International Scholarly Research Network
ISRN Probability and Statistics
Volume 2012, Article ID 186348, 24 pages
doi:10.5402/2012/186348

Research Article

Uniform Asymptotics for the Finite-Time Ruin Probability of a Time-Dependent Risk Model with Pairwise Quasiasymptotically Independent Claims

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Received 11 April 2012; Accepted 10 May 2012

Academic Editors: N. Chernov, F. Fagnola, and P. E. Jorgensen

We consider a generalized time-dependent risk model with constant interest force, where the claim sizes are of pairwise quasiasymptotical independence structure, and the claim size and its interclaim time satisfy a dependence structure defined by a conditional tail probability of the claim size given the interclaim time before the claim occurs. As the claim-size distribution belongs to the dominated variation class, we establish some weakly asymptotic formulae for the tail probability of discounted aggregate claims and the finite-time ruin probability, which hold uniformly for all times in a relevant infinite interval.

1. Introduction

In the paper we will consider a generalized risk model of an insurance company, in which the claim sizes $\{X_i, i \geq 1\}$ are nonnegative, identically distributed, but not necessarily independent random variables (r.v.s) with common distribution F and generic r.v. X, and their interarrival times $\{\theta_i, i \geq 1\}$ are other independent, identically distributed (i.i.d.), and nonnegative r.v.s with generic r.v. θ. To avoid triviality, X and θ are assumed not to be degenerate at 0. Denote the claim arrival times by $\tau_0 = 0$, $\tau_n = \sum_{i=1}^{n} \theta_i$, $n \geq 1$, which constitute a renewal counting process as follows:

$$N(t) = \sup\{n \geq 1, \tau_n \leq t\}, \quad t \geq 0, \tag{1.1}$$

with a finite-mean function $\lambda(t) = EN(t) = \sum_{i=1}^{\infty} P(\tau_i \leq t)$, $t \geq 0$. Assume that, for every $i \geq 2$, X_i and τ_{i-1} are mutually independent. Meanwhile, as mentioned by Wang [1], the total amount of premiums accumulated before $t \geq 0$, denoted by $C(t)$ with $C(0) = 0$ and $C(t) < \infty$

almost surely for any fixed $t > 0$, is a nonnegative and nondecreasing stochastic process. Let $r \geq 0$ be the constant interest force and $x \geq 0$ be the insurer's initial reserve. Hence, the total reserve up to $t \geq 0$ of the insurance company, denoted by $U_r(t)$, satisfies as following:

$$U_r(t) = xe^{rt} + \int_0^t e^{r(t-s)}C(ds) - \int_0^t e^{r(t-s)}S(ds), \tag{1.2}$$

and the discounted aggregate claims up to $t \geq 0$ are expressed as

$$D_r(t) = \int_0^t e^{-rs}S(ds) = \sum_{i=1}^{\infty} X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \leq t\}}, \tag{1.3}$$

where $S(t) = \sum_{i=1}^{N(t)} X_i$ is the aggregate claim amount before $t \geq 0$ with $S(t) = 0$ if $N(t) = 0$. As usual, the ruin probability within a finite time $t > 0$ is defined by

$$\psi_r(x,t) = P(U_r(s) < 0 \text{ for some } 0 \leq s \leq t), \tag{1.4}$$

and the infinite-time ruin probability is

$$\psi_r(x,\infty) = P(U_r(t) < 0 \text{ for some } 0 \leq t < \infty). \tag{1.5}$$

For the renewal risk model with i.i.d. claim sizes $\{X_i, i \geq 1\}$ and i.i.d. interarrival times $\{\theta_i, i \geq 1\}$, in which $\{X_i, i \geq 1\}$ and $\{\theta_i, i \geq 1\}$ are mutually independent, there are many related works on ruin theory with a constant interest force $r > 0$, for example, see Klüppelberg and Stadtmüller [2], Kalashnikov and Konstantinides [3], Konstantinides et al. [4], Tang [5, 6], and Hao and Tang [7], among others. However, the independence assumptions above are made mainly not for practical relevance but for theoretical interest.

In recent years, various extensions to the renewal risk model have been proposed to appropriately relax these independence assumptions. Generally, there are two directions to discuss the extensions. One is that a certain dependence structure is imposed on the claim sizes $\{X_i, i \geq 1\}$ and/or their interarrival times $\{\theta_i, i \geq 1\}$, but $\{X_i, i \geq 1\}$ are assumed to be independent of $\{\theta_i, i \geq 1\}$. See, for example, Chen and Ng [8], Li et al. [9], Yang and Wang [10], Wang et al. [11], and Liu et al. [12], and references therein. The other is that the claim size X and its interarrival time θ follow a certain dependence structure, but $\{(X_i, \theta_i), i \geq 1\}$ are i.i.d. random pairs. In this direction, many researchers considered some ruin-related problems of a generalized risk model with a certain dependence between X and θ when the claim-sized distribution is light tailed, for example, Albrecher and Teugels [13], Boudreault et al. [14], Cossette et al. [15], and Badescu et al. [16]. Besides, Asimit and Badescu [17] introduced a general dependence structure for (X, θ) and presented the tail behavior of discounted aggregate claims $D_r(t)$ for the compound Poisson model with constant interest force and heavy-tailed claims. Under the dependence structure of Asimit and Badescu [17], Li et al. [18] extended the tail behavior of $D_r(t)$ to the renewal risk model.

The dependence structure between X and θ introduced by Asimit and Badescu [17] satisfies that the relation

$$P(X > x \mid \theta = t) \sim \overline{F}(x)h(t), \quad t \geq 0, \tag{1.6}$$

holds for some measurable function $h(\cdot) : [0,\infty) \mapsto (0,\infty)$, where the symbol $\sim$ means that the quotient of both sides tends to 1 as $x \to \infty$. When t is not a possible value of θ, the conditional probability in (1.6) is understood as an unconditional one, and then $h(t) = 1$. If relation (1.6) holds uniformly for all $t \in [0,\infty)$, then, by conditioning on τ_{i-1} and θ_i, $i \geq 1$, it holds uniformly for all $t \in [0,\infty)$ that

$$\begin{aligned} P\left(X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \leq t\}} > x\right) &\sim \int_0^t \int_0^{t-v} P\left(X_i e^{-r(u+v)} > x\right) P(\tau_{i-1} \in du) h(v) G(dv) \\ &= P\left(X_i e^{-r(\tau_{i-1}+\theta^*)}\mathbf{1}_{\{\tau_{i-1}+\theta^* \leq t\}} > x\right), \end{aligned} \tag{1.7}$$

where θ^* is a r.v. independent of X and θ, with a proper distribution given by

$$P(\theta^* \in dv) = h(v) G(dv). \tag{1.8}$$

Remark 1.1. Note that the general dependence structure defined by (1.6) can cover both positive and negative dependence and is also easily verifiable for many common bivariate copulas, which can be found in Li et al. [18]. Furthermore, practitioners in insurance industry often meet the following situations: for autoinsurance or fire insurance, if the claim interarrival time is longer, then more measures can be taken to reduce the forthcoming losses of property, while if the deductible of the insured is raised, then the claim interarrival time will increase since some small claims can avoid. These situations tell us that the claim size and its interarrival time are interacted on each other, and the dependence structure defined by (1.6) is realistic in actuarial environments.

Based on the two study directions above to extend the renewal risk model, this paper will consider a more generalized risk model with the claim sizes following some dependence structure as well as the random pair (X,θ) satisfying relation (1.6) and establish the weakly asymptotic formulae for the tail probability of discounted aggregate claims and the finite-time ruin probability, which hold uniformly for all times in a relevant infinite interval. The method used in the paper is different from that in the above mentioned literatures, and the obtained results can extend and improve some existing results.

The rest of this paper is organized as follows: Section 2 will present the main results of this paper after preparing some preliminaries, and Section 3 will give some lemmas which are helpful to prove our main results in Section 4.

2. Preliminaries and Main Results

All limit relationships in the paper are for $x \to \infty$ unless mentioned otherwise. For two positive functions $a(\cdot)$ and $b(\cdot)$, we write $a(x) = O(1)b(x)$ if $\limsup a(x)/b(x) = c < \infty$, write $a(x) \lesssim b(x)$ or $b(x) \gtrsim a(x)$ if $c \leq 1$, write $a(x) \sim b(x)$ if $a(x) \lesssim b(x)$ and $b(x) \lesssim a(x)$, write $a(x) = o(1)b(x)$ if $c = 0$, and write $a(x) \asymp b(x)$ if $a(x) = O(1)b(x)$ and $b(x) = O(1)a(x)$. Further, for two positive bivariate functions $a(\cdot,\cdot)$ and $b(\cdot,\cdot)$, we write $a(x,t) \sim b(x,t)$ uniformly for all $t \in \Delta \neq \emptyset$ if $\limsup_{t\in\Delta} |a(x,t)/b(x,t) - 1| = 0$.

