Using Life Settlements to Hedge the Mortality Risk of Life Insurers: An Asset-Liability Management Approach*

Paper Presentation for 2012 ARIA Meeting
(Draft of July 2012)

Jennifer L. Wang* (corresponding author)
Department of Risk Management and Insurance
Risk and Insurance Research Center
College of Commerce, National Chengchi University
64, Sec. 2, Chihnan Rd, Taipei, Taiwan, 11605, R.O.C.
E-mail: jenwang@nccu.edu.tw

Ming-Hua Hsieh
Department of Risk Management and Insurance
Risk and Insurance Research Center
College of Commerce, National Chengchi University
64, Sec. 2, Chihnan Rd, Taipei, Taiwan, 11605, R.O.C.
E-mail: mhsieh@nccu.edu.tw

Chenghsien Tsai
Department of Risk Management and Insurance
Risk and Insurance Research Center
College of Commerce, National Chengchi University
64, Sec. 2, Chihnan Rd, Taipei, Taiwan, 11605, R.O.C.
E-mail: ctasi@nccu.edu.tw

*The authors are grateful to the Risk and Insurance Research Center of National Chengchi University for its financial support.

Abstract

Life settlements have attracted increasing attentions of investors and scholars. This paper extends the literature by conducting the first analysis of life settlements as a hedging vehicle for life insurers. We employ real-case data from Coventry to calibrate the parameters of the mortality rate models and consider the variations of the deviations from life expectancy. We further propose a new approach to construct an optimal hedging strategy with regard to certain risk measures. Our numerical results show that life settlements can be an effective hedging tool to significantly reduce the insurer's mortality risk. The results are robust across risk measures, correlation specifications, and mortality rate models.

1. Introduction

Life insurance companies are exposed to both longevity risk and mortality risk. On the one hand, Benjamin and Soliman (1993) and McDonald et al. (1998) confirmed that unprecedented improvements in population longevity have occurred around the world. Decreasing mortality rates have created a major risk for life insurance companies selling annuities. On the other hand, we have witnessed an increasing frequency of pandemics and catastrophes that have given rise to sudden and significant payments of death benefits from life insurances. Different products sold by life insurers are thus exposed to longevity and mortality risk to different degrees. These uncertain cash flows may lead not only to short-term liquidity shocks but also to long-term solvency threats to the life insurers.

The literature has proposed a number of ways to mitigate the longevity and mortality risks of life insurers. They can be classified into three categories. The first is capital market solutions including mortality securitization (see, for example, Dowd, 2003; Lin and Cox, 2005; Blake et al., 2006a, 2006b; Cox et al., 2006), survivor bonds (e.g., Blake and Burrows, 2001; Denuit et al., 2007), and survivor swaps (e.g., Dowd et al., 2006). The second way is through internal self-insurance that takes place within the industry such as the natural hedging strategy of Cox and Lin (2007), the duration matching strategy of Wang et al. (2010), and the reinsurance swap of Lin and Cox (2005). The third method, known as mortality projections, aims to provide accurate estimations of mortality processes (e.g., Milevsky and Promislow, 2001; Dahl, 2004; Biffis, 2005; Schrager, 2006; Brouhns et al., 2002; Renshaw and Haberman, 2003; Cairns et al., 2006b).

Among the industry's self-insurance solutions, the natural hedging strategy suggests that life insurance can serve as a hedging vehicle against the longevity risk of annuity products with low cost. Wang et al. (2010) demonstrated that an optimal product mix between life insurance and annuities could effectively reduce the longevity risk faced by life insurers. However, life insurers have difficulties in implementing this kind of strategy because they may not be able to allocate or change their product portfolios accordingly. The sales of insurance products are not only directed by the insurance companies but are also controlled by their distribution channels. Since life insurers do not have full control over their sales, they may be unable to achieve the optimal product mix. In addition, changing the commission schemes may enhance the control power, but the expenses incurred may offset the benefits of the natural hedging.

Hedging the longevity and mortality risks from the asset side may be more flexible and cost-effective than through the liability side. An emerging area for longevity investments is the life settlements market, and its related investment products may further help life insurers to achieve this goal. Life settlements are transactions to trade life policies in the secondary market for life insurance and also known as "traded life policies" (TLP). The life insurance policy holder can sell a policy and assign the death benefit to the purchaser. These contracts differ in their premium payment methods and are attractive to various investors[1]. Life settlements are becoming an increasingly popular asset class, offering good returns[2] that are largely unaffected by financial crises and market downturns like those of 2000 and 2008. Moreover, life settlements have many important characteristics, such as uncorrelated performance to the capital market, potentially attractive risk/return profiles, relatively low volatility and superior credit quality[3].

Since life settlements are a rather young asset class, there are only few early studies which focus on this topic and mainly analyzed their investment characteristics and economic impacts (e.g., Giacalone, 2001; Doherty and Singer 2002; Ingraham and Salani, 2004; Kamath and Sledge, 2005; Ziser, 2006 and 2007; Smith and Washington, 2006; Seitel, 2006 and 2007; Conning and Company, 2007; Freeman, 2007; Sherman, 2007; Blake and Harrison, 2008; Leimberg et al., 2008). More recently, Gatzert (2010) analyzes related risk and return performance in the United Kingdom, Germany, and the United States. Braun, Gatzert, and Schmeiser (2011) suggest that life settlements can be good investments to life insurance companies since they offer good yields with near-zero betas.[4] As a result, life settlements

---

[1] In a life settlement transaction, the policyholder may receive a payment that exceeds the surrender value but is less than the death benefit. The life settlement provider offers a price by actuarial valuation which largely depends on the insured's estimated life expectancy. From the investor's perspective, the investment return is determined by the quality of the life expectancy estimates provided by medical underwriters.

[2] With the structure similar to hedge funds, the open-end life settlement funds usually targeted absolute returns of between 8 and 15 percent per annum.

[3] Gatzert (2010) gives an excellent review of various settlement products and recent life settlement markets.

[4] Braun, Gatzert, and Schmeiser (2011) provide a comprehensive analysis of the risk and return performance of life settlements. Their result supports that life settlements offer attractive returns with low volatility and uncorrelated with other asset classes.

are regarded as a strong market, which has the potential[5] to exceed $140 billion by 2016. Another stream of research focuses on the actuarial modeling and valuation of life settlements (e.g., Zollars, Grossfeld, and Day, 2003; Deloitte, 2005; Russ, 2005; Perera and Reeves, 2006; Milliman, 2008; Mason and Singer, 2008). Stone and Zissu (2006) and Ortiz, Stone, and Zissu (2008) further discuss the securitization of life settlements. Perera and Reeves (2006) and Stone and Zissu (2007) investigate the sensitivity of the investment returns of life settlements to life expectancy estimates and possibilities of risk mitigation. From the perspective of asset-liability management, life settlements not only can enhance the investment return but also provide hedging benefits to life insurers' cash flows since their payoffs are related to the mortality rate.

