Solvency II Solvency Capital Requirement for life insurance companies based on Expected Shortfall

Tim J. Boonen*
University of Amsterdam

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Abstract

This paper examines the consequences for a life annuity insurance company if the Solvency II Solvency Capital Requirements (SCR) are calibrated based on Expected Shortfall (ES) instead of Value-at-Risk (VaR). We focus on the risk modules of the SCRs for the three risk classes equity risk, interest rate risk and longevity risk. The stress scenarios are determined as proposed by EIOPA in 2014. We apply the stress-scenarios for these three risk classes to a fictitious life annuity insurance company. We find that for EIOPA's current quantile 99.5% of the Value-at-Risk, the stress scenarios of the various risk classes based on Expected Shortfall are close to the stress scenarios based on Value-at-Risk. Might EIOPA choose to calibrate the stress scenarios on a smaller quantile, the longevity SCR is relatively larger and the equity SCR is relatively smaller if Expected Shortfall is used instead of Value-at-Risk. We derive the same conclusion if stress scenarios are determined with empirical stress scenarios.

Keywords: Solvency II; Solvency Capital Requirement; Expected Shortfall; Value-at-Risk

1 Introduction

Solvency II is a new supervisory framework that will be in force from 2016 for insurers and reinsurers in Europe. It puts demands on the required economic capital, risk management and reporting standards of insurance companies. Solvency II focuses on an enterprize risk management approach towards required capital standards. Its main objective is to ensure that insurance companies hold sufficient economic capital to protect the policyholder as it aims to reduce the risk that an insurance company is unable to meet its financial claims (see, e.g., Doff, 2008). The capital is adjusted to the risks that the insurance company incurs. In this way, Solvency II describes the quantitative requirements. This capital requirement is called the Solvency Capital Requirement (SCR) and covers all the risks that an insurer faces. EIOPA (2014a) defines the SCR of an insurance or reinsurance company as the Value-at-Risk (VaR) of the basic own funds subject to a confidence level of 99.5% over a one-year period. In this paper, we base our model on the proposals for Solvency II of EIOPA (2014a).

The Value-at-Risk is a widely used risk measure and can be described as the maximum loss within a certain confidence level. In the case of the SCR, the confidence level of 99.5% tells us that the company can expect to lose no more than VaR in the next year with 99.5% confidence,

*Email address: t.j.boonen@uva.nl, Amsterdam School of Economics, University of Amsterdam, Roetersstraat 11, 1018 WB, Amsterdam, The Netherlands

so, on average, only once every 200 years the VaR loss level will be exceeded. The Value-at-Risk is however being criticized for not being sub-additive (Artzner et al., 1999). Sub-additivity of a risk measure ensures that diversification is rewarded. Besides not being sub-additive, the VaR does also not consider the shape of the tail beyond the confidence level. This means that the VaR does not take into account what happens beyond the confidence level, so it does not consider the worst case scenarios. This is discussed by Acerbi and Tasche (2002) and Yamai and Yoshiba (2005). The fact that VaR is not sub-additive and that it does not consider the tail beyond the confidence level might make it not very suitable to use for a capital requirement calculation.

There are alternative risk measures for VaR which are sub-additive and consider the shape of the tail beyond the confidence level, these risk measurers are called coherent risk measurers. Coherent risk measurers are defined by Artzner et al. (1999) as the risk measures satisfying the four axioms of translation invariance, sub-additivity, positive homogeneity, and monotonicity. The most popular coherent risk measure is the Expected Shortfall (ES). This risk measure is equal to the expected value of the loss, given that the loss is larger than the Value-at-Risk, therefore the Expected Shortfall also depends on the quantile used. Two other regulatory frameworks for financial institutions, the Swiss Solvency Test and the Basel III framework, both use the Expected Shortfall as risk measure.

The contribution of this paper is not to claim that the Expected Shortfall is a better risk measure than the Value-at-Risk. Instead, we exemplary analyze the effects on three main risk factors of the total SCR for a life annuity insurance company if the Solvency II SCR estimation is based on Expected Shortfall instead of Value-at-Risk. The use of Expected Shortfall for insurance stress testing is also suggested by CEIOPS (2006), Sandström (2007), and Wagner (2014). We hereby consider the interaction of the change in regulation for three major risk classes, namely equity risk, interest rate risk, and longevity risk. Moreover, we analyze what would happen if all SCRs are determined via empirical stress scenarios.

The standard model of Solvency II explicitly assumes a Gaussian distribution for some risk classes. If a risk is Gaussian, the Value-at-Risk, as prescribed by Solvency II, and Expected Shortfall provide similar information (e.g., Yamai and Yoshiba, 2005). If risks are non-Gaussian, which is observed for the most important classes of risk, the use of Value-at-Risk might lead to a mismatch of the SCR and the underlying riskiness. In this paper, we examine what the effects are of using such a standard model when risks are calibrated using the Expected Shortfall instead. We find that for EIOPA's current quantile 99.5% of the Value-at-Risk, the stress scenarios of the various risk classes based on Expected Shortfall are close to the stress scenarios based on Value-at-Risk. We show that if EIOPA aims to keep the solvency capital requirement the same if it would switch to Expected Shortfall, it should set the confidence level at approximately 98.8%. So, applying a 99% confidence level for the Expected Shortfall leads to an increase in the solvency capital requirement. Moreover, might EIOPA choose to calibrate the stress scenarios on a smaller quantile, the longevity SCR is relatively larger and the equity SCR is relatively smaller if Expected Shortfall is used instead of Value-at-Risk.

2 Risk measures to estimate solvency capital

Risk measures are used to estimate the amount of sufficient economic capital to be kept in reserve in order to protect a company for any negative risky impacts that may arise in the future. A risk measure is a function $\rho : L^\infty \to \mathbb{R}$, where L^∞ is the class of bounded random variables. In this paper we discuss two well known risk measures: the Value-at-Risk (VaR) and the Expected Shortfall (ES).

The Value-at-Risk can be described as a bad-case realization of a risk within a certain confidence level. The VaR is the α-quantile, i.e.,

$$VaR_\alpha(X) = \inf(x \in \mathbb{R} \colon \mathbb{P}(X < x) \geq 1 - \alpha), \tag{1}$$

for all $\alpha \in (0,1]$ and all $X \in L^\infty$.

Coherence of risk measurers is introduced by Artzner et al. (1999). A risk measures ρ is coherent if it satisfies the four axioms translation invariance, sub-additivity, positive homogeneity, and monotonicity. The relevance of these properties is widely discussed by Artzner et al. (1999). Particularly, sub-additivity of a risk measure implies that the risk measure weakly decreases if risks are pooled. It also implies that there is no incentive for a company to split its risk into pieces and evaluate them separately.

The Expected Shortfall (ES) is given by

$$ES_\alpha(X) = \frac{1}{\alpha} \int_0^\alpha VaR_\tau(X) d\tau, \tag{2}$$

for all $\alpha \in (0,1]$ and all $X \in L^\infty$. If X is continuously distributed, the Expected Shortfall may be even more intuitively expressed as conditional VaR or the tail conditional expectation:

$$ES_\alpha(X) = E(X \mid X \leq VaR_\alpha(X)). \tag{3}$$

For numerical convenience, the expression in (3) is often used in historical simulations. For the same reason, we use (3) as definition of Expected Shortfall as well.

CEIOPS (2006) acknowledges the theoretical advantages of using the Expected Shortfall to calculate the SCR. In the current literature, the is an increasing support for the Expected Shortfall (see, e.g., Acerbi and Tasche, 2002, Tasche, 2002; Frey and McNeil, 2002; Yamai and Yoshiba, 2005).

When risk is Gaussian, the same information is given by the Value-at-Risk and Expected Shortfall. In this case, the Value-at-Risk and Expected Shortfall are multiples of the standard deviation. For example, VaR at 99.5% confidence level is 2.58 times the standard deviation, while Expected Shortfall at the same confidence level is 2.89 times the standard deviation. The assumption that financial risk is Gaussian is often criticized and it is said that the it does lead to an underestimation of the risk being faced (see, e.g., Sandström, 2007). It is a clear observation (e.g., Cont, 2001) that asset returns are fat-tailed and asymmetric and, therefore, not Gaussian.

3 Solvency II

In this section, we describe the Solvency II regulations. First, we introduce this regulatory framework in Subsection 3.1. In Subsection 3.2 we specify all formulas of the Solvency Capital Requirement (SCR) according to EIOPA (2014a) for the relevant risk classes.

3.1 Introduction

Solvency II is a new regulatory framework that is expected to be in force in 2016 for the European insurance industry and puts demands on the required economic capital, risk management and reporting standards of insurance companies. The underlying quantitative regulation mechanism is that insurers should hold an amount of capital that enables them to absorb unexpected losses and meet the obligations towards policy-holders at a high level of equitableness. In the European

Union, there are on-going discussions about applying such a framework to all European pension funds as well (EIOPA, 2012). For instance, see Doff (2008), Eling et al. (2007), Sandström (2007), Steffen (2008) and Wagner (2014) for an overview and critical discussion of the Solvency II framework.

This paper focusses on the first pillar of the Solvency II framework. This pillar prescribes the quantitative requirements which an insurer must meet. It contains the SCR, which is to be fulfilled by using the standard formula or an internal model. These quantitative requirements are supported by the so-called Quantitative Impact Studies (QIS).

EIOPA (2014a) describes how the assets and liabilities of an insurance company should be valued. Assets should be valued at the amount for which they could be exchanged between knowledgeable willing parties in an arm's length transaction. Liabilities should be valued at the amount for which they could be transferred, or settled, between parties in an arm's length transaction. Valuing assets on a market-consistent basis is generally straightforward, whereas valuing liabilities is generally more complicated. Generally, perfect replication of expected cash flows is not possible for the liabilities of an insurance company. In that case, the Quantitative Impact Study 5 (QIS 5) prescribes to use the best estimate and the risk margin to value the liabilities. The Best Estimate of the Liabilities (BEL) corresponds to the present value of the expected future liability payments. The risk margin is calculated by determining the cost of holding an amount of own funds equal to the SCR over the lifetime of the insurance obligation. To determine the cost of holding that amount of own funds a so-called cost-of-capital rate is used. The sum of the BEL and the risk margin is called the technical provisions.

There are three ways to estimate the SCR: by using an internal model, by using the standard formula or by using a combination of the two. EIOPA (2014a) prescribes that the SCR should correspond to the Value-at-Risk of the basic own funds of an insurance company subject to a confidence level of 99.5% over a one-year period. Many companies use the standard formula. The principle of the standard formula is to apply a set of shocks to certain risk drivers and calculate the impact on the value of the assets and liabilities for various risks. These shocks are calibrated using the VaR with a confidence level of 99.5%. Predefined correlation matrices are used to aggregate to total SCR for all risks together. The standard formula of the SCR is divided into the modules shown in Figure 1.