As everyone knows, in the insurance industry how to model the dangerous claims is one of the main worries of the practicing actuaries, and actually most practitioners choose the claim-size distribution from the heavy-tailed distribution class, one of which is the dominated variation class. By definition, a distribution F belongs to the dominated variation class, denoted by $F \in \mathcal{D}$, if $\overline{F}(xy) = O(1)\overline{F}(x)$ for all $y > 0$, where $\overline{F}(x) = 1 - F(x)$; belongs to the extended regular variation (ERV) class if there exist some $0 < \alpha \le \beta < \infty$ such that $y^{-\beta} \le \overline{F}_*(y) \le \overline{F}^*(y) \le y^{-\alpha}$ for all $y \ge 1$, where $\overline{F}_*(y) = \liminf \overline{F}(xy)/\overline{F}(x)$ and $\overline{F}^*(y) = \limsup \overline{F}(xy)/\overline{F}(x)$; belongs to the consistent variation class, denoted by $F \in \mathcal{C}$, if $L_F = \lim_{y \searrow 1} \overline{F}_*(y) = 1$; belongs to the subexponential class, denoted by $F \in \mathcal{S}$, if $\overline{F}(x) > 0$ for all $x > 0$ and $\overline{F^{*2}}(x) \sim 2\overline{F}(x)$, where F^{*2} denotes the 2-fold convolution of F. Note that if $F \in \mathcal{S}$ then F is long tailed, denoted by $F \in \mathcal{L}$ and characterized by $\overline{F}(x+y) \sim \overline{F}(x)$ for all $y \neq 0$.

For a distribution F and any $y > 0$, we set $J_F^+ = -\lim_{y\to\infty} \log \overline{F}_*(y)/\log y$ and $J_F^- = -\lim_{y\to\infty} \log \overline{F}^*(y)/\log y$. It is well known that the following relationships hold, namely:

$$\mathrm{ERV} \subset \mathcal{C} \subset \mathcal{L} \cap \mathcal{D} \subset \mathcal{S} \subset \mathcal{L}, \quad \mathcal{D} \not\subset \mathcal{L}, \mathcal{L} \not\subset \mathcal{D}. \tag{2.1}$$

For more details of heavy-tailed distributions and their applications to insurance and finance, the readers are referred to Bingham et al. [19] and Embrechts et al. [20]. Additionally, some reviews on the class $\mathcal{D}$ and its application can be found in Shneer [21], Wang and Yang [22], and others.

Following the first trend to extend the renewal risk model, in the paper we will discuss a risk model with the claim sizes satisfying the following dependence structure.

Definition 2.1. Say that the r.v.s $\{\xi_i, i \ge 1\}$ with their respective distributions $V_i, i \ge 1$ are pairwise quasiasymptotically independent if

$$P(\xi_i > x, \xi_j > x) = o(1)\left(\overline{V_i}(x) + \overline{V_j}(x)\right) \quad \text{for } i \neq j,\ i, j \ge 1. \tag{2.2}$$

Remark 2.2. The term "pairwise quasiasymptotic independence" is borrowed from Chen and Yuen [23]. Clearly, if $\{\xi_i, i \ge 1\}$ are identically distributed, relation (2.2) is equivalent to

$$P(\xi_i > x, \xi_j > x) = o(1)P(\xi_i > x) \quad \text{for } i \neq j,\ i, j \ge 1, \tag{2.3}$$

which means that $\{\xi_i, i \ge 1\}$ are pairwise asymptotically independent or bivariate upper tail independent; see Zhang et al. [24] and Gao and Wang [25]. Also remark that the sequence of pairwise quasiasymptotically independent r.v.s can cover the sequence of widely upper orthant dependent/widely lower orthant dependent r.v.s (see Wang et al. [11]), extended negatively dependent r.v.s (see Liu [26]), negatively upper orthant dependent/negatively lower orthant dependent r.v.s (NUOD/NLOD, see Block et al. [27]), pairwise negatively quadrant dependent r.v.s (NQD, see Lehmann [28]), and even it can cover some sequences of positive dependent r.v.s.

For statement convenience, we denote by Λ the set of all t such that $0 < \lambda(t) \leq \infty$. Let $\underline{t} = \inf\{t : P(\tau_1 \leq t) > 0\}$, it is clear that

$$\Lambda = \begin{cases} [\underline{t}, \infty], & \text{if } P(\tau_1 = \underline{t}) > 0; \\ (\underline{t}, \infty], & \text{if } P(\tau_1 = \underline{t}) = 0. \end{cases} \tag{2.4}$$

Also we write $\Lambda_T = [0, T] \cap \Lambda$ for any finite $T \in \Lambda$.

Under the case that the claim sizes and/or interarrival times follow some dependence structure, Li et al. [9] obtained a weakly asymptotic formula for the finite-time ruin probability as follows.

Theorem 2.3. *Consider the insurance risk model introduced in Section 1, in which the claim sizes are pairwise NQD with common distribution $F \in \mathfrak{D}$ such that $J_F^- > 0$, the interarrival times $\{\theta_i, i \geq 1\}$ are NLOD, and the premium process $\{C(t), t \geq 0\}$ is a deterministic linear function. If $\{X_i, i \geq 1\}$ and $\{\theta_i, i \geq 1\}$ are mutually independent, then, for every fixed $t \in \Lambda$,*

$$L_F \int_0^t \overline{F}(xe^{rs}) d\lambda(s) \lesssim \psi_r(x, t) \lesssim L_F^{-2} \int_0^t \overline{F}(xe^{rs}) d\lambda(s). \tag{2.5}$$

In addition, if $\{C(t), t \geq 0\}$ is a general stochastic process and $\{N(t), t \geq 0\}$ is a delayed renewal counting process, Yang and Wang [10] also gave the formula (2.5) for $\psi_r(x, t)$.

Following the second extension direction, Li et al. [18] have investigated the tail behavior of discounted aggregate claims $D_r(t)$ for the renewal risk model with X and θ meeting the dependence structure defined by (1.6).

Theorem 2.4. *Consider the discounted aggregate claims defined in* (1.3) *with $r > 0$, in which $\{(X_i, \theta_i), i \geq 1\}$ are i.i.d. random pairs, and relation* (1.6) *holds uniformly for all $t \in \Lambda$. If $F \in ERV(-\alpha, -\beta)$ for some $0 < \alpha \leq \beta < \infty$, and $\inf_{0 \leq t \leq t^*} h(t) > 0$ for some $t^* \in \Lambda$, then*

$$P(D_r(t) > x) \sim \int_0^t \overline{F}(xe^{rs}) d\tilde{\lambda}(s) \tag{2.6}$$

holds uniformly for $t \in \Lambda$, where

$$\tilde{\lambda}(t) = \sum_{i=1}^{\infty} P(\tau_{i-1} + \theta^* \leq t). \tag{2.7}$$

Inspired by the results of Theorems 2.3 and 2.4, in this paper we will further discuss the following issues.

(1) For the structure assumptions on the claim sizes, we will study the more general case of pairwise quasiasymptotically independence assumption.

(2) We will extend the mutual independence between the claim sizes and their inter-times to a general dependence structure defined by relation (1.6).

(3) We will extend the condition $F \in$ ERV of Theorem 2.4 to the conditions $F \in \mathfrak{D}$ (or $F \in \mathcal{C}$).

(4) We will cancel the condition that $\inf_{0\le t\le t^*} h(t) > 0$ for some $t^* \in \Lambda$ of Theorem 2.4.

(5) We do not confine this paper only to the case that $r > 0$, but discuss the case that $r \geq 0$.

(6) We will discuss the case when the premium process $\{C(t), t \geq 0\}$ is not necessarily independent of $\{X_i, i \geq 1\}$ and $\{N(t), t \geq 0\}$.

The following are the main results of this paper, among which the first two theorems discuss the tail behavior of the discounted aggregate claims $D_r(t)$ described by (1.3).

Theorem 2.5. *Consider the discounted aggregate claims* (1.3) *described in Section 1 with* $r \geq 0$. *If the claim sizes* $\{X_i, i \geq 1\}$ *are pairwise quasiasymptotically independent r.v.s with common distribution* $F \in \mathfrak{D}$ *such that* $J_F^- > 0$, *and relation* (1.6) *holds uniformly for all* $t \in \Lambda_T$, *then it holds uniformly for all* $t \in \Lambda_T$ *that*

$$\int_0^t \overline{F}(xe^{rs})d\tilde{\lambda}(s) \lesssim P(D_r(t) > x) \lesssim L_F^{-2} \int_0^t \overline{F}(xe^{rs})d\tilde{\lambda}(s). \tag{2.8}$$

Additionally if $F \in \mathcal{C}$, *then relation* (2.6) *holds uniformly for all* $t \in \Lambda_T$.