However, little research has been done on developing an efficient hedging mechanism for life insurers using life settlements. To fill up this gap, we extend Stone and Zissu (2006; 2008) to incorporate the variation of deviations from life expectancy within and between life settlements and insurance portfolios. We further propose an approach to calculate the optimal investment amount of life settlements for hedging the mortality risk[6] of life insurance products with respect to certain risk measures. In calibrating the mortality rate models, we employ real-case data from Coventry. We then use simulations to quantify the hedging benefits of life settlements. To the best of our knowledge, this is the first attempt to

[5] According to Conning (2011), life settlement market potential is the percentage of the total of in-force life insurance face amounts that meet the criteria used by life settlement buyers and investors, where the policies' owners would consider settling their policies.

[6] We use the term "mortality risk" in a more general way starting here to represent both the longevity and mortality risks mentioned above.

investigate the potential of life settlements as a hedging vehicle for life insurers by using real empirical data in the literature. The numerical results of the paper demonstrate that life settlements can be regarded as an effective hedging vehicle to significantly reduce the aggregate risk and such hedging activities can be arranged with low costs for life insurance companies. We find that life settlements to a significant extent reduce the mortality risk of a life insurer's liability portfolio in most cases. The reduction could reach 50% under reasonable speculations on the correlation coefficients between life settlements and insurance contracts. Our results are robust across risk measures, mortality rate models, and the correlation specifications between life settlements and insurance contracts. Life settlements could therefore serve as a good hedging vehicle in addition to an alternative investment.

The remainder of this article is organized as follows. In Section 2, we introduce the research models and propose an approach to calculate optimal hedge ratio for life insurers. In Section 3, we describe our data and demonstrate how our proposed models can be implemented by using different numerical examples for various mortality correlation settings and risk measures. Finally, we analyze the simulation results in Section 4, and we then conclude in Section 5.

2. Research Models

In this section, we first provide brief descriptions of the mortality models and related assumptions. Then we describe the proposed hedging strategy using life settlements. A life insurer that sells whole life insurance to senior people should consider an investment plan to hedge the mortality risk since hedging the risk through the liability side itself may be infeasible.

Suppose that insurer's product portfolio contains m whole life contracts and the life expectancy of the life contract j is τ_j, $j = 1, 2, \ldots, m$. Further assume that the insurer's may purchase n senior life settlements that have life expectancy t_i ($i = 1, 2, \ldots, n$) with the current value $V_i(t_i)$. According to Stone and Zissu (2006; 2008), the values of a life settlement when considering the life extension or contraction from expectancy T_i can be expressed as follows:

$$V_i(t_i + T_i) \approx V_i + d_i T_i + \frac{1}{2} c_i T_i^2, \qquad i = 1, 2, \ldots, n \quad (1)$$

The constants d_i and c_i represent modified life-extension[7] duration (le-duration) and life-extension convexity (le-convexity), respectively.

Stone and Zissu (2006; 2008) consider only a "static" life extension and thus assume that all life settlements have the same life extension. However, the above assumption may

[7] For the convenience of the reading, we shorten the term "life extension or contraction from expectancy" to "life extension".

not be appropriate when comparing the mortality risk between life settlements and insurance policies. When measuring the hedging effect between life settlements and the insurer's liabilities associated with life insurance contracts, we should further consider the variation in life extensions in various life settlements and insurance contracts. Therefore, we model the future lifetimes (t_1+T_1, $t_2+T_{2, \ldots,} t_n+T_n$) and ($\tau_1$, $\tau_{2, \ldots,} \tau_m$) using random vectors with known marginal distributions[8]. This setting enables us to have the mortality tables corresponding to each life settlement and insurance contract. More precisely, the morality table corresponding to life settlement *i* describes the marginal distribution of $t_i + T_i$, and the morality table for insurance contract *j* describes the marginal distribution of τ_j.

The marginal distribution function of t_i+T_i is denoted by $F_i(.)$ and their joint behaviors are described by a normal factor copula[9] (see Burtschell, Gregory, and Laurent, (2009); Hull, (2011). Burtschell, Gregory, and Laurent, (2009) select normal factor copula to model the dependence structure of times to the defaults of bonds in a credit portfolio. Following their idea, we select normal factor copula to model the dependence structure of times to the deaths of policy holders in a life settlement pool in this paper. In particular, we assume that

$$T_i = F_i^{-1}\big(N(X_i)\big) - t_i, \quad \mathrm{i} = 1, \ldots, \mathrm{n}, \tag{2}$$

[8] We do not make any special assumptions of the marginal distribution of t_i+T_i and τ_j. For valuation purpose, the marginal distribution of t_i+T_i is based on the mortality table provided by medical underwriters and the marginal distribution of τ_j is based on insurers' internal mortality table.

[9] As suggested by Asmussen and Glynn (2007), copulas provide a possible approach for modeling multivariate distributions in which one has a rather well-defined idea of the marginal distributions but a rather vague one of the dependence structure.

where $N(.)$ is the cumulative distribution function of the standard normal random variable and X_i are the latent variables used to model the joint distributions of T_i. The correlations among X_i are induced through common factors M and M_{ls} as follows:

$$X_i = aM + bM_{ls} + \sqrt{1 - a^2 - b^2}Z_i, \quad i = 1, 2, \dots, n, \tag{3}$$

where M, M_{ls}, $Z_1,\dots, Z_n$ are independent standard normal random variables and a, b denote constant factor loadings. The common factor M represents the global trend in age improvement while M_{ls} reflects improvement trend of the life settlement pool[10]. On the other hand, $Z_1,\dots, Z_n$ are specific factors pertaining to each life settlement.

The marginal distribution function of τ_j is denoted by $G_j(.)$ and their joint behaviors are also described by a normal factor copula. In particular, we assume that

$$\tau_j = G_j^{-1}\left(N(Y_j)\right), \quad j = 1, \dots, m, \tag{4}$$

where Y_j are the latent variables used to model the joint distributions of τ_j. The correlations among Y_j are induced through common factors M and M_{wl} as follows:

$$Y_j = cM + dM_{wl} + \sqrt{1 - c^2 - d^2}W_j, \quad j = 1, 2, \dots, m, \tag{5}$$

where M_{wl}, $W_1,\dots, W_m$ are independent standard normal random variables and constants c and d denote factor loadings. M_{wl} represents the factor of age improvements for the insurance contract portfolio as a whole and $W_1,\dots, W_m$ are specific factors pertaining to each insurance contract. They are also independent of M, M_{ls}, $Z_1,\dots, Z_n$.

[10] We use Maximum Likelihood approach to estimate the factor loadings from the related samples of life settlements pool.

According to above settings, the correlation coefficient ρ_{ik} between life settlements i and k is a^2+b^2. The correlation coefficient κ_{jl} between life contracts j and l is c^2+d^2, and the correlation coefficient between life settlement i and insurance contract j is ac.

