Reserve Risk Modelling: Theoretical and Practical Aspects

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ERM and Financial Modelling Seminar

EMB and The Israeli Association of Actuaries
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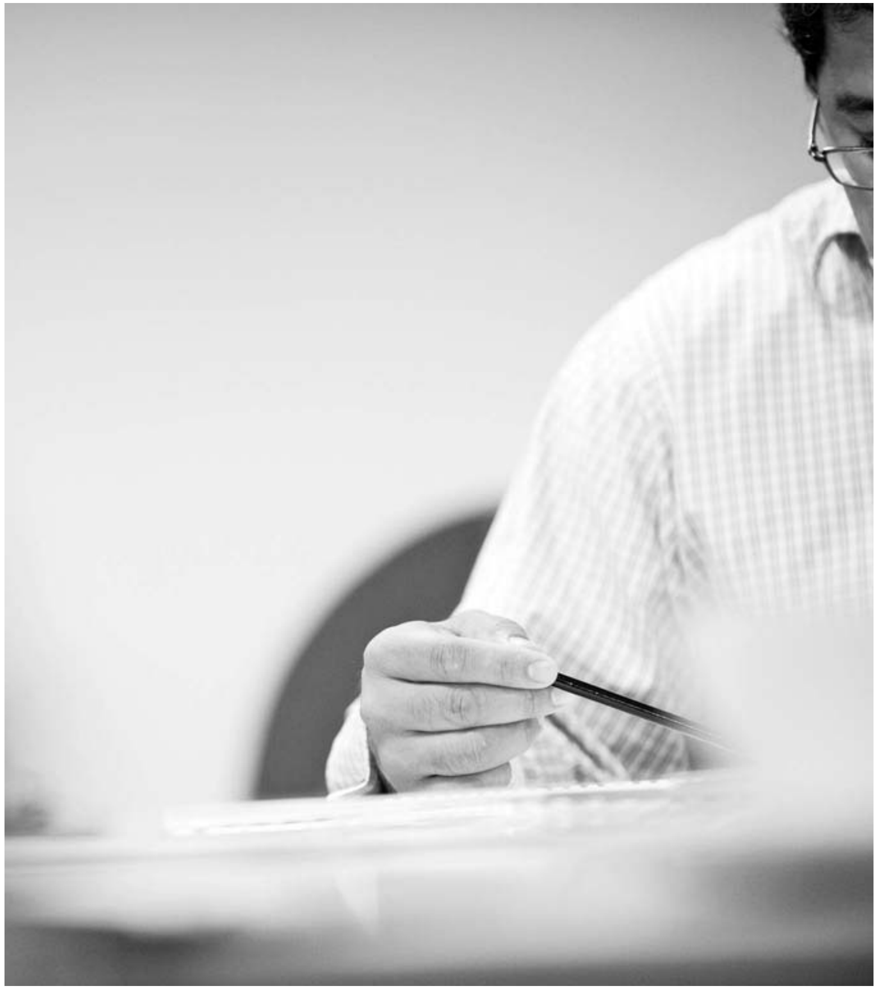
Agenda

- Motivation
- A quick summary
- Basic concepts: uncertainty when forecasting
- A stochastic reserving model: Mack's model
  - Analytic
  - Using bootstrapping
- The "1 year" view of reserving risk



