Agenda

- Motivation
- A quick summary
- Basic concepts: uncertainty when forecasting
- A stochastic reserving model: Mack’s model
  - Analytic
  - Using bootstrapping
- The “1 year” view of reserving risk
Solvency 2 Requirements

- The **best estimate** is equal to the **expected present value** of all future potential cash-flows (**probability weighted average** of distributional outcomes)…
  - See Groupe Consultatif Interim Report Nov 2008
- Risk margins: A **cost-of-capital** methodology should be used
  - The precise mechanics of the cost-of-capital methodology, with approved simplifications, have not yet been published
  - A **notional** capital amount is required by (Solvency II) line of business
  - CP71 and 75 clarify that the profit/loss on the (expected) reserves over 1 year can be used to help estimate the notional capital required
Reserving Risk

- Reserving is concerned with forecasting outstanding liabilities
- There is uncertainty associated with any forecast
- Reserving risk attempts to capture that uncertainty
- We are interested in the predictive distribution of ultimate losses AND the associated cash-flows
  - Cash-flows are required for discounting
- We need methods that can provide a distribution of cash-flows
- The methods are still evolving
Conceptual Framework

Reserve Estimate (Measure of Location)

Variability (Prediction Error)

Predictive Distribution

Traditional deterministic methods

Statistical assumptions required

Prediction Error = SD of Forecast

Can be estimated analytically

Usually cannot be obtained analytically

Simulation methods required
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Paid Claims 01/01/2001

Development Year

0 1 2 3 4 5 6 7 8 9 10 11 12

0 500,000 1,000,000 1,500,000 2,000,000 2,500,000 3,000,000 3,500,000 4,000,000 4,500,000 5,000,000 5,500,000 6,000,000 6,500,000 7,000,000
Basic Concepts

Uncertainty when Forecasting: Prediction errors and Predictive distributions
A Simple One Parameter Problem

- Number of large claims in each of the last 10 years = [3, 8, 5, 9, 5, 8, 4, 8, 7, 3]
- Best estimate of the number of large claims next year? Expected value = 6
- Standard error of the mean?
- Prediction error of a new forecast value?
- Distributed as a Poisson random variable?
- Predictive distribution of a new forecast value?
A Simple One Parameter Problem

- Number of large claims in each of the last 10 years = [3,8,5,9,5,8,4,8,7,3]
- Best estimate of the number of large claims next year?
- Expected value = 6
- Standard error of the mean (if i.i.d)?

\[
Var(\mu) = Var\left(\frac{\sum_{i=1}^{n} x_i}{n}\right) = \frac{1}{n^2} Var\left(\sum_{i=1}^{n} x_i\right) = \frac{1}{n^2} \left(\sum_{i=1}^{n} Var(x_i)\right) = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}
\]

\[
SE(\mu) = \frac{\sigma}{\sqrt{n}}
\]

\[0.68\]
Variability of a Forecast

- Includes estimation variance and process variance

- Analytic solution: estimate the two components

\[
\text{prediction error} = (\text{process variance} + \text{estimation variance})^{\frac{1}{2}}
\]

\[
\text{estimation variance} = \frac{\sigma^2}{n}
\]

\[
\text{process variance (Poisson)} = \mu
\]

\[
\text{prediction error of forecast} = \left( \mu + \frac{\sigma^2}{n} \right)^{\frac{1}{2}}
\]

\[
0.68^2 = 0.46 \quad 6.0 \quad 2.54
\]
Bootstrapping is a simple but effective way of obtaining a distribution of the parameters. The method involves creating many new data sets from which the parameters are estimated. The new data sets are created by sampling with replacement from the observed data. Results in a (“simulated”) distribution of the parameters.
Simple Example
Bootstrapping the Mean

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<th>Observed Data</th>
<th>Mean</th>
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Bootstrap standard error 0.68
Forecasting

Simulate a forecast observation, conditional on each bootstrap mean

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<td>5.5</td>
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Assuming a Poisson process distribution

Standard Error 0.68  Prediction error 2.54
Important Lessons

- We could calculate the SD of the forecast ("prediction error") analytically, taking account of parameter uncertainty.