The second theorem extends the set over which relations (2.6) and (2.8) hold uniformly to the whole set Λ. It is well known that if $r = 0$, then $D_r(t) \to \infty$ almost surely as $t \to \infty$, and hence it is not impossible to establish the uniformity of (2.6) and (2.8) for all $t \in \Lambda$. So we assume that $r > 0$ in the following result.

Theorem 2.6. *Consider the discounted aggregate claims* (1.3) *described in Section 1 with* $r > 0$. *If the claim sizes* $\{X_i, i \geq 1\}$ *are pairwise quasiasymptotically independent r.v.s with common distribution* $F \in \mathfrak{D}$ *such that* $J_F^- > 0$, *and relation* (1.6) *holds uniformly for all* $t \in \Lambda$, *then relation* (2.8) *still holds uniformly for all* $t \in \Lambda$. *Further assume that* $F \in \mathcal{C}$, *then relation* (2.6) *still holds uniformly for all* $t \in \Lambda$.

In what follows, we will deal with the asymptotic behavior of the finite-time and infinite-time ruin probabilities, where we will discuss two cases: one is that the premium process $\{C(t), t \geq 0\}$ is independent of $\{X_i, i \geq 1\}$ and $\{N(t), t \geq 0\}$, and the other is that $\{C(t), t \geq 0\}$ is not necessarily independent of $\{X_i, i \geq 1\}$ or $\{N(t), t \geq 0\}$. For later use, we write the discounted value of premiums accumulated before time t as

$$\widetilde{C}(t) = \int_0^t e^{-rs} C(ds). \tag{2.9}$$

Clearly, by the conditions on $C(t)$, it holds that $0 \leq \widetilde{C}(t) < \infty$ almost surely for any fixed $0 < t < \infty$.

Theorem 2.7. *Consider the insurance risk model introduced in Section 1 with* $r \geq 0$. *Under the conditions of Theorem 2.5, it holds uniformly for* $t \in \Lambda_T$ *that*

$$L_F \int_0^t \overline{F}(xe^{rs})d\tilde{\lambda}(s) \lesssim \psi_r(x,t) \lesssim L_F^{-2} \int_0^t \overline{F}(xe^{rs})d\tilde{\lambda}(s), \tag{2.10}$$

if either of the following conditions holds:

(1) *the premium process* $\{C(t), t \geq 0\}$ *is independent of* $\{X_i, i \geq 1\}$ *and* $\{N(t), t \geq 0\}$*;*

(2) *the discounted value of premiums accumulated by time* t *satisfies that*

$$P\left(\widetilde{C}(t) > x\right) = o(1)\overline{F}(x). \tag{2.11}$$

Particularly, if $F \in \mathcal{C}$*, it holds uniformly for* $t \in \Lambda_T$ *that*

$$\psi_r(x,t) \sim \int_0^t \overline{F}(xe^{rs})d\widetilde{\lambda}(s). \tag{2.12}$$

Applying Theorem 2.7, we now propose a corollary for the case when $r = 0$.

Corollary 2.8. *Consider the insurance risk model in Section 1 with* $r = 0$*; if the conditions of Theorem 2.7 are valid, then it holds uniformly for all* $t \in \Lambda_T$ *that*

$$L_F\overline{F}(x)\widetilde{\lambda}(t) \lesssim \psi_0(x,t) \lesssim L_F^{-2}\overline{F}(x)\widetilde{\lambda}(t), \tag{2.13}$$

$$\kappa^{-1}L_F\int_x^{x+\kappa\widetilde{\lambda}(t)}\overline{F}(y)dy \lesssim \psi_0(x,t) \lesssim \kappa^{-1}L_F^{-3}\int_x^{x+\kappa\widetilde{\lambda}(t)}\overline{F}(y)dy, \tag{2.14}$$

where κ *is any positive number. Further assume that* $F \in \mathcal{C}$*, and then*

$$\psi_0(x,t) \sim \overline{F}(x)\widetilde{\lambda}(t) \sim \kappa^{-1}\int_x^{x+\kappa\widetilde{\lambda}(t)}\overline{F}(y)dy \tag{2.15}$$

holds uniformly for all $t \in \Lambda_T$*.*

Theorem 2.9. *Consider the insurance risk model introduced with* $r > 0$*. Under the conditions of Theorem 2.6, relation* (2.10) *holds uniformly for* $t \in \Lambda$*, if either of the following conditions holds:*

(1) *the premium process* $\{C(t), t \geq 0\}$ *is independent of* $\{X_i, i \geq 1\}$ *and* $\{N(t), t \geq 0\}$*;*

(2) *the total discounted amount of premiums satisfies the following:*

$$0 \leq \widetilde{C}(\infty) < \infty \quad \textit{almost surely}, \qquad P\left(\widetilde{C}(\infty) > x\right) = o(1)\overline{F}(x). \tag{2.16}$$

Particularly, if $F \in \mathcal{C}$*, then relation* (2.12) *still holds uniformly for* $t \in \Lambda$*.*

According to the uniformity of $\psi_r(x,t)$ for all $t \in \Lambda$ in Theorem 2.9, we can immediately derive the corresponding result on the infinite-time ruin probability $\psi_r(x,\infty)$.

Corollary 2.10. *Under conditions of Theorem 2.9, one has*

$$L_F\int_0^\infty \overline{F}(xe^{rs})d\widetilde{\lambda}(s) \lesssim \psi_r(x,\infty) \lesssim L_F^{-2}\int_0^\infty \overline{F}(xe^{rs})d\widetilde{\lambda}(s). \tag{2.17}$$

If $F \in \mathcal{C}$, then

$$\psi_r(x,\infty) \sim \int_0^\infty \overline{F}(xe^{rs})d\widetilde{\lambda}(s). \tag{2.18}$$

Remark 2.11. We would like to make some explanations of conditions $F \in \mathfrak{D}$ with $J_F^- > 0$ in the main results. Wang and Yang [22] proposed the following assertions.

(i) $\mathfrak{D} = \mathfrak{D}_1 \cup \mathfrak{D}_2 \cup \mathfrak{D}_3$, where

$$\begin{aligned}
\mathfrak{D}_1 &= \left\{F \text{ on } [0,\infty) : \overline{F}_*(y) = 1 \ \forall y > 1\right\},\\
\mathfrak{D}_2 &= \left\{F \text{ on } [0,\infty) : 0 < \overline{F}_*(y) < \overline{F}^*(y) = 1 \ \forall y > 1\right\},\\
\mathfrak{D}_3 &= \left\{F \text{ on } [0,\infty) : 0 < \overline{F}_*(y) \le \overline{F}^*(y) < 1 \ \forall y > 1\right\},
\end{aligned} \tag{2.19}$$

and $\mathfrak{D}_1$, $\mathfrak{D}_2$, and $\mathfrak{D}_3$ are pairwise disjoint sets.

(ii) $F \in \mathfrak{D}$ *with* $J_F^- > 0 \Leftrightarrow F \in \mathfrak{D}_3$.

(iii) Denote that

$$\mathfrak{D}(-\alpha,-\beta,A,B) = \left\{F \text{ on } [0,\infty) : By^{-\beta} \le \overline{F}_*(y) \le \overline{F}^*(y) \le Ay^{-\alpha} \ \forall y > 1\right\}, \tag{2.20}$$

where $0 \le \alpha \le \beta < \infty$, $0 < B \le 1 \le A < \infty$, and then

$$\begin{aligned}
\mathfrak{D} &= \bigcup_{0\le\alpha\le\beta<\infty,0<B\le 1\le A<\infty} \mathfrak{D}(-\alpha,-\beta,A,B),\\
\mathfrak{D}_3 &= \bigcup_{0<\alpha\le\beta<\infty,0<B\le 1\le A<\infty} \mathfrak{D}(-\alpha,-\beta,A,B).
\end{aligned} \tag{2.21}$$

In the particular case where $A = B = 1$, the class ERV $\subset \mathfrak{D}_3$ and this inclusion are proper. For example, the Peter and Paul distribution (see Example 1.4.2 in Embrechts et al. [20]) belongs to $\mathfrak{D}_3$, but it does not belong to $\mathcal{L}$; thus, it does not belong to the class ERV.

Remark 2.12. By the expression of (2.7), one can easily see that $\widetilde{\lambda}(t)$ is exactly the mean function of a delayed renewal process constituted by $\{\theta^*, \theta_i, i \ge 2\}$.

