provides the results of allocating capital pro-rata.

FIGURE 14

PORTFOLIO	PRO-RATA ALLOCATION
X	12.4
Y	6.9
Z	17.7
XYZ	37.0

7.2.2. Discrete marginal contributions

Discrete marginal contributions are calculated by first determining the capital of the total portfolio, excluding the portfolio in question. For example, the capital of portfolio YZ is 25.9. This capital is then subtracted from the capital of the total portfolio to arrive at the marginal risk amount the portfolio

Discrete marginal contributions are calculated by first determining the capital of the total portfolio, excluding the portfolio in question.

in question contributes to the total portfolio. So, the discrete marginal contribution of portfolio X is therefore $37.0 - 25.9 = 11.1$.

Often the marginal contribution is scaled (in this example by the factor $37/30$) to get scaled marginal contributions that add up to the total portfolio risk. Discrete marginal contributions are an approximation to the continuous marginal contributions approach.

The following table shows the discrete marginal contribution and the scaled marginal contribution.

FIGURE 15

PORTFOLIO	VaR OF TOTAL PORTFOLIO EXCLUDING THIS ONE	DISCRETE MARGINAL CONTRIBUTION	SCALED MARGINAL CONTRIBUTION
X	25.9	11.1	13.6
Y	31.4	5.6	6.9
Z	23.6	13.4	16.5
XYZ		30.0	37.0

7.2.3. Continuous marginal contributions

The continuous marginal contributions approach is also known as the Euler method. This method calculates the derivative of the total portfolio risk with respect to each individual portfolio's risk. The derivatives multiplied by the individual portfolio's risk measures give the continuous marginal contribution of that portfolio.

For example, for a 1% increase in the VaR of portfolio X, portfolio XYZ's capital requirement will increase 0.81%. Therefore, the continuous marginal contribution of portfolio X is $0.81 \times 15.9 = 12.9$. Note, the continuous marginal contributions of the portfolios add up the total capital of portfolio XYZ. The drawback of this approach is that there can be negative contributions where negative correlations exist between the individual portfolios.

The following table shows the continuous marginal contribution.

FIGURE 16

PORTFOLIO	CHANGE VAR / CHANGE PORTFOLIO VAR	CONTINUOUS MARGINAL CONTRIBUTION
X	0.81	12.9
Y	0.71	6.2
Z	0.79	17.9
XYZ		37.0

7.2.4. Myers-Read allocation method

Myers-Read has the advantage over the other marginal methods in that the marginal increments add up to the total capital.

Shareholders of a firm hold a put option on the insurer's losses that exceed the capital the insurer has available (the shareholders can put these losses to the policyholders as the shareholders are only liable for losses while the insurer is solvent.) Myers-Read usually assumes a normal or lognormal

total loss distribution. The value of the default put option can then be determined via the Black-Scholes option pricing formula.

Increasing risk exposure on any portfolio leads to a larger value of the default put option on that portfolio, while adding capital to the portfolio decreases the value of the default put option. Myers-Read calculates the cost of the last unit of exposure added to a portfolio in terms of the addition to the capital required to support that unit of exposure. The additional capital required in a portfolio when an additional unit of risk is added is calculated such that the value of the default put option as a percentage of expected losses remains constant.

The additional cost for the last unit added is applied to all the units in the portfolio to arrive at the total risk capital for the portfolio.

7.3. Game theory

Game theory is a widely used approach to decision making in conflict situations. In risk allocation, the conflict is how to share the diversification benefit between each sub portfolio. Each sub-portfolio benefits from being part of a larger diversified portfolio, but also gives up some diversification benefit. Lloyd Shapley introduced the *Shapley Value* as a stable solution to coalition games. While there can be any number of players, the limitation is that the number of players needs to be a whole number (see more on this under Aumann-Shapley contributions.)

The Shapley Value is based on the average first in, last in and all the intermediate *ins*. In the example above, second in is the only intermediate *in* as there are only three portfolios. The first in values are the same as the pro-rata values in the previous section. The last in values are the same as the discrete marginal allocations – the marginal allocation if the particular portfolio was the last to enter the XYZ portfolio.

The second in calculations are calculated as if the portfolio was the second entrant into each possible combination of the portfolios. For example, for portfolio X, X can be second into portfolio XY or XZ. The second-in calculations determine the second-in contribution that X makes to each of these portfolios.

- Portfolio XY allocation (23.6, from the table in the previous section) less portfolio Y's first-in contribution (6.9) gives X's second-in contribution to portfolio XY ($23.6 - 6.9 = 16.7$).
- X's second-in contribution to portfolio XZ (13.7) is calculated as XZ's allocation (31.4, from the table in the previous section) less the first-in allocation for Z (17.7).

