## Political Risk Reinsurance Pricing – A Capital Market Approach

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#### Abstract:

Political risk insurance is concerned with the risk associated with government intervention and restriction of trade into emerging markets. It may encompass long-term perils (investment related), such as the confiscation, expropriation or nationalization of an infrastructure project in an emerging market or short term perils (export trade related), such as contract frustration, embargo or currency inconvertibility.

Today exposure to the political will of a host country is being challenged by economic events blurring the landscape for political risk, which is creating a growing need for better informed econometrics, and advanced analysis of risk vs. reward, as well as a broader interpretation of covered perils and events.

The need for reinsurance capacity in Political Risk is higher than it has been in a decade. However, more and more reinsurers are either restricting the coverage or entirely leaving the practice. This paper proposes an objective reinsurance pricing methodology to assess the risk of writing a large political risk reinsurance portfolio based on country risk ratings, sovereign ceilings, political risk default rates and severity assumptions based on historical data. The paper also incorporates a mechanism to indicate the effects of regional and trade correlations based on the advanced credit modeling. Having such an objective methodology to assess the risk of reinsuring political risk should broaden support from traditional reinsurance markets and attract risk capital from the capital markets.

Key Words: Confiscation, Contract Frustration, Contract Repudiation, Correlation, Country Risk Ratings, Currency Inconvertibility, Default Rates, Exchange Transfer, Expropriation, Gaussian Copula, Nationalization, Recovery Rates, Merton Model, Reinsurance, Sovereign Ceiling, Wrongful Calling of Guarantee Organizations Participating in Willis Analytics Political Risk Study

Ace Global Markets (Ace) American International Group (AIG) Beazley Group Plc Coface Export Development Corporation (EDC Canada) EXPORTKREDITNÄMNDEN (ekn) Hiscox Syndicate MIGA (World Bank) Overseas Private Investment Corporation (OPIC) XL Insurance

## Section 1: Introduction

The underwriting evaluation of a political risk portfolio to provide reinsurance capacity is one of the toughest and most complex decisions made by reinsurers today. There is enormous uncertainty in assessing the risk-return profile of any reinsurance cover. The sheer complexity of the underlying business makes it nearly impossible to quantify profitability in political risk reinsurance using existing actuarial risk evaluation methodologies such as trending and developing past losses to estimate future claims. The Argentinean crisis in 2001 forced several reinsurers to either reduce the reinsurance capacity they provided or leave the political risk reinsurance arena altogether. The crisis was a serious wake up call to many reinsurers regarding the need for consistent and objective risk evaluation and management processes - even if they were not appropriately awakened by the 1997 Asian financial crisis and the 1999 Russian debt crisis.

The Argentinean crisis alerted reinsurers to the massive political risk exposures in developing and emerging markets. The media assertions of large losses without understanding the probabilities of default and the relatively small percentage of loss given default added greater pressure on senior management to make tough business decisions. The reinsurers who decided to remain in the political risk business were required to justify to senior management not only that the business is profitable in the long run but also there are objective criteria to quantify the risk.

The primary goal of this paper is to propose an objective methodology to quantify the risk-return profile of a political risk insurer for the purpose of providing reinsurance capacity. This methodology will also allow political risk insurance and reinsurance carriers to evaluate their own portfolios for overall profitability and capital requirements to support the on-going business.

Political Risk can be defined as the company's exposure to the risk of a political event that would diminish the value of an investment or a loan. The major political risk covers (classes) are:

- 1. Currency Inconvertibility (CI) and Exchange Transfer (FX)
- 2. Confiscation, Expropriation and Nationalization (CEN)
- 3. Political Violence (PV) or War (including revolution, insurrection, politically motivated civil strife, terrorism)
- 4. Breach of Contract, Contract Frustration (CF), Contract Repudiation (CR)
- 5. Wrongful Calling of Guarantee (WCG)

## 1. Currency Inconvertibility (CI) and Exchange Transfer (FX):

- Inability of an investor/lender to convert profits, investment returns and debt service from local currency to hard currency (\$ € £)
- Inability of an investor/lender to transfer hard currency out of the country of risk

## 2. Confiscation, Expropriation and Nationalization (CEN):

• Loss of funds or assets due to confiscation, expropriation, or nationalization by the host government of the country of risk

• Any unlawful action by the host government depriving the investor of fundamental rights in a project (creeping expropriation)

# 3. Political Violence (PV) or War (including revolution, insurrection, politically motivated civil strife, terrorism)

• Loss of funds or assets due to political violence or war

#### 4. Breach of Contract, Contract Frustration (CF), Contract Repudiation (CR)

- Loss of funds or assets due to arbitrary non-honoring of a contract by a foreign government (or a semi government entity) or breach of contract by a private business entity due to an arbitrary act of a foreign government
- Loss of funds due to non-payment of a loan or guarantee

## 5. Wrongful Calling of Guarantee (WCG)

• Loss of funds or assets due to the host government arbitrarily calling its bonds or a business entity being forced to call guarantees due to political events (the bonds are generally backed by irrevocable letters of credit which are callable on demand)

## Section 2: Data

Willis Analytics has created a comprehensive database combining the data contributed by the industry participants. The database contains over 15,000 records covering 180 countries. The loss data covers years 1966 through 2004 while the corresponding policy limits are not always available. The data is available by year, country, and risk type (form). The loss data is also available in its component parts (paid loss, loss reserve, recoveries) on a limited basis. The policy information (policy counts and aggregate limits by country and year) is available for the most part on an aggregate basis.