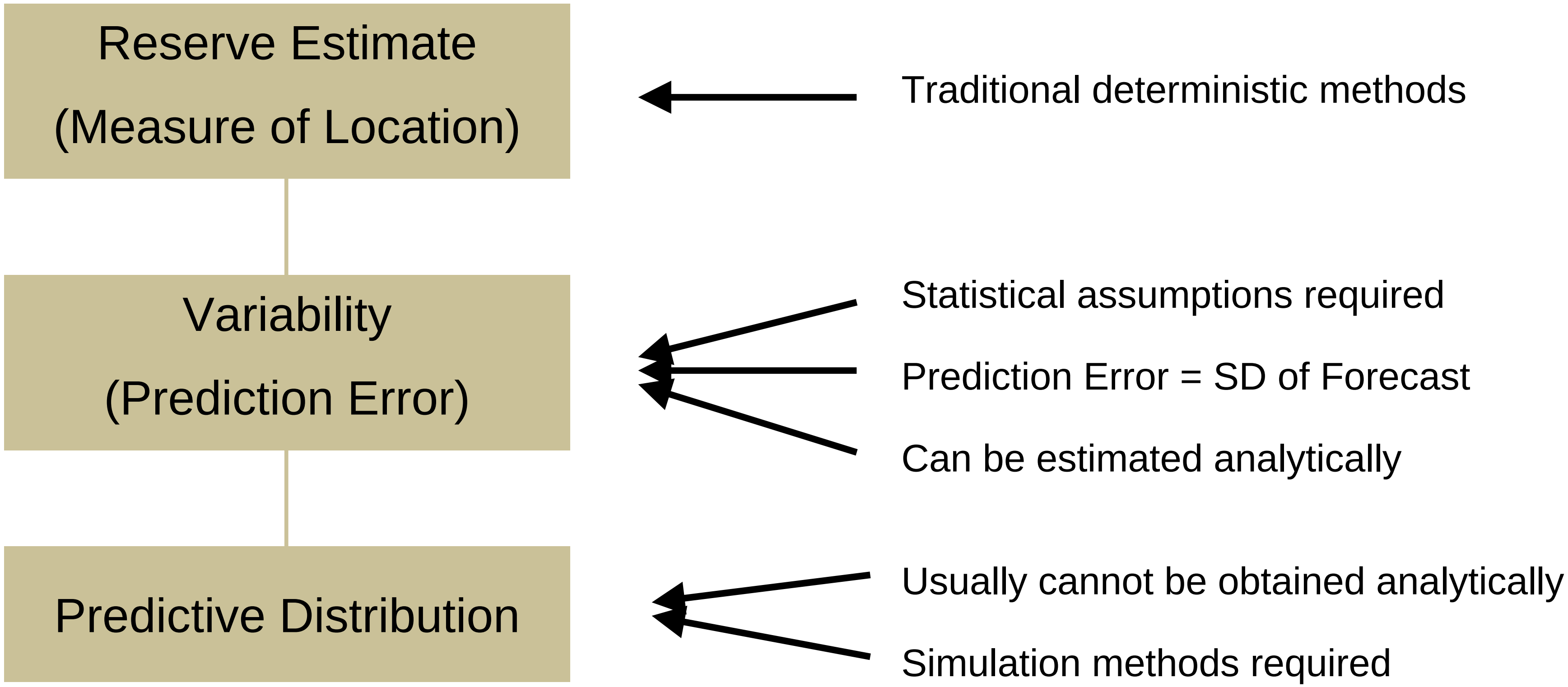
Solvency 2 Requirements

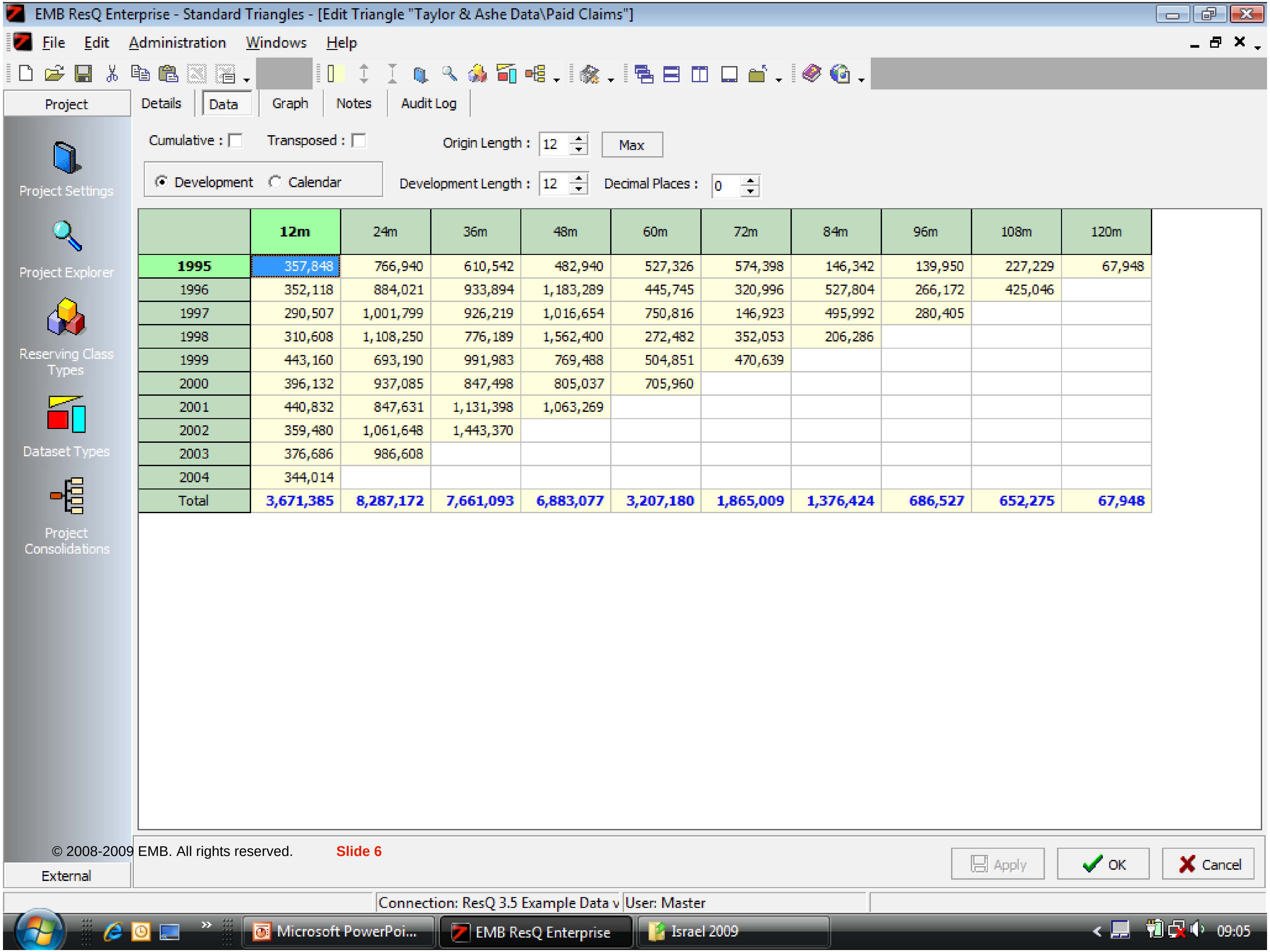
- The best estimate is equal to the expected present value of all future potential cash-flows (probability weighted average of distributional outcomes)…
  - See Groupe Consultatif Interim Report Nov 2008
- Risk margins: A cost-of-capital methodology should be used
  - The precise mechanics of the cost-of-capital methodology, with approved simplifications, have not yet been published
  - A notional capital amount is required by (Solvency II) line of business
  - CP71 and 75 clarify that the profit/loss on the (expected) reserves over 1 year can be used to help estimate the notional capital required