Let the rate of return for the life settlement and required investment return of insurance contracts be r_{ls} and r_{wl}, respectively. Assume that the premium payment of life settlement i at time t is $P_i(t)$, its death benefit is B_i, the premium payment of insurance contract j at time t is $Q_j(t)$, and its death benefit is A_j. Then the value of the life settlement pool can be expressed as Equation (6):

$$V_{ls} = \sum_{i=1}^{n} V_{ls}(i), \tag{6}$$

where $V_{ls}(i) = \frac{B_i}{(1+r_{ls})^{t_i+T_i}} - \sum_{t=1}^{t_i+T_i} \frac{P_i(t)}{(1+r_{ls})^t}$.

and the value of the insurance liability portfolio is Equation (7):

$$V_{wl} = \sum_{j=1}^{m} V_{wl}(j), \tag{7}$$

where $V_{wl}(j) = \frac{-A_j}{(1+r_{wl})^{\tau_j}} + \sum_{t=1}^{\tau_j} \frac{Q_j(t)}{(1+r_{wl})^t}$.

Suppose that the insurer would like to use life settlements as a hedging tool. By implementing a hedging program, the total value of the hedged liabilities V_h can be expressed as:

$$V_h = V_{wl} + h\, V_{ls}, \tag{8}$$

in which h denotes the hedge ratio. In this paper, we assume the life settlement portfolio is a closed-end fund and the insurer can decide to purchase a portion of the life settlement fund. Therefore, h is a number between 0 and 1. Note that, according to Equations (6) and (7), V_{wl} is negative (liability) and hV_{ls} is positive which represents the investment in life settlements (asset). Equation (8) implies that the hedging effect between the liabilities and life settlements depends on the correlation parameters (a, b, c, and d) as well as the hedge ratio h. We can calculate the optimal hedge ratio h under a specified risk measure and correlation structure. The optimality is defined by minimizing a certain risk measure on V_h. The risk measures that we consider in this paper include standard deviation, Value at Risk (VaR) and expected shortfall (ES). We use Monte Carlo simulation to generate the distributions of V_{wl} and V_{ls} and determine the optimal hedge ratio h based on the simulated distributions of V_h. The insurers can then establish their hedging programs according to different liability portfolios or needs.

3. Data and Numerical Illustrations

3.1 Data

We employ real-case data from Coventry to calibrate the parameters of the mortality rate models, which is the first hedging analysis using real life settlement cases in the literature. Our data on a pool of life settlements are from Coventry and comprise more than 5,000 life policies that they originate for a real case investor. Coventry is one of the major originators in the US life settlement market[11]. The samples are a subset of the policies purchased over a 21-month period (from July 2009 to April 2011). Considering the purpose of the numerical analysis in this paper, we select 250 universal life policies[12] that were sold to senior males[13]. Our approach can be used to analyze different kinds of life insurance policies and insured groups according to the investors' needs when they construct life settlement funds. The data provide the life expectancies of individual policies estimated by one of the Coventry major medical underwriters.

3.2 Model Assumptions

Assume that an insurance company selling whole life contracts would like to use life settlements to hedge the mortality risk. The liability portfolio consists of 500 homogeneous whole life policies with the insured being senior males at age 65 now. Each policy has death benefits of $500,000 and there are no future premiums to be collected.

We adopt two mortality rate assumptions for the alternative marginal distributions of future lifetimes to explore the hedging effects of life settlements. The first assumption is that the age-specific mortality rate follows the Heligman-Pollard law suggested by Heligman

---

[11] Coventry is a global financial services firm leading the development of a robust longevity market. Based in Philadelphia with offices in London, New York and Sydney, Coventry has been named the fastest growing privately-held company in the Philadelphia region by the annual Philadelphia 100 ranking. Coventry indeed created the secondary market for life insurance in the US.

[12] According to the study of LPD (2007a, 2007b), the share of universal life among purchased policies is approximately 80–85 percent and is by far the largest segment of current life settlements market.

[13] For demonstration purposes, we focus only on universal life policies that are sold to senior males in this paper.

and Pollard (1980) and Pitacco et al. (2007).[14] Its functional form is as follows:

$$q_x \approx \frac{G \times H^x}{1 + G \times H^x},$$

where G and H are constants. The second mortality assumption is that the mortality rates are the same as those in the SSA population mortality table published in the 2008 VBT report of Experience Studies of SOA (as used in Bahna-Nolan et al., 2008).

For each life policy contract in the hedged liabilities, we assume age-specific mortality rate follows the Heligman-Pollard law with parameters G = 0.000002 and H = 1.13451 (as suggested in Example 1.2 of Pitacco et al., 2007).

We calibrate the mortality rates of each life settlement policy based on the data from Coventry. For the first mortality rate assumption based on the Heligman-Pollard law, we first set the value of G as 0.000002 (as suggested in Example 1.2 of Pitacco et al., 2007) and then estimate the values of H based on the life expectancy of each life settlement policy estimated by the medical underwriter. The estimated H ranges from 1.1296 to 1.1699. For the second mortality rates assumption based on SSA table, we scale the mortality rates q_x in the SSA table so that the life expectancies are equal to those estimated by the medical underwriter. The resulting scaling factors range from 0.522 to 3.706.

[14] Heligman and Pollard (1980) propose formulae to represent the age-pattern of mortality over the whole span of life. The original Heligman–Pollard laws are defined in terms of the odds. Pitacco et al. (2007) show that at higher ages, the annual probability of death q_x approximates the simple functional form above. Since the most important and unique information provided by medical underwriters is life expectancy, we use this information and the simple functional form to estimate the age-pattern of mortality of each policy holder in the life settlement pool.

3.3 Optimal Hedge Ratio and Hedging Effectiveness Index

Under the above assumptions, we simulate *N* scenarios of T_i and τ_j with different values of *a*, *b*, *c* and *d* (the parameters used to model the correlation structures among the future lifetimes of settlers and policyholders). Assume the rate of return for the life settlement r_{ls} is 12 % and required investment return of insurance contracts r_{wl} is 8%. Then we construct the distributions of V_{wl} (the value of the insurance liability portfolio) and V_{ls} (the value of the life settlement) using the simulated scenarios of future lifetimes. Based on the constructed distributions, we compute the optimal hedge ratios *h* with respect to the standard deviation σ, 95% value at risk (VaR), and 95% expected shortfall (ES) of V_h. If the values of the insurer's liability portfolio and life settlement pool are independent of each other, then

$$\sigma_h^2 \approx \sigma_{wl}^2 + h^2\sigma_{ls}^2.$$

Therefore, we report η as a proxy of the effectiveness of hedging.

$$\eta = 1 - \left(\frac{\sigma_h^2}{\sigma_{wl}^2 + h^2\sigma_{ls}^2}\right)$$

We further investigate how alternative correlation parameters affect the hedging effects. The results can help insurers to implement a successful hedging program against the mortality risk via life settlements.

3.4 Analysis of Hedging Benefits

The results of the hedging effects of life settlements under the first mortality assumption of Heligman-Pollard law are summarized in Tables 1-3. Table 1A shows that the hedging benefits (as measures of σ_h / σ_{wl} and η) of life settlements to insurance are material in almost all times and can be rather significant. The standard deviation would be reduced by about 10% even when one of the common factor loadings is as low as 0.1. When both life settlements and insurance have medium loadings of 0.5 with regard to the mortality common factor, the standard deviation is reduced by almost 50%. The hedging benefits are immaterial only when the correlation coefficient between the life settlement and insurance contract is as low as 0.01. The observations that the mortality rates in many countries exhibit downward trends imply that the loadings would not be small. Our simulation results therefore indicate that life settlements will render significant hedging benefits to life insurers.