Remark 2.13. The Lemma 3.3 below tells us that $X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \le t\}}$ and $X_j e^{-r\tau_j}\mathbf{1}_{\{\tau_j \le t\}}$ are quasiasymptotically independent for all $t \in \Lambda_T$ and every fixed $i \ne j \ge 1$ under the following assumptions: the claim sizes $\{X_i, i \ge 1\}$ are pairwise quasiasymptotically independent with common distribution $F \in \mathfrak{D}$, and their interarrival times $\{\theta_i, i \ge 1\}$ are independent, identically distributed and such that (1.6) holds uniformly for all $t \in \Lambda_T$. Hence, the dependence structures among claim sizes and that between claim size and its interarrival time in the paper are technically feasible.

Remark 2.14. In Theorems 2.7–2.9 and Corollaries 2.8–2.10, the independence between the premium process and the claim process in condition 1 has been extensively considered by Wang [1], Yang and Wang [10], Wang et al. [11], Liu et al. [12], and many others, while condition 2, which does not require the independence between the premium process and the claim process, allows for a more realistic case that the premium rate varies as a deterministic or stochastic function of the insurer's current reserve, as that considered by Petersen [29], Michaud [30], Jasiulewicz [31], and Tang [5].

3. Some Lemmas

In order to prove the main results, we need the following lemmas, among which the first lemma is a combination of Proposition 2.2.1 of Bingham et al. [19] and Lemma 3.5 of Tang and Tsitsiashvili [32].

Lemma 3.1. *For a distribution F on $(-\infty, \infty)$, the following assertions hold:*

(1) $F \in \mathfrak{D} \Leftrightarrow L_F > 0 \Leftrightarrow J_F^+ < \infty$,

(2) *If $F \in \mathfrak{D}$, then for any $p_1 < J_F^-$ and any $p_2 > J_F^+$, there are positive numbers C_i and D_i, $i = 1, 2$, such that*

$$\frac{\overline{F}(y)}{\overline{F}(x)} \geq C_1 \left(\frac{x}{y}\right)^{p_1} \quad \forall x \geq y \geq D_1, \tag{3.1}$$

$$\frac{\overline{F}(y)}{\overline{F}(x)} \leq C_2 \left(\frac{x}{y}\right)^{p_2} \quad \forall x \geq y \geq D_2. \tag{3.2}$$

Lemma 3.2. *Consider the insurance risk model introduced in Section 1 with $r \geq 0$ and $F \in \mathfrak{D}$, where a generic pair (X, θ) is such that relation (1.6) holds uniformly for all $t \in \Lambda_T$, and then*

(1) *the distribution of the $X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}}$ belongs to the class $\mathfrak{D}$ for every fixed $i \geq 1$ and all $t \in \Lambda_T$; moreover, if $F \in \mathcal{C}$, then the distribution of the $X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}}$ still belongs to the class $\mathcal{C}$ for every fixed $i \geq 1$ and all $t \in \Lambda_T$;*

(2) *for every fixed $i \geq 1$ and all $t \in \Lambda_T$, it holds that*

$$\overline{F}(x) \asymp P\left(X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}} > x\right). \tag{3.3}$$

Proof. (1) If $F \in \mathfrak{D}$, then we have from (1.7) and Theorem 3.3(ii) of Cline and Samorodnitsky [33] that, for all $y > 0$,

$$\limsup_{x \to \infty} \frac{P\left(X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}} > xy\right)}{P\left(X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}} > x\right)} = \limsup_{x \to \infty} \frac{P\left(X_i e^{-r(\tau_{k-1}+\theta^*)} \mathbf{1}_{\{\tau_{k-1}+\theta^* \leq t\}} > xy\right)}{P\left(X_i e^{-r(\tau_{k-1}+\theta^*)} \mathbf{1}_{\{\tau_{k-1}+\theta^* \leq t\}} > x\right)} < \infty \tag{3.4}$$

holds for every fixed $i \geq 1$ and all $t \in \Lambda_T$, which implies that the distribution of the $X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}}$ belongs to the class $\mathfrak{D}$ for every fixed $i \geq 1$ and all $t \in \Lambda_T$. Moreover, if $F \in \mathcal{C}$, then by (1.7) and Lemma 2.5 of Wang et al. [34], we derive that, for every fixed $i \geq 1$ and all $t \in \Lambda_T$,

$$\lim_{y \searrow 1} \liminf_{x \to \infty} \frac{P\left(X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}} > xy\right)}{P\left(X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}} > x\right)} = \lim_{y \searrow 1} \liminf_{x \to \infty} \frac{P\left(X_i e^{-r(\tau_{k-1}+\theta^*)} \mathbf{1}_{\{\tau_{k-1}+\theta^* \leq t\}} > xy\right)}{P\left(X_i e^{-r(\tau_{k-1}+\theta^*)} \mathbf{1}_{\{\tau_{k-1}+\theta^* \leq t\}} > x\right)} = 1, \tag{3.5}$$

which also implies that the distribution of the $X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}}$ belongs to the class $\mathfrak{D}$ for every fixed $i \geq 1$ and all $t \in \Lambda_T$.

(2) By $F \in \mathfrak{D}$, (1.7) and Theorem 3.3(iv) of Cline and Samorodnitsky [33], we can prove that (3.3) holds. □

Lemma 3.3. *Under the conditions of Theorem 2.5, if $F \in \mathfrak{D}$, then $X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}}$ and $X_j e^{-r\tau_j} \mathbf{1}_{\{\tau_j \leq t\}}$ are still quasiasymptotically independent for all $t \in \Lambda_T$ and every fixed $i \neq j \geq 1$.*

Proof. Without loss of generality, we assume that $j > i \geq 1$. By conditioning on τ_{i-1} and θ_i, $i \geq 1$, and using (1.6) and (1.7), it holds for all $t \in \Lambda_T$ that

$$\begin{aligned}
&P\left(X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}} > x, X_j e^{-r\tau_j} \mathbf{1}_{\{\tau_j \leq t\}} > x\right) \\
&\quad = \int_0^t \int_0^{t-v} P\left(X_i e^{-r(u+v)} > x, X_j e^{-r(u+v+\theta_{i+1}+\cdots+\theta_j)} \mathbf{1}_{\{\theta_{i+1}+\cdots+\theta_j \leq t-(u+v)\}} > x \mid \tau_{i-1} = u, \theta_i = v\right) \\
&\qquad\qquad \times P(\tau_{i-1} \in du) G(dv) \\
&\quad = \int_0^t \int_0^{t-v} P\left(X_i e^{-r(u+v)} > x \mid \theta_i = v\right) \\
&\qquad\qquad \times P\left(X_j e^{-r(u+v+\theta_{i+1}+\cdots+\theta_j)} \mathbf{1}_{\{\theta_{i+1}+\cdots+\theta_j \leq t-(u+v)\}} > x \mid X_i e^{-r(u+v)} > x\right) P(\tau_{i-1} \in du) G(dv) \\
&\quad \sim \int_0^t \int_0^{t-v} P\left(X_i e^{-r(u+v)} > x, X_j e^{-r(u+v+\theta_{i+1}+\cdots+\theta_j)} \mathbf{1}_{\{\theta_{i+1}+\cdots+\theta_j \leq t-(u+v)\}} > x\right) \\
&\qquad\qquad \times P(\tau_{i-1} \in du) P(\theta^* \in dv) \\
&\quad = P\left(X_i e^{-r(\tau_{i-1}+\theta^*)} \mathbf{1}_{\{\tau_{i-1}+\theta^* \leq t\}} > x, X_j e^{-r(\tau_{j-1}+\theta^*)} \mathbf{1}_{\{\tau_{i-1}+\theta^* \leq t\}} > x\right).
\end{aligned} \tag{3.6}$$

Denote $W_i = e^{-r(\tau_{i-1}+\theta^*)} \mathbf{1}_{\{\tau_{i-1}+\theta^* \leq t\}}$ and $W_j = e^{-r(\tau_{j-1}+\theta^*)} \mathbf{1}_{\{\tau_{j-1}+\theta^* \leq t\}}$, which are clearly independent of (X_i, X_j). Thus, by (2.2) and (1.7), we will complete this proof if we can show that

$$P\left(W_i X_i > x, W_j X_j > x\right) = o(1)\left(P(W_i X_i > x) + P\left(W_j X_j > x\right)\right) \quad \text{for } i \neq j \geq 1. \tag{3.7}$$

Let H_i and H_j be the distributions of W_i and W_j, respectively, and $H(s,t)$ be the joint distribution of W_i and W_j, then we see that

$$\begin{aligned}
&P(W_iX_i > x, W_jX_j > x) \\
&\quad \le \iint_{\{0<s\le t\le 1\}} P\left(X_i > \frac{x}{t}, X_j > \frac{x}{t}\right) dG(s,t) + \iint_{0<t<s\le 1} P\left(X_i > \frac{x}{s}, X_j > \frac{x}{s}\right) dG(s,t) \\
&\quad = A_1(x) + A_2(x).
\end{aligned} \tag{3.8}$$

