A WAVELET-BASED ASSESSMENT OF MARKET RISK:
THE EMERGING MARKETS CASE

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A wavelet-based assessment of market risk: The emerging markets case

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Abstract

The measurement of market risk poses major challenges to researchers and different economic agents. On one hand, it is by now widely recognized that risk varies over time. On the other hand, the risk profile of an investor, in terms of investment horizon, makes it crucial to also assess risk at the frequency level. We propose a novel approach to measuring market risk based on the continuous wavelet transform. Risk is allowed to vary both through time and at the frequency level within a unified framework. In particular, we derive the wavelet counterparts of well-known measures of risk. One is thereby able to assess total risk, systematic risk and the importance of systematic risk to total risk in the time-frequency space. To illustrate the method we consider the emerging markets case over the last twenty years, finding noteworthy heterogeneity across frequencies and over time, which highlights the usefulness of the wavelet approach.

Keywords: Market risk; Variance; CAPM; Beta coefficient; Continuous wavelet transform; Emerging markets.

JEL classification: C40, F30, G15.

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1

1 Introduction

The assessment of market risk has long posed a challenge to many types of economic agents and researchers (see, for instance, Granger (2002) for an overview). Market risk arises from the random unanticipated changes in the prices of financial assets and measuring it is crucial for investors. Besides its interest to portfolio managers, the assessment of market risk is relevant for the overall risk management in banks and bank supervisors. Although bank failures are traditionally related with an excess of non-performing loans (the so-called credit risk), the failure of the Barings Bank in 1995 showed how market risk can lead to bankruptcy. Furthermore, market risk has received increasing attention in recent years as banks' financial trading activities have grown.

Although the measurement of market risk has a long tradition in finance, there is still no universally agreed upon definition of risk. The modern theory of portfolio analysis dates back to the pioneering work of Harry Markowitz in the 1950s. The starting point of portfolio theory rests on the assumption that investors choose between portfolios on the basis of their expected return, on the one hand, and the variance of their return, on the other. The investor should choose a portfolio that maximizes expected return for any given variance, or alternatively, minimizes variance for any given expected return. The portfolio choice is determined by the investor's preferred trade-off between expected return and risk. Hence, in his seminal paper, Markowitz (1952) implicitly provided a mathematical definition of risk, that is, the variance of returns. In this way, risk is thought in terms of how spread-out the distribution of returns is.

Later on, the Capital Asset Pricing Model (CAPM) emerged through the contributions of Sharpe (1964) and Lintner (1965a, 1965b). According to the CAPM, the relevant risk measure in holding a given asset is the systematic risk, since all other risks can be diversified away through portfolio diversification. The systematic risk, measured by the beta coefficient,

is a widely used measure of risk. In statistical terms, it is assumed that the variability in each stock's return is a linear function of the return on some larger market with the beta reflecting the responsiveness of an asset to movements in the market portfolio. For instance, in the context of international portfolio diversification, the country risk is defined as the sensitivity of the country return to a world stock return. Traditionally, it is assumed that beta is constant through time. However, empirical research has found evidence that betas are time varying (see, for example, the pioneer work of Blume (1971, 1975)). Such a finding led to a surge in contributions to the literature (see, for example, Fabozzi and Francis (1977, 1978), Sunder (1980), Alexander and Benson (1982), Collins *et al.* (1987), Harvey (1989, 1991), Ferson and Harvey (1991, 1993) and Ghysels (1998) among others). One natural implication of such a result is that risk measurement must be able to account for this time-varying feature.

Besides the time-variation, risk management should also take into account the distinction between the short and long-term investor (see, for example, Candelon *et al.* (2008)). In fact, the first kind of investor is naturally more interested in risk assessment at higher frequencies, that is, short-term fluctuations, whereas the latter focuses on risk at lower frequencies, that is, long-term fluctuations. Analysis at the frequency level provides a valuable source of information, considering that different financial decisions occur at different frequencies. Hence, one has to resort to the frequency domain analysis to obtain insights into risk at the frequency level.