[Insert Table 1A Here]

From Table 1A, We further observe that both measures of the hedging effect, σ_h / σ_{wl} and η , decrease with *a* and *c*, respectively. The hedge ratio increases with these two loadings as well. The risk of the insurance portfolio (σ_{wl}) also rises significantly with both loadings, which further demonstrates the hedging benefits of life settlements. For instance, σ_{wl} rises from $9,285,807 to $23,222,054 when *c* is 0.5 instead of 0.1 holding *a* as 0.5, and life settlements can reduce the mortality risk to $12,488,106. The above results imply that the hedging benefits are there when the insurer needs them.

Comparing Tables 1B and 1C with Table 1A, we see that the hedging benefits of life settlements to insurance remain intact, if they are not more significant, when we change the risk measure from standard deviation to VaR and expected shortfall. The extent of the risk reduction is even larger when we adopt VaR or expected shortfall rather than standard deviation σ. For instance, the risk of the hedged portfolio is 51.9% and 48.6% of the un-hedged portfolio in terms of VaR and expected shortfall, respectively, given that both *a* and *c* are 0.5. The ratio in terms of standard deviation σ in Tables 1A is 53.8%. The results of Tables 1B and 1C support that life settlements seem to provide more hedging benefits for downside risks than for the deviation risk.

[Insert Tables 1B and 1C Here]

The set of Tables 2 and 3 summarize the hedging effects of life settlements by fitting mortality rates with the Heligman-Pollard law but with different values of *b* and *d*, the risk loadings specific to life settlement and in the insurance portfolio, respectively. In comparing of Table 2 with that of Table 1, we notice that the hedging benefits of life settlements decrease with *b* and *d*. For instance, $\mathrm{VaR}_h / \mathrm{VaR}_{wl}$ increases from 0.519 to 0.694 when *b* and *d* increase from 0.02 to 0.03, given that *a* and *c* are 0.5. The reduction in hedging benefits is reasonable due to the increase in the specific risk factor relative to the common factor. The numbers in the set of Table 3 further confirms the above findings and speculations.

[Insert the sets of Table 2 and 3 here]

The sets of Tables 1-3 result from fitting mortality rates with the Heligman-Pollard law while the sets of Tables 4-6 are obtained by scaling the mortality rates of the SSA table. All the above observations support that our results are robust across mortality rate models.

The results of Tables 1-3 are very similar to those of Tables 4-6. For example, using life settlements can reduce the expected shortfall by more than 50% given that *a* and *c* are 0.5 as we can see from Table 4C. The results also demonstrate that the hedging benefits rendered by life settlements are similar under alternative mortality rate models.

[Insert the sets of Tables 4 to 6 here]

4. Conclusions and Future Works

This article investigates the hedging potential of the life settlements to life insurance companies. This paper adds a new contribution to the insurance literature by conducting the first analysis of life settlements as a hedging vehicle and measuring the hedging benefits for life insurers. We calibrate our mortality rate models using the real-case data provided by Coventry and consider the variations in the deviations from life expectancy within and between life settlements and insurance products. We then propose a hedging approach to calculate the optimal investment amount for life settlements so that the mortality risk of a life insurance portfolio can be minimized. The hedging benefits rendered by life settlements are also quantified under various settings of correlation structures with respect to different risk measurements including standard deviation, value at risk and expected shortfall. The results from our numerical examples demonstrate that the proposed hedging strategy could help life insurers to effectively mitigate the mortality risk to a meaningful extent in most cases. We speculate that the hedging benefits would be substantial due to the global trends in mortality improvements that suggested medium to high loadings of the common factor between life settlements and insurance products even across countries. Our numerical results demonstrate that adding life settlements to an insurer's investment asset can reduce the standard deviation of insurance portfolio by about 10-50% under different settings of correlation parameters. In addition, the results of risk measurements of value at risk and expected shortfall support that life settlements provide more hedging benefits for downside risks than for the deviation risk. The findings of this paper are robust across risk measures, parameter values, and mortality rate models. Life settlements, therefore, can be used as an effective hedging vehicle in addition to their good risk-return characteristics as identified in Braun, Gatzert, and Schmeiser (2011).

We will conduct some more future works to add important insights into the paper. In the numerical experiments, we will replace the hypothetical life policy portfolio with an actual portfolio consisting about 500 whole life policies from a major life insurer in Taiwan. The required investment returns and mortality tables for computing actuarial values of these life policies are also available from real practices. We believe that using the real data from an insurer's liability will enhance the reliability of the hedge effect for our numerical results.

Furthermore, in order to model the dependence structure of the survival times or default times, we will adopt the Gaussian factor copula model. In survival analysis, a common assumption is that of conditional independence in a Cox-Process setup. These types of models correspond to the Clayton copula (a special case of Archimedean copula). The assumptions of these models may affect results considerably. Therefore, we will also test the hedging effectiveness under one of these models.

Third, we will conduct sensitivity tests on the rate of return as well as the mortality improvements for the life settlements. In this version, we assume the rate of return for the life settlement is assumed at 12%, which was suggested by Coventry. However, as indicated in Braun et al. (2011), the actual realized annual return of life settlement index from Dec. 2003 to June 2010 is only about 4.85%. In addition, it is reasonable to assume that the mortality improvement trend for the sub-group of life settlements may be considerably different from that for the general insurance policyholder pool since settlement companies usually target senior policyholders with below average life expectancies,. Therefore, we will test additional assumptions on loading factors (b and d) for mortality improvements in the life settlement pool and insurance contract pool, to account for the impacts of this heterogeneity.

References:

1. Asmussen, S., and Glynn, P. W., 2007, Stochastic Simulation. Algorithms and Analysis. Springer Verlag.
2. Bahna-Nolan, Mary and Chuck Ritzke, 2008 Valuation Basic Table (VBT) Report, SOA. http://www.soa.org/research/experience-study/ind-life/valuation/2008-vbt-report-tables.aspx
3. Benjamin, B., and Soliman., A. S. (eds.), 1993, Mortality on the Move, Oxford: Institute of Actuaries.
4. Biffis, E., 2005, 'Affine Processes for Dynamic Mortality and Actuarial Valuations,' Insurance: Mathematics and Economics, 37, 443-468.
5. Blake, D. and W. Burrows, 2001, Survivor Bonds: Helping to Hedge Mortality Risk, Journal of Risk and Insurance, 68 (2), 339-348.
6. Blake, D., A. J. G. Cairns, and K. Dowd, 2006a, Living with Mortality: Longevity Bonds and Other Mortality-Linked Securities, British Actuarial Journal, 12, 153-197.
7. Blake, D., A. J. G. Cairns, K. Dowd, and R. MacMinn, 2006b, Longevity Bonds: Financial Engineering, Valuation, and Hedging. Journal of Risk and Insurance, 73 (4), 647-672.
8. Braun, A., N. Gatzert, and H. Schmeiser, 2011, Performance and Risks of Open-End Life Settlement Funds, Journal of Risk and Insurance, forthcoming.
9. Brouhns, N., Denuit, M., and Vermunt, J. K., 2002, 'A Poisson Log-Bilinear Regression Approach to the Construction of Projected Life Tables,' Insurance: Mathematics and Economics, 29, 75-108.
10. Burtschell, X., and J. Gregory et al., 2009, A Comparative Analysis of CDO Pricing Models under the Factor Copula Framework. The Journal of Derivatives, 16(4), 9-37.
11. Hull, J. C., 2011, Options, Futures, and Other Derivatives, 8th ed., Prentice Hall.
12. Cairns, A. J. G., D. Blake, and K. Dowd, 2006a, Pricing Death: Frameworks for the Valuation and Securitization of Mortality Risk, ASTIN Bulletin, 36, 79-120.
13. Cairns, A. J. G., Blake, D., and Dowd, K., 2006b, 'A Two-Factor Model for Stochastic Mortality with Parameter Uncertainty: Theory and Calibration,' Journal of Risk and Insurance, 73, 687-718.
14. Conning & Company, 2007, Conning Research: Annual Life Settlement Volume Rises to $6.1 Billion in 2006. World Wide Web: www.conning.com (accessed October 4, 2010).
15. Cowley, A. and J. D. Cummins, 2005, Securitization of Life Insurance Assets and Liabilities, Journal of Risk and Insurance, 72, 193-226.
16. Cox, J. C., J. E. Ingersoll, and S. A. Ross, 1985, A Theory of the Term Structure of Interest Rates, Econometrica 53: 385-407.
17. Cox, S. H. and Y. Lin, 2007, Natural Hedging of Life and Annuity Mortality Risks,

North American Actuarial Journal, 11 (3), 1-15.
18. Cox, S. H., Y. Lin, and S.N. Wang, 2006, Multivariate Exponential Tilting and Pricing Implications for Mortality Securitization. Journal of Risk and Insurance, 73, 719-736.
19. Dahl, M., 2004, ' Stochastic Mortality in Life Insurance: Market Reserves and Mortality-Linked Insurance Contracts,' Insurance: Mathematics and Economics, 35, 113-136.
20. Dahl, M. and T. Miller, 2006, Valuation and Hedging of Life Insurance Liabilities with Systematic Mortality Risk. Insurance: Mathematics and Economics, 39, 193-217.
21. Dahl, M., M. Melchior, and T. Miller, 2008, On Systematic Mortality Risk and Risk-minimization with Survivor Swaps. Scandinavian Actuarial Journal, 2(3), 114-146.
22. Deloitte, 2005, The Life Settlements Market, An Actuarial Perspective on Consumer Economic Value, Research Report with the University of Connecticut.
23. Denuit, M., P. Devolder, and A. C. Goderniaux, 2007, Securitization of Longevity Risk: Pricing Survivor Bonds with Wang Transform in the Lee-Carter Framework. Journal of Risk and Insurance, 74, 87-113.
24. Doherty, N. A., and H. J. Singer, 2002, The Benefits of a Secondary Market for Life Insurance Policies, Working Paper, Wharton Financial Institutions Center.
25. Dowd, K., 2003, 'Survivor Bonds: A Comment on Blake and Burrows,' Journal of Risk and Insurance, 70, 339-348.
26. Dowd, K., D. Blake, A. J. G. Cairns, and P. Dawson, 2006, Survivor Swaps. Journal of Risk and Insurance, 73, 1-17.
27. Embrechts, P., 2009, Copulas: A Personal View, Journal of Risk and Insurance, 76(3): 639-650.
28. Freedman, M., 2007, STOLI: Fact and Fiction—Combating STOLI Without Violating Consumer Rights, California Broker, December.
29. Gatzert, N., G. Hoermann, and H. Schmeiser, 2009, The Impact of the Secondary Market on Life Insurers' Surrender Profits, Journal of Risk and Insurance, 76(4): 887-908.
30. Gatzert, N., 2010, The SecondaryMarket for Life Insurance in the U.K., Germany, and the U.S.: Comparison and Overview, Risk Management and Insurance Review, 13(2): 279-301.
31. Gerstner, T., M. Griebel, M. Holtz, R. Goschnick, and M. Haep, 2008, A General Asset-liability Management Model for the Efficient Simulation of Portfolios of Life Insurance Policies, Insurance: Mathematics and Economics, 42, 704-716.
32. Giacalone, J. A., 2001, Analyzing an Emerging Industry: Viatical Transactions and the Secondary Market for Life Insurance Policies, Southern Business Review, 27(1): 1-7.
33. Grundl, H., T. Post., and R. N. Schulze, 2006, To Hedge or Not to Hedge: Managing Demographic Risk in Life Insurance Companies, The Journal of Risk and Insurance, 73 (1), 19-41.

34. Heligman, L. and J. H. Pollard, 1980, The Age Pattern of Mortality, Journal of the Institute of Actuaries, 107, 49-80.
35. Ingraham, H. G., and S. S. Salani, 2004, Life Settlements as a Viable Option, Journal of Financial Service Professionals, 58(5): 72-76.
36. Jalen L. and R. Mamon, 2009, Valuation of Contingent Claims with Mortality and Interest Rate Risks, Mathematical and Computer Modelling, 49, 1893-1904.
37. Kamath, S., and T. Sledge, 2005, Life Insurance Long View—Life Settlements Need Not Be Unsettling, Bernstein Research Call, March 4, 2005 (New York: Sanford C. Bernstein & Co., LLC).
38. Leimberg, S. R., M. D. Weinberg, B. T. Weinberg, and C. J. Callahan, 2008, Life Settlements: Know When to Hold and Know When to Fold, Journal of Financial Service Professionals, 62(5): 61-72.
39. Lin, Y. and Cox, S.H., 2005, Securitization of Mortality Risks in Life Annuities, Journal of Risk and Insurance, 72, 227-252.
40. Mason, J., and H. J. Singer, 2008,AReal Options Approach to Valuing Life Settlements Transactions, Journal of Financial Transformation, 23: 61-68.
41. Marceau, E. and P. Gaillardetz, 1999, On Life Insurance Reserve in a Stochastic Mortality and Interest Rates Environment, Insurance: Mathematics and Economics 25 (3), 261-80.
42. McDonald, A. S., Cairns, A. J. G., Gwilt, P. L., and Miller, K. A., 1998, 'An International Comparison of Recent Trends in the Population Mortality,' British Actuarial Journal, 3, 3-141.
43. Mileysky, M. A., and Promislow, S. D., 2001, 'Mortality Derivatives and the Option to Annuities,' Insurance: Mathematics and Economics, 29, 299-318.
44. Milliman Inc., 2008, Life Settlement Characteristics and Mortality Experience for Two Providers, Milliman Research Report, April 2008. World Wide Web: www.milliman.com (accessed October 4, 2010).
45. Ortiz, C. E., C. A. Stone, and A. Zissu, 2008, Securitization of Senior Life Settlements: Managing Interest Rate Risk With a Planned Duration Class, Journal of Financial Transformation, 23: 35-41.
46. Pitacco, E, M. Denuit, S. Haberman, and A. Olivieri, 2007, Modelling Longevity Dynamics for Pensions and Annuity Business. Oxford University Press.
47. Perera, N., and B. Reeves, 2006, Risk Mitigation for Life Settlements, Journal of Structured Finance, Summer 2006: 55-60.
48. Renshaw, A. E. and S. Haberman, 2003, Lee-Carter Mortality Forecasting with Age Specific Enhancement, Insurance: Mathematics and Economics, 33, 255-272.
49. Russ, J., 2005, Wie gut sind die Lebenserwartungsgutachten bei US-Policenfonds? World Wide Web: www.ifa-ulm.de (accessed October 4, 2010).
50. Schrager, D. F., 2006, Affine Stochastic Mortality, Insurance: Mathematics and