In this paper we do not take into account the SCR for operational risk and the adjustment for the risk absorbing effect of technical provisions and deferred taxes, therefore BSCR (Basic SCR, see Figure 1) equals SCR. Moreover, we only consider equity risk, interest rate risk and longevity risk. In Figure 2 we provide an overview of our *reduced* total SCR calculation and its different risk modules.

Market risk and life risk account together for approximately 91.1% of the BSCR for life insurance companies (EIOPA, 2011). Moreover, EIOPA (2013) shows that the market risk predominantly consists of equity and interest rate risk.

Market risk arises from the volatility of market prices of financial instruments. Exposure to market risk is measured by the impact of movements in the level of financial variables such as equity prices, interest rates, real estate prices and exchange rates. Market risk is the largest component of the SCR and for life insurance companies market risk accounts for approximately 67.4% of the diversified Basic SCR (EIOPA, 2011). Equity risk arises from the volatility of market prices for equities. Exposure to equity risk refers to all assets and liabilities whose value is sensitive to changes in equity prices. Interest rate risk exists for all assets and liabilities for which the net asset value is sensitive to changes in the term structure of interest rates.

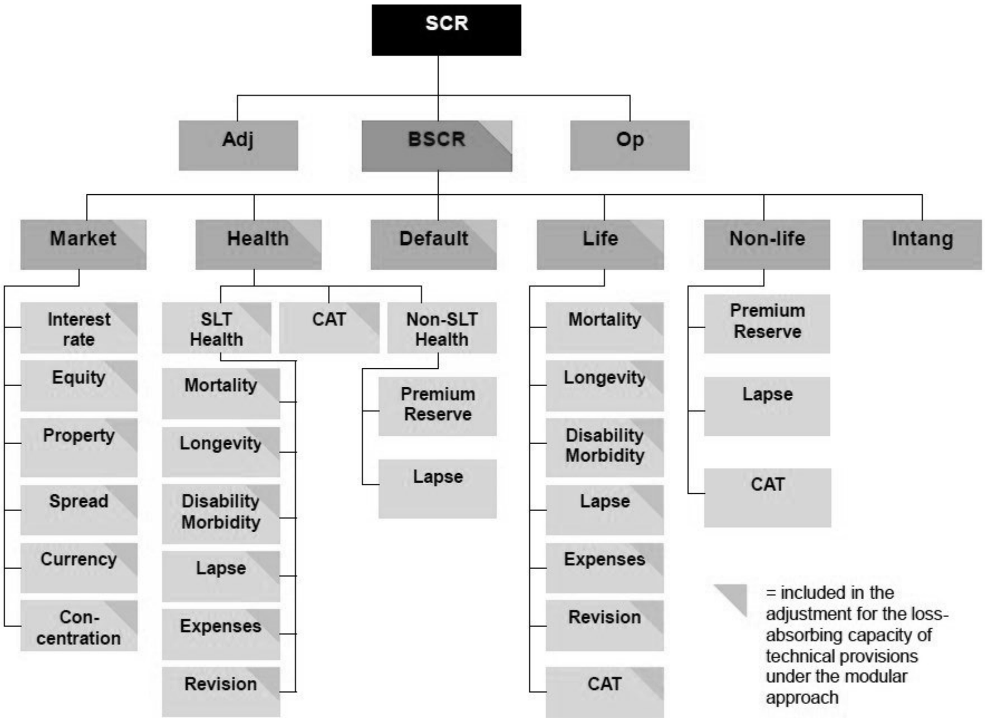

Figure 1: The different modules of the standard formula SCR (EIOPA, 2014a). Here, Adj is the adjustment for the risk absorbing effect of technical provisions and deferred taxes, and Op is the operational risk.

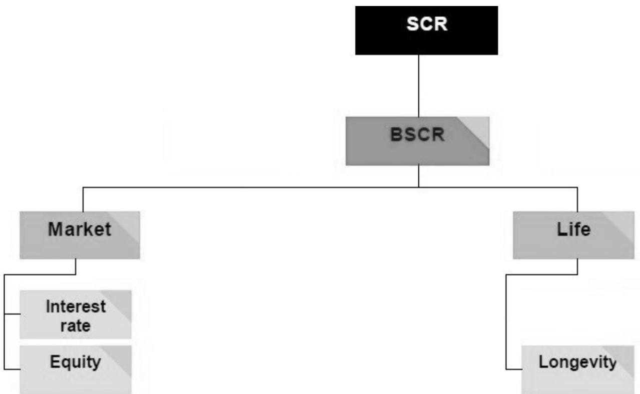

Figure 2: The different risk modules that we calibrate in this paper for the reduced total SCR.

Life risk covers the risk arising from the underwriting of life insurance, associated with both the perils covered and the processes followed in the conduct of the business. Life risk is the second largest risk class and it accounts for approximately 23.7% of the diversified BSCR for life insurance companies (EIOPA, 2011). The most important components for a life annuity insurance company are longevity risk and lapse risk. Longevity risk covers approximately 44% of the life SCR (EIOPA, 2011), and is associated with insurance obligations where an insurance company guarantees to make recurring series of payments until the death of the policyholder and where a decrease in mortality rates leads to an increase in the technical provisions. Longevity risk is associated to higher than expected pay-outs because of increasing life expectancy. In this paper, we ignore lapse risk for simplicity.

3.2 Calculation of the SCR

In this subsection, we describe the Solvency II regulations for calculating the reduced total SCR. The market SCR is a combination of the different market risks, in this case equity risk and interest rate risk. The market SCR, denoted by SCR_{mkt}, is defined as follows:

$$SCR_{mkt} = \sqrt{\sum_{(i,j)\in\{eq,int\}^2} CorrMkt_{i,j} \cdot Mkt_i \cdot Mkt_j}, \tag{4}$$

where index eq denotes equity risk, index int denotes interest rate risk, Mkt_i is the solvency capital requirement for the individual market risk $i \in \{eq, int\}$, and the predefined correlation matrix $CorrMkt$ is given by:

CorrMkt	Equity	Interest
Equity	1	A
Interest	A	1

where $A = 0$ if the interest rate shocks are determined via the interest rate up scenario, and $A = 0.5$ otherwise. We clarify the up and down shocks in interest rates later in (7)-(9).

EIOPA (2014a) provides a standard model where the "Global" equity shock is -39% and the "Other" equity shock is -49%. There is a symmetric adjustment applied by EIOPA (2014a) that corrects for pro-cyclicality in equity returns (see Eling and Pankoke, 2013).[1]

The Basic Own Funds (BOF, net value of assets minus liabilities) is defined by CEIOPS (2010) as the asset value minus the Best Estimate of the Liabilities (BEL).[2] The solvency capital requirement for equity risk is determined by the decrease of the BOF after a negative shock has been given to the equity. This negative shock implies that the value of equity decreases with a certain percentage. The shock differs for "Global" equity and "Other" equity. "Global" equity includes equities listed in EEA or OECD countries. The "Other" equities include equities listed in other than EEA or OECD countries, hedge funds, private equities and other alternative investments. For each category $i \in \{$"Global", "Other"$\}$ this yields:

$$Mkt_{eq,i} = \max\{\triangle BOF \,|equity\ shock_i, 0\}, \tag{5}$$

where $\triangle BOF$ is the BOF before the equity shock minus the BOF after the equity shock. The equity SCR, denoted by Mkt_{eq}, is given by

$$Mkt_{eq} = \sqrt{\sum_{(i,j)\in\{\text{“Global”},\text{“Other”}\}^2} CorrEQ_{i,j} \cdot Mkt_{eq,i} \cdot Mkt_{eq,j}}, \tag{6}$$

where the predefined correlation matrix $CorrEQ$ is given by

[1] On December 31st, 2013, the symmetric adjustment equals +7.5%, leading to the "Global" equity shock of -46.5% and the "Other" equity shock of -56.5% (see EIOPA, 2014a).

[2] Adding a risk margin (based on the total SCR) would lead to a circularity argument. This is noted by Coppola and D'Amato (2014).

CorrEQ	"Global"	"Other"
"Global"	1	0.75
"Other"	0.75	1

To estimate the solvency capital requirement for interest rate risk, an upward and a downward shock are given to the interest term structure. The altered term structures are derived by $\max\{(1+\mathrm{s}^{up})\cdot r_m, r_m+1\%\}$ or $(1+s^{down})\cdot r_m$, where r_m is the current interest rate with maturity m, and s^{up} and s^{down} are the up- and down-shocks to the interest rates. So, in addition to the calibration of the relative stress factor, a minimum shock of 1% is applied for the interest rate in the upward scenario (EIOPA, 2014b).[3] Using an alternative term structure results in a change of the value of the assets and liabilities. The solvency capital requirements for the downward and upward shock are determined by the changes in the basic own funds if the shocked interest rate curve is used instead of the nominal term structure. The interest rate shock is the worst of the up and down shock. This leads to the following definitions:

$$Mkt_{up,int} = \triangle BOF\,|_{up}, \tag{7}$$

$$Mkt_{down,int} = \triangle BOF\,|_{down}, \tag{8}$$

$$Mkt_{int} = \max\{Mkt_{up,int}, Mkt_{down,int}\}. \tag{9}$$

In this paper, we are only considering the module longevity risk as life risk, i.e., life SCR equals longevity SCR. The longevity SCR is estimated by the change in net value of assets minus liabilities after a permanent percentage decrease in mortality rates for all ages. This decrease resembles the risk that people live longer than expected and this leads to an increase in the present value of the liabilities from annuity products. This shock therefore leads to a decrease in the value of *BOF*. The definition of longevity SCR is given by:

$$SCR_{life} = Life_{long} = \triangle BOF\,|longevity\ shock. \tag{10}$$

EIOPA (2014a) provides a standard model where all mortality rates are reduced by 20%.