- Bootstrapping gives a distribution of parameters, hence an estimate of the estimation error, without the hard maths.

- When supplemented by a second simulation step incorporating the process error, a distribution of the forecast is generated.
An example of a stochastic reserving model

Mack’s model
Mack’s Model


\[ D_{ij} = \text{Cumulative claims in origin year } i \text{ and development year } j \]

Specified mean and variance only:

\[
E(D_{ij}) = \lambda_j D_{i,j-1}
\]

\[
V(D_{ij}) = \sigma_j^2 D_{i,j-1}
\]

Expected value proportional to previous cumulative

Variance proportional to previous cumulative
Mack’s Model

\[ \hat{\lambda}_j = \frac{\sum_{i=1}^{n-j+1} w_{ij} f_{ij}}{\sum_{i=1}^{n-j+1} w_{ij}} \]

Estimator for lambda

\[ \hat{\sigma}_j^2 = \frac{1}{n-j} \sum_{i=1}^{n-j+1} w_{ij} \left( f_{ij} - \hat{\lambda}_j \right)^2 \]

Estimator for sigma squared

\[ w_{ij} = D_{i,j-1} \text{ and } f_{ij} = \frac{D_{ij}}{D_{i,j-1}} \]
Variability in Claims Reserves

- Variability of a forecast
- Includes estimation variance and process variance

prediction error = (process variance + estimation variance) \( \frac{1}{2} \)

- Problem reduces to estimating the two components. For example, for the reserves in origin year \( i \):

\[
RMSEP \left[ \hat{R}_i \right] \approx \sqrt{ \hat{D}_{in}^2 \sum_{k=n-i+1}^{n-1} \frac{\hat{\sigma}_{k+1}^2}{\hat{\lambda}_{k+1}^2} \left( \frac{1}{\hat{D}_{ik}} + \frac{1}{\sum_{q=1}^{n-k} D_{qk}} \right) }
\]
An example of bootstrapping a stochastic reserving model

Bootstrapping Mack’s model (England & Verrall 2006)
Bootstrapping assumes the data are independent and identically distributed.

With regression type problems, the data are often assumed to be independent but are not identically distributed (the means are different for each observation).

However, the residuals are usually *i.i.d*, or can be made so.

Therefore, with regression problems, it is common to bootstrap the (standardised) residuals instead.
Reserving and Bootstrapping

Any model that can be clearly defined can be bootstrapped.
Bootstrapping Mack: 9 Steps

1. Create standard DFM
   - Data
   - Ultimate
   - Link Ratios
   - Selected Link Ratios

2. Generate crude residuals
   - Crude Residuals

3. Normalize residuals
   - Normalized Residuals

4. Sample with replacement
   - Normalized Residuals

5. Convert residuals back to crude
   - Crude Residuals

6. Convert crude residuals back to link ratios
   - Link Ratios

7. Re-calculate average pattern
   - Link Ratios
   - Selected Link Ratios

8. Square up triangle of losses using link ratios and incorporating process variance
   - Data
   - Selected Link Ratios
   - Simulated Ultimate

9. Repeat steps 4-8 10,000 times
“Reserving risk” and Solvency II

The one-year view of reserving risk
Solvency 2 is notionally projecting a balance sheet, and requires a distribution of “Net Assets” over a one year time horizon.

Solvency 2 requires a view of the distribution of expected liabilities in one year.

For reserving risk, this requires a distribution of the profit/loss on reserves over one year.