Reserving Risk

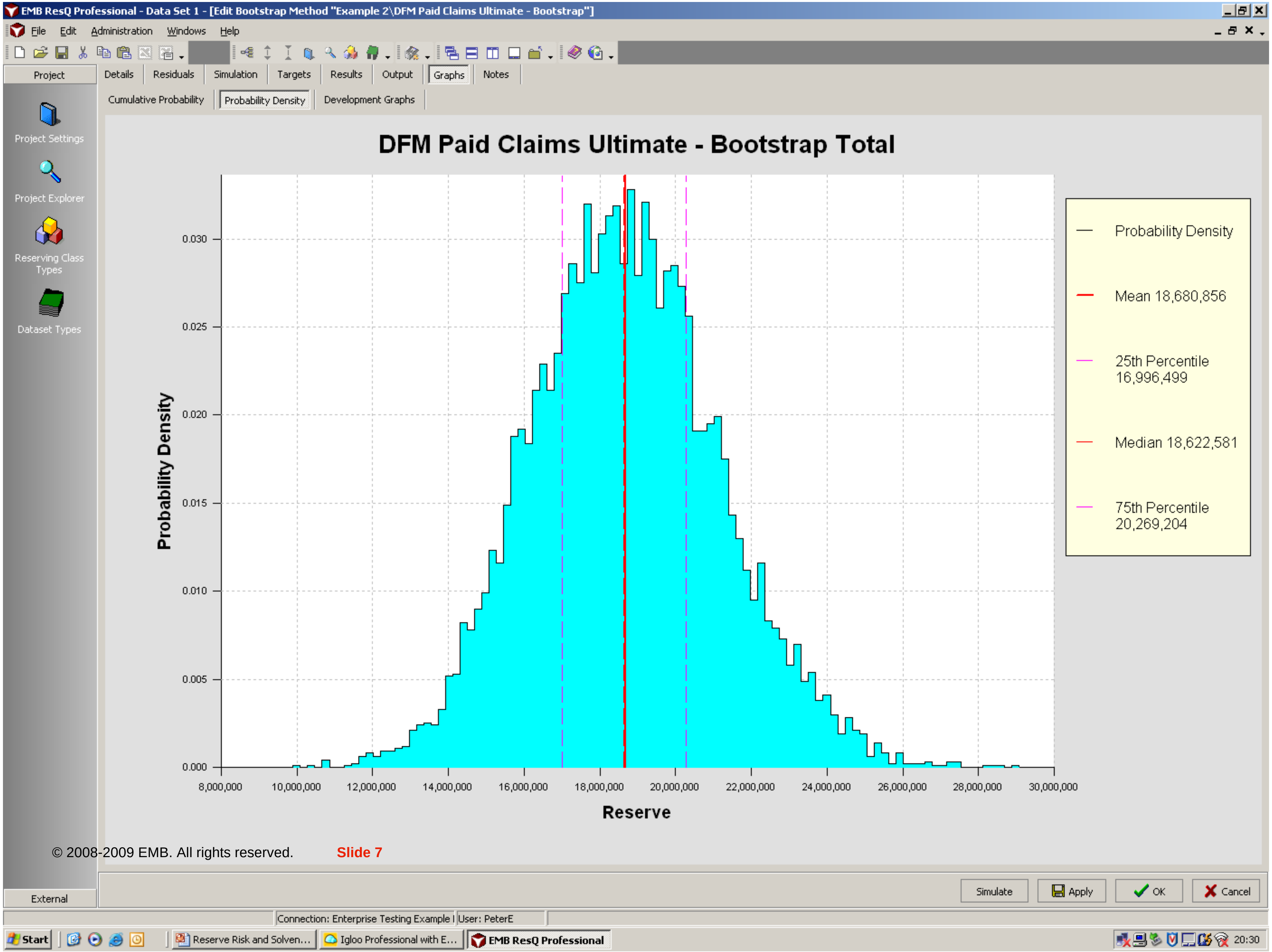
- Reserving is concerned with forecasting outstanding liabilities
- There is uncertainty associated with any forecast
- Reserving risk attempts to capture that uncertainty
- We are interested in the predictive distribution of ultimate losses AND the associated cash-flows
  - Cash-flows are required for discounting
- We need methods that can provide a distribution of cash-flows
- The methods are still evolving