The market risk and life risk SCRs are combined to estimate the reduced total SCR. The reduced total SCR is calculated by the following formula:

$$SCR = \sqrt{\sum_{(i,j)\in\{mkt,life\}^2} CorrTot_{i,j}\cdot SCR_i\cdot SCR_j}, \tag{11}$$

where SCR_i is the solvency capital requirement for the individual risk class $i\in\{mkt, life\}$, and the predefined correlation matrix *CorrTot* is given by:

CorrTot	Market	Life
Market	1	0.25
Life	0.25	1

The structure of the solvency capital requirements in Solvency II is partially derived from a formula developed by the German Insurance Association (see, e.g., Eling et al., 2007; Schubert

[3] This was different in EIOPA (2010), where the upward shock was given by $(1+\mathrm{s}^{up})\cdot r_m$, and the downward shock by $\max\{\min\{(1+s^{down})\cdot r_m, r_m-1\%\}, 0\}$.

and Griessmann, 2007). The structure of a square-root formula as in (4), (6) and (11) is derived from the assumption that the individual SCRs are Gaussian and the dependence is linear. In general, linear dependence is sufficient to describe dependencies between elliptical distributions. However, there arise problems with this formula if the individual risks are not Gaussian, or dependencies are non-linear. For instance, skewness or excess kurtosis of the marginal distributions may lead to very irregular outcomes (see Sandström, 2007). Moreover, even if the different risk classes are Gaussian, their influence on the aggregate loss will in general not be determined via a square-root formula (see Bauer et al., 2010). Non-linear dependence structures may yield situations where the square-root formula severely underestimates the total SCR in (11) (see Pfeifer and Strassburger, 2008). In this paper, we do not analyze alternative risk aggregation methods.

4 Methodology

In this section, we calibrate the SCR stress scenarios for the three risk classes based on Value-at-Risk and Expected Shortfall following the calibration methods used in Solvency II. We compare the estimated SCR calibrated on Value-at-Risk with the estimated SCR calibrated on Expected Shortfall for a fictitious life annuity insurance company. To show the impact of the stress scenarios on this fictitious life annuity insurance company, we first define this company in Subsection 4.1. In Subsection 4.2, we define the method we use to compare the stress scenarios based on VaR with the stress scenarios based on ES. In Subsection 4.3, we show the stress scenarios for all three risk classes.

4.1 Calibrating the fictitious life annuity insurance company

The liability portfolio of the fictitious insurance company consists of life annuity products only. The portfolio is normalized such that it consists of 100,000 male policyholders. The policyholders in this portfolio have an average age of 50 years, the youngest group is 21 years old and the oldest group has an age of 79 years. In Table 6 in Appendix A, we provide a precise overview of the liability portfolio. We assume that all policyholders are born on January 1st.

The single-life annuity will be paid at the end of every year starting from age 65 if alive. For the youngest age cohort 21, this amount equals 0.067. This annuity amount increases linearly over the age cohorts up to 1 for age cohort 65 and above. We hereby use age 65 as a retirement age, where the retirement benefits are normalized to 1 for every individual in retirement. We refer to Hári et al. (2008) for an overview of the actuarial valuation of the annuity liabilities.

We assume that the fictitious insurance company has a funding ratio of 100%. The asset portfolio consists for 25% out of "Global" equity and for 75% out of government bonds. We pick the 5-year and 30-year UK Gilt as bond mix, which are bonds issued by the British government, and has a AAA credit rating. The 5-year UK Gilt is a 2.13% coupon bond with a remaining time to maturity of 5 years and the 30-year UK Gilt is a 3.45% coupon bond with a remaining time to maturity of 30 years. The coupon rates are based on the market data of July 2nd, 2014. We determine the bond portfolio as follows. The duration of the bonds equals 50% of the duration of the liabilities. Since 75% of the assets is invested in bonds, this coincides with a duration of the assets of $75\% \cdot 50\% = 37.5\%$ of the duration of the liabilities. Via this duration, we determine the amount invested in the 5-year and 30-year UK Gilts. For some both the age characteristics of the portfolio as the composition of the asset portfolio, we will later perform some robustness checks.

To discount cash flows, we use the nominal interest rate term structure of the European central bank and a Smith-Wilson extrapolation (2001) towards an Ultimate Forward Rate (UFR) for interest rates after 20 years (see Figure 8 in Appendix A).[4] We set the present at December 31st, 2014.

The appropriate life tables that are to be used are determined by the regulator. We use the mortality table of the Dutch Actuarial Society: AG 2012-2062,[5] developed in 2012, which contains a longevity trend. Whereas we know that longevity risk and investment risk are heavily correlated (see, e.g., Stevens et al., 2010), the modular approach in Solvency II does not include any correlation factor.

4.2 Matching the total SCR value

In this paper, we calculate the SCR based on VaR (resp. ES) stress scenarios, which we denote by SCR(VaR_α) (resp. SCR(ES_θ)), for different quantiles α and θ. We focus on the relative impact on the SCR for the three risk classes. Therefore, to compare the SCR(VaR_α) properly with the SCR(ES_θ), the confidence level for the ES, θ, is chosen based on matching the reduced total SCR value, i.e.,

$$SCR(VaR_\alpha) = SCR(ES_\theta). \tag{12}$$

In this way, we define the function $\theta : [95\%, 100\%] \to [0, 1]$ as a strictly increasing function such that (12) is satisfied for all $\alpha \in [95\%, 100\%]$. For instance, we find that $\theta(99.5\%) \approx 98.78\%$ and $\theta(98.5\%) \approx 97.00\%$ if we calibrate the SCR risk scenarios as we will specify in Subsection 4.3. For the fictitious life annuity insurance company in the base case, we display the function θ in Figure 3. We get that the function θ is not affine, which might be due to heavy tails in some risks.[6]

Using the function θ in (12), we compare the stress scenarios for equity risk, interest rate risk and longevity risk. Those three scenarios lead to the reduced total SCR as described in Subsection 3.2. We define the vector of the SCRs for all three risk classes as an allocation.

4.3 Calibrating the SCR stress scenarios

In this subsection, we calibrate stress scenarios for equity risk, interest rate risk, and longevity risk based on the Value-at-Risk and the Expected Shortfall. The derived stress scenarios based on the Value-at-Risk with quantile 99.5% are not identical to the stress scenarios as given by EIOPA (2014a). The data we use in this paper is similar to the data used of CEIOPS (2010) for the QIS 5 stress scenarios calibration, except that we modify the horizons.

The calibration method for equity risk is discussed by CEIOPS (2010). The distribution of annual holding period returns is derived from the Morgan Stanley Capital International (MSCI) World Developed (Market) Price Equity Index. This index is used to calculate the empirical VaR and empirical ES. The data from Bloomberg spans a daily period of 41 years, starting in June 1973 until June 2014. The empirical VaR and empirical ES serve as the stress rate for "Global" equity. We follow the technical specifications of EIOPA (2014a) with an adjustment of 7.5% to decrease the equity risk shock. With this symmetric adjustment included, the stress rates for equity risk are provided in Table 1. If $\alpha = 99.5\%$, the differences between using VaR and ES are small. When the quantile $\alpha = 98.5\%$ is chosen, the use of VaR leads to a fiercer equity risk shock.

[4] See http://www.toezicht.dnb.nl/en/binaries/51-226788.pdf (last accessed: March 5th, 2015) or CEIOPS (2010) for more information on the procedure to calibrate the interest rate term structure.

[5] For the detailed mortality table that we use, see http://www.ag-ai.nl/download/14127-Prognosetafel+AG2012-2062.xls (last accessed: March 5th, 2015).

[6] If all risks are Gaussian, we would have $\theta(\alpha) \approx 1 - 2.58 \cdot (1 - \alpha)$ for all $\alpha \in [95\%, 100\%]$ and, hence, $\theta(99.5\%) \approx 98.71\%$.

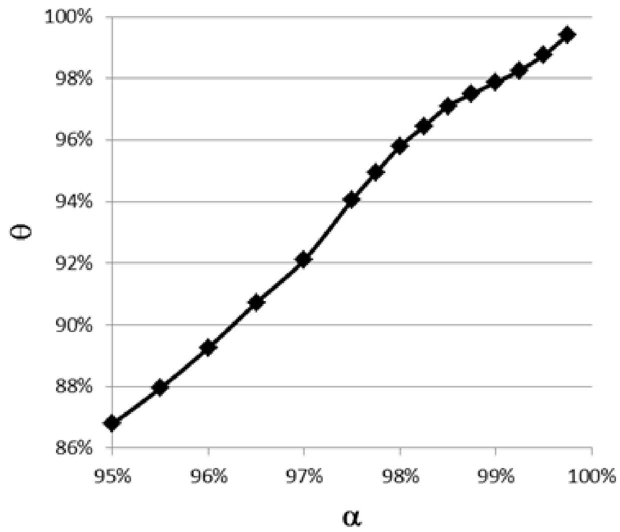

Figure 3: The function θ as defined in Subsection 4.2.

α	VaR_α	$ES_{\theta(\alpha)}$
99.5%	-51.66%	-51.69%
98.5%	-46.75%	-45.22%

Table 1: Stress rates for "Global" equity risk derived from VaR and ES.

For the calibration method for interest rate risk, we use the following four datasets as also used by CEIOPS (2010) and EIOPA (2014b), but with shifted horizons in order to include present data:

- Euro area government bond yield curve, with maturities from 1 year to 15 years, spaced out in annual intervals. The daily data spans a period of approximately 10 years and runs from September 2004 to July 2014. The data is from the European Central Bank;[7]
- the UK government liability curve. The data is daily and is from the Bank of England.[8] The data covers a period from January 1998 to June 2014, so that the longer maturities (i.e., beyond 15 years) are all available. It contains rates of maturities starting from 1 year up until 20 year whilst the in between data points are spaced on annual intervals;
- Euro vs Euribor IR swap rates. The daily data is downloaded from Datastream and covers a period from 1999 to 2014. The data contains the 1 to 10 year rates spaced out in one year intervals, as well as the 12 year, 15 year and 20 year rates;
- UK (GBP) 6m IRS swap rates. The daily data is downloaded from Datastream and covers a period from 1999 to 2014. The data contains the 1 to 10 year rates spaced out in one year intervals, as well as the 12 year, 15 year and 20 year rates.

[7]See https://www.ecb.europa.eu/stats/money/yc/html/index.en.html (last accessed: March 5th, 2015).
[8]See http://www.bankofengland.co.uk/statistics/yieldcurve/archive.htm.

The data sets represent the most liquid markets for interest rate-sensitive instruments in the European area.

We calibrate the stress interest rate scenarios using Principal Component Analysis (PCA) as prescribed by EIOPA. To transform the principal components and eigenvectors into VaR and ES based interests rate scenarios, we use the method described by Fiori and Iannotti (2006). We describe and discuss the method we use in Appendix B. For every dataset, we derive a shock scenario for the annualized interest rate. For each maturity, the overall interest rate shock is the average of the four shock scenarios (EIOPA, 2014b).