This is different from the standard approach to reserving risk, which considers the distribution of the ultimate cost of claims (e.g., Mack 1993, England & Verrall 1999, 2002, 2006).
The one-year run-off result (undiscounted)
(the view of profit or loss on reserves after one year)

For a particular origin year, let:

The opening reserve estimate be \( R_0 \)

The reserve estimate after one year be \( R_1 \)

The payments in the year be \( C_1 \)

The run-off result (claims development result) be \( CDR_1 \)

Then

\[
CDR_1 = R_0 - C_1 - R_1 = U_0 - U_1
\]

Where the opening estimate of ultimate claims and the estimate of the ultimate after one year are \( U_0, U_1 \)
The one-year run-off result
(the view of profit or loss on reserves after one year)

Merz & Wuthrich (2008) derived analytic formulae for the standard deviation of the claims development result after one year assuming:

- The opening reserves were set using the pure chain ladder model (no tail)
- Claims develop in the year according to the assumptions underlying Mack’s model
- Reserves are set after one year using the pure chain ladder model (no tail)
- (The mathematics is quite challenging)

The M&W method is gaining popularity, but has limitations. What if:

- We need a tail factor to extrapolate into the future?
- Mack’s model is not used – other assumptions are used instead?
- We want another risk measure (say, VaR @ 99.5%)?
- We want a distribution of the CDR (not just a standard deviation)?
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<th>Accident Year</th>
<th>12m</th>
<th>24m</th>
<th>36m</th>
<th>48m</th>
<th>60m</th>
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The one-year run-off result in a simulation model
(the view of profit or loss on reserves after one year)

For a particular origin year, let:

The opening reserve estimate be \( R_0 \)

The expected reserve estimate after one year be \( R_1^{(i)} \)

The payments in the year be \( C_1^{(i)} \)

The run-off result (claims development result) be \( CDR_1^{(i)} \)

Then

\[
CDR_1^{(i)} = R_0 - C_1^{(i)} - R_1^{(i)} = U_0 - U_1^{(i)}
\]

Where the opening estimate of ultimate claims and the expected ultimate after one year are \( U_0, U_1^{(i)} \)

for each simulation \( i \)
The one-year run-off result in a simulation model

Modus operandi

1. Given the opening reserve triangle, simulate all future claim payments to ultimate using a bootstrap or Bayesian MCMC technique.

2. Now forget that we have already simulated what the future holds.

3. Move one year ahead. Augment the opening reserve triangle by one diagonal, that is, by the simulated payments from step 1 in the next calendar year only. An actuary only sees what emerges in the year.

4. For each simulation, estimate the outstanding liabilities, conditional only on what has emerged to date. (The future is still “unknown”).

5. A reserving methodology is required for each simulation – an “actuary-in-the-box” is required*. We call this re-reserving.

6. For a one-year model, this will underestimate the true volatility at the end of that year (even if the mean across all simulations is correct).

* The term “actuary-in-the-box” was coined by Esbjörn Ohlsson
EMB ResQ Example
### Analytic vs Simulated Accident

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<th>Analytic Prediction Errors</th>
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Re-reserving in Simulation-based Capital Models

The advantage of investigating the claims development result (using re-reserving) in a simulation environment is that the procedure can be generalised:

- Not just the chain ladder model
- Not just Mack’s assumptions
- Can include curve fitting and extrapolation for tail estimation
- Can incorporate a Bornhuetter-Ferguson step
- Can be extended beyond the 1 year horizon to look at multi-year forecasts
- Provides a distribution of the CDR, not just a standard deviation
- Provides a link between the traditional “ultimate” view of risk and the “1 year” view
- Can be used to help calibrate Solvency 2 internal models
A simple risk margin method

1. Apply bootstrapping in the usual way
2. Generate a distribution of the one-year CDR (using re-reserving)
3. Estimate opening capital required by applying a risk measure to the one-year CDR distribution (e.g., VaR @ 99.5%)
4. Apply the proportional proxy for future capital requirements
5. Multiply by the cost-of-capital loading
6. Discount and sum

Issues:
- UPR, dependencies, aggregation, paid vs incurred, gross vs net, non-bootstrapped lines, discounting/investment income, non-annual analysis dates, accident vs underwriting year issues, attritional/large claims split, scaling, operational risk, credit risk and market risk…
References


CP71 and CP75 (2009). http://www.ceiops.eu/content/view/14/18/