In this paper, we re-examine risk measurement through a novel approach, wavelet analysis. Wavelet analysis constitutes a very promising tool as it represents a refinement in terms of analysis in the sense that both time and frequency domains are taken into account. In particular, one can resort to wavelet analysis to provide a unified framework to measure risk in the time-frequency space. As both time and frequency domains are encompassed, one is able to capture the time-varying feature of risk while disentangling its

3

behavior at the frequency level. In this way, one can simultaneously measure the evolving risk exposure and distinguish the risk faced by short and long-term investors. Although wavelets have been more popular in fields such as signal and image processing, meteorology, and physics, among others, such analysis can also shed fruitful light on several economic phenomena (see, for example, the pioneering work of Ramsey and Zhang (1996, 1997) and Ramsey and Lampart (1998a, 1998b)). Recent work using wavelets includes that of, for example, Kim and In (2003, 2005), who investigate the relationship between financial variables and industrial production and between stock returns and inflation, Gençay *et al.* (2003, 2005) and Fernandez (2005, 2006), who study the CAPM at different frequency scales, Connor and Rossiter (2005) focus on commodity prices, In and Kim (2006) examine the relationship between the stock and futures markets, Gallegati and Gallegati (2007) provide a wavelet variance analysis of output in G-7 countries, Gallegati *et al.* (2008) and Yogo (2008) resort to wavelets for business cycle analysis, Rua (2011) focuses on forecasting GDP growth in the major euro area countries, and others (see Crowley (2007) for a survey). However, up to now, most of the work drawing on wavelets has been based on the discrete wavelet transform. In this paper we focus on the continuous wavelet transform to assess market risk (see also, for example, Raihan *et al.* (2005), Crowley and Mayes (2008), Rua and Nunes (2009), Rua (2010, 2012), Tonn *et al.* (2010), and Aguiar-Conraria and Soares (2011a, 2011b, 2011c)).

We provide an illustration by considering the emerging markets case. The new equity markets that have emerged around the world have received considerable attention in the last two decades, leading to extensive recent literature on this topic (see, for example, Harvey (1995), Bekaert and Harvey (1995, 1997, 2000, 2002, 2003), Garcia and Ghysels (1998), Estrada (2000), De Jong and De Roon (2005), Chambet and Gibson (2008), Dimitrakopoulos *et al.* (2010), among others). The fact that the volatility of stock prices changes over time has long been known (see, for example, Fama (1965)),

4

and such features have also been documented for the emerging markets. The time variation of risk comes even more naturally in these countries due to the changing economic environment resulting from capital market liberalizations or the increasing integration with world markets and the evolution of political risks. In fact, several papers have acknowledged time varying volatility and betas for the emerging markets (see, for example, Bekaert and Harvey (1997, 2000, 2002, 2003), Santis and Imrohoroglu (1997), and Estrada (2000)). Moreover, the process of market integration is a gradual one, as emphasized by Bekaert and Harvey (2002). Therefore, methods that allow for gradual transitions at changing speeds, such as wavelets, are preferable to segmenting the analysis into various subperiods. Hence, the emerging markets case makes an interesting example for measuring risk with the continuous wavelet transform.

This paper is organized as follows. In section 2, the main building blocks of wavelet analysis are presented. In section 3, we provide the wavelet counterpart of well-known risk measures. In section 4, an application to the emerging markets case is provided. Section 5 concludes.

2 Wavelet analysis

The wavelet transform decomposes a time series in terms of some elementary functions, the daughter wavelets or simply wavelets $\psi_{\tau,s}(t)$. Wavelets are "small waves" that grow and decay in a limited time period. These wavelets result from a mother wavelet $\psi(t)$ that can be expressed as a function of the time position τ (translation parameter) and the scale s (dilation parameter), which is related with the frequency. While the Fourier transform decomposes the time series into infinite length sines and cosines (see, for example, Priestley (1981)), discarding all time-localization information, the basis functions of the wavelet transform are shifted and scaled versions of the time-localized mother wavelet. In fact, wavelet analysis can be seen as a refinement of Fourier analysis. More explicitly, wavelets are defined as