#### Section 3: Proposed Methodology

The proposed methodology will create a consistent and objective framework to conduct a portfolio level analysis of a political risk book of business. It is based on developing a distribution of political risk losses that would support the determination of Value at Risk (VAR) and/or Tail Value at Risk (TVAR) to determine the capital requirements to conduct the business that is being analyzed. In addition, the methodology facilitates analysis of underwriting profitability, return on capital, reinsurance pricing, and many other underwriting and financial measurements. The development of loss distributions at portfolio and policy level is based on the entity's political risk default rates, severity rates, and regional/trade correlation assumptions. This level of granularity provides the ability to establish the portfolio and marginal rate of returns in order to objectively determine the price for political risk.

## The conceptual framework

- Application of Generalized Linear Models (GLM) to derive expected default rates by form/class based on historical default rates and statistically significant country specific macro economic factors such as gross domestic product, inflation, unemployment rates, political stability indexes et cetera.
- Analysis of country specific macro economic factors to derive the political risk default dependence between countries to develop input assumptions for the correlation matrix
- Usage of the Generalized Linear Models (GLM) framework to derive recovery rates by form/class
- Combination of derived default rates by form and event correlation assumptions by country/region to achieve reasonable pair-wise correlation factors
- Aggregation of loss distributions by simulating correlated default events and corresponding simulated losses by country and form

## The Process

The aggregate loss distribution at policy and portfolio level will be produced by simultaneously simulated processes:

- Simulation of the correlated econometric and political scenarios
- Recovery rates simulations
- Loss aggregation across portfolio

## Loss Distribution = f(A, D, C, N, L)

A = Projected Country Aggregate Exposure by class of risk

- D = Political Risk Default Rate by class of risk
- C = Correlation Matrix

N = Percentage of Defaults within a Sovereign Nation (given that the country is in default)

L = Percentage of Loss Net of Recoveries (given that the limit is in default)

- The country aggregate exposure by class of risk per a given country represents the sum of all limits within a sovereign nation for a given class of risk (CEN, CI, etc.)
- The political risk default rate by class of risk represents default rates calculated based on historical data or prospective default rates based on non-linear regression.
- The correlation matrix contains the dependence structure between countries in the political risk portfolio being analyzed.
- The percentage of defaults within a sovereign nation represents sum of limits in default compared to the total aggregate exposure in the entire country
- The percentage of loss net of recoveries is estimated by the ultimate loss compared to the total limit that is in default

#### Exhibit 1

## Willis Analytics Political Risk – Analytical Process



The analytical process shown in Exhibit 1 is applied to a portfolio of political risks in a simulation environment with the goal of producing a distribution of political risk losses. The calculation of reinsurance pricing and the estimate of return on capital as well as other portfolio analysis becomes a routine exercise when the loss distribution is produced. In Section 4, there is a detailed and technical description of the modeling procedure for inclined practitioners. Section 5 contains a copy of the output from a test run of the model for one class of business. [The actual default rates by class are not shown due to confidentiality issues.]

## Section 4: Modeling a Political Risk Portfolio

The modeling of a political risk portfolio consists of several key steps:

- 1. Development of default rates
- 2. Development of percentage of loss within a policy limit (i.e. 1 recovery rate)
- 3. Development of percentage of defaults within a sovereign nation given that the country is in default
- 4. Development of a correlation matrix

#### 1. <u>The development of expected default rates</u>

The development of default rates consist of four parts:

a) default rates based on historical data

The authors define "default rate" as number of claims divided by number of policies. The "country year" is defined as one sovereign nation with any type of political risk coverage for one full year. Moreover, the "defaulted country year" is defined as a country year with at least one political risk default within the country of risk.

There are two sets of default rates calculated in this study.

- default rates by class (CI/FX, CEN, PV/War, CF/CR, WCG)
- default rate per country year

In addition, the ratio of number of defaulted country years divided by total country years is compared to the ratio of total claims divided by total policy counts.