In Table 7 in Appendix A, we provide the resulting stress scenarios of interest up and interest down scenarios with quantiles $\alpha = 98.5\%, 99.5\%$ for various maturities of the bonds. Similar as for equity risk, the differences between using VaR and ES are small when $\alpha = 99.5\%$. Even though the differences are small, calibrating with VaR always leads to larger stress rates. When the quantile $\alpha = 98.5\%$ is chosen, the differences are larger. We get for the fictitious insurance company that the down-shock is more harmful than the up-shock. This is partially due to the fact that duration is not fully hedged.[9] Hence, the down-shock is generally used for the interest rate SCR.

We follow the calibration method for longevity shocks as introduced by CEIOPS (2010). From the Human Mortality Database, we use unisex mortality tables from 1992 until 2009 with age bands of five years from nine countries. These nine countries are Denmark, France, United Kingdom, Estonia, Italy, Sweden, Poland, Hungary, and Czech Republic. We calibrate the longevity stress scenarios by assuming that annual mortality rate improvements follow a Gaussian distribution as prescribed by CEIOPS (2010). The same shock in mortality rates is used for all different ages.

Annual mortality rate changes are calculated per country, per age band and per year based on the data from the Human Mortality Database. We compute the means and standard deviations of the annual mortality rate improvements, and we assume that all annual mortality rate improvements follow a Gaussian distribution. In this case we have $9 \cdot 22 = 198$ Gaussian distributions, since we have nine countries and twenty-two different age bands. This results in 198 different VaRs or ESs. The average of these VaRs or ESs is the longevity shock for all different ages. Due to the Gaussian distribution of mortality improvements, we could define $\hat{\theta}$ analytically for every α such that $\mathrm{SCR}_{life}(VaR_\alpha) = \mathrm{SCR}_{life}(ES_{\hat{\theta}})$ for any insurance portfolio. Differences in the longevity SCR are due to the function θ.

In Table 2, we display the stress scenarios for longevity risk. For both quantiles $\alpha = 98.5\%, 99.5\%$, calibrating with ES leads to fiercer shock rates. The effect is larger when the quantile $\alpha = 98.5\%$ is used.

α	VaR_α	$ES_{\theta(\alpha)}$
99.5%	-18.78%	-18.90%
98.5%	-16.19%	-16.90%

Table 2: Stress rates for longevity risk derived from VaR and ES.

When we apply the stress scenarios, we obtain that the reduced total SCR of the fictitious life annuity insurance company is approximately 23.24% of the BEL. The allocation of this

[9]If the fictitious life annuity insurer applies duration matching, we still observe that the down-shock is typically more harmful than the up-shock.

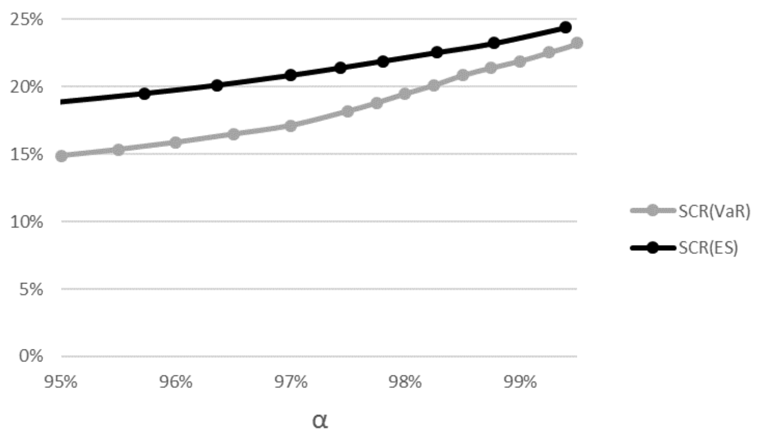

Figure 4: Comparing SCR(VaR_α) with SCR(ES_α) of the fictitious life annuity insurance company for various α.

SCR($VaR_{99.5\%}$) to the three different risk classes in shown in Table 3. In Figure 4 we display SCR(VaR_α) and SCR(ES_α). By construction, it holds that SCR(VaR_α) $\leq$ SCR(ES_α).

Reduced total SCR	23.24%
Interest rate SCR	10.05%
Equity SCR	14.85%
Market SCR	21.70%
Longevity SCR	4.49%

Table 3: The allocation of this SCR($VaR_{99.5\%}$) for the fictitious life annuity insurance company.

5 Results

In this section, we show the base case results. In Subsection 5.1, we compare the allocation of SCR(VaR_α) with the allocation of SCR($ES_{\theta(\alpha)}$) for a fictitious life annuity insurance company. In Subsection 5.2, we provide some sensitivity analysis and show that the results of Subsection 5.1 are robust to changes to the asset and liability portfolio of the fictitious life annuity insurance company.

5.1 Comparing the SCR(VaR_α) with the SCR($ES_{\theta(\alpha)}$) for the fictitious life annuity insurance company

In this subsection, we focus on comparing the SCR(VaR_α) with the SCR($ES_{\theta(\alpha)}$) for the fictitious life annuity insurance company. Since θ is chosen such that the total SCR(VaR_α) equals SCR(ES_θ), we focus on the allocation of the reduced total SCR over the three risk classes: equity SCR, interest rate SCR and longevity SCR. By comparing these allocations for using VaR and ES, we can see if the current method, which uses VaR, underestimates or overestimates certain risks. This is done for different quantiles α used for the VaR.

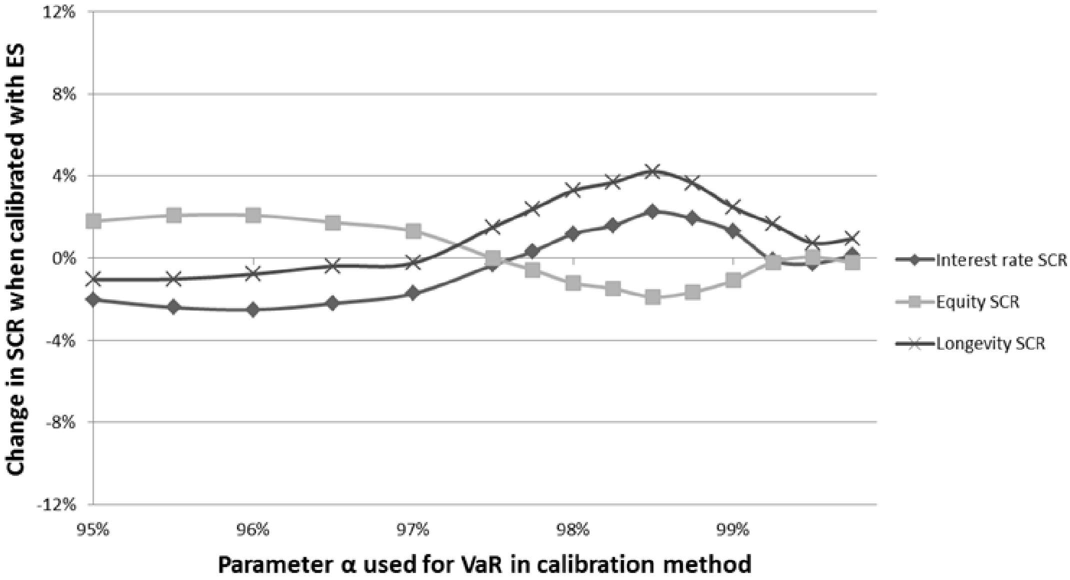

Figure 5: Comparing the allocation of SCR(VaR_α) with the allocation of SCR($ES_{\theta(\alpha)}$) for the fictitious life annuity insurance company, when the stress scenarios are determined in a similar way as EIOPA prescribes (base case). The horizontal axis represents the quantile α used for VaR in the calibration method. The vertical axis represents the change in allocation of the SCR when the stress scenarios are calibrated with ES instead of VaR.

5.1.1 The changes in allocation of the reduced total SCR

Figure 5 shows the change in allocation of the reduced total SCR over the three different risk classes when ES is used to calibrate the shock scenarios instead of VaR. The stress scenarios used to estimate the SCR are derived as prescribed by EIOPA, that means that the interest rate stress scenarios are estimated by using PCA and the longevity stress scenarios are estimated by assuming Gaussian distributions.

Figure 5 shows for the quantile $\alpha = 99.5\%$, as used in Solvency II, that the differences in allocation are small. When the quantile α decreases to 98.5%, the differences become more significant. By using ES to determine the shocks, the longevity risk and the interest rate shocks are more harmful and the equity risk shock is milder. When we use ES instead of VaR to determine the stress shocks for $\alpha = 98.5\%$, the longevity SCR would grow with 4.22%, the interest rate SCR would grow with 2.27% and equity SCR would decline with 1.77%.

These changes do not add up to zero since the amounts of SCR for the different risk measures differ. Therefore, a decrease of 2% for the equity SCR leads to a larger change in the actual amount of SCR than an increase of 2% for the interest rate SCR, because the equity SCR is larger than the interest rate SCR. When the quantile α is smaller than 98.5%, the differences are getting smaller. For values of α smaller than 97.5%, we get that the equity SCR is relatively large. This follows from the observation that the tails for equity returns are fat (see, e.g., Cont, 2001), but the extreme events occur more interest rate shocks. We get that relative effects get fairly constant in α gets smaller, as tail risk events will have a smaller impact on both the ES as the VaR.

5.1.2 The changes in allocation of the reduced total SCR with empirical stress scenarios

Historical simulation is a popular method in practice. The empirical performance of historical simulation has been examined by, e.g., Beder (1995), Hendricks (1996) and Pritsker (2006). Historical simulation is a resampling method which does not assume any distribution about the underlying risk process. For this non-parametric model, we assume that the distribution of past returns is a perfect representation of the expected future returns. The advantage is that we do not impose any assumptions about the underlying probability distributions. A disadvantage is that risk estimates can become biased when the past returns turn out to be not representative for the future returns. Historical simulation is heavily sensitive to the length of the sample of past returns (see, e.g., Pritsker, 2006).

If we apply historical simulation to the three risk classes, the equity SCR remains unchanged as the equity shock was already determined empirically. For the interest rate SCR, we use the annual absolute interest rate changes of the four datasets as described in Subsection 4.3. Per dataset and for each maturity, we calculate the empirical VaR and ES and this leads to a vector of absolute up and down shocks. The percentage interest rate stress vectors per dataset are then computed by dividing it by the average interest rates for each maturity. Since the swap rates are not defined for all maturities between 1 year and 20 years, linear interpolation is used to fill in shocks for this maturities. The average of the four up and down shock vectors has been taken to determine the overall up and down shock vector. For the longevity SCR, we aggregate all annual mortality rate changes for all nine countries used by EIOPA (see Subsection 4.3) and all age cohorts for the period 1992-2009. For this dataset, we calculate an empirical VaR or ES. The empirical VaR or empirical ES serves as the stress rate for longevity risk for all age cohorts.