X's average second-in value (15.2) is the average across the second in contribution that X makes in all possible portfolios – in this case XY and XZ (so 15.2 is the average of 16.7 and 13.7).

The table below presents the results of allocating via the Shapley Value.

Increasing risk exposure on any portfolio leads to a larger value of the default put option on that portfolio, while adding capital to the portfolio decreases the value of the default put option. Myers-Read calculates the cost of the last unit of exposure added to a portfolio in terms of the addition to the capital required to support that unit of exposure.

FIGURE 17

PORTFOLIO	1ST IN	2ND IN	LAST IN	AVERAGE	2ND IN CALCULATIONS	
X	12.4	15.2	11.1	12.9	16.7	13.7
Y	6.9	9.7	5.6	7.4	11.1	8.2
Z	17.7	19.0	13.4	16.7	18.9	19.0
XYZ				37.0		

No scaling is needed. The allocations to each portfolio naturally add up to the total. However, there are a couple of drawbacks. First, the calculation is computation intensive, meaning, the number of calculations increase exponentially for more portfolios. Second, there is the issue of needing a whole number of players. If, for example, portfolio X was split into portfolios X1 and X2 and the calculations were done again on 4 portfolios, the allocation to Y and Z would change. This is not a desirable property of an allocation method.

Aumann and Shapley developed the Aumann-Shapley value, which allows for fractional players. The Aumann-Shapley value represents the rate of increase in risk – how much additional risk a portfolio adds for a small increase in size. The risk is calculated based on the risk measure adopted, which can be any of those mentioned previously. The Myers-Read approach above is considered a special case of Aumann-Shapley, where the risk measure used is the default put option.

The Aumann-Shapley values can be calculated using simulation techniques. This is done using the Ruhm-Mango-Kreps (RMK) conditional risk algorithm (detailed in the paper by Ruhm & Mango.) The RMK algorithm can be applied to a number of different risk measures and can, therefore, also be used to calculate Myers-Read values too.

In that paper, 10,000 simulations are run and the capital requirement (based on Myers-Read) for each simulation is determined. These simulations are then ordered with the simulation requiring the least capital in first position, and the simulation requiring the most capital in last position. A ruin probability of 2.275% is selected as tolerable. Therefore, an interval is chosen from ordered simulation 9,723 up to simulation number 9,822. Note, this is an interval around simulations 9,772 and 9,773, the point at which ruin would occur ($1 - 2.275\%$.) The capital amounts required for the simulations in this interval are averaged to arrive at the simulated capital requirement for the portfolio.

7.4. Other approaches

Co-measures were developed as a way of allocating capital in an additive manner that is still consistent with the risk measure used to define total capital. For example, if the risk measure is TVaR, then the co-TVaR for a portfolio would be the average of the losses (for example, across the simulations where losses exceeded VaR) that the portfolio contributes to the total TVaR. The co-TVaR's naturally add up to the total TVaR.

Capital could also be allocated so that it equalizes relative risk: each portfolio would be allocated risk such that, when viewed in isolation, each portfolio has the same ratio of risk to expected losses.

8. OPERATIONAL IMPLEMENTATION OF AN ECONOMIC CAPITAL FRAMEWORK

This chapter discusses the requirements for implementing an economic capital framework.

8.1. Output requirements (in an ideal world)

As capital markets are continuously fluctuating (e.g. forex), the asset liability risks that a company is exposed to and, by extension, their economic capital, change on a continuous basis. In an ideal world with unlimited resources, this risk and economic capital information would be available on a live basis. Managers responsible for managing risk and making strategic decisions based upon economic capital information rely on this information being as accurate and as up to date as possible.

Initially this may seem like overkill, particularly since insurance companies are traditionally risk-managed and capital is assessed on much longer frequencies. However, given the new recognition of the market risks inherent in guarantee products such as variable annuities and the dynamic hedging strategies that support them or traditional products with profit sharing, there is a real need to obtain such information on a real-time basis. Furthermore, in time of market stress, net economic risk exposures can change dramatically, and the need for this information on a highly frequent basis (e.g. live), increases significantly. Questions such as *how much risk exposure do we have?* and *are we still solvent?* and *how much does the market need to move by before we become insolvent?* become increasingly common and urgent. Being prepared for them in advance is the only way to adequately answer them.

Although this may sound like a pipe dream, in fact it is becoming more of a reality with some of the largest players setting global best standards in this area. The introduction of variable annuities in recent years across Europe has helped to drive innovation. For those adopting dynamic hedging programs, it is common for liabilities and their risk sensitivities to be calculated every night on a per policy seriatim basis with the risks being monitored on a live basis in conjunction with their hedge assets. The hedge assets are then rebalanced whenever residual risks become out of balance due to market movements. For a small few, this process is continuous, as these tasks are moved from one time zone to another following the opening trading hours of capital markets around the globe.