The historical data is provided by the participating entities listed at the beginning of this paper.

b) Prospective default rates based on macro economic variables and political indicators

Prospective political risk events can be modeled based on the binary situation of occurrence or nonoccurrence of a political risk event. Limited Dependent Variable (LDV) theory presents a methodology to map continuous characteristics into a discrete set of variables via nonlinear regression. The most used examples of this methodology are logit and probit models. Academic researchers suggest that the countryspecific political risks are highly correlated with a country's macroeconomic variables and political stability indicators. Among the suggested list of explanatory variables are GDP per capita, inflation, current account deficit, exchange rate, political stability indicators, and government effectiveness indicators. For the purpose of calibrating the predictive model, the historical default rates of political risk events are calculated from the data base provided by participating entities. The statistical data of the macroeconomic variables and the political indicators come from the International Monetary Fund (World Economic Outlook database) as well as the World Bank (World Development Indicators database).

Logistic regression analysis describes how a binary response variable is associated with a set of explanatory variables. It is a member of the family of Generalized Linear Models (GLM) in which the mean of a response variable is related to explanatory variables through a regression equation:

$$g(\mu) = \beta_0 + \beta_1 X_1 + \ldots + \beta_p X_p,$$

where  $g(\mu)$  is the link function that is chosen according to the distribution of the response variable. In general, the natural link for a binary response variable is the logit function. The predictive model that is fitted to forecast probabilities of the future occurrences of political risk events is shown below:

$$\log(\frac{d}{1-d}) = \beta_0 + \beta_1 X_1 + ... + \beta_p X_p,$$

where d is the political risk default rate and  $X_t$  through  $X_p$  are economic variables and political indicators that explain the occurrence of the political risk events.

An alternative to logistic regression for binary responses is to choose g(d) to be the inverse of the cumulative standard normal probability distribution function, i.e., g(d) equals the 100 dth percentile in the standard normal distribution. The default rate d is assumed to follow a logistic distribution under the logistic regression or a normal distribution under the probit regression. MIGA (World Bank) has used statistical analysis and non-linear regression to project default rates based on past MIGA experience and judgment (Hamada, J. 2004) similar to the process outlined above.

c) Distribution of average default rates by rating category

Discriminant Function Analysis is used to determine the linear combinations of variables that best discriminate the subjects in different categories. For example, Discriminant Function Analysis can be used to investigate which variables discriminate between countries that fall into the country rating categories from Aaa to D similar to Moody's sovereign ceilings.

In the two-group case, the Discriminant Function Analysis can also be thought of as a multiple regression. The grouping category is used as the dependent variable and the economic variables are used as independent variables in a multiple regression analysis, then the results are analogous to those obtained via Discriminant Function Analysis. In general, the two group discriminant function is illustrated as:

$$Group = a + b_1 X_1 + b_2 X_2 + \dots + b_p X_p$$
,

where *Group* is the country rating category and  $X_t$  through  $X_p$  are macroeconomic variables and political indicators.

The number of discriminant functions that is needed to solve the generalized version is one less than the number of groups. The relevant output from discriminant analysis consists of the coefficients of the discriminant functions, the discriminant function scores for each subject, measures of the percentage of cases correctly classified by the analysis, and an overall measure of group differences. The discriminant

functions and scores that are estimated from historical data can be applied to classify future country-year combination into different rating categories based on the projection of macroeconomic and political indicators of a country.

d) Distribution of average default rates based on duration

Curves are fitted to political risk default rates by class based on the time of default (short term, medium term, and long term). The fitted curves are used to distribute the average default rates by class into a table of default rates that vary by duration.

The outcome of this process is a table of expected default rates by rating class and duration. In addition, this methodology presents an opportunity to simulate future default rates based on country specific (or region specific) macro economic factors and correlations derived from the same factors.

## 2. <u>The development of percentage of loss within a policy limit (i.e. severity rate)</u>

The severity rate can be defined as the ratio of ultimate loss divided by the policy limit.

There are two sets of severity rates calculated in this study that are based on data provided by participating entities.

- 1. severity rates by class (CI/FX, CEN, PV/War, CF/CR, WCG)
- 2. severity rate per country year

# 3. <u>The development of percentage of defaults within a sovereign nation given that the country is in default</u>

The percentage of defaults within a country can be defined as total policy limits in default within the country divided by the total aggregate policy limits. The available data to estimate this ratio is sparse. Therefore, a confidence interval is established and the ratio is allowed to fluctuate within this range during the simulation process.

#### 4. <u>The development of a correlation matrix</u>

A key feature of the proposed methodology is the inclusion of correlated events, either regional, or trade, in the estimation of losses. The global economy is inter-related as never before. In addition to obvious political and economic influences between countries that are geographically connected, there are strong and long-standing trade relationships between these countries. An economic downturn or a political upheaval in one country could adversely affect another country that is connected via proximity or trade. It is the authors' attempt to model the potential adverse effect of correlated political risk events to the portfolio under consideration by utilizing a correlation matrix in Monte Carlo simulation. The easier method is to consider a fixed time horizon model and use a Gaussian copula to incorporate the correlation matrix. The harder and perhaps more appropriate method is to use a multi-year model with a Gaussian copula or an

Archimedean copula where the time of loss is taken into account in simulation (in addition to whether there is a loss event or not). The key idea is to model several large political risk losses within the portfolio while capturing the timing risk of defaults to assess the capital need to withstand an extreme event. The input for the correlation matrix would come from two sources.

