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# Optimal hedging of demographic risk in life insurance

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**Abstract** A Markov chain model is taken to describe the development of a multi-state life insurance policy or portfolio in a stochastic economic-demographic environment. It is assumed that there exists an arbitrage-free market with tradeable securities derived from demographic indices. Adopting a mean-variance criterion, two problems are formulated and solved. First, how can an insurer optimally hedge environmental risk by trading in a given set of derivatives? Second, assuming that insurers perform optimal hedging strategies in a given derivatives market, how can the very derivatives be designed in order to minimize the average hedging error across a given population of insurers? The paper comes with the caveat emptor that the theory will find its prime applications, not in securitization of longevity risk, but rather in securitization of catastrophic mortality risk.

**Keywords** Stochastic mortality · Mortality derivatives · Mean-variance hedging · Optimal design of derivatives

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## 1 Introduction

*A. Environmental risk in insurance.* The contract period of a typical life insurance or pension policy is long enough to see significant changes in mortality. Although such changes are perceived as random and referred to as *stochastic mortality*, the insurance risk associated with them is often referred to as

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*systematic mortality risk*. Accordingly, the risk associated with random differences in individual life lengths within a cohort is called *non-systematic mortality risk*. Stochastic mortality is an example of an *environmental* or *collective* risk factor that impacts all or a large number of policies in a portfolio. Other examples are catastrophes in general insurance and life insurance and uncertain yields on invested reserves. Increasing the size of the portfolio aggravates rather than alleviates such forms of risk, and insurers have therefore invented other ways of managing them. In the following three paragraphs we describe the main categories briefly.

*B. Internal risk management.* At the level of the direct insurer, collective risk can be managed through the design of the individual contracts. The idea is to make the payments depend, not only on what happens to the individual policy, but also on the performance of the entire portfolio.

In life insurance the traditional way of doing this is the *with-profit* scheme, which works as follows. The payments stipulated in an insurance policy depend only on the life history of the insured individual, but the premium is set sufficiently high to be adequate under all likely economic-demographic scenarios. Systematic surpluses emerging from the prudent assumptions are redistributed in arrears as bonuses to the policyholders. For a modern account of this scheme the reader is referred to [10].

An alternative idea, much to the same effect, is perfectly *index-linked* insurance, whereby the very contractual payments are linked to indices for economic and demographic factors like interest and mortality, see [12].

Either scheme can be designed in such a manner that solvency of the insurer is secured, the crux of the matter being that the economic-demographic risk is left to the customers. With-profit and index-linked schemes in their ideal forms are not common in practice: usually they come with various forms of guarantees that reintroduce solvency risk on the part of the insurer. This risk can only be managed through risk exchanges with third parties or market operations to be described in the following two paragraphs.

*C. Reinsurance.* Direct insurers can transfer collective risk to reinsurance companies. Reinsurance of catastrophe risk is common in both general insurance and life insurance. Only in recent years and only to a limited extent have traditional reinsurance companies been engaging in coverage of long term economic-demographic risk.

*D. Alternative risk transfer.* Insurers can exchange risk with parties outside the insurance industry. We shall describe two ways in which this can be done.

An insurer can enter into a so-called swap with some counter-party, whereby the former pays the expected net liabilities to the latter (the fixed leg) and receives in return the value of the liabilities based on statistical experience (the floating leg). Swaps are akin to reinsurance contracts, and designing the fixed leg is essentially a pricing problem. Being over-the-counter contracts, swaps are not liquidly traded and do not leave a record of the "market price" of mortality risk.

A different form of alternative risk transfer is *securitization*, which means creating tradeable *derivative securities* with payoffs depending on indices related to collective risk factors. Such derivatives offer, on the one hand, diversification opportunities for investors and, on the other hand, hedging opportunities for insurers, and can thus serve to transfer collective risk to the financial markets.

The past two decades have seen the advent of catastrophe derivatives with pay-offs linked to indices for natural catastrophes (floods, earthquakes, hurricanes). Such "CAT" securities exist in many different forms, are liquidly traded, and serve the combined purpose of diversification and hedging.