For $A_1(x)$, by (2.2), it follows that

$$\begin{aligned}
A_1(x) &\le \int_0^1 P\left(X_i > \frac{x}{t}, X_j > \frac{x}{t}\right) dH_2(t) \\
&\le \left(P(X_i > x) + P(X_j > x)\right) \int_0^1 \frac{P(X_i > (x/t), X_j > (x/t))}{P(X_i > (x/t)) + P(X_j > (x/t))} dH_2(t) \\
&= o(1)\left(P(X_i > x) + P(X_j > x)\right).
\end{aligned} \tag{3.9}$$

Then, by Theorem 3.3(iv) of Cline and Samorodnitsky [33], we have

$$\begin{aligned}
A_1(x) &= o(1)\left(P(X_i > x) + P(X_j > x)\right) \\
&= o(1)\left(P(W_iX_i > x) + P(W_jX_j > x)\right) \quad \text{for } i \ne j \ge 1.
\end{aligned} \tag{3.10}$$

Similarly, we also have

$$A_2(x) = o(1)\left(P(W_iX_i > x) + P(W_jX_j > x)\right) \quad \text{for } i \ne j \ge 1. \tag{3.11}$$

Therefore, we attain (3.7), and then we complete the proof. □

Lemma 3.4. *If the claim sizes $\{X_i, i \ge 1\}$ are pairwise quasiasymptotically independent r.v.s with common distribution $F \in \mathfrak{D}$, and relation (1.6) holds uniformly for all $t \in \Lambda_T$, and then, for $r \ge 0$ and for every fixed $n \ge 1$, it holds uniformly for all $t \in \Lambda_T$ that*

$$\sum_{i=1}^{n} P\left(X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \le t\}} > x\right) \lesssim P\left(\sum_{i=1}^{n} X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \le t\}} > x\right) \lesssim L_F^{-1} \sum_{i=1}^{n} P\left(X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \le t\}} > x\right). \tag{3.12}$$

Additionally, if $F \in \mathcal{C}$, then it holds uniformly for all $t \in \Lambda_T$ that

$$P\left(\sum_{i=1}^{n} X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \le t\}} > x\right) \sim \sum_{i=1}^{n} P\left(X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \le t\}} > x\right). \tag{3.13}$$

Proof. Clearly, for every fixed $n \geq 1$ and all $t \in \Lambda_T$, we have

$$P\left(\sum_{i=1}^{n} X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \leq t\}} > x\right) \geq \sum_{i=1}^{n} P\left(X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \leq t\}} > x\right) - \sum_{1 \leq i \neq j \leq n} P\left(X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \leq t\}} > x, X_j e^{-r\tau_j}\mathbf{1}_{\{\tau_j \leq t\}} > x\right) = \sum_{i=1}^{n} P\left(X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \leq t\}} > x\right) - A_3. \tag{3.14}$$

Form Lemma 3.3, we see that

$$\begin{aligned} &\limsup_{x\to\infty} \frac{A_3}{\sum_{i=1}^{n} P\left(X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \leq t\}} > x\right)} \\ &\quad\leq \limsup_{x\to\infty} \sum_{1 \leq i \neq j \leq n} \frac{P\left(X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \leq t\}} > x, X_j e^{-r\tau_j}\mathbf{1}_{\{\tau_j \leq t\}} > x\right)}{P\left(X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \leq t\}} > x\right) + P\left(X_j e^{-r\tau_j}\mathbf{1}_{\{\tau_j \leq t\}} > x\right)} \\ &\quad= 0, \end{aligned} \tag{3.15}$$

which, along with (3.14), leads to

$$P\left(\sum_{i=1}^{n} X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \leq t\}} > x\right) \gtrsim \sum_{i=1}^{n} P\left(X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \leq t\}} > x\right) \tag{3.16}$$

that holds uniformly for all $t \in \Lambda_T$. On the other hand, we have that, for any fixed $0 < v < 1$,

$$\begin{aligned} P\left(\sum_{i=1}^{n} X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \geq t\}} > x\right) &\leq P\left(\bigcup_{i=1}^{n}\left(X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \leq t\}} > vx\right)\right) \\ &\quad + P\left(\sum_{i=1}^{n} X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \leq t\}} > x, \bigcap_{i=1}^{n}\left(X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \leq t\}} \leq vx\right)\right) \\ &= A_4 + A_5. \end{aligned} \tag{3.17}$$

For A_4, it follows from $F \in \mathfrak{D}$ that for any fixed $\varepsilon > 0$, there exists a $x_1 > 0$ such that, for all $x \geq x_1$ and all $t \in \Lambda_T$, we have

$$\begin{aligned}
A_4 &\leq \sum_{i=1}^{n} P\left(X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}} > vx\right) \\
&\sim \sum_{i=1}^{n} P\left(X_i e^{-r(\tau_{i-1}+\theta^*)} \mathbf{1}_{\{\tau_{i-1}+\theta^* \leq t\}} > vx\right) \\
&= \sum_{i=1}^{n} \int_0^t \overline{F}(vxe^{rs}) P(\tau_{i-1} + \theta^* \in ds) \\
&\leq (1+\varepsilon)\left(\overline{F_*}\left(v^{-1}\right)\right)^{-1} \sum_{i=1}^{n} \int_0^t \overline{F}(xe^{rs}) P(\tau_{i-1} + \theta^* \in ds) \\
&= (1+\varepsilon)\left(\overline{F_*}\left(v^{-1}\right)\right)^{-1} \sum_{i=1}^{n} P\left(X_i e^{-r(\tau_{i-1}+\theta^*)} \mathbf{1}_{\{\tau_{i-1}+\theta^* \leq t\}} > x\right) \\
&\sim (1+\varepsilon)\left(\overline{F_*}\left(v^{-1}\right)\right)^{-1} \sum_{i=1}^{n} P\left(X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}} > x\right).
\end{aligned} \tag{3.18}$$

Hence, we can derive by the arbitrariness of $\varepsilon > 0$ and $0 < v < 1$ that

$$A_4 \lesssim L_F^{-1} \sum_{i=1}^{n} P\left(X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}} > x\right) \tag{3.19}$$

holds uniformly for all $t \in \Lambda_T$, where L_F^{-1} is of sense from Lemma 3.1(1). For A_5, it holds uniformly for all $t \in \Lambda_T$ that

$$\begin{aligned}
A_5 &= P\left(\sum_{i=1}^{n} X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}} > x, \frac{x}{n} < \max_{1 \leq k \leq n} X_k e^{-r\tau_k} \mathbf{1}_{\{\tau_k \leq t\}} \leq vx\right) \\
&\leq \sum_{k=1}^{n} P\left(\sum_{i=1, i \neq k}^{n} X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}} > (1-v)x, X_k e^{-r\tau_k} \mathbf{1}_{\{\tau_k \leq t\}} > \frac{x}{n}\right) \\
&\leq \sum_{k=1}^{n} \sum_{i=1, i \neq k}^{n} P\left(X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}} > \frac{(1-v)x}{n}, X_k e^{-r\tau_k} \mathbf{1}_{\{\tau_k \leq t\}} > \frac{(1-v)x}{n}\right) \\
&= o(1) \sum_{i=1}^{n} \sum_{i=1, i \neq k}^{n} \left(P\left(X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}} > \frac{(1-v)x}{n}\right) + P\left(X_k e^{-r\tau_k} \mathbf{1}_{\{\tau_k \leq t\}} > \frac{(1-v)x}{n}\right)\right) \\
&= o(1) \sum_{i=1}^{n} P\left(X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}} > x\right),
\end{aligned} \tag{3.20}$$

where in the second last step we used Lemma 3.3, and in the last step we used $F \in \mathfrak{D}$ and Lemma 3.2(1). Hence, substituting (3.19) and (3.20) into (3.17), it holds uniformly for all $t \in \Lambda_T$ that

$$P\left(\sum_{i=1}^{n} X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \ge t\}} > x\right) \lesssim L_F^{-1}\sum_{i=1}^{n} P\left(X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \le t\}} > x\right). \tag{3.21}$$

This, along with (3.16), proves the uniformity of (3.12) for all $t \in \Lambda_T$.

Additionally, if $F \in \mathcal{C}$, then $L_F = 1$, and thus we get the uniformity of (3.13) by (3.12). □

Lemma 3.5. *Under the conditions of Theorem 2.6, relation* (2.8) *holds for every fixed* $t \in \Lambda$. *Additionally, if* $F \in \mathcal{C}$, *then relation* (2.6) *holds for every fixed* $t \in \Lambda$.