- The dependence quantified in the Generalized Linear Modeling of macro economic factors
- Judgment based on empirical data

Technical details on correlation and copulas are provided in Appendix A.

## Section 5: Sample Output

A sample portfolio containing 130 countries is tested for one class of business:

# Willis Analytics Political Risk Reinsurance Model

## Entity XYZ

Currency: €

Average Life	5
Default Stress Factor	1
Number of Accounts	130
Number of Sectors	6
Number of Simulations	50,000
layers	4

				Original	Adjusted	Loss as %		Stressed	
Index	Account Name	Exposure	Sector	Rating	Rating	of Exposure	Std. Dev.	Default Rate	IG Flag
1	Turkey	400,000,000	6	Ba3	Ba3				0
2	China	300,000,000	3	Baa2	Baa2				1
3	South Korea	250,000,000	3	A2	A2				1
4	Iran	240,000,000	5	Ba3	Ba3				0
5	Dubai	240,000,000	5	Baa1	Baa1				0
6	BRAZIL	200,000,000	2	Ba3	Ba3				0
7	Iraq	180,000,000	5	A1	A1				1
8	Russia	175,000,000	4	Ba1	Ba1				0
9	INDIA	150,000,000	3	Ba1	Ba1				0
10	INDONESIA	150,000,000	3	B1	B1				0
11	ROMANIA	125,000,000	4	Ba3	Ba3				0
12	ANGOLA	110,000,000	1	Caa3	Caa3				0
13	MOROCCO	105,000,000	5	Ba2	Ba2				1
14	MEXICO	100,000,000	2	Baa3	Baa3				1
15	SUDAN	100,000,000	5	Caa1	Caa1				0
16	VIET NAM	95,000,000	3	Ba3	Ba3				0
17	SAUDI ARABIA	90,000,000	5	Ba1	Ba1				1
18	NIGERIA	85,000,000	1	B3	B3				0
19	Libiya	75,000,000	5	B2	B2				0
20	QATAR	75,000,000	5	Baa1	Baa1				1
21	EGYPT	65,000,000	5	Ba2	Ba2				0
22	GHANA	60,000,000	1	B1	B1				0
23	Oman	60,000,000	5	Baa2	Baa2				0
24	CUBA	55,000,000	2	Caa1	Caa1				0
25	ABU-DHABI	55,000,000	5	Baa1	Baa1				1
26	CAMEROON	50,000,000	1	B2	B2				0
27	TUNISIA	50,000,000	1	Baa3	Baa3				1
28	CHILE	50,000,000	2	Baa1	Baa1				1
29	PHILIPPINES	50,000,000	3	Ba3	Ba3				0
30	THAILAND	45,000,000	3	Baa3	Baa3				1
31	KAZAKHSTAN	45,000,000	4	Ba1	Ba1				0
32	COSTA RICA	40,000,000	2	Ba1	Ba1				0
33	POLAND	40,000,000	4	Baa1	Baa1				1
34	BAHRAIN	35,000,000	5	Baa3	Baa3				1
35	SOUTH AFRICA	35,000,000	1	Baa3	Baa3				1
36	SYRIA	30,000,000	5	Caa1	Caa1				0
37	MALAYSIA	30,000,000	3	Baa3	Baa3				1
38	UNITED ARAB EMIRATES	30,000,000	5	Baa1	Baa1				1
39	BULGARIA	25,000,000	4	Ba2	Ba2				0
40	MOZAMBIQUE	25,000,000	1	B1	B1				0

# **Willis Analytics Political Risk Reinsurance Model**

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Currency: €

Reinsurance Terms	Gross	XOL 1	XOL 2	XOL 3
Per Risk Limit	999,999,999,999	25,000,000	50,000,000	100,000,000
Per Risk Attachment	-	25,000,000	50,000,000	100,000,000
Aggregate Limit Aggregate Deductible	999,999,999,999,999 -	50,000,000 -	100,000,000	100,000,000