Securitization of systematic mortality and longevity risk is currently gaining momentum in practice and is drawing increasing attention from contributors to the professional and scientific actuarial press. A fairly up to date overview is offered in Section 6 of [2]. *CAT Mortality bonds*, aiming to offer hedging opportunities for life offices against catastrophic mortality risk in the short term, have fared reasonably well. A *longevity bond*, aiming to offer hedging opportunities for annuity businesses and pension funds against uncertain long term mortality developments, did not sell at sustainable prices and was taken off the market. The mixed experience reflects certain differences between mortality risk and longevity risk to be described next.

*E. On the feasibility of securitization of systematic mortality and longevity risk.* The CAT mortality bonds have much in common with their CAT predecessors in general insurance. They are short term and related to limited losses that can comfortably be absorbed by the global capital markets. The underlying risk is caused by purely random phenomena that can be reliably described by a mathematical model calibrated to historical data.

Longevity bonds are different. They need to be exceedingly long term and are related to potential losses at a macroeconomic scale. The underlying longevity risk is the resultant of complex demographic, societal, and technological processes causing fluctuations, trends, and shifts that cannot be inferred from statistical data. In the shorter term they can to some extent be anticipated from current collateral information to which investors will not have equal access. Thus, to the usual pros for securitization of longevity risk there are certain cons to be added that cast doubt on its practical feasibility and on any attempt to analyze its working with traditional financial mathematics models. Since the present study is based on the assumption that systematic mortality risk can be adequately modeled for the entire term of the derivatives, we anticipate that it will find its prime applications in securitization of catastrophic mortality and, possibly, some similar forms of risk.

*F. Risk management starts with designing sustainable contracts.* And that is where it could also end if it were done properly. Since longevity risk can be managed internally as described in Paragraph B above, there is no compelling reason why it should be passed on to the market. Market management of longevity risk has become an issue because life offices and pension funds created non-diversifiable risk through the very design of their schemes: they promised too much.

*G. Related literature and outline of the present study.* The theoretical studies published up to the present worked mainly with single-factor models for describing stochastic mortality. Dahl [3] and Cairns et al. [1] considered a complete market model with a money market account and a mortality bond where valuation of systematic mortality risk is a matter of arbitrage pricing. Dahl and Møller [4] considered an incomplete market with a money market account and a zero coupon bond, but no mortality related securities. In this framework they derived risk minimizing hedging strategies for systematic mortality risk. Dahl et al. [5] were the first to study hedging of systematic mortality risk in an incomplete market with mortality related securities. They considered two mortality swaps on different mortality indices driven by square-root diffusions that can be correlated (a two-factor model), and (assuming the swaps can be freely traded) worked out optimal hedging strategies in the framework of risk minimization.

The present paper discusses hedging of systematic risk associated with mortality and possibly other vital rates like cause-specific mortality, disability, recovery, etc. Section 2 introduces a multi-factor model apt to describe the joint random development of a life insurance policy (non-systematic risk) and its economic-demographic environment (systematic risk), the latter comprising demographic indices for the vital rates as well as market indices like interest and stock prices. The driving processes are point processes allowing for stochastic calculus based on compensated counting processes. For the sake of simplicity, they are assumed to be time-continuous Markov chains as in [10]. The financial market is assumed to comprise a finite number of securities with pay-offs dependent on the economic and demographic factors. Some possible designs of mortality derivatives are discussed in a multiple decrement scenario with cause-specific stochastic mortality rates. The market is assumed to be free of arbitrage but incomplete due to the high complexity of the random development of the vital rates. Section 3 derives the optimal hedging strategy for a given agent (life office or pension fund) using a simple mean-variance criterion that amounts to minimizing the expected squared difference between the total insurance payments and the terminal value of a self-financing portfolio of mortality-related derivatives. The theory is akin to but different from that in [11], which considered risk minimization and could only deal with simple claims. Section 4 presents a worked example based on a multiple decrement model. In Section 5 it is assumed that all agents adopt the optimal strategy for a given set of available derivatives, and it is then shown how to design the derivatives in such a manner as to minimize the overall mean hedging error across the population of agents. Section 6 adds some substance to the introductory discussion of the very securitization idea, and finishes with a brief discussion of the present theoretical approach.