The SCR($VaR_{99.5\%}$) increases from approximately 23.24% of the BEL under the EIOPA standard model to approximately 24.40% if the stress scenarios for the three risk classes are determined empirically. We show the differences of the SCR of all classes of risk in Table 4. From this table we get that the longevity shocks are more harmful if the SCR is determined empirically. This effect dominates the increase in the reduced total SCR. The interest rate SCR decreases by approximately 13.09% if it is determined empirically.

Change reduced total SCR	5.01%
Change interest rate SCR	−13.09%
Change equity SCR	0.00%
Change market SCR	−4.82%
Change longevity SCR	96.11%

Table 4: Overview of the SCR changes of different risk classes using $VaR_{99.5\%}$, if we switch from PCA analysis for interest shocks and the standard longevity shocks to empirical interest and longevity shocks. All changes in SCR are expressed in percentage of the BEL.

Figure 6 shows the change in allocation of the reduced total SCR over the three different risk classes when ES is used to calibrate the shock scenarios instead of VaR. The stress scenarios used to estimate the SCR are this time determined empirically. Similar as when EIOPA stress scenarios are used, the difference in individual risk modules is small when $\alpha = 99.5\%$. Again, the differences are largest when α is approximately 98.5%. At the quantile $\alpha = 98.5\%$, the differences are considerably larger compared to the base case, since the longevity SCR is approximately 11.26% higher and the equity SCR is approximately 2.85% lower for SCR($ES_{\theta(\alpha)}$) when compared with SCR(VaR_α). For smaller values of the quantile α until approximately 97.5% we see that the differences are getting smaller. The changes in interest rate SCR differ substantially when empirical stress scenarios are used compared to when EIOPA stress scenarios are used. For values of the quantile α smaller than 97.5%, we see that the differences are stable, and differences in the interest rate SCR are smallest. We also see this in Figure 5 this in Subsection 5.5.1, where EIOPA stress scenarios are used.

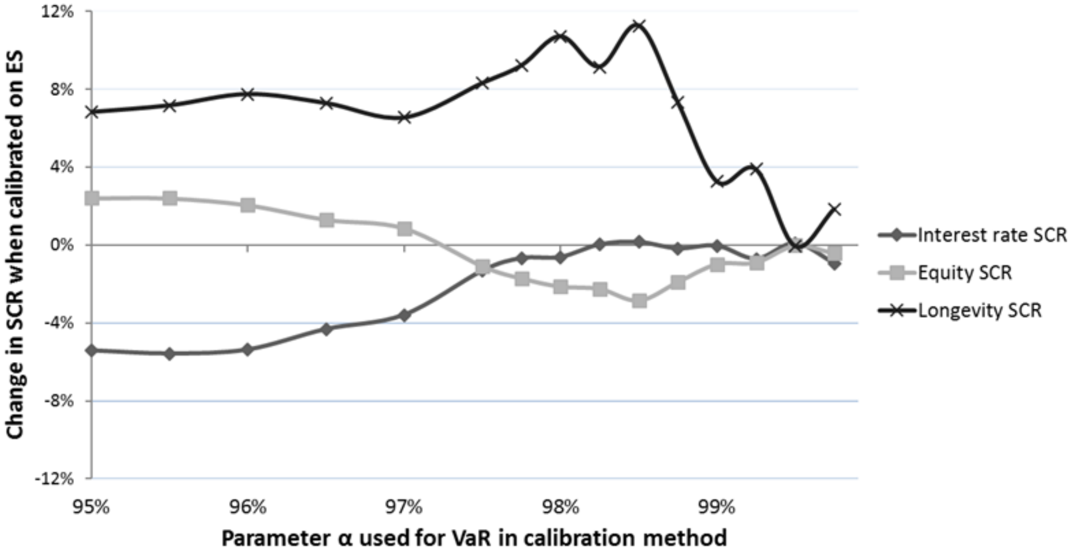

Figure 6: Comparing the allocation of SCR(VaR_α) with the allocation of SCR($ES_{\theta(\alpha)}$) for the fictitious life annuity insurance company, when the stress scenarios are determined empirically. The horizontal axis represents the quantile α used for VaR in the calibration method. The vertical axis represents the change in allocation of the SCRs when the stress scenarios are calibrated with ES instead of VaR.

Comparing the scenarios in Figures 5 and 6, we get that the differences in SCR allocation when ES is used instead of VaR are considerably larger when the stress scenarios are determined empirically. The longevity SCR will increase more when ES is used instead of VaR and when the quantile α is below 99%. This increase is at the expense of a lower interest rate SCR.

For both using EIOPA stress scenarios and empirical stress scenarios we see that the changes in allocation of the reduced total SCR are small for the quantile $\alpha = 99.5\%$ as used in Solvency II. When the quantile α decreases, the differences between the SCR allocation when ES is used instead of VaR get larger. We get that by using ES to determine the shocks, longevity risk is more harmful and equity risk is milder, especially at the quantile $\alpha = 98.5\%$. When we decrease the quantile α until approximately 97.5% these effects are getting smaller.

By comparing the empirical distribution of the tail of the data used for equity risk and longevity risk we can explain why the differences are largest at approximately 98.5%. The downside tails of the data used for equity risk and longevity risk are shown in Figure 7. In this figure, both graphs have two horizontal axis. The first horizontal axis represents the annual holding period return for equity risk and the annual mortality rate changes for longevity risk. On the second horizontal axis, we display the survival distribution (1 - cumulative distribution).

One of the main characteristics of VaR is that it does not consider the shape of the tail. When the tail is not heavy and, e.g., shaped like the tail of a Gaussian distribution, this does

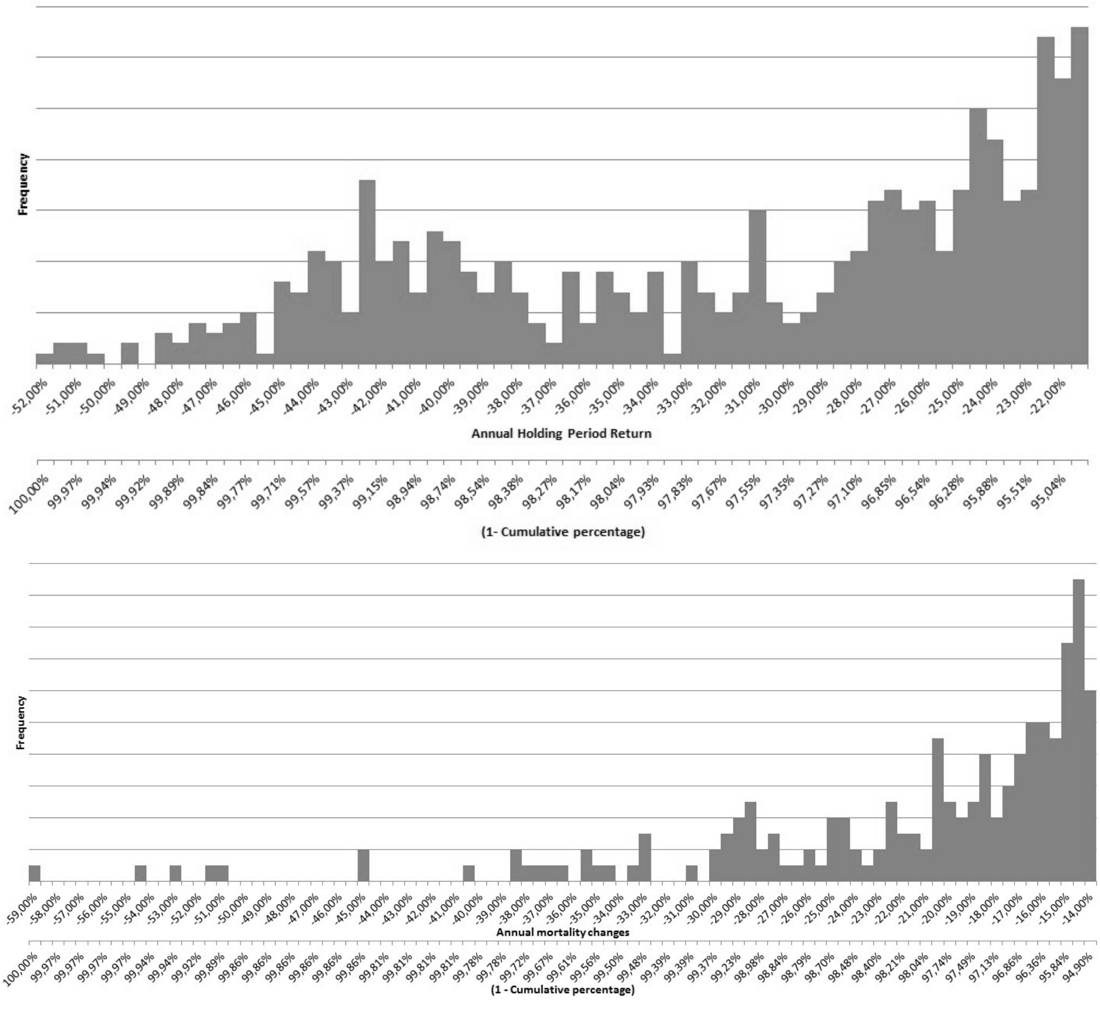

Figure 7: The left tail (smallest 5%) of the empirical distribution of the calibration data used in this paper for equity risk (upper graph) and longevity risk (lower graph). The first horizontal axis represents the annual holding period return (upper graph) or the annual longevity changes (lower graph), the second horizontal axis displays the survival distribution (1 - cumulative distribution) and the vertical axis represents the frequency.

not lead to problems. However, when the distribution has a heavy tail beyond the VaR level, then this might lead to an underestimation of the risk.

From Figure 7, we get that the distribution of longevity risk has a heavy tail and that the distribution of equity returns is less heavy than the tail of the distribution of survival probabilities risk. This explains why the longevity SCR is smaller and the equity SCR is larger if we use the VaR. To understand why the difference is largest if $\alpha \approx 98.5\%$, we focus on the empirical survival distribution. For equity risk the $VaR_{98.5\%}$ is situated in the middle of the heavy tail, therefore the equity SCR is relatively big when VaR is used. For longevity risk, however, the $VaR_{98.5\%}$ is situated just before the start of the heavy tail, and this leads to a much smaller longevity SCR when VaR is used.