	Losses			Counts				
Percentiles of Ceded	Cross		XOL 2		Cross		VOL 2	
LOSSes	10 007 700				G1055			XUL 3
Std Dov	10,907,790	1 765 629	0,100	0	0.00	0.03	0.00	-
	19,001,021	1,700,020	317,413 20070/		0.30	0.10	0.03	- #DIV/0I
U.V. Modian	2 /20 902	031/0	3001 /0	#DIV/0!	100 /0	0,000	0, 1013	#DIV/0:
Min	2,430,093	0	0	0	ے د	0	0	0
Max	232 613 520	26 014 554	16 456 288	0	04	2	1	0
5.0%	202,010,020	20,314,004	10,400,200					
10.0%	-	-		-			-	
15.0%	_	-	-	-	-	-	-	-
20.0%		-	-	-	-	-	-	
25.0%	156 148	_	-	_	1	-	_	-
30.0%	422,908	_	-	-	1	-	-	-
35.0%	717,449	-	-	-	1	-	-	-
40.0%	1.118.020	-	-	-	2	-	-	-
45.0%	1.688.936	-	-	-	2	-	-	-
50.0%	2.430.893	-	-	-	2	-	-	-
55.0%	3,531,002	-	-	-	3	-	-	-
60.0%	5,131,817	-	-	-	4	-	-	-
65.0%	7,206,008	-	-	-	5		-	-
70.0%	9,882,275	-	-	-	6	-	-	-
75.0%	13,075,373	-	-	-	7	-	-	-
80.0%	17,283,128	-	-	-	9	-	-	-
85.0%	22,899,489	-	-	-	11	-	-	-
90.0%	31,924,810	-	-	-	15	-	-	-
91.0%	34,301,808	-	-	-	16	-	-	-
92.0%	37,306,699	-	-	-	17	-	-	-
93.0%	40,404,309	-	-	-	19	-	-	-
94.0%	45,048,767	-	-	-	20	-	-	-
95.0%	49,137,774	-	-	-	22	-	-	-
96.0%	54,679,148	-	-	-	24	-	-	-
97.0%	63,374,955	-	-	-	27	-	-	-
98.0%	75,791,591	1,441,589	-	-	32	1	-	-
99.0%	94,423,873	8,323,393	-	-	40	1	-	-
99.5%	115,214,334	13,600,798	-	-	47	1	-	-

The loss and counts distributions provide all the necessary claim information for reinsurance pricing and return on capital calculations.

## Return on Capital Exhibits

Required capital to cover 1 in 100 year loss:

## Willis Analytics Entity XYZ Political Risk - Gross Return on Capital (1 in 100 year scenario) <sub>Currency:</sub> €

Selecte	d Capital	68,423,873
	1 in 100 year Loss - (Premium - Expenses)	
Income		20,809,000
	Premium - Loss - Commission - Expense + Interest Income + Investment Income	
Return	on Capital	30.4%
	Income/Capital	

Notes:	1 in 100 year loss = 99th percentile loss:	94,423,873
	Gross Premium:	40,000,000
	Commission, Expense and Capital Charge:	14,000,000
	Loss @ 27.3 % Ratio	10,907,790
	Commission and Expense:	35.0% of Premium
	Capital Charge:	0.0% of intermediate capital
	Interest Income:	4.5% of 1/2 of (Premium - Expense)
	Investment Income:	7.5% of Capital
	Tax Rate	0.0% of Pre-Tax Income

The Value at Risk is calculated as the unexpected portion of the 99<sup>th</sup> percentile loss from the loss distribution. Please note that the Tail Value at Risk can be calculated similarly from the loss distribution.

Required capital to cover 1 in 200 year loss:

## Willis Analytics Entity XYZ Political Risk - Gross Return on Capital (1 in 200 year scenario) <sup>Currency:</sup> €

Selected Capital	89,214,334
1 in 200 year Loss - (Premium - Expenses)	
Income	22,368,285
Premium - Loss - Commission - Expense + Interest Income + Investment Income	
Return on Capital	25.1%
Income/Capital	

Notes:	1 in 200 year loss = 99.5th percentile loss:	115,214,334
	Gross Premium:	40,000,000
	Commission, Expense and Capital Charge:	14,000,000
	Loss @ 27.3 % Ratio	10,907,790
	Commission and Expense:	35.0% of Premium
	Capital Charge:	0.0% of intermediate capital
	Interest Income:	4.5% of 1/2 of (Premium - Expense)
	Investment Income:	7.5% of Capital
	Tax Rate	0.0% of Pre-Tax Income

The Value at Risk is calculated as the unexpected portion of the 99.5<sup>th</sup> percentile loss from the loss distribution. The authors believe that the allocation of capital to cover a 1 in 100 year loss is reasonable. However, the capital allocation to cover a 1 in 200 year loss is demonstrated to facilitate regulatory requirements in different jurisdictions.

#### Underwriting profitability Exhibit

## Willis Analytics

Entity XYZ Political Risk - Gross Underwriting Profit currency: €

Gross Premium	40,000,000
Commission + Expense	35.0%
Gross Capital	68,423,873
Probability of Gross UW Upside Average Gross UW Upside	87.1% 21,205,556
Probability of Gross UW Downside Average Gross UW Downside	12.9% (26,038,237)
Average Gross UW Profit/(Loss) Capital Charge @ 0 %	15,092,210 -
Gross Profit Net of Capital Charge	15,092,210
Gross Profit Margin Net of Capital Charge Gross Combined Ratio	37.7% 62.3%

#### Variability in Gross Profits



The Underwriting Profitability Exhibit shows both the upside and the downside of reinsuring a political risk portfolio in a systematic way. The extreme right hand side of the curve reflects the effect of correlated high severity events that are generated by this methodology.