Proof. Clearly, for every integer $i \ge 2$ and every fixed $t \in \Lambda$, $X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \le t\}} \le X_i e^{-r\tau_{i-1}}$, where X_i and $e^{-r\tau_{i-1}}$ are independent, note that, for any $n \ge 1$ and all $t \in \Lambda_T$,

$$\begin{aligned}
P\left(\sum_{i=n+1}^{\infty} X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i \le t\}} > x\right) &\le P\left(\sum_{i=n+1}^{\infty} X_i e^{-r\tau_{i-1}} > x\right) \\
&= P\left(\bigcup_{i=n+1}^{\infty} \left(X_i e^{-r\tau_{i-1}} > x\right)\right) \\
&\quad + P\left(\sum_{i=n+1}^{\infty} X_i e^{-r\tau_{i-1}} > x, \bigcap_{i=n+1}^{\infty} \left(X_i e^{-r\tau_{i-1}} \le x\right)\right) \\
&\le \sum_{i=n+1}^{\infty} P\left(X_i e^{-r\tau_{i-1}} > x\right) \\
&\quad + P\left(\sum_{i=n+1}^{\infty} X_i e^{-r\tau_{i-1}}\mathbf{1}_{\{X_i e^{-r\tau_{i-1}} \le x\}} > x\right) \\
&= A_6 + A_7.
\end{aligned} \tag{3.22}$$

For A_6, it follows by (3.1) that, for all $x \ge D_1$,

$$A_6 = \sum_{i=n+1}^{\infty} \int_0^1 \overline{F}\left(\frac{x}{y}\right) dP\left(e^{-r\tau_{i-1}} \le y\right) \le C_1^{-1}\overline{F}(x)\sum_{i=n}^{\infty}\left(Ee^{-rp_1\tau_1}\right)^i. \tag{3.23}$$

Then for any given $\varepsilon > 0$, there exists some positive integer n_0 such that for all $n \ge n_0$,

$$A_6 \le \varepsilon\overline{F}(x). \tag{3.24}$$

For A_7, by Markov's inequality, we obtain that, for some $p_2 > J_F^+$,

$$A_7 \le x^{-p_2} E\left(\sum_{i=n+1}^{\infty} X_i e^{-r\tau_{i-1}} \mathbf{1}_{\{X_i e^{-r\tau_{i-1}} \le x\}}\right)^{p_2}. \tag{3.25}$$

On the one hand when $0 < J_F^+ < 1$, applying the inequality $|a+b|^r \le |a|^r + |b|^r$ for $0 < r < 1$ and any number a, b, we have

$$\begin{aligned} A_7 &\le x^{-p_2} \sum_{i=n+1}^{\infty} E\left(X_i e^{-r\tau_{i-1}}\right)^{p_2} \mathbf{1}_{\{X_i e^{-r\tau_{i-1}} \le x\}} \\ &= x^{-p_2} \sum_{i=n+1}^{\infty} \int_0^{\infty} \int_0^{xe^{rs}} y^{p_2} e^{-rsp_2} dF(y) dP(\tau_{i-1} \le s) \\ &\le x^{-p_2} \sum_{i=n+1}^{\infty} \int_0^{\infty} \int_0^{xe^{rs}} p_2 e^{-rsp_2} y^{p_2-1} \overline{F}(y) dy\, dP(\tau_{i-1} \le s). \end{aligned} \tag{3.26}$$

If $xe^{rs} < D_2$, then

$$\int_0^{xe^{rs}} p_2 e^{-rsp_2} y^{p_2-1} \overline{F}(y) dy \le x^{p_2} \le x^{p_2} \frac{\overline{F}(xe^{rs})}{\overline{F}(D_2)}. \tag{3.27}$$

If $xe^{rs} \ge D_2$, then by (3.2) we get that

$$\begin{aligned} \int_0^{xe^{rs}} p_2 e^{-rsp_2} y^{p_2-1} \overline{F}(y) dy &= \left(\int_0^{D_2} + \int_{D_2}^{xe^{rs}}\right) p_2 e^{-rsp_2} y^{p_2-1} \overline{F}(y) dy \\ &\le \frac{C_2}{\overline{F}(D_2)} x^{p_2} \overline{F}(xe^{rs}) + C_2 \int_{D_2}^{xe^{rs}} p_2 x^{p_2} \overline{F}(xe^{rs}) y^{-1} dy \\ &\le \left(\frac{C_2}{\overline{F}(D_2)} + C_2 p_2 \ln\left(\frac{xe^{rs}}{D_2}\right)\right) x^{p_2} \overline{F}(xe^{rs}). \end{aligned} \tag{3.28}$$

Hence, combining (3.27) and (3.28) and letting $C = \max\{1/\overline{F}(D_2), C_2/\overline{F}(D_2) + C_2 p_2 \ln(xe^{rt}/D_2)\}$ can show that

$$\int_0^{xe^{rs}} p_2 e^{-rsp_2} y^{p_2-1} \overline{F}(y) dy \le C x^{p_2} \overline{F}(xe^{rs}). \tag{3.29}$$

Substituting (3.29) into (3.26) and by (3.24), we deduce that, for all $n \ge n_0$,

$$A_7 \le C \sum_{i=n+1}^{\infty} P\left(X_i e^{-r\tau_{i-1}} > x\right) \le \varepsilon \overline{F}(x). \tag{3.30}$$

On the other hand when $J_F^+ > 1$, by Minkowski's inequality and along with the similar lines of the proof of the case when $0 < J_F^+ < 1$, we also attain that, for some constant $C > 0$,

$$\begin{aligned}
A_7 &\le x^{-p_2}\left(\sum_{i=n+1}^{\infty}\left(E\left(X_i e^{-r\tau_{i-1}}\right)^{p_2}\mathbf{1}_{\{X_i e^{-r\tau_i}\le x\}}\right)^{(1/p_2)}\right)^{p_2} \\
&\le C\left(\sum_{i=n+1}^{\infty}\left(P\left(X_i e^{-r\tau_{i-1}} > x\right)\right)^{(1/p_2)}\right)^{p_2} \\
&\le C\left(\sum_{i=n+1}^{\infty}\left(\int_0^1 \overline{F}\left(\frac{x}{y}\right)dP\left(e^{-r\tau_{i-1}}\le y\right)\right)^{(1/p_2)}\right)^{p_2} \\
&\le C\left(\sum_{i=n}^{\infty}\left(C_1^{-1}\overline{F}(x)\left(Ee^{-rp_1\tau_1}\right)^i\right)^{(1/p_2)}\right)^{p_2} \\
&\le CC_1^{-1}\overline{F}(x)\left(\sum_{i=n}^{\infty}\left(Ee^{-r\tau_1 p_1}\right)^{(i/p_2)}\right)^{p_2},
\end{aligned} \tag{3.31}$$

where in the second last step we used (3.1). Therefore, for all $n \ge n_0$, it still holds that

$$A_7 \le \varepsilon\overline{F}(x). \tag{3.32}$$

Consequently, from (3.22), (3.24), (3.30), and (3.32), we prove that, for some positive integer n_0 and every fixed $t \in \Lambda$,

$$P\left(\sum_{i=n_0+1}^{\infty} X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i\le t\}} > x\right) \le \varepsilon\overline{F}(x), \tag{3.33}$$

$$\sum_{i=n_0+1}^{\infty} P\left(X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i\le t\}} > x\right) \le \sum_{i=n_0+1}^{\infty} P\left(X_i e^{-r\tau_{i-1}} > x\right) \le \varepsilon\overline{F}(x). \tag{3.34}$$

Let n_0 be fixed as above. Applying (3.12) in Lemma 3.4, (3.34), and Lemma 3.2 (2) in turn, we find that, for every fixed $t \in \Lambda$,

$$\begin{aligned}
P(D_r(t) > x) &\ge P\left(\sum_{i=1}^{n_0} X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i\le t\}} > x\right) \\
&\sim \left(\sum_{i=1}^{\infty} - \sum_{i=n_0+1}^{\infty}\right)P\left(X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i\le t\}} > x\right) \\
&\ge (1-\varepsilon)\sum_{i=1}^{\infty}P\left(X_i e^{-r\tau_i}\mathbf{1}_{\{\tau_i\le t\}} > x\right),
\end{aligned} \tag{3.35}$$

which, along with (1.7) and (2.7), proves that for every fixed $t \in \Lambda$,

$$P(D_r(t) > x) \gtrsim (1-\varepsilon) \int_0^t \overline{F}(xe^{rs}) d\tilde{\lambda}(t). \tag{3.36}$$

By contrast, for n_0 as above and any fixed $0 < v < 1$, we obtain from (3.12) and (3.33) that, for every fixed $t \in \Lambda$,

$$\begin{aligned} P(D_r(t) > x) &\leq P\left(\sum_{i=1}^{n_0} X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}} > (1-v)x\right) \\ &\quad + P\left(\sum_{i=n_0+1}^{\infty} X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}} > vx\right) \\ &\lesssim L_F^{-1} \sum_{i=1}^{n_0} P\left(X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}} > (1-v)x\right) + \varepsilon \overline{F}(vx) \\ &= A_8 + A_9. \end{aligned} \tag{3.37}$$