The information produced by these programs is very similar in nature to what is required to derive economic capital. Additional runs can be incorporated into these frameworks in order to produce the additional information required. These programs can and are being extended to cover multiple blocks of business, different geographic areas, and various product classes. Economic capital models can then produce economic capital results on a highly frequent basis for use in regular day-to-day decision making. With suitably designed infrastructure and systems in place, economic capital can be decomposed into individual product lines, business units, and risk factors for dissemination.

Companies with such processes in place should, in theory, be able to create competitive advantages through more robust and efficient management of risk and economic capital.

8.2. Operational Process Requirements

For those companies seeking to implement such a framework, there are various operational requirements in order to do so. These can be broken down into three main areas: liability modelling, asset modelling and economic capital modelling. The training of people, the implementation of systems and the designing of processes need to be established in each of these areas.

Liability Modelling

The starting point is to design and build a model of the company's liabilities. There are generally two approaches for achieving this. The first is to model the full block of all policies on a per policy seriatim basis very frequently (e.g. nightly). For some companies with VA blocks of business, this will be relatively straightforward, however for others, with more traditional product, it is a significant

As capital markets are continuously fluctuating (e.g. forex), the asset liability risks that a company is exposed to and, by extension, their economic capital, change on a continuous basis.

undertaking. In order to produce the necessary suite of liability valuations, policy data needs to be captured from administration systems, capital market and other input assumptions need to be captured in order to calibrate models, and valuation routines need to be run on distributed grid computing systems.

The alternative method is to use replicating portfolio techniques to simplify the above process. Replicating portfolios attempt to model a block of liabilities, usually in the form of model points as a simple financial instrument (e.g. an option) that captures the broad characteristics / risk sensitivities of the liabilities. These instruments can then be used as a proxy (i.e. a replicating portfolio) for the liabilities. Their valuation is usually relatively simple, since they are based upon closed form formulae. The trade-off between accuracy and computational efficiency is critical here. Some product classes such as fixed annuities can be modelled quite accurately this way, whilst others, such as some variable annuity products, do not lend themselves to this type of approach.

When replicating portfolio techniques, a portfolio of assets is chosen to represent the liabilities. An alternative to this is known as cluster modelling. This involves using a subset of the liabilities with an appropriate scalar applied to each cell to represent the entire liability portfolio. To accomplish this, policies are grouped into a relatively small number of relatively homogeneous clusters based on their similarity along a set of variables considered appropriate by the modeller. Once each cluster of policies is determined, the most representative policy in the cluster is scaled up to represent the entire cluster.

Some of the variables used to measure similarity can be based on the in-force file (such the distribution of account value across fund types); others can be based on policy-level results from one or a few calibration scenarios (such as the present value of future cash flows on each policy under a specified scenario). The variables should be chosen with a focus on the intended use of the model, which may include matching across economic scenarios, or matching with changes to mortality and lapse assumptions, for example.

Clearly systems and infrastructure play a critical role in both of these approaches. The automation of policy and capital market data extracts, data validation, valuation runs and checks are critical. Specialists such as actuaries are necessary to manage this process, particularly for the calibration of models and validation of results.

Asset Modelling

In essence, the same activities need to be undertaken for the assets. However, compared to the liabilities, modelling assets is generally easier. Physical instruments such as equities and fixed income securities can be modelled through a direct mapping to risk factors such as equities indices or duration and credit exposures. Derivative instruments such as futures, swaps and options can be valued in most cases using closed form formulae, although stochastic valuation models may also be required for more exotic derivatives. Asset databases will need to be kept, and capital market data will need to be sourced or linked directly through a market data system such as Bloomberg. Valuation of assets and their risk sensitivities can then be undertaken on a live basis, and by combining this with the liability information, an accurate picture of the net risk exposures of the company can be calculated. This information is what is used in the risk management of dynamic hedging programs. The benefit of this approach is that by using the same system and framework, economic capital can be tightly aligned to risk management, meeting the Solvency II use test for an internal model.

Economic Capital Modelling

In addition to the usual stresses applied in order to calculate risk information for risk management purposes, the shocks used to derive economic capital also need to be applied. An economic capital model then takes this information and calculates economic capital. Ideally, the model will be designed such that economic capital be aggregated and allocated to each business unit, product class and geographic region as necessary.

Compared to the liabilities, modelling assets is generally easier. Physical instruments such as equities and fixed income securities can be modelled through a direct mapping to risk factors such as equities indices or duration and credit exposures.