Conceptual Framework





	12m	24m	36m	48m	60m	72m	84m	96m	108m	120m
1995	357,848	766,940	610,542	482,940	527,326	574,398	146,342	139,950	227,229	67,948
1996	352,118	884,021	933,894	1,183,289	445,745	320,996	527,804	266,172	425,046	
1997	290,507	1,001,799	926,219	1,016,654	750,816	146,923	495,992	280,405		
1998	310,608	1,108,250	776,189	1,562,400	272,482	352,053	206,286			
1999	443,160	693,190	991,983	769,488	504,851	470,639				
2000	396,132	937,085	847,498	805,037	705,960					
2001	440,832	847,631	1,131,398	1,063,269						
2002	359,480	1,061,648	1,443,370							
2003	376,686	986,608								
2004	344,014									
Total	3,671,385	8,287,172	7,661,093	6,883,077	3,207,180	1,865,009	1,376,424	686,527	652,275	67,948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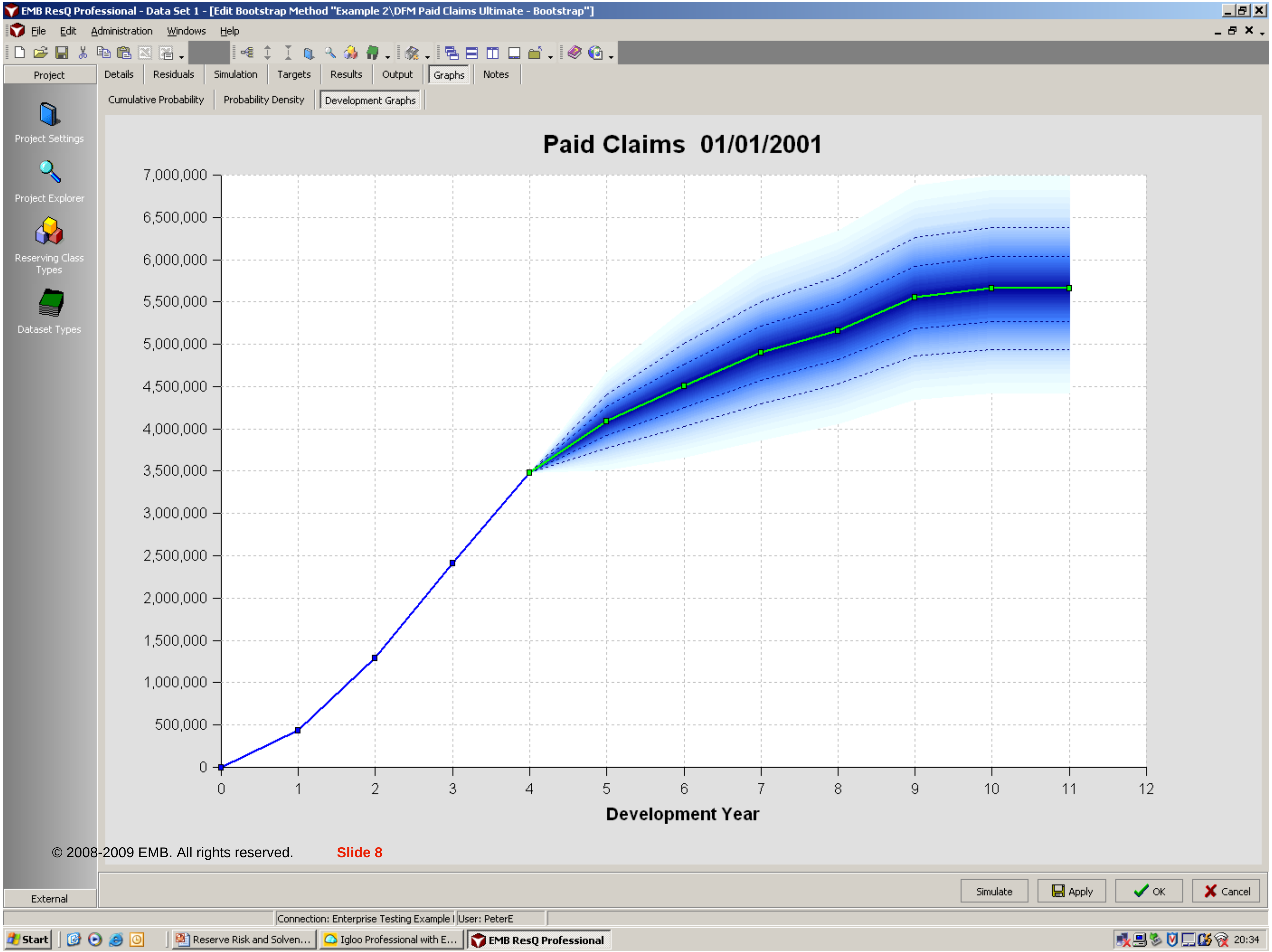
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Basic Concepts

Uncertainty when Forecasting: Prediction errors and Predictive distributions

A Simple One Parameter Problem

- Number of large claims in each of the last 10 years = [3,8,5,9,5,8,4,8,7,3]
- Best estimate of the number of large claims next year?
- Expected value = 6
- Standard error of the mean?
- Prediction error of a new forecast value?
- Distributed as a Poisson random variable?
- Predictive distribution of a new forecast value?