# Willis Analytics Political Risk Reinsurance Model

#### Contribution to the 99th percentile

				Percentage
Index	Country	Exposure	Rating	of Loss
1	Turkey	400,000,000	Ba3	11.36%
9	INDIA	150,000,000	Ba1	31.25%
12	ANGOLA	110,000,000	Caa3	24.84%
13	MOROCCO	105,000,000	Ba2	5.70%
18	NIGERIA	85,000,000	B3	13.88%
26	CAMEROON	50,000,000	B2	1.32%
34	BAHRAIN	35,000,000	Baa3	1.30%
56	DJIBOUTI	15,000,000	B2	1.69%
67	GABON	10,000,000	B1	1.50%
80	NIGER	10,000,000	B2	0.82%
81	RWANDA	10,000,000	Caa1	1.50%
85	ALGERIA	5,000,000	B1	0.06%
86	SEYCHELLES	5,000,000	B1	0.61%
90	MACEDONIA	5,000,000	Caa1	0.47%
100	CHAD	5,000,000	B2	0.72%
103	GUINEA	5,000,000	B3	0.24%
113	BURKINA FASO	5,000,000	Ba3	0.24%
115	BURUNDI	5,000,000	Caa1	0.45%
116	CAMBODIA	5,000,000	Caa1	0.51%
117	DOMINICA	5,000,000	Caa1	0.58%
122	MALAWI	5,000,000	B3	0.71%
123	SAO TOME & PRINCIPE	5,000,000	Caa1	0.25%

This exhibit demonstrates the countries that are likely to use most of the reinsurance capacity provided there is an extreme event. The idea is to generate questions regarding the risk management and individual long-term profitability of contracts in these countries.

#### Section 6: Conclusion

It is our belief that the proposed methodology will provide actuaries and financial analysts with the ability to consistently and objectively quantify the risk of writing a political risk portfolio. It will facilitate the estimation of Value at Risk, Tail Value at Risk, Required Capital, Return on Capital, and Underwriting Profitability for a given political risk portfolio by reinsurers and capital markets. Given that the analytical processes and country rating systems of the primary political risk carriers are already in place, the data and analytical tools are available to apply this methodology to evaluate the risk-return profile of a political risk portfolio.

The authors strongly believe that applying this approach would give organizations involved in political risk insurance and reinsurance a solid foundation for making strategic and business decisions.

#### Appendix A

#### The fixed-time horizon model

Correlation is the degree to which two or more quantities are linearly associated. In a two-dimensional plot, the degree of correlation between the values on the two axes is quantified by the so-called correlation coefficient.

According to Li (1999): the linear correlation of default for two securities *i* and *j*,  $\rho_{ij}$  satisfies the following equation

$$\rho_{ij} = \frac{Cov(i,j)}{\sqrt{Var(i) \cdot Var(j)}} = \frac{Cov(i,j)}{\sqrt{u_i(1-u_i)u_j(1-u_j)}}$$
(1),

where  $u_i, u_j$  are corresponding default probabilities. The approach the authors used to incorporate correlation into the simulation engine is described below. Any attempt to simulate credit events without giving appropriate regard to the effects of correlation would severely underestimate the tail of the distribution.

It is necessary to compute Cov(i, j) using the within and between correlations assumptions determined at the outset of the analysis. The authors use Merton's approach to calculate Cov(i, j). The Merton approach to the firm's value suggests that a default occurs when the value of assets is below certain threshold (Merton (1974)). In other words, default takes place when a random variable representing firm's assets  $X_i$  (with CDF  $P(X_i)$ ) is below a certain level. Two companies are in default if  $X_i < P^{-1}(u_i)$ ,  $X_j < P^{-1}(u_j)$ . Then the covariance equals to

$$P(P^{-1}(u_i), P^{-1}(u_j)) - u_i u_j$$

In order to generate credit events (defaults), Monte Carlo simulation is applied. A set of independent normal random variables are transformed to a correlated standard normal random variables by introducing a correlation matrix to the process. The correlated standard normal random variables are compared to the thresholds based on default rates. The correlation matrix needs to be decomposed based on the Cholesky decomposition prior to creating a matrix of correlated standard random variables. There are two key adjustments that are necessary to obtain a reasonable set of outcomes. They are outlined below.

#### The Merton Adjustment

The most straightforward approach in the calculation of a copula in the above equation is to assume that  $X_i$ ,  $X_i$  are normally distributed. Then, one obtains for coefficient of correlation (Pugachevsky (2002))

$$\rho_{ij} = \frac{N^{(2)}(N^{-1}(u_i), N^{-1}(u_j), \rho_{ij}^M) - u_i u_j}{\sqrt{u_i(1 - u_i)u_j(1 - u_j)}}$$
(2).