5.2 Sensitivity analysis

In this subsection we test the sensitivity of our results. We do this by changing the composition of the asset portfolio and the liability portfolio of the fictitious life annuity insurance company.

5.2.1 Changing the asset portfolio

As described in Subsection 4.1, the fictitious life annuity insurance company has an asset portfolio which consists for 25% of equity and the remainder are government bonds. To test if our results are sensitive to the composition of the asset portfolio we vary with the percentage of equity in the portfolio and compare the allocation of SCR(VaR_α) with the allocation of SCR($ES_{\theta(\alpha)}$).

We analyze the following asset portfolios:

- 100% equity;
- 50% equity and 50% government bonds;
- 25% equity and 75% government bonds (base case);
- 10% equity and 90% government bonds;
- 100% government bonds.

When the asset portfolio contains government bonds, the amounts invested in the 5-year and 30-year UK Gilts are determined by matching the duration of the bond portfolio with 50% of the duration of the liabilities. If 100% of the assets are invested in equity, there is duration zero of the assets.

Figures 10, 11, 12 and 13 in Appendix C show the change in allocation of the reduced total SCR for the three different risk classes when ES is used to calibrate the shock scenarios instead of VaR for the different asset portfolios. To see how sensitive our results are to the composition of the asset portfolio we can compare these figures with Figure 5.

When the equity holdings in the portfolio increase, (Figures 10 and 11) we see the same trend as in the base case. The changes in allocation of the reduced total SCR are again small for $\alpha = 99.5\%$, i.e., using VaR or ES leads to similar stress scenarios for all three risk classes. When the quantile α decreases, this difference becomes more significant and is largest for $\alpha \approx 98.5\%$. For this quantile, the longevity SCR and the interest rate SCR are larger and the equity SCR is smaller when the SCR is based on ES. When we decrease the value α even more until 97.5% the differences are getting smaller.

Figure 12 displays a similar trend as the base case, except for one difference. The similarity with the base case is that the change in allocation of the reduced total SCR is small for the quantile $\alpha = 99.5\%$, is largest for the quantile $\alpha \approx 98.5\%$, and if we decrease the quantile α

until 97.5% the differences are getting smaller again. This figure differs from the base case if we consider the percentage change in SCR per risk class. When an asset portfolio of 10% equity and 90% government bonds is used, the largest percentage change is for the equity SCR. Whereas the relative change in equity SCR grows in comparison with the base case, the change in longevity SCR and interest rate SCR declines in comparison with the base case. Overall, we still conclude that the longevity SCR is larger and the equity SCR is smaller when ES is used.

Figure 13 looks somewhat different. In this case, the asset portfolio consists for 100% of government bonds, meaning there is no equity risk. We see however that the interest rate SCR is smaller for all quantiles when ES is used.

From comparing Figures 5, 11 and 12, we get that for different asset portfolios it still holds that the differences in the SCR allocation are largest at $\alpha \approx 98.5\%$ and that the longevity SCR is larger and the equity SCR is smaller when ES is used. The percentage change per risk class differs if the equity holdings change in the portfolio, but the main trends do not change significantly.

5.2.2 Changing the liability portfolio

To test if our results are sensitive to the composition of the liability portfolio we vary with the distribution of the policyholders and compare the allocation of SCR(VaR_α) with the allocation of SCR($ES_{\theta(\alpha)}$). We compare the allocation of SCR(VaR_α) with the allocation of SCR($ES_{\theta(\alpha)}$) for the following liability portfolios:

- young life annuity insurance company: policyholders with an average age of 35;
- base case life annuity insurance company: policyholders with an average age of 50;
- old life annuity insurance company: policyholders with an average age of 65.

All other assumptions made in Section 4 still hold. For a precise description of the liability portfolios, see Table 6 in Appendix A. The number of policyholders per liability portfolio is chosen based on matching the BEL to the BEL of the base case. Similar to the base case, the asset portfolio consists of 25% equity and 75% government bonds. We again match 37.5% of the duration of the liabilities to determine the amounts invested in the 5-year and 30-year UK Gilts.

Figures 14 and 15 in Appendix C show the change in allocation of the reduced total SCR over the three different risk classes when ES is used to calibrate the shock scenarios instead of VaR for the different liability portfolios. To see how sensitive our results are to the composition of the liability portfolio we can compare Figures 14 and 15 with Figure 5. For all liability portfolios, the change in allocation of the reduced total SCR is small when $\alpha = 99.5\%$., This difference is largest for $\alpha \approx 98.5\%$; for smaller α the differences are getting smaller until approximately 97.5%. We observe that for all liability portfolios that the longevity SCR is larger and the equity SCR is smaller when ES is used. For the value $\alpha = 98.5\%$, it holds for all three different liability portfolios that the longevity SCR increases with approximately 4% and the equity SCR declines with approximately 2.5% if ES was used instead of VaR. If we compare Figures 14 and 15 we get that relative interest rate SCR changes are amplified for an older composition of the liability portfolio.

6 Other regulatory frameworks

The Swiss Solvency Test (SST) is a regulatory framework for insurance companies in Switzerland. The SST uses a holistic approach by taking all risks into the capital requirement calculations, while Solvency II has a modular approach. Moreover, the stress scenarios for the SST are calibrated using Expected Shortfall with confidence level 99%.

The differences in SCR($ES_{99\%}$) compared to the solvency capital requirement of Solvency II, which are determined via SCR($VaR_{99.5\%}$), are displayed in Table 5. All other assumptions are as in the base case in Section 4. We here ignore all other differences of the SST regulation. The reduced total SCR increases from approximately 23.24% of the BEL under the EIOPA standard model to an SCR($ES_{99\%}$) of approximately 23.73% under $ES_{99\%}$.

Change reduced total SCR	2.14%
Change interest rate SCR	3.14%
Change equity SCR	1.37%
Change market SCR	2.03%
Change longevity SCR	3.36%

Table 5: Overview of the SCR changes of different risk classes, if we switch from using $VaR_{99.5\%}$ to $ES_{99\%}$. All changes in SCR are expressed in percentage of the BEL.

From Table 5, we get that the difference in the reduced total SCR is substantial if $ES_{99\%}$ is applied. This difference is driven by increases in all the three classes of risk.

The Basel III framework is a global regulatory framework for banks which is planned to be implemented in 2018. Basel III was set up in a different manner than Solvency II since it is a regulatory framework for a different part of the financial industry. The regulation does however also use stress scenarios to see the impact of shocks on certain risk drivers. The current Basel II framework uses stress scenarios calibrated on Value-at-Risk with quantile $\alpha = 99\%$, but the Basel III framework will be calibrated using Expected Shortfall with parameter $\theta = 97.5\%$. The parameter $\theta = 97.5\%$ of ES is set such that the ES corresponds to the $VaR_{99\%}$ if the risks are Gaussian. Our analysis for the classes interest rate risk and equity risk risk suggests that $ES_{97.5\%}$ might lead to lower capital requirements than $VaR_{99\%}$, since $\theta(99\%) = 97.81\%$ and $\text{SCR}_{life}(VaR_{99\%}) < \text{SCR}_{life}(ES_{\theta(99\%)})$. Note that longevity risk is much less severe for banks. The Committee on Banking Supervision acknowledged the incoherence of the VaR as a risk measurement (Basel Committee on Banking Supervision, 2011).

7 Conclusion

This paper examines the consequences for a life annuity insurance company if the Solvency II SCR estimation is based on Expected Shortfall (ES) instead of Value-at-Risk (VaR). First, we calibrate the SCR stress scenarios for equity risk, interest rate risk and longevity risk based on Value-at-Risk and Expected Shortfall. Thereafter, we compare the SCR(VaR_α) with the SCR($ES_{\theta(\alpha)}$) for a fictitious life annuity insurance company.

Since $\theta(\alpha)$ is defined such that the total SCR(VaR_α) equals SCR($ES_{\theta(\alpha)}$), we focused on the allocation of the reduced total SCR over the three risk classes: equity SCR, interest rate SCR and longevity SCR. For the quantile $\alpha = 99.5\%$, as used in Solvency II, the difference in allocation is small between SCR(VaR_α) with the SCR($ES_{\theta(\alpha)}$), i.e., the equity SCR, interest rate SCR and longevity SCR differ little is we use the VaR or ES. Of course, this only applies if the confidence level $\theta(99.5\%) = 98.78\%$ is applied for determining the Expected Shortfall. When $\alpha \approx 98.5\%$, the differences are largest. If we use the ES instead of the VaR to determine the shocks, the longevity SCR is larger and equity SCR is smaller. For smaller values of α, the differences become smaller.

These results are robust in terms of variation with the composition of the asset and liability portfolio of the life annuity insurance company. To test the sensitivity of our results to the estimation methods used by EIOPA, we compare the results with the SCR allocations when the stress scenarios are determined empirically. We find that when empirical stress scenarios are used, the reduced total SCR increases due to a higher longevity SCR. Moreover, the interest rate SCR is smaller compared to when the estimation methods are as determined by EIOPA (2014a).

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A Data

Age	Number of policies young	Number of policies base case	Number of policies old	Accrued yearly payment starting at age 65
21	7634	361	0	0.067
23	9141	531	0	0.109
25	10646	762	0	0.152
27	12061	1063	0	0.194
29	13289	1441	0	0.236
31	14242	1902	88	0.279
33	14847	2440	138	0.321
35	15054	3046	213	0.364
37	14847	3698	318	0.406
39	14242	4367	462	0.448
41	13289	5016	654	0.491
43	12061	5605	899	0.533
45	10646	6090	1203	0.576
47	9141	6437	1565	0.618
49	7634	6618	1981	0.661
51	6202	6618	2438	0.703
53	4900	6437	2919	0.745
55	3766	6090	3400	0.788
57	2815	5605	3852	0.830
59	2047	5016	4244	0.873
61	1448	4367	4548	0.915
63	996	3698	4742	0.958
65	666	3046	4808	1
67	434	2440	4742	1
69	274	1902	4548	1
71	169	1441	4244	1
73	101	1063	3852	1
75	59	762	3400	1
77	33	531	2919	1
79	18	361	2438	1
81	0	0	1981	1
83	0	0	1565	1
85	0	0	1203	1
87	0	0	899	1
89	0	0	654	1

Table 6: The liability portfolio of the young, base case, and old life annuity insurance company corresponding to Subsections 4.1 and 5.2.