A Simple One Parameter Problem

- Number of large claims in each of the last 10 years = [3,8,5,9,5,8,4,8,7,3]
- Best estimate of the number of large claims next year?
- Expected value = 6
- Standard error of the mean (if *i.i.d*)?

$$Var(\mu) = Var\left(\frac{\sum_{i=1}^{n} x_i}{n}\right) = \frac{1}{n^2} Var\left(\sum_{i=1}^{n} x_i\right) = \frac{1}{n^2}\left(\sum_{i=1}^{n} Var(x_i)\right) = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

$$SE(\mu) = \frac{\sigma}{\sqrt{n}} \longleftarrow \boxed{0.68}$$

Variability of a Forecast

- Includes estimation variance and process variance

- Analytic solution: estimate the two components

$$\text{prediction error} = (\text{process variance} + \text{estimation variance})^{1/2}$$

$$\text{estimation variance} = \frac{\sigma^2}{n} \quad \longleftarrow \quad 0.68^2 = 0.46$$

$$\text{process variance (Poisson)} = \mu \quad \longleftarrow \quad 6.0$$

$$\text{prediction error of forecast} = \left(\mu + \frac{\sigma^2}{n}\right)^{\frac{1}{2}} \quad \longleftarrow \quad 2.54$$

Parameter Uncertainty- Bootstrapping

- Bootstrapping is a simple but effective way of obtaining a distribution of the parameters
- The method involves creating many new data sets from which the parameters are estimated
- The new data sets are created by sampling with replacement from the observed data
- Results in a (“simulated”) distribution of the parameters

Simple Example
Bootstrapping the Mean

Observed Data										Mean
3	8	5	9	5	8	4	8	7	3	6

	Bootstrap Samples										Mean
1	4	3	3	8	7	5	7	7	3	8	5.5
2	4	3	9	5	7	5	8	5	8	8	6.2
3	8	5	7	8	4	9	3	8	5	7	6.4
4	8	5	8	9	4	8	8	8	8	8	7.4
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
10,000	5	3	5	8	8	3	4	8	8	3	5.5
										Bootstrap standard error	0.68

Forecasting

Simulate a forecast observation, conditional on each bootstrap mean

Mean	Simulated Forecast
5.5	4
6.2	7
6.4	5
7.4	9
.	.
.	.
.	.
5.5	6

Assuming a Poisson process distribution → Simulated Forecast

Standard Error → 0.68

2.54 ← Prediction error

Important Lessons

- We could calculate the SD of the forecast ("prediction error") analytically, taking account of parameter uncertainty
- Bootstrapping gives a distribution of parameters, hence an estimate of the estimation error, without the hard maths
- When supplemented by a second simulation step incorporating the process error, a distribution of the forecast is generated

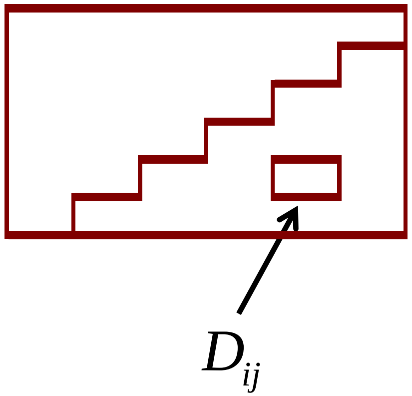
An example of a stochastic reserving model

Mack's model

Mack's Model

Mack, T (1993), *Distribution-free calculation of the standard error of chain-ladder reserve estimates*. ASTIN Bulletin, 22, 93-109

D_{ij} = Cumulative claims in origin year i and development year j



Specified mean and variance only:

$$E(D_{ij}) = \lambda_j D_{i,j-1}$$

← Expected value proportional to previous cumulative

$$V(D_{ij}) = \sigma_j^2 D_{i,j-1}$$

← Variance proportional to previous cumulative

Mack's Model

$$\hat{\lambda}_j = \frac{\sum_{i=1}^{n-j+1} w_{ij} f_{ij}}{\sum_{i=1}^{n-j+1} w_{ij}}$$

← Estimator for lambda

$$\hat{\sigma}_j^2 = \frac{1}{n-j} \sum_{i=1}^{n-j+1} w_{ij} \left(f_{ij} - \hat{\lambda}_j \right)^2$$

← Estimator for sigma squared

$$w_{ij} = D_{i,j-1} \text{ and } f_{ij} = \frac{D_{ij}}{D_{i,j-1}}$$

Variability in Claims Reserves

- Variability of a forecast
- Includes estimation variance and process variance

$$\text{prediction error} = (\text{process variance} + \text{estimation variance})^{1/2}$$

- Problem reduces to estimating the two components. For example, for the reserves in origin year *i*:

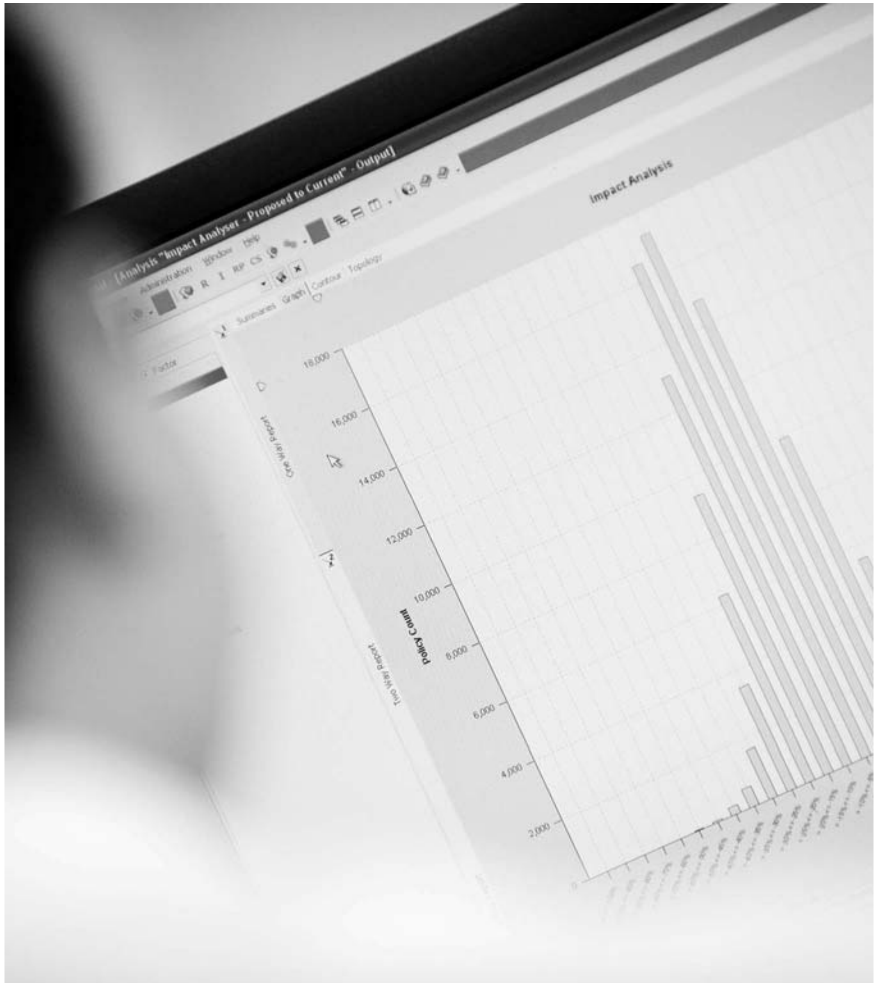
$$RMSEP\left[\hat{R}_i\right] \approx \sqrt{\hat{D}_{in}^2 \sum_{k=n-i+1}^{n-1} \frac{\hat{\sigma}_{k+1}^2}{\hat{\lambda}_{k+1}^2} \left(\frac{1}{\hat{D}_{ik}} + \frac{1}{\sum_{q=1}^{n-k} D_{qk}} \right)}$$

An example of bootstrapping a stochastic reserving model

Bootstrapping Mack's model (England & Verrall 2006)

Stochastic Reserving: Bootstrapping

- Bootstrapping assumes the data are independent and identically distributed
- With regression type problems, the data are often assumed to be independent but are not identically distributed (the means are different for each observation)
- However, the residuals are usually *i.i.d*, or can be made so
- Therefore, with regression problems, it is common to bootstrap the (standardised) residuals instead