In equation (2),  $N^{(2)}$  is the cumulative bivariate normal distribution function with pair-wise correlation coefficient  $\rho_{ij}^{M}$  and  $N^{(-1)}$  is the inverse of standard normal distribution. The matrix  $\rho_{ij}^{M}$  is determined numerically from Eq.(2) and is used in the loss simulation. After pair-wise correlation coefficients are computed, the simulation engine can produce a correlated multi-variate distribution. According to Pugachevsky (2002), the main advantage of this method is that it is easy to define correlations between random variables in a simulation environment.

The resulting correlation  $\rho_{ij}^{M}$  is materially higher than the discrete events correlation  $\rho_{ij}$ . It should be noted that the use of events correlation  $\rho_{ij}$  in simulation would lead to substantial underestimation of the correlation effect.

#### How to make the correlation matrix positive-definite

The resulting correlation matrix obtained from Eq. (2) is not necessarily positive-definite. The positive-definiteness is a requirement that guarantees the ability to decompose the correlation matrix after the application of Merton adjustment. There are several known techniques that would help transform the correlation matrix into a positive definite matrix. The authors chose the approach suggested by Rebonato and Jackel (1999) to revise the matrix  $\rho_{ij}^{M}$ . The adjustment procedure involves three steps. First, eigenvalues and eigenvectors of the matrix of pair-wise correlations  $\Sigma^{2}$  are defined,

$$\Sigma^2 S = \Lambda S ,$$

where  $\Lambda$ , S are matrices of eigenvalues and eigenvectors, respectively. Second, zero or negative eigenvalues are replaced by very small positive numbers. Third step involves the production of the correlation matrix using modified eigenvalues  $\lambda'$  and eigenvectors of initial correlation matrix. Taking into account that diagonal elements of the correlation matrix have to be equal to one, the resulting modified matrix equals

$$\sqrt{TS\Lambda'S^T}\sqrt{T'}$$

where the matrix T is required for the normalization.

#### Copulas required for a multi-year model

According to Nelsen (1999), *d*-dimensional  $(d \ge 2)$  distribution function with marginals uniformly distributed in I (I is a [0,1] interval) is called copula. For two-dimensional case, let X, Y be continuous random variables with CDFs F(x), G(y) and joint distribution H(x,y). For every point (x,y) there is a point in  $\mathbf{I}$  with coordinates (F(x), G(y), H(x,y)). This mapping from  $\mathbf{I}$  to I is copula.

Sklar's theorem clarifies importance of copulas in statistical modeling. It states existence of copula C for H, F, G: H(x,y)=C(F(x),G(y)), and existence of two-dimensional distribution H given C,F,G (Nelsen). Based on Sklar's theorem,

$$H(x_1,...,x_n) = \Pr(X_1 \le x_1,...,X_n \le x_n) = C(F_1(x_1),...,F_n(x_n)).$$

Copulas provide a natural way to measure dependence between random variables. A generalization of equation (2) for linear correlation between times of defaults of two entities A, B can be expressed as

$$\rho_{A,B} = \frac{\Pr(\tau_A \le T_A, \tau_B \le T_B) - \Pr(\tau_A \le T_A) \Pr(\tau_B \le T_B)}{\sqrt{\Pr(\tau_A \le T_A)(1 - \Pr(\tau_A \le T_A)) \Pr(\tau_B \le T_B)(1 - \Pr(\tau_B \le T_B)))}}$$

Concordance/discordance is another dependence measure. Two observations (x,y) of pair of continuous random variables (X,Y) are concordant if  $(x_1 - x_2)(y_1 - y_2) > 0$ , and discordant otherwise.

Kendall's tau is given by  $\tau = \Pr((X_1 - X_2)(Y_1 - Y_2) > 0) - \Pr((X_1 - X_2)(Y_1 - Y_2) < 0).$  If joint distribution of *X*, *Y* is represented by copula *C* then Kendall's tau equals  $\tau = 4 \iint C(u, v) dC(u, v) - 1$ .

Copulas are instrumental in studying tail dependence , which is defined for upper tail dependence (see Embrechts) as

$$\Pr(Y > G^{-1}(u) \mid X > F^{-1}(u)) = \lambda_U.$$

 $\lambda_{U}$  can be expressed through copula as limit of ratio,

$$\lambda_U \underset{u \to 1}{=} \frac{(1 - 2u + C(u, u))}{1 - u}$$

A comparison of pair-wise correlation to event correlation is provided in Appendix B.