For A_8, arguing as the proof of (3.19) leads to

$$A_8 \lesssim L_F^{-2} \sum_{i=1}^{\infty} P\left(X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}} > x\right). \tag{3.38}$$

For A_9, by Lemmas 3.2 (2) and 3.2 (1) successively, we have

$$A_9 = \varepsilon P\left(X_1 e^{-r\tau_1} \mathbf{1}_{\{\tau_1 \leq t\}} > x\right). \tag{3.39}$$

From (3.37) to (3.39) and by the arbitrariness of $\varepsilon > 0$, we obtain that, for every fixed $t \in \Lambda$,

$$\begin{aligned} P(D_r(t) > x) &\lesssim L_F^{-2} \sum_{i=1}^{\infty} P\left(X_i e^{-r\tau_i} \mathbf{1}_{\{\tau_i \leq t\}} > x\right) \\ &= L_F^{-2} \int_0^t \overline{F}(xe^{rs}) d\tilde{\lambda}(t), \end{aligned} \tag{3.40}$$

where in the last step we used (1.7) and (2.7) again. Hence, combining (3.36) and (3.40) can yield that relation (2.8) holds for every fixed $t \in \Lambda$.

Additionally, if $F \in \mathcal{C}$, then $L_F = 1$, and so we attain by (2.8) that relation (2.6) holds for every fixed $t \in \Lambda$. □

4. Proofs of Main Results

Proof of Theorem 2.5. From the proof of Lemma 3.5, we can see that the relations (3.33)–(3.40) still hold uniformly for all $t \in \Lambda_T$, and then we get the uniformity of (2.8) for all $t \in \Lambda_T$ immediately. As $F \in \mathcal{C}$, the uniformity of (2.6) over all $t \in \Lambda_T$ is clear. □

Proof of Theorem 2.6. According to the proof of Lemma 4.2 of Hao and Tang [7] (or see the proof of (4.3) of Tang [6]), we know that, for an arbitrarily fixed $\varepsilon > 0$, there exists some $T_0 \in \Lambda$ such that

$$\int_{T_0}^{\infty} \overline{F}(xe^{rs}) d\tilde{\lambda}(s) \leq \varepsilon \int_0^{T_0} \overline{F}(xe^{rs}) d\tilde{\lambda}(s). \tag{4.1}$$

Combining with Theorem 2.5, it suffices to prove that relation (2.8) holds uniformly for all $t \in (T_0, \infty]$. On the one hand, by Lemma 3.5 with $t = T_0$ and (4.1), it holds uniformly for all $t \in (T_0, \infty]$ that

$$\begin{aligned} P(D_r(t) > x) &\geq P(D_r(T_0) > x) \\ &\gtrsim \int_0^{T_0} \overline{F}(xe^{rs}) d\tilde{\lambda}(s) \\ &\geq \left(\int_0^t - \int_{T_0}^{\infty} \right) \overline{F}(xe^{rs}) d\tilde{\lambda}(s) \\ &\geq (1 - \varepsilon) \int_0^t \overline{F}(xe^{rs}) d\tilde{\lambda}(s). \end{aligned} \tag{4.2}$$

On the other hand, by Lemma 3.5 and (4.1), it holds uniformly for all $t \in (T_0, \infty]$ that

$$\begin{aligned} P(D_r(t) > x) &\lesssim L_F^{-2} \int_0^{\infty} \overline{F}(xe^{rs}) d\tilde{\lambda}(s) \\ &\leq L_F^{-2} \left(\int_0^t + \int_{T_0}^{\infty} \right) \overline{F}(xe^{rs}) d\tilde{\lambda}(s) \\ &\leq (1 + \varepsilon) L_F^{-2} \int_0^t \overline{F}(xe^{rs}) d\tilde{\lambda}(s). \end{aligned} \tag{4.3}$$

By relations (4.2), (4.3), and the arbitrariness of $\varepsilon > 0$, we get the uniformity of relation (2.8) for all $t \in (T_0, \infty]$.

As $F \in \mathcal{C}$, we have $L_F = 1$, and hence relation (2.6) holds uniformly for all $t \in \Lambda$. □

Proof of Theorem 2.7. Recalling the insurer's surplus process (1.2), we can attain its discounted value as

$$\tilde{U}_r(t) = e^{-rt} U_r(t) = x + \tilde{C}(t) - D(t), \quad t \geq 0. \tag{4.4}$$

Following the definition (1.4) of the finite-time ruin probability, we have

$$\begin{aligned}\psi_r(x,t) &= P\Big(\widetilde{U}_r(s) < 0 \text{ for some } 0 < s \le t\Big) \\ &= P\left(\sup_{0<s\le t}\Big\{D_r(s) - \widetilde{C}(s)\Big\} > x\right).\end{aligned} \tag{4.5}$$

Then, it follows that

$$\psi_r(x,t) \le P(D_r(t) > x), \tag{4.6}$$

$$\begin{aligned}\psi_r(x,t) &= P\left(\bigcup_{0<s\le t}\Big\{D_r(s) - \widetilde{C}(s)\Big\} > x\right) \\ &\ge P\Big(D_r(t) > x + \widetilde{C}(t)\Big).\end{aligned} \tag{4.7}$$

By Theorem 2.5 and (4.6), it holds uniformly for all $t \in \Lambda_T$ that

$$\psi_r(x,t) \lesssim L_F^{-2} \int_0^t \overline{F}(xe^{rs})d\widetilde{\lambda}(s). \tag{4.8}$$

In the following, we establish the uniform asymptotic lower bound of $\psi_r(x,t)$ for all $t \in \Lambda_T$. Under the condition 1 of Theorem 2.7, we deduce from (4.7), Theorem 2.5, and $F \in \mathfrak{D}$ that, for any fixed $0 < w < 1$ and any given $\varepsilon > 0$, there exists some $x_2 > 0$ such that for all $x \ge x_2$ and uniformly for all $t \in \Lambda_T$,

$$\begin{aligned}\psi_r(x,t) &\ge P\Big(D_r(t) > x + \widetilde{C}(T)\Big) = \int_0^\infty P(D_r(t) > x + y)P\Big(\widetilde{C}(T) \in dy\Big) \\ &\gtrsim \int_0^\infty \int_0^t \overline{F}((x+y)e^{rs})d\widetilde{\lambda}(s)P\Big(\widetilde{C}(T) \in dy\Big) \\ &\ge \int_0^\infty \int_0^t \overline{F}((1+w)xe^{rs})d\widetilde{\lambda}(s)P\Big(\widetilde{C}(T) \in dy\Big) \\ &\ge (1-\varepsilon)\overline{F}_*(1+w)\int_0^t \overline{F}(xe^{rs})d\widetilde{\lambda}(s),\end{aligned} \tag{4.9}$$

which, along with the arbitrariness of $\varepsilon > 0$ and $0 < w < 1$, can give that, uniformly for all $t \in \Lambda_T$,

$$\psi_r(x,t) \gtrsim L_F \int_0^t \overline{F}(xe^{rs})d\widetilde{\lambda}(s). \tag{4.10}$$

Under the condition 2 of Theorem 2.7, it holds from (4.7) that, for any fixed $0 < \delta < 1$ and all $t \in \Lambda_T$,

$$\psi_r(x,t) \geq P(D_r(t) > (1+\delta)x) - P\left(\widetilde{C}(T) > \delta x\right) = I_1 - I_2. \tag{4.11}$$

For I_1, using Theorem 2.5 and $F \in \mathfrak{D}$, there exists some $x_3 > 0$ such that for all $x \geq x_3$ and uniformly for all $t \in \Lambda_T$,

$$\begin{aligned} I_1 &\gtrsim \int_0^t \overline{F}((1+\delta)xe^{rs})d\widetilde{\lambda}(s) \\ &\geq (1-\varepsilon)\overline{F}_*(1+\delta)\int_0^t \overline{F}(xe^{rs})d\widetilde{\lambda}(s). \end{aligned} \tag{4.12}$$

For I_2, by the condition 2 of Theorem 2.7 and $F \in \mathfrak{D}$, we have

$$\limsup_{x\to\infty}\frac{I_2}{\overline{F}(x)} = \limsup_{x\to\infty}\frac{I_2}{\overline{F}(\delta x)}\cdot\frac{\overline{F}(\delta x)}{\overline{F}(x)} = 0. \tag{4.13}$$