Reserving and Bootstrapping

Define and fit statistical model

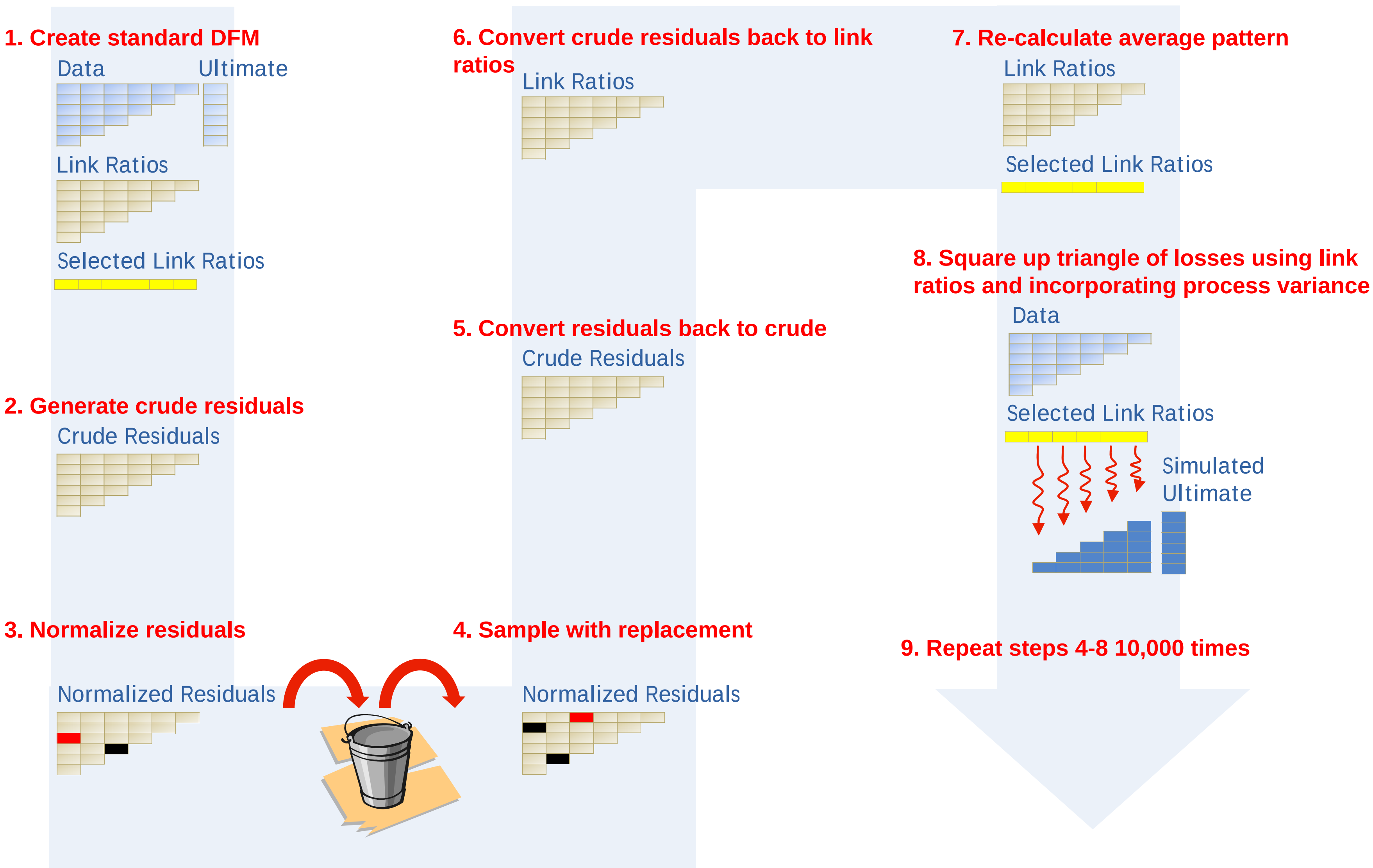
Obtain residuals and pseudo data

Re-fit statistical model to pseudo data

Obtain forecast, including process error

Any model that can be clearly defined can be bootstrapped

Bootstrapping Mack: 9 Steps

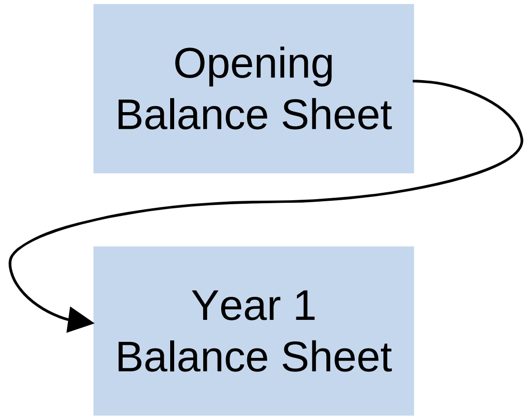


“Reserving risk” and Solvency II

The one-year view of reserving risk

Solvency 2

- Solvency 2 is notionally projecting a balance sheet, and requires a distribution of "Net Assets" over a one year time horizon.
- Solvency 2 requires a view of the distribution of expected liabilities in one year
- For reserving risk, this requires a distribution of the profit/loss on reserves over one year
- This is different from the standard approach to reserving risk, which considers the distribution of the ultimate cost of claims (eg Mack 1993, England & Verrall 1999, 2002, 2006)



The one-year run-off result (undiscounted)
(the view of profit or loss on reserves after one year)

For a particular origin year, let:

The opening reserve estimate be R_0

The reserve estimate after one year be R_1

The payments in the year be C_1

The run-off result (claims development result) be CDR_1

Then

$$CDR_1 = R_0 - C_1 - R_1 = U_0 - U_1$$

Where the opening estimate of ultimate claims and the estimate of the ultimate after one year are U_0, U_1

The one-year run-off result
(the view of profit or loss on reserves after one year)

Merz & Wuthrich (2008) derived analytic formulae for the standard deviation of the claims development result after one year assuming:

- The opening reserves were set using the pure chain ladder model (no tail)
- Claims develop in the year according to the assumptions underlying Mack's model
- Reserves are set after one year using the pure chain ladder model (no tail)
- (The mathematics is quite challenging)

The M&W method is gaining popularity, but has limitations. What if:

- We need a tail factor to extrapolate into the future?
- Mack's model is not used – other assumptions are used instead?
- We want another risk measure (say, VaR @ 99.5%)?
- We want a distribution of the CDR (not just a standard deviation)?

Merz & Wuthrich (2008)

Data Triangle

Accident Year	12m	24m	36m	48m	60m	72m	84m	96m	108m
0	2,202,584	3,210,449	3,468,122	3,545,070	3,621,627	3,644,636	3,669,012	3,674,511	3,678,633
1	2,350,650	3,553,023	3,783,846	3,840,067	3,865,187	3,878,744	3,898,281	3,902,425	
2	2,321,885	3,424,190	3,700,876	3,798,198	3,854,755	3,878,993	3,898,825		
3	2,171,487	3,165,274	3,395,841	3,466,453	3,515,703	3,548,422			
4	2,140,328	3,157,079	3,399,262	3,500,520	3,585,812				
5	2,290,664	3,338,197	3,550,332	3,641,036					
6	2,148,216	3,219,775	3,428,335						
7	2,143,728	3,158,581							
8	2,144,738								