#### A list of copulas employed by practitioners

Gaussian copula is the most popular example of the family of elliptical copulas and shown as  $N^{(2)}(u,v) = N^{(2)}(N^{-1}(u), N^{-1}(v)).$ 

A copula is called Archimedean if it has the form

 $C(x_1,...,x_d) = \psi^{[-1]}(\sum_{i=1}^d \psi(x_i)), \text{ where the function } \psi \text{ is called generator of the copula.}$ The most intuitive example of Archimedean copulas is the bivariate independent copula  $[C(u_1, u_2) = u_1 u_{2]}, with the generator <math>-\ln(u)$ . The widely used Archimedean copulas include Clayton, Gumbel, Frank copulas, etc. (Nelsen). For example, Gumbel copula has a generator equal to  $\psi(t) = (-\ln(t))^{\vartheta}, \quad \vartheta \ge 1$ . This generator yields following expression for bivariate copula,  $C(u,v) = \exp\{-[(-\ln(u))^{\vartheta} + (-(\ln(v))^{\theta}]^{1/\vartheta}\}$ . The tail dependence for this copula can be obtained by substitution of expression for the copula into formula for  $\lambda_U$  and computing limit using L'Hopital's rule. The Gumbel copula has positive tail dependence for  $\vartheta \ge 1$ ,  $\lambda_U = 2 - 2^{1/\theta}$ .

A key attraction of Archimedean copulas is that they can handle asymmetric situations such as stronger dependence between big losses than between big gains (Embrechts - 2003). For example, Gumbel copula has an upper tail dependence for  $\vartheta \ge 1$  and lower tail independence,  $\lambda_L = 0$ .

In this paper, authors selected a Gaussian copula in the multi-period model to account for correlation of defaults. The advantages are as follows:

- 1. Gaussian copulas are easy to calculate.
- 2. Transparency of effects of correlation.

The disadvantages are as follows:

- 1. There is a school of thought that the Gaussian copulas are better suited for equity modeling.
- 2. Gaussian copulas may not be suited to model correlation of extreme events in the tail.

#### Multi-period analysis

The authors believe that a fixed time horizon model is not a sufficient tool to model a multi-year political risk portfolio. Thus, it is imperative to introduce a multi-year model that contains a reasonable structure to reflect dependence between and within sovereign nations.

The standard approach to default in interval [t-1, t] would be to assume that the probability of default prior to t equals probability of default prior to t-1 plus conditional probability of default between [t, t-1]

i.e. 
$$u_t = u_{t-1} + (1 - u_{t-1})u_{t-1,t}$$
,

Then,

$$u_{t-1,t} = \frac{u_t - u_{t-1}}{1 - u_{t-1}}.$$

#### Appendix B:



Event Correlation = 20% Upper Curve = Baa2 Lower Curve = B2 Acknowledgements



The authors would like to express their gratitude and appreciation to the following industry participants for providing valuable historical data and guidance.

<u>Ace Global Markets (Ace)</u> Julian Edwards, Steve Capon, and Peter Sprent

<u>American International Group (AIG)</u> John Hegeman, John Salinger

<u>Beazley Group</u> Adrian Lewers, Simon Brickman

<u>Coface</u> Arnaud Tisseyre, Didier Collet, Xavier Joseph

Export Development Corporation (EDC Canada) Norman Kimber

**EXPORTKREDITNÄMNDEN (ekn)** Eva Cassel

Hiscox Syndicate Andrew Underwood

#### MIGA (World Bank)

Jotaro Hamada

## **Overseas Private Investment Corporation (OPIC)**

Edith Quintrell, Dr. Stephen Everhart

#### XL Insurance

David Wright, Elizabeth Edwards

The authors wish to also acknowledge the support and contributions from our colleagues at Willis Re. Ge Zhang made major contributions to the selection of techniques for predictive modeling and Discriminant Function analysis and did the preliminary work on creating a default rates data base. The authors appreciate the efforts from Grace Qiu to continue and expand the database related work started by Ge Zhang.

Rick Bowering and Mark Jenkins, Willis Re Political Risk Practice Leaders for North America and Continental Europe, respectively, provided strong support and direction to this project, while David Neckar, Andrew Pace, and James Vickers were generous with their help in the initial phase of gathering data and organizing the project team.

The authors are grateful for the time and efforts of Bill Panning, Yves Provencher, Michael Parker, Ian Cook, Jurgen Gaiser-Porter, David Neckar, Ellen Friedman, Peter Szendro, Ron Carlson, and Mark Hvidsten in reviewing the various drafts of this paper and providing valuable feedback.

Finally, we want to express our appreciation for the commitment of Peter Hearn, CEO, Willis Re Inc., and his management team to this project and to long-term research and development at Willis Re.

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#### Glossary

**Copula** A function that joins univariate distribution functions to form multivariate distribution functions. A copula of a multivariate distribution can be thought of as the instrument that describes the dependence structure.

**Credit Risk** is the risk due to uncertainty in a counterparty's (also called an obligor or credit's) ability to meet it's obligations. Because there are many types of counterparties, from individuals to sovereign governments and many different types of obligations, from auto loans to derivatives transactions, credit risk takes many forms

**Credit Spread** for a bond equals to difference between yield on a risky bond and yield on a default-free government bond with a similar maturity

**Recovery Rate** In the event of a default, the recovery rate is the fraction of the exposure that may be recovered through bankruptcy proceedings or some other form of settlement

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