Economics, 38, 81-97.
51. Seitel, C. L., 2006, Inside the Life Settlement Industry: An Institutional Investor’s Perspective, Journal of Structured Finance 12(12): 38-40.
52. Smith, B. B., and S. L. Washington, 2006, Acquiring Life Insurance Portfolios: Diversifying and Minimizing Risk, Journal of Structured Finance, 12(12): 41-45.
53. Stallard, E., 2006, Demographic Issues in Longevity Risk Analysis, The Journal of Risk and Insurance, 73 (4), 575-609.
54. Stone, C. A. and A. Zissu, 2006, Securitization of Senior Life Settlements: Managing Extension Risk, The Journal of Derivatives, 13(3): 66-72.
55. Stone, C. A., and A. Zissu, 2007, The Return on a Pool of Senior Life Settlements, Journal of Structured Finance, 13(2): 62-69.
56. Stone, C. A. and A. Zissu, 2008, Using Life Extension-Duration and Life Extension-Convexity to Value Senior Life Settlement Contracts, The Journal of Alternative Investments, 11, 94-108.
57. Wang, J. L. and H. C. Huang et al., 2010, An Optimal Product Mix for Hedging Longevity Risk in Life Insurance Companies: The Immunization Theory Approach, Journal of Risk and Insurance, 77(2), 473-497.
58. Ziser, B., 2006, Life Settlements Today: A Secret No More, Journal of Structured Finance 7(2): 35-37.
59. Ziser, B., 2007, An Eventful Year in the Life Settlement Industry, Journal of Structured Finance 8(2): 1-4.
60. Zollars, D., S. Grossfeld, and D. Day, 2003, The Art of the Deal—Pricing Life Settlements, Contingencies, January/February: 34-38.

Table 1A: Optimal Hedge Ratios and Hedging Benefits in terms of Standard Deviations

b = 0.02, d = 0.02, N = 2000, under the Heligman-Pollard law

a	c	ac	h^*	σ_{wl}	σ_h	σ_h / σ_{wl}	η
0.1	0.1	0.01	0.024	9,491,567	9,365,670	0.987	0.051
0.1	0.5	0.05	0.131	22,655,941	21,030,499	0.928	0.243
0.1	0.9	0.09	0.239	40,773,507	37,780,622	0.927	0.248
0.5	0.1	0.05	0.026	9,285,807	8,528,297	0.918	0.271
0.5	0.5	0.25	0.135	23,222,054	12,488,106	0.538	0.831
0.5	0.9	0.45	0.252	41,664,659	21,824,656	0.524	0.841
0.9	0.1	0.09	0.016	9,292,551	8,421,576	0.906	0.303
0.9	0.5	0.45	0.083	22,892,218	10,446,745	0.456	0.884
0.9	0.9	0.81	0.155	42,061,265	15,781,899	0.375	0.924

Table 1B: Optimal Hedge Ratios and Hedging Benefits in terms of Value at Risk

b = 0.02, d = 0.02, N = 2000, under the Heligman-Pollard law

a	c	ac	h^*	VaR_{wl}	VaR_h	$\text{VaR}_h / \text{VaR}_{wl}$
0.1	0.1	0.01	0.012	16,834,276	16,538,419	0.982
0.1	0.5	0.05	0.150	43,031,904	37,535,019	0.872
0.1	0.9	0.09	0.434	88,908,259	76,268,407	0.858
0.5	0.1	0.05	0.035	16,026,572	14,305,153	0.893
0.5	0.5	0.25	0.162	42,304,751	21,953,491	0.519
0.5	0.9	0.45	0.310	92,298,997	38,029,541	0.412
0.9	0.1	0.09	0.013	16,458,316	14,551,552	0.884
0.9	0.5	0.45	0.092	43,166,030	16,623,541	0.385
0.9	0.9	0.81	0.168	88,318,614	22,074,876	0.250

Table 1C: Optimal Hedge Ratios and Hedging Benefits in terms of Expected Shortfall

b = 0.02, d = 0.02, N = 2000, under the Heligman-Pollard law

a	c	ac	h^*	ES_{wl}	ES_h	$\text{ES}_h / \text{ES}_{wl}$
0.1	0.1	0.01	0.027	21,974,677	21,806,067	0.992
0.1	0.5	0.05	0.203	56,528,290	50,789,457	0.898
0.1	0.9	0.09	0.572	120,473,504	103,585,158	0.860
0.5	0.1	0.05	0.033	21,323,188	19,165,686	0.899
0.5	0.5	0.25	0.174	59,155,619	28,768,671	0.486
0.5	0.9	0.45	0.317	118,142,091	49,285,888	0.417
0.9	0.1	0.09	0.021	21,673,765	19,000,109	0.877
0.9	0.5	0.45	0.102	58,237,610	23,697,101	0.407
0.9	0.9	0.81	0.195	121,545,823	30,133,862	0.248

Table 2A: Optimal Hedge Ratios and Hedging Benefits in terms of Standard Deviations

b = 0.03, d = 0.03, N = 2000, under the Heligman-Pollard law

a	c	ac	h^*	σ_{wl}	σ_h	σ_h / σ_{wl}	η
0.1	0.1	0.01	0.013	13,337,970	13,287,707	0.996	0.015
0.1	0.5	0.05	0.071	24,490,213	23,701,556	0.968	0.119
0.1	0.9	0.09	0.132	41,993,813	40,409,019	0.962	0.138
0.5	0.1	0.05	0.022	12,997,462	12,554,442	0.966	0.126
0.5	0.5	0.25	0.116	25,253,717	17,560,336	0.695	0.681
0.5	0.9	0.45	0.214	43,026,973	28,166,120	0.655	0.727
0.9	0.1	0.09	0.015	13,016,888	12,458,241	0.957	0.155
0.9	0.5	0.45	0.077	24,728,007	15,174,929	0.614	0.768
0.9	0.9	0.81	0.146	43,328,549	21,029,536	0.485	0.866

Table 2B: Optimal Hedge Ratios and Hedging Benefits in terms of Value at Risk

b = 0.03, d = 0.03, N = 2000, under the Heligman-Pollard law

a	c	ac	h^*	VaR_{wl}	VaR_h	$\text{VaR}_h / \text{VaR}_{wl}$
0.1	0.1	0.01	0.033	24,104,670	23,910,003	0.992
0.1	0.5	0.05	0.101	46,287,920	44,072,645	0.952
0.1	0.9	0.09	0.359	92,034,282	81,374,320	0.884
0.5	0.1	0.05	0.033	23,042,204	21,354,340	0.927
0.5	0.5	0.25	0.142	46,596,371	32,354,828	0.694
0.5	0.9	0.45	0.300	96,491,083	49,618,316	0.514
0.9	0.1	0.09	0.017	23,128,623	21,889,004	0.946
0.9	0.5	0.45	0.086	47,001,950	25,386,989	0.540
0.9	0.9	0.81	0.184	92,375,589	32,287,721	0.350