When $\underline{t} \in \Lambda$, by (4.13) and (3.2) in Lemma 3.1, there exists some $x_4 > 0$ such that for all $x \geq \max\{D_2, x_4\}$ and uniformly for all $t \in \Lambda_T$,

$$I_2 \leq \varepsilon\overline{F}(x) \leq C\varepsilon\int_0^{\underline{t}} \overline{F}(xe^{rs})d\widetilde{\lambda}(s) \leq C\varepsilon\int_0^t \overline{F}(xe^{rs})d\widetilde{\lambda}(s), \tag{4.14}$$

where $C = C_2 e^{rp_2\underline{t}}/\widetilde{\lambda}(\underline{t})$. When $\underline{t} \notin \Lambda$, choose some $0 < \sigma < 1$ such that $\underline{t} + \sigma \leq t$, again by (4.13), (3.2), and arguing as (4.14), there exists some $x_4' > 0$ such that for all $x \geq \max\{D_2, x_4'\}$ and uniformly for all $t \in \Lambda_T$,

$$I_2 \leq \varepsilon\overline{F}(x) \leq C'\varepsilon\int_0^{\underline{t}+\sigma} \overline{F}(xe^{rs})d\widetilde{\lambda}(s) \leq C'\varepsilon\int_0^t \overline{F}(xe^{rs})d\widetilde{\lambda}(s), \tag{4.15}$$

where $C' = C_2 e^{rp_2(\underline{t}+\sigma)}/\widetilde{\lambda}(\underline{t}+\sigma)$. Hence, from (4.11) to (4.15) and by the the arbitrariness of $\varepsilon > 0$ and $0 < \delta < 1$, we still obtain the uniformity of (4.10) for all $t \in \Lambda_T$. This ends the proof for the uniformity of (2.10) over all $t \in \Lambda_T$.

If $F \in \mathcal{C}$, then $L_F = 1$. Therefore, we conclude from (2.10) that relation (2.12) uniformly holds for all $t \in \Lambda_T$. □

Proof of Corollary 2.8. Clearly, by Theorem 2.7, we know that relation (2.13) holds uniformly for all $t \in \Lambda_T$. Note that for all finite $t \in \Lambda$,

$$\widetilde{\lambda}(t)\overline{F}\left(x+\kappa\widetilde{\lambda}(t)\right) \leq \kappa^{-1}\int_x^{x+\kappa\widetilde{\lambda}(t)} \overline{F}(y)dy \leq \widetilde{\lambda}(t)\overline{F}(x). \tag{4.16}$$

By $F \in \mathfrak{D}$, it follows that, for any given $\varepsilon > 0$ and any fixed $0 < \alpha < 1$, there exists some $x_5 > 0$ such that for all $x \geq x_5$ and uniformly for all $t \in \Lambda_T$,

$$\begin{aligned} \widetilde{\lambda}(t)\overline{F}\left(x + \kappa\widetilde{\lambda}(t)\right) &\geq \widetilde{\lambda}(t)\overline{F}\left(x + \kappa\widetilde{\lambda}(T)\right) \\ &\geq \widetilde{\lambda}(t)\overline{F}((1+\alpha)x) \\ &\geq (1-\varepsilon)\overline{F}_*(1+\alpha)\widetilde{\lambda}(t)\overline{F}(x), \end{aligned} \tag{4.17}$$

which, along with the arbitrariness of $\varepsilon > 0$ and $0 < \alpha < 1$, yields that

$$\widetilde{\lambda}(t)\overline{F}\left(x + \kappa\widetilde{\lambda}(t)\right) \gtrsim L_F\widetilde{\lambda}(t)\overline{F}(x) \tag{4.18}$$

holds uniformly for all $t \in \Lambda_T$. Thus, we obtain from (4.16) and (4.18) that, uniformly for all $t \in \Lambda_T$,

$$L_F\widetilde{\lambda}(t)\overline{F}(x) \lesssim \kappa^{-1}\int_x^{x+\kappa\widetilde{\lambda}(t)} \overline{F}(y)dy \lesssim \widetilde{\lambda}(t)\overline{F}(x). \tag{4.19}$$

So, combining (2.13) and (4.19) leads to the uniformity of (2.14) for all $t \in \Lambda_T$.

If $F \in \mathcal{C}$, then $L_F = 1$, and hence the uniformity of (2.15) for all $t \in \Lambda_T$ is proved immediately. □

Proof of Theorem 2.9. Applying (4.6) and Theorem 2.6, relation (4.8) still holds uniformly for all $t \in \Lambda$. On the other hand, by Theorem 2.7, we also attain that relation (4.10) holds uniformly for all $t \in \Lambda_T$ under conditions 1 and 2 of Theorem 2.9. Hence, we only need to show the uniformity of (4.10) for all $t \in (T, \infty]$. Under the condition 1 of Theorem 2.9, similarly to the proof of (4.9), we prove that, uniformly for all $t \in (T, \infty]$,

$$\begin{aligned} \psi_r(x,t) &\geq P\left(D_r(T) > x + \widetilde{C}\right) \\ &= \int_0^\infty P(D_r(T) > x + y)P\left(\widetilde{C} \in dy\right) \\ &\gtrsim \int_0^\infty \int_0^T \overline{F}((x+y)e^{rs})d\widetilde{\lambda}(s)P\left(\widetilde{C} \in dy\right) \\ &\geq \int_0^\infty \int_0^T \overline{F}((1+w)xe^{rs})d\widetilde{\lambda}(s)P\left(\widetilde{C} \in dy\right) \\ &\geq (1-\varepsilon)\overline{F}_*(1+w)\int_0^T \overline{F}(xe^{rs})d\widetilde{\lambda}(s) \\ &\geq (1-2\varepsilon)\overline{F}_*(1+w)\int_0^t \overline{F}(xe^{rs})d\widetilde{\lambda}(s), \end{aligned} \tag{4.20}$$

where the last step is due to (4.1). Because $\varepsilon > 0$ and $0 < w < 1$ are arbitrary, the relation (4.10) holds uniformly for all $t \in (T, \infty]$.

Under the condition 2 of Theorem 2.9, we derive from (4.7) that, for the fixed $0 < \delta < 1$ as above and all $t \in (T, \infty]$,

$$\begin{aligned}\psi_r(x,t) &\geq P\left(D_r(T) > x + \widetilde{C}\right)\\ &\geq P(D_r(T) > (1+\delta)x) - P\left(\widetilde{C} > \delta x\right)\\ &= I_3 - I_4.\end{aligned} \tag{4.21}$$

For I_3, by Theorem 2.6 with $t = T$ and the similar proof of (4.12), it holds uniformly for all $t \in (T, \infty]$ that

$$\begin{aligned}I_3 &\sim \int_0^T \overline{F}((1+\delta)xe^{rs})d\widetilde{\lambda}(s)\\ &\geq (1-\varepsilon)\overline{F}_*(1+\delta)\int_0^T \overline{F}(xe^{rs})d\widetilde{\lambda}(s)\\ &\geq (1-2\varepsilon)\overline{F}_*(1+\delta)\int_0^t \overline{F}(xe^{rs})d\widetilde{\lambda}(s),\end{aligned} \tag{4.22}$$

where the last step is also due to (4.1). For I_4, by condition 2 of Theorem 2.9 and $F \in \mathfrak{D}$, we get that, for all $t \in (T, \infty]$,

$$\begin{aligned}\limsup_{x\to\infty}\frac{I_4}{\int_0^t \overline{F}(xe^{rs})d\widetilde{\lambda}(s)} &\leq \limsup_{x\to\infty}\frac{I_4}{\int_0^T \overline{F}(xe^{rs})d\widetilde{\lambda}(s)}\\ &\leq \limsup_{x\to\infty}\frac{I_4}{\overline{F}(\delta x)}\cdot\frac{\overline{F}(\delta x)}{\overline{F}(xe^{rT})\widetilde{\lambda}(T)}\\ &= 0,\end{aligned} \tag{4.23}$$

which implies that, for all $t \in (T, \infty]$ and all large x,

$$I_4 \leq \varepsilon\int_0^t \overline{F}(xe^{rs})d\widetilde{\lambda}(s). \tag{4.24}$$

Consequently, using (4.21)–(4.24) and by the arbitrariness of $\varepsilon > 0$ and $0 < \delta < 1$, we can obtain the uniformity of (4.10) for all $t \in (T, \infty]$, and then we prove the uniformity of (2.10) over all $t \in \Lambda$.

From $F \in \mathcal{C}$ and (2.10), it is easy that relation (2.12) holds uniformly for all $t \in \Lambda_T$. $\square$

Acknowledgments

The author wishes to thank the referees and the editors for their very valuable comments on an earlier version of this paper. This research is supported by Research Start-up Foundation of Nanjing Audit University (no. NSRC10022), Natural Science Foundation of Jiangsu Province of China (no. BK2010480), and Natural Science Foundation of Jiangsu Higher Education Institutions of China (no. 11KJD110002).

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