Maturity	$SCR(VaR_{99.5\%})$ Up	Down	$SCR(ES_{\theta(99.5\%)})$ Up	Down	$SCR(VaR_{98.5\%})$ Up	Down	$SCR(ES_{\theta(98.5\%)})$ Up	Down
1	49.1%	-83.0%	49.5%	-82.6%	42.0%	-74.7%	43.8%	-77.7%
2	51.0%	-84.5%	51.1%	-84.9%	43.3%	-76.4%	45.1%	-78.7%
3	47.3%	-75.6%	47.4%	-75.5%	40.4%	-65.6%	42.0%	-67.9%
4	44.2%	-66.3%	44.3%	-66.6%	37.6%	-59.1%	39.1%	-60.7%
5	41.3%	-59.8%	41.2%	-60.1%	35.0%	-54.0%	36.5%	-55.2%
6	39.6%	-55.1%	39.6%	-55.3%	33.6%	-49.8%	35.0%	-51.1%
7	37.6%	-51.1%	37.6%	-51.2%	32.5%	-46.7%	33.5%	-47.6%
8	35.3%	-48.5%	35.2%	-48.5%	30.4%	-43.6%	31.4%	-44.8%
9	33.8%	-47.2%	33.6%	-46.9%	28.9%	-42.3%	30.0%	-43.2%
10	32.4%	-44.9%	32.6%	-45.2%	27.9%	-40.8%	29.0%	-41.6%
11	32.8%	-44.0%	32.7%	-44.5%	27.9%	-40.0%	29.1%	-40.9%
12	32.7%	-43.6%	32.7%	-43.8%	28.0%	-39.6%	29.1%	-40.3%
13	32.6%	-42.6%	32.6%	-42.7%	27.8%	-38.5%	28.8%	-39.2%
14	32.2%	-41.3%	32.4%	-41.6%	27.5%	-37.3%	28.6%	-38.2%
15	31.8%	-40.3%	32.2%	-40.5%	27.1%	-36.3%	28.2%	-37.2%
16	29.5%	-39.2%	29.7%	-39.4%	24.9%	-35.7%	25.9%	-36.4%
17	29.6%	-38.9%	29.9%	-38.8%	24.9%	-35.1%	25.9%	-35.8%
18	29.8%	-38.5%	30.0%	-38.3%	24.9%	-34.5%	26.0%	-35.3%
19	30.0%	-37.8%	30.2%	-37.8%	24.9%	-34.0%	26.1%	-34.7%
20	30.5%	-37.4%	30.4%	-37.3%	25.5%	-33.4%	26.2%	-34.2%

Table 7: Simulated interest rate stress scenarios corresponding to Subsection 4.3.

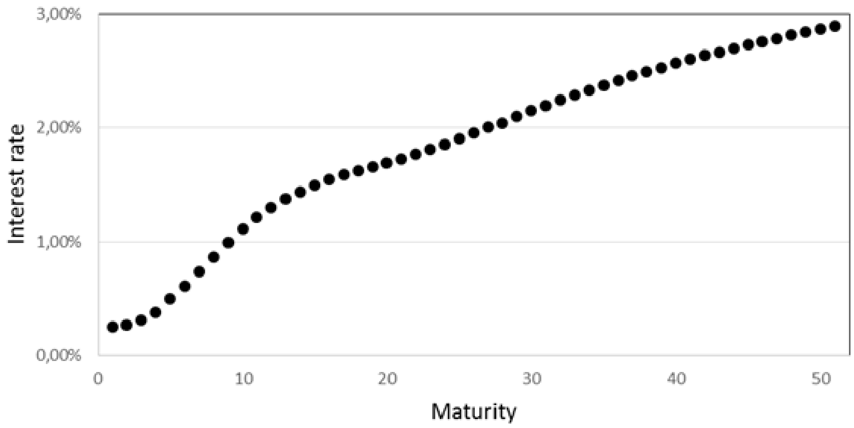

Figure 8: The nominal interest rate term structure from the European Central Bank of December 31st, 2014, used in Subsection 4.3. Source: https://www.statistics.dnb.nl/index.cgi?lang=nl&todo=Rentes (Last accessed on August 28th, 2015).

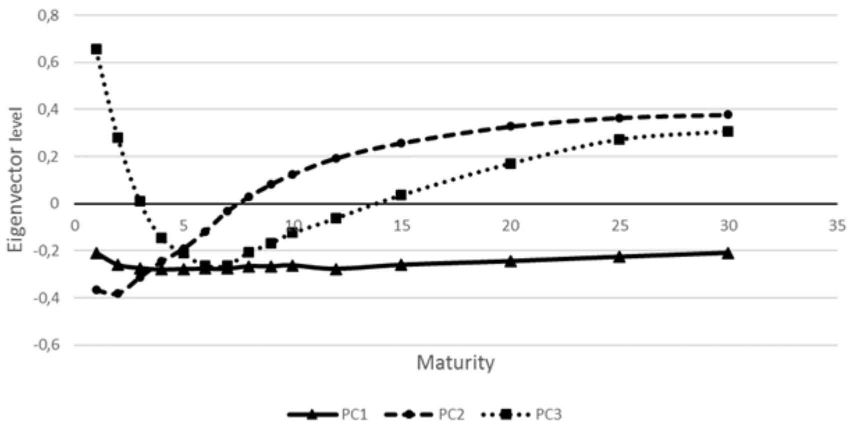

Figure 9: The shift (PC1), twist (PC2) and butterfly (PC3) components of the yield curve of the Euro vs Euribor IR swap rates. The horizontal axis represents the term to maturity and the vertical axis represents the level of the eigenvector component.

B Interest rate risk stress scenarios by using Principal Component Analysis

In this appendix, we describe the technique Principal Component Analysis (PCA) to calibrate the interest rate shocks. PCA is used to describe movements of the yield curve and is explained by Fiori and Iannotti (2006) and Barber and Copper (2012). It applies an orthogonal linear transformation that converts interest rate changes data of correlated variables into a set of values of linearly uncorrelated variables. A yield curve change, for m maturities, can then be represented exactly as a linear combination of m vectors:

$$X_t = c_t + b_{1,t} U_1 + \cdots + b_{m,t} U_m \tag{13}$$

where X_t is the absolute change in the annualized yield curve at time $t = 1, \ldots, T$, c_t is a time varying constant, and U_k is a time independent, zero-mean and orthogonal $m \times 1$ vector.

PCA transforms the data into a new coordinate system such that the first coordinate (i.e., the principal component $b_{1,t}$) explains the largest share of the variance by any projection of the data, the second coordinate has the second largest explanation of the variance, etcetera. The objective of PCA is to determine a small set of components that best explain the total variance of the data. The number of components is then small $K \ll m$, but with high explanatory power:

$$X_t = c_t + b_{1,t} U_1 + \cdots + b_{K,t} U_K + \varepsilon_t$$

where ε_t is the zero-mean error term.

When PCA is used to describe interest rate movements, the factors $U_k, k = 1, \ldots, K$ are the eigenvectors of the covariance matrix of the original data. The first six components explain between 99.2% and 99.5% of the variance with a 90% confidence interval (cf. Barber and Copper, 2010). The principal components describe the different yield curve movements and the first three are interpreted as the shift, twist and butterfly moves of the yield curve (Novosyolov and Satchkov, 2008). Figure 9 shows the first three eigenvectors of the Euro vs Euribor IR swap rates. In this figure, we observe the shift, twist and butterfly moves.

The principal components are derived via the following matrix multiplication:

$$b_{k,t} = U_k' X_t, \text{ for } k = 1, 2, \ldots, K. \tag{14}$$

We use the annual interest rate changes to describe the yield curve movements. To transform the principal components and eigenvectors into VaR and ES based interests rate stress scenarios, we use a method of Fiori and Iannotti (2006). For a maturity k, we regress the derived annual percentage rate changes on the principal components to derive the "beta" sensitivity of each rate to each principal component via (14). The combined sum returns the stress factor for maturity k (EIOPA, 2010). We use the empirical distribution of each principal component vector b_k. We use the historical simulation of the principal components to derive a down risk stress scenario and an up risk stress scenario for both VaR and or ES as follows:

$$VaR_\gamma = VaR_\gamma \left(\sum_{k=1}^{K} \hat{b}_k \, U_k \right), \text{ for } \gamma = \alpha, (1-\alpha); \tag{15}$$

$$ES_\gamma = ES_\gamma \left(\sum_{k=1}^{K} \hat{b}_k \, U_k \right), \text{ for } \gamma = \theta(\alpha), 1-\theta(\alpha). \tag{16}$$

Since the eigenvectors $U_k, k = 1, \ldots, K$ are orthogonal, we can randomly draw the marginal distributions of $\hat{b}_k, k = 1, \ldots, K$ and determine the empirical distribution of $\sum_{k=1}^{K} \hat{b}_k \, U_k$ accordingly (see, e.g., Fiori and Iannotti, 2006). We use 4,000 simulations for each dataset.

In our four datasets, we set $K = 3$ or $K = 4$, which is determined such that the principal components describe at least 95% of the variance. Since we have four datasets, we have four different up and down shock vectors. In line with EIOPA (2014a), we set the up and down shock at maturity of 90 years and longer at +20% and -20%. Moreover, the swap rates are not always defined for every maturity, and therefore linear interpolation is used to fill in shocks for these maturities. The mean result of the four different up and down shock vectors has been taken to arrive at a generalized up and down shock vector.

C Figures sensitivity analysis

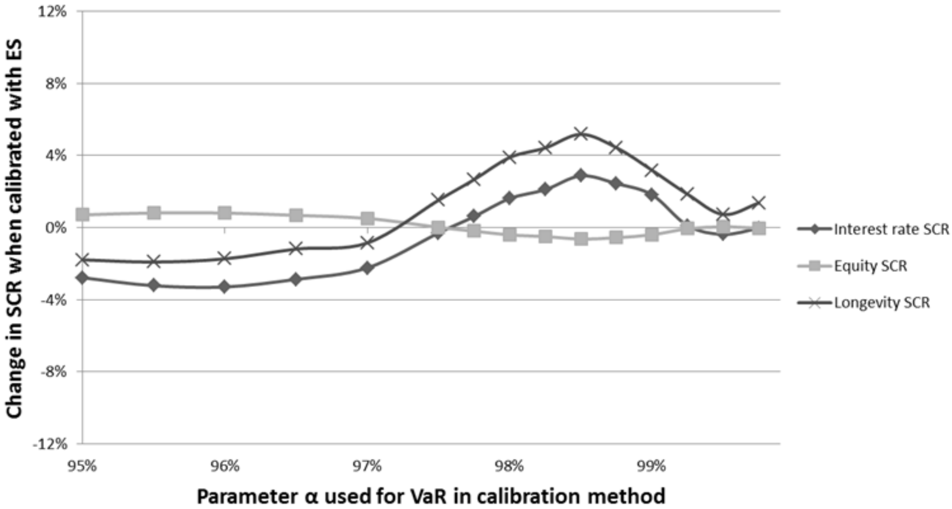

Figure 10: Comparing the allocation of SCR(VaR_α) with the allocation of SCR($ES_{\theta(\alpha)}$) for the fictitious life annuity insurance company, with an asset portfolio consisting of 100% equity. The horizontal axis represents the quantile α used for VaR in the calibration method. The vertical axis represents the change in allocation of the SCRs when the stress scenarios are calibrated with ES instead of VaR.