Table 2C: Optimal Hedge Ratios and Hedging Benefits in terms of Expected Shortfall

b = 0.03, d = 0.03, N = 2000, under the Heligman-Pollard law

a	c	ac	h^*	ES_{wl}	ES_h	$\text{ES}_h / \text{ES}_{wl}$
0.1	0.1	0.01	0.012	31,933,088	31,872,256	0.998
0.1	0.5	0.05	0.132	61,535,464	58,750,062	0.955
0.1	0.9	0.09	0.290	125,353,826	114,887,781	0.917
0.5	0.1	0.05	0.031	30,713,189	29,275,501	0.953
0.5	0.5	0.25	0.166	65,936,957	42,603,359	0.646
0.5	0.9	0.45	0.319	122,858,869	66,931,177	0.545
0.9	0.1	0.09	0.022	31,056,986	29,299,366	0.943
0.9	0.5	0.45	0.104	63,449,871	35,778,039	0.564
0.9	0.9	0.81	0.201	126,162,483	40,409,681	0.320

Table 3A: Optimal Hedge Ratios and Hedging Benefits in terms of Standard Deviations

b = 0.04, d = 0.04, N = 2000, under the Heligman-Pollard law

a	c	ac	h^*	σ_{wl}	σ_h	σ_h / σ_{wl}	η
0.1	0.1	0.01	0.009	17,411,501	17,385,931	0.999	0.006
0.1	0.5	0.05	0.042	26,892,418	26,474,979	0.984	0.060
0.1	0.9	0.09	0.081	43,709,872	42,784,859	0.979	0.080
0.5	0.1	0.05	0.018	16,925,589	16,647,991	0.984	0.063
0.5	0.5	0.25	0.097	27,910,440	22,465,114	0.805	0.521
0.5	0.9	0.45	0.175	44,982,724	34,141,421	0.759	0.595
0.9	0.1	0.09	0.014	16,988,150	16,600,617	0.977	0.086
0.9	0.5	0.45	0.070	27,198,922	19,953,676	0.734	0.632
0.9	0.9	0.81	0.135	45,060,944	26,496,281	0.588	0.791

Table 3B: Optimal Hedge Ratios and Hedging Benefits in terms of Value at Risk

b = 0.04, d = 0.04, N = 2000, under the Heligman-Pollard law

a	c	ac	h^*	VaR_{wl}	VaR_h	$\text{VaR}_h / \text{VaR}_{wl}$
0.1	0.1	0.01	0.012	30,876,865	30,735,758	0.995
0.1	0.5	0.05	0.086	54,244,338	50,258,852	0.927
0.1	0.9	0.09	0.197	95,577,715	90,298,066	0.945
0.5	0.1	0.05	0.028	30,299,720	30,374,993	1.002
0.5	0.5	0.25	0.113	51,012,367	41,141,023	0.806
0.5	0.9	0.45	0.254	99,466,110	59,903,653	0.602
0.9	0.1	0.09	0.007	32,165,310	31,980,403	0.994
0.9	0.5	0.45	0.109	53,642,866	37,292,131	0.695
0.9	0.9	0.81	0.203	106,362,084	39,980,267	0.376

Table 3C: Optimal Hedge Ratios and Hedging Benefits in terms of Expected Shortfall

b = 0.04, d = 0.04, N = 2000, under the Heligman-Pollard law

a	c	ac	h^*	ES_{wl}	ES_h	$\text{ES}_h / \text{ES}_{wl}$
0.1	0.1	0.01	0.008	41,737,680	41,706,679	0.999
0.1	0.5	0.05	0.078	70,589,500	68,722,708	0.974
0.1	0.9	0.09	0.150	128,439,304	125,621,929	0.978
0.5	0.1	0.05	0.023	39,811,901	39,263,418	0.986
0.5	0.5	0.25	0.142	69,894,647	54,188,033	0.775
0.5	0.9	0.45	0.290	133,322,152	81,064,137	0.608
0.9	0.1	0.09	0.015	43,276,689	42,552,021	0.983
0.9	0.5	0.45	0.107	70,984,451	48,389,161	0.682
0.9	0.9	0.81	0.203	137,275,963	55,112,338	0.401

Table 4A: Optimal Hedge Ratios and Hedging Benefits in terms of Standard Deviations

b = 0.02, d = 0.02, N = 2000, with Underwriter's Estimates of Life Expectancy and the SAA Table

a	c	ac	h^*	σ_{wl}	σ_h	σ_h / σ_{wl}	η
0.1	0.1	0.01	0.023	9,491,567	9,364,167	0.987	0.052
0.1	0.5	0.05	0.124	22,655,941	21,023,566	0.928	0.244
0.1	0.9	0.09	0.226	40,773,507	37,777,405	0.927	0.248
0.5	0.1	0.05	0.025	9,285,807	8,527,446	0.918	0.271
0.5	0.5	0.25	0.127	23,222,054	12,522,377	0.539	0.830
0.5	0.9	0.45	0.236	41,664,659	21,990,088	0.528	0.838
0.9	0.1	0.09	0.015	9,292,551	8,428,054	0.907	0.301
0.9	0.5	0.45	0.078	22,892,218	10,568,018	0.462	0.881
0.9	0.9	0.81	0.145	42,061,265	16,495,380	0.392	0.917

Table 4B: Optimal Hedge Ratios and Hedging Benefits in terms of Value at Risk

b = 0.02, d = 0.02, N = 2000, with Underwriter's Estimates of Life Expectancy and the SAA Table

a	c	ac	h^*	VaR_{wl}	VaR_h	$\text{VaR}_h / \text{VaR}_{wl}$
0.1	0.1	0.01	0.011	16,834,276	16,521,231	0.981
0.1	0.5	0.05	0.142	43,031,904	37,445,347	0.870
0.1	0.9	0.09	0.376	88,908,259	76,233,419	0.857
0.5	0.1	0.05	0.034	16,026,572	14,253,960	0.889
0.5	0.5	0.25	0.156	42,304,751	22,210,484	0.525
0.5	0.9	0.45	0.277	92,298,997	38,303,308	0.415
0.9	0.1	0.09	0.020	16,458,316	14,598,226	0.887
0.9	0.5	0.45	0.087	43,166,030	16,397,837	0.380
0.9	0.9	0.81	0.158	88,318,614	22,265,314	0.252