Again, technology will play an important role in being able to produce this information efficiently. Data links from the asset-liability model need to be built such that the economic capital model captures the most up-to-date risk positions. A sufficient number of specialists with the right level of expertise and experience are necessary to manage this process, and most importantly, to disseminate and explain the results to the relevant stakeholders.

8.3. How Milliman Can Help

Designing and implementing the above framework, systems and processes is no simple task. Expertise and experience are essential ingredients for achieving this in a timely and cost efficient way. Milliman consultants have a wealth of expertise and experience in the economic capital field that clients are able to leverage. As we work with a number of clients around the globe, we know what is needed in order to meet global best practice standards.

EC Framework

As a first step, Milliman can assist clients in developing an overall framework for Economic Capital management. Decisions regarding theoretical approach and process will be influenced by a large number of factors, including the nature of our client's business, corporate culture, existing processes, regulation, and the demands of markets, analysts, and auditors.

An overriding goal is to assure that methods employed are theoretically sound, can be easily explained to management and outside parties, possess adequate transparency, and can be linked to support management decisions.

Milliman supports clients in both the initial development of an Economic Capital framework, and the fine-tuning of an existing approach.

Model Design and Construction

Once a broad framework has been established, Milliman can support clients in model design and construction. Our support may vary from high level discussions of modelling approaches, to peer review or hands-on assistance.

In the course of our work, we can assist with the design of model processes, model structure, integration with data sources, coding, design of reports, and the download and sharing of results among applications.

Integrated Functionality

As described in this report, Economic Capital has a bearing on a very large number of management functions, including interface with supervisors, product pricing, risk assessment, risk management and hedging, capital allocation / project financing, performance management, and financial reporting.

Often, as a result of legacy systems and processes, these diverse functions are handled on a piecemeal basis with implications for efficiency and integrity of results. Although there may be compelling reasons to employ multiple systems, Milliman can work with clients to streamline and integrate processes, thereby assuring that results are efficiently developed, audited, and shared among key constituents.

Analysis of Results

As results become available, Milliman can provide assistance to clients in the analysis of results, and the presentation of results to management or outside parties.

Integration with Enterprise Risk Management

Milliman assists clients in the integration of Economic Capital and Enterprise Risk Management activities.

An overriding goal is to assure that methods employed are theoretically sound, can be easily explained to management and outside parties, possess adequate transparency, and can be linked to support management decisions.

Milliman supports clients in both the initial development of an Economic Capital framework, and the fine-tuning of an existing approach.

While Economic Capital management is typically viewed as one part of a broader Enterprise Risk Management agenda, it can be useful to regard EC and ERM as separate but intrinsically linked processes.

Analysis of enterprise risk requires the holistic evaluation of financial and non-financial risks together, allowing for the interactions between risk factors.

CRisALIS is the industry's most advanced approach to the holistic analysis of enterprise risk. By contextualising all risk components in terms of their interactions, it not only allows explicit modelling and evaluation allowing for risk interactions but helps managers to understand the evaluation in real business terms.

While Economic Capital management is typically viewed as one part of a broader Enterprise Risk Management agenda, it can be useful to regard EC and ERM as separate but intrinsically linked processes. Indeed, choices made in the course of ERM affect EC, and decisions related to EC have a strong bearing on ERM.

An organization's view of risk, their risk appetite, and decisions made with regard to the management of risk will affect levels of required capital as well as the manner in which this capital is allocated to business units. These decisions, in turn, will affect reported rates of return, solvency ratios, and the perceptions of regulators, analysts, and investors, with important implications for a company's vitality and operations.

Analysis of enterprise risk requires the holistic evaluation of financial and non-financial risks together, allowing for the interactions between risk factors. Milliman's CRisALIS methodology applies techniques from complexity theory to the holistic analysis of enterprise risk and helps management to understand how financial, operational and strategic risks interact. This is an essential component of ERM activities, and will directly influence management decisions regarding economic capital calculation and aggregation.

CRisALIS is the industry's most advanced approach to the holistic analysis of enterprise risk. By contextualising all risk components in terms of their interactions, it not only allows explicit modelling and evaluation allowing for risk interactions but helps managers to understand the evaluation in real business terms. This enables management to gain better insight into both qualitative and quantitative aspects of risk aggregation than has previously been possible. It also enables the creation and monitoring of integrated complexity-based warning signals which can spot emerging risks earlier than traditional methods. Understanding the holistic dynamics of risk exposure enables more realistic simulation and forecasting than a *re-aggregated* silo approach. Overall CRisALIS helps companies to integrate risk calculation and management and make the investment in risk management more valuable.

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