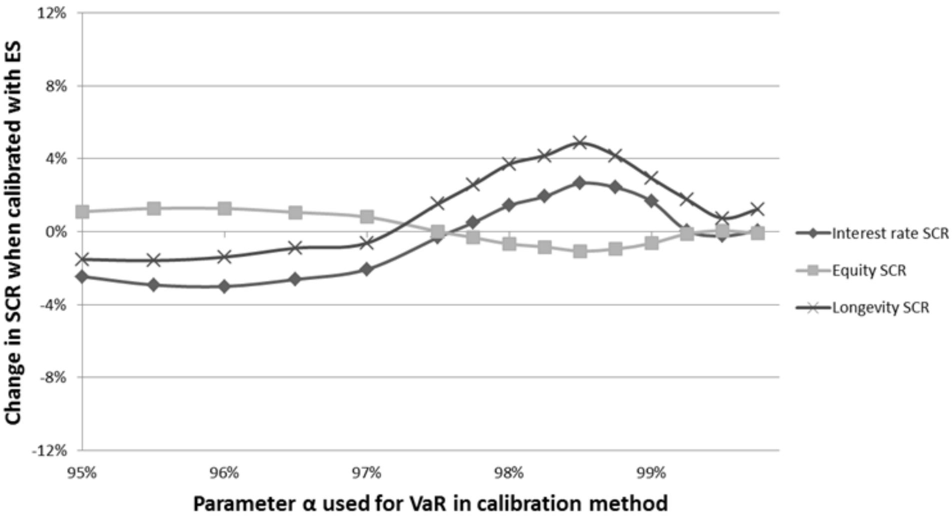

Figure 11: Comparing the allocation of SCR(VaR_α) with the allocation of SCR($ES_{\theta(\alpha)}$) for the fictitious life annuity insurance company, with an asset portfolio consisting of 50% equity and 50% government bonds. The horizontal axis represents the quantile α used for VaR in the calibration method. The vertical axis represents the change in allocation of the SCRs when the stress scenarios are calibrated on ES instead with VaR.

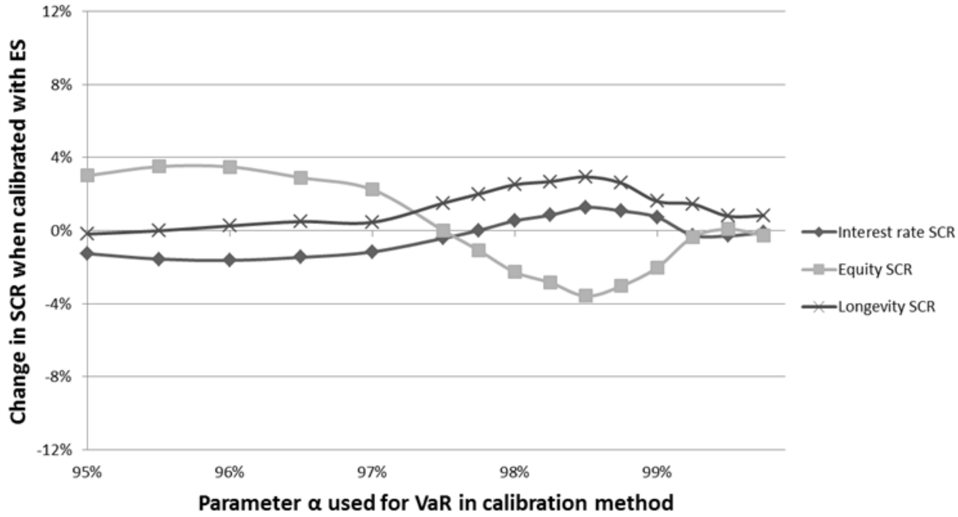

Figure 12: Comparing the allocation of SCR(VaR_α) with the allocation of SCR($ES_{\theta(\alpha)}$) for the fictitious life annuity insurance company, with an asset portfolio consisting of 10% equity and 90% government bonds. The horizontal axis represents the quantile α used for VaR in the calibration method. The vertical axis represents the change in allocation of the SCRs when the stress scenarios are calibrated with ES instead of VaR.

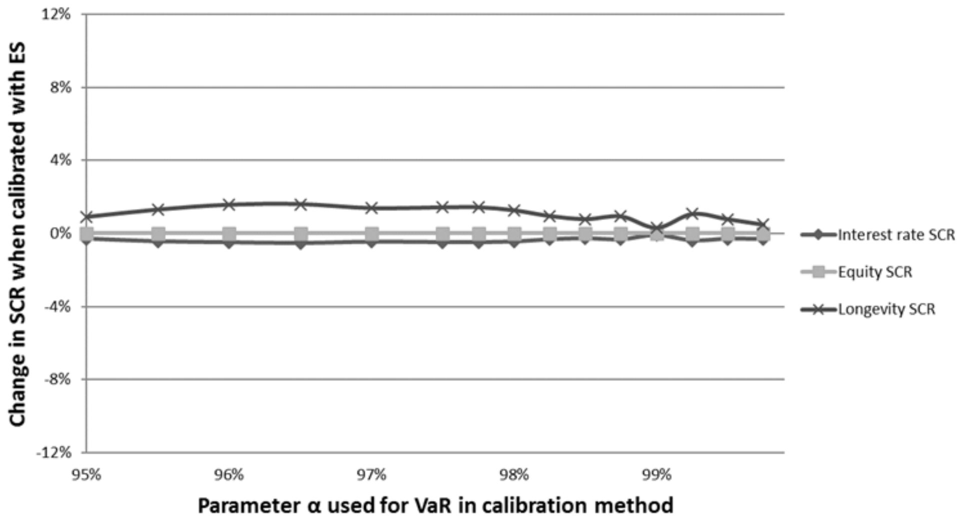

Figure 13: Comparing the allocation of SCR(VaR_α) with the allocation of SCR($ES_{\theta(\alpha)}$) for the fictitious life annuity insurance company, with an asset portfolio consisting of 100% government bonds. The horizontal axis represents the quantile α used for VaR in the calibration method. The vertical axis represents the change in allocation of the SCRs when the stress scenarios are calibrated with ES instead of VaR.

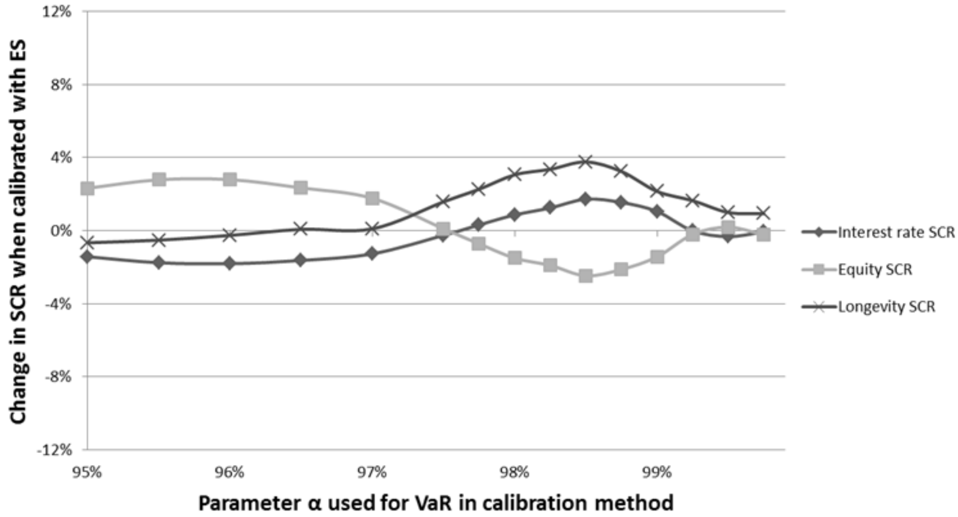

Figure 14: Comparing the allocation of SCR(VaR_α) with the allocation of SCR($ES_{\theta(\alpha)}$) for the young life annuity insurance company. The horizontal axis represents the quantile α used for VaR in the calibration method. The vertical axis represents the change in allocation of the SCRs when the stress scenarios are calibrated with ES instead of VaR.

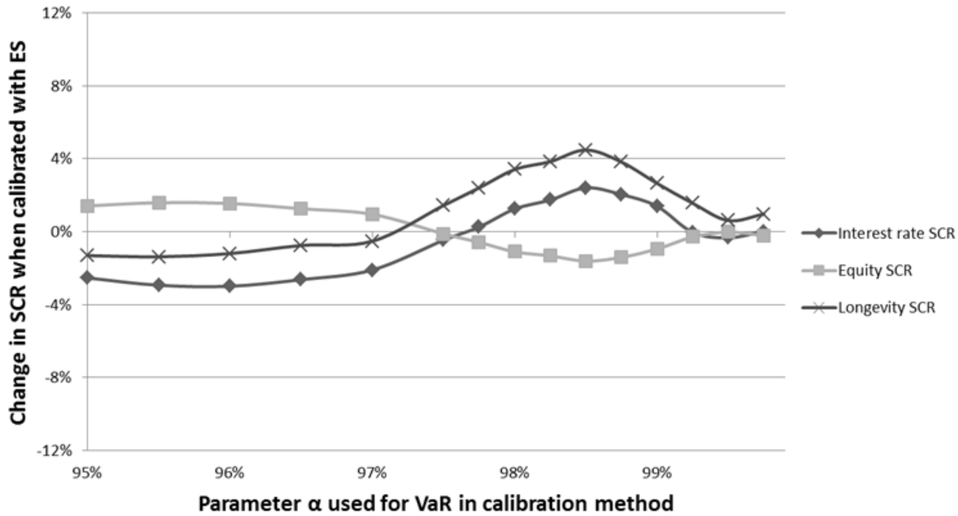

Figure 15: Comparing the allocation of SCR(VaR_α) with the allocation of SCR($ES_{\theta(\alpha)}$) for the old life annuity insurance company. The horizontal axis represents the quantile α used for VaR in the calibration method. The vertical axis represents the change in allocation of the SCRs when the stress scenarios are calibrated with ES instead of VaR.