Table 4C: Optimal Hedge Ratios and Hedging Benefits in terms of Expected Shortfall

b = 0.02, d = 0.02, N = 2000, with Underwriter's Estimates of Life Expectancy and the SAA Table

a	c	ac	h^*	ES_{wl}	ES_h	$\text{ES}_h / \text{ES}_{wl}$
0.1	0.1	0.01	0.025	21,974,677	21,796,917	0.992
0.1	0.5	0.05	0.188	56,528,290	50,775,187	0.898
0.1	0.9	0.09	0.540	120,473,504	103,530,147	0.859
0.5	0.1	0.05	0.031	21,323,188	19,158,280	0.898
0.5	0.5	0.25	0.163	59,155,619	28,871,046	0.488
0.5	0.9	0.45	0.298	118,142,091	49,659,537	0.420
0.9	0.1	0.09	0.020	21,673,765	19,009,151	0.877
0.9	0.5	0.45	0.098	58,237,610	23,943,215	0.411
0.9	0.9	0.81	0.187	121,545,823	32,481,890	0.267

Table 5A: Optimal Hedge Ratios and Hedging Benefits in terms of Standard Deviations

b = 0.03, d = 0.03, N = 2000, with Underwriter's Estimates of Life Expectancy and the SAA Table

a	c	ac	h^*	σ_{wl}	σ_h	σ_h / σ_{wl}	η
0.1	0.1	0.01	0.013	13,337,970	13,286,822	0.996	0.015
0.1	0.5	0.05	0.067	24,490,213	23,698,650	0.968	0.120
0.1	0.9	0.09	0.125	41,993,813	40,405,605	0.962	0.138
0.5	0.1	0.05	0.021	12,997,462	12,554,581	0.966	0.126
0.5	0.5	0.25	0.109	25,253,717	17,578,158	0.696	0.680
0.5	0.9	0.45	0.201	43,026,973	28,255,818	0.657	0.725
0.9	0.1	0.09	0.014	13,016,888	12,465,322	0.958	0.153
0.9	0.5	0.45	0.072	24,728,007	15,254,893	0.617	0.765
0.9	0.9	0.81	0.137	43,328,549	21,557,247	0.498	0.859

Table 5B: Optimal Hedge Ratios and Hedging Benefits in terms of Value at Risk

b = 0.03, d = 0.03, N = 2000, with Underwriter's Estimates of Life Expectancy and the SAA Table

a	c	ac	h^*	VaR_{wl}	VaR_h	$\text{VaR}_h / \text{VaR}_{wl}$
0.1	0.1	0.01	0.031	24,104,670	23,857,515	0.990
0.1	0.5	0.05	0.096	46,287,920	44,129,064	0.953
0.1	0.9	0.09	0.346	92,034,282	81,184,912	0.882
0.5	0.1	0.05	0.032	23,042,204	21,345,872	0.926
0.5	0.5	0.25	0.131	46,596,371	32,376,265	0.695
0.5	0.9	0.45	0.265	96,491,083	49,747,063	0.516
0.9	0.1	0.09	0.015	23,128,623	21,836,475	0.944
0.9	0.5	0.45	0.083	47,001,950	25,288,540	0.538
0.9	0.9	0.81	0.168	92,375,589	32,973,732	0.357

Table 5C: Optimal Hedge Ratios and Hedging Benefits in terms of Expected Shortfall

b = 0.03, d = 0.03, N = 2000, with Underwriter's Estimates of Life Expectancy and the SAA Table

a	c	ac	h^*	ES_{wl}	ES_h	$\text{ES}_h / \text{ES}_{wl}$
0.1	0.1	0.01	0.011	31,933,088	31,867,409	0.998
0.1	0.5	0.05	0.125	61,535,464	58,737,931	0.955
0.1	0.9	0.09	0.266	125,353,826	114,873,684	0.916
0.5	0.1	0.05	0.030	30,713,189	29,276,709	0.953
0.5	0.5	0.25	0.156	65,936,957	42,669,256	0.647
0.5	0.9	0.45	0.300	122,858,869	66,998,956	0.545
0.9	0.1	0.09	0.021	31,056,986	29,324,438	0.944
0.9	0.5	0.45	0.099	63,449,871	35,869,777	0.565
0.9	0.9	0.81	0.192	126,162,483	41,578,567	0.330

Table 6A: Optimal Hedge Ratios and Hedging Benefits in terms of Standard Deviations

b = 0.04, d = 0.04, N = 2000, with Underwriter's Estimates of Life Expectancy and the SAA Table

a	c	ac	h^*	σ_{wl}	σ_h	σ_h / σ_{wl}	η
0.1	0.1	0.01	0.008	17,411,501	17,385,382	0.998	0.006
0.1	0.5	0.05	0.040	26,892,418	26,473,122	0.984	0.060
0.1	0.9	0.09	0.077	43,709,872	42,783,753	0.979	0.080
0.5	0.1	0.05	0.017	16,925,589	16,648,425	0.984	0.063
0.5	0.5	0.25	0.091	27,910,440	22,472,807	0.805	0.520
0.5	0.9	0.45	0.165	44,982,724	34,184,785	0.760	0.594
0.9	0.1	0.09	0.013	16,988,150	16,607,942	0.978	0.085
0.9	0.5	0.45	0.066	27,198,922	20,012,991	0.736	0.629
0.9	0.9	0.81	0.127	45,060,944	26,902,339	0.597	0.783

Table 6B: Optimal Hedge Ratios and Hedging Benefits in terms of Value at Risk

b = 0.04, d = 0.04, N = 2000, with Underwriter's Estimates of Life Expectancy and the SAA Table

a	c	ac	h^*	VaR_{wl}	VaR_h	$\text{VaR}_h / \text{VaR}_{wl}$
0.1	0.1	0.01	0.011	31,912,384	31,508,195	0.987
0.1	0.5	0.05	0.046	50,757,743	49,312,193	0.972
0.1	0.9	0.09	0.185	94,956,887	90,181,225	0.950
0.5	0.1	0.05	0.036	30,310,700	29,009,669	0.957
0.5	0.5	0.25	0.101	52,902,857	42,072,261	0.795
0.5	0.9	0.45	0.275	100,966,825	62,153,192	0.616
0.9	0.1	0.09	0.010	30,969,877	29,799,677	0.962
0.9	0.5	0.45	0.090	51,800,011	34,144,710	0.659
0.9	0.9	0.81	0.196	96,639,360	42,985,209	0.445

Table 6C: Optimal Hedge Ratios and Hedging Benefits in terms of Expected Shortfall

b = 0.04, d = 0.04, N = 2000, with Underwriter's Estimates of Life Expectancy and the SAA Table

a	c	ac	h^*	ES_{wl}	ES_h	$\text{ES}_h / \text{ES}_{wl}$
0.1	0.1	0.01	0.002	42,870,599	42,866,298	1.000
0.1	0.5	0.05	0.057	68,795,413	67,471,660	0.981
0.1	0.9	0.09	0.203	131,972,957	123,934,548	0.939
0.5	0.1	0.05	0.026	41,216,863	40,302,616	0.978
0.5	0.5	0.25	0.138	74,374,046	57,041,738	0.767
0.5	0.9	0.45	0.294	130,954,790	86,773,214	0.663
0.9	0.1	0.09	0.019	41,899,626	40,648,703	0.970
0.9	0.5	0.45	0.096	70,778,366	48,946,365	0.692
0.9	0.9	0.81	0.196	132,390,722	54,522,029	0.412