Merz & Wuthrich (2008)
Prediction errors

	Analytic Prediction Errors	
Accident Year	1 Year Ahead CDR	Mack Ultimate
0	0	0
1	567	567
2	1,488	1,566
3	3,923	4,157
4	9,723	10,536
5	28,443	30,319
6	20,954	35,967
7	28,119	45,090
8	53,320	69,552
Total	81,080	108,401

The one-year run-off result in a simulation model

(the view of profit or loss on reserves after one year)

For a particular origin year, let:

The opening reserve estimate be R_0

The expected reserve estimate after one year be $R_1^{(i)}$

The payments in the year be $C_1^{(i)}$

The run-off result (claims development result) be $CDR_1^{(i)}$

Then

$$CDR_1^{(i)} = R_0 - C_1^{(i)} - R_1^{(i)} = U_0 - U_1^{(i)}$$

Where the opening estimate of ultimate claims and the expected ultimate after one year are $U_0, U_1^{(i)}$

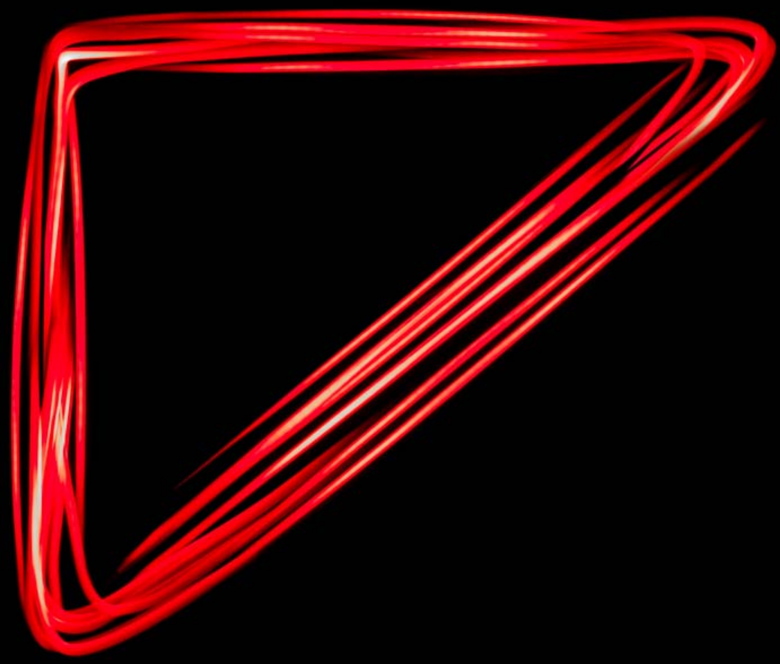
for each simulation *i*

The one-year run-off result in a simulation model
Modus operandi

1. Given the opening reserve triangle, simulate all future claim payments to ultimate using a bootstrap or Bayesian MCMC technique.
2. Now forget that we have already simulated what the future holds.
3. Move one year ahead. Augment the opening reserve triangle by one diagonal, that is, by the simulated payments from step 1 in the next calendar year only. An actuary only sees what emerges in the year.
4. For each simulation, estimate the outstanding liabilities, conditional only on what has emerged to date. (The future is still "unknown").
5. A reserving methodology is required for each simulation – an "actuary-in-the-box" is required*. We call this re-reserving.
6. For a one-year model, this will underestimate the true volatility at the end of that year (even if the mean across all simulations is correct).

** The term "actuary-in-the-box" was coined by Esbjörn Ohlsson*

EMB ResQ Example



Merz & Wuthrich (2008)

Analytic vs Simulated

	Analytic Prediction Errors		Simulated Prediction Errors	
Accident Year	**1 Year Ahead CDR**	**Mack Ultimate**	**1 Year Ahead CDR**	**Mack Ultimate**
0	0	0	0	0
1	567	567	569	569
2	1,488	1,566	1,494	1,571
3	3,923	4,157	3,903	4,144
4	9,723	10,536	9,687	10,518
5	28,443	30,319	28,363	30,393
6	20,954	35,967	20,924	35,772
7	28,119	45,090	28,358	45,668
8	53,320	69,552	53,591	69,999
Total	81,080	108,401	81,159	108,442

Re-reserving in Simulation-based Capital Models

The advantage of investigating the claims development result (using re-reserving) in a simulation environment is that the procedure can be generalised:

- Not just the chain ladder model
- Not just Mack's assumptions
- Can include curve fitting and extrapolation for tail estimation
- Can incorporate a Bornhuetter-Ferguson step
- Can be extended beyond the 1 year horizon to look at multi-year forecasts
- Provides a *distribution* of the CDR, not just a standard deviation
- Provides a link between the traditional "ultimate" view of risk and the "1 year" view
- Can be used to help calibrate Solvency 2 internal models

A simple risk margin method

1. Apply bootstrapping in the usual way
2. Generate a distribution of the one-year CDR (using re-reserving)
3. Estimate opening capital required by applying a risk measure to the one-year CDR distribution (eg VaR @ 99.5%)
4. Apply the proportional proxy for future capital requirements
5. Multiply by the cost-of capital loading
6. Discount and sum

Issues:

- UPR, dependencies, aggregation, paid vs incurred, gross vs net, non-bootstrapped lines, discounting/investment income, non-annual analysis dates, accident vs underwriting year issues, attritional/large claims split, scaling, operational risk, credit risk and market risk…

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