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right) \tag{1}$$

where $\frac{1}{\sqrt{s}}$ is a normalization factor to ensure that wavelet transforms are comparable across scales and time series. To be a mother wavelet, $\psi(t)$ must meet several conditions (see, for example, Percival and Walden (2000)): it must have zero mean, $\int_{-\infty}^{+\infty}\psi(t)dt = 0$; its square integrates to unity, $\int_{-\infty}^{+\infty}\psi^2(t)dt = 1$, which means that $\psi(t)$ is limited to an interval of time; and it should also satisfy the so-called admissibility condition, $0 < C_\psi = \int_0^{+\infty}\frac{|\widehat{\psi}(\omega)|^2}{\omega}d\omega < +\infty$ where $\widehat{\psi}(\omega)$ is the Fourier transform of $\psi(t)$, that is, $\widehat{\psi}(\omega) = \int_{-\infty}^{+\infty}\psi(t)e^{-i\omega\tau}dt$. The last condition allows the reconstruction of a time series $x(t)$ from its continuous wavelet transform, $W_x(\tau, s)$. Thus, it is possible to recover $x(t)$ from its wavelet transform through the following formula

$$x(t) = \frac{1}{C_\psi}\int_0^{+\infty}\left[\int_{-\infty}^{+\infty}\frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)W_x(\tau,s)d\tau\right]\frac{ds}{s^2} \tag{2}$$

The continuous wavelet transform of a time series $x(t)$ with respect to $\psi(t)$ is given by the following convolution

$$W_x(\tau, s) = \int_{-\infty}^{+\infty}x(t)\psi_{\tau,s}^*(t)dt = \frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty}x(t)\psi^*\left(\frac{t-\tau}{s}\right)dt \tag{3}$$

where * denotes the complex conjugate. For a discrete time series, $x(t)$, $t = 1, ..., N$ we have

$$W_x(\tau, s) = \frac{1}{\sqrt{s}}\sum_{t=1}^{N}x(t)\psi^*\left(\frac{t-\tau}{s}\right) \tag{4}$$

Although it is possible to compute the wavelet transform in the time domain using equation (4), a more convenient way to implement it is to carry out the wavelet transform in Fourier space (see, for example, Torrence and Compo

(1998)).The most commonly used continuous mother wavelet is the Morlet wavelet[1] and is defined as

$$\psi(t) = \pi^{-\frac{1}{4}} \left(e^{i\omega_0 t} - e^{-\frac{\omega_0^2}{2}} \right) e^{\frac{-t^2}{2}} \tag{5}$$

Since the term $e^{-\frac{\omega_0^2}{2}}$ becomes negligible for an appropriate ω_0, the Morlet wavelet is simply defined as

$$\psi(t) = \pi^{-\frac{1}{4}} e^{i\omega_0 t} e^{\frac{-t^2}{2}} \tag{6}$$

with the corresponding Fourier transform given by

$$\widehat{\psi}(\omega) = \pi^{\frac{1}{4}} \sqrt{2} e^{-\frac{1}{2}(\omega - \omega_0)^2} \tag{7}$$

One can see that the Morlet wavelet consists of a complex sine wave, given by the term $e^{i\omega_0 t}$, within a Gaussian envelope captured by the term $e^{\frac{-t^2}{2}}$. The exponential decay of the Gaussian distribution makes this wavelet localized in time. Since about 99% of the mass of the Gaussian distribution is contained in the interval (-3, 3), the Morlet wavelet is basically limited to an interval including 3 time units to the left and 3 time units to the right. In particular, this implies that when using monthly data, if the continuous wavelet transform $W_x(\tau, s)$ was computed at some point in time τ for a cycle of one year, an interval containing basically 72 monthly observations would be used. For a cycle of 0.25 years, the interval of data used would consist of roughly 18 monthly observations.

The parameter ω_0 controls the number of oscillations within the Gaussian envelope. By increasing (decreasing) the wavenumber one achieves better (poorer) frequency localization but poorer (better) time localization. In practice, ω_0 is set to 6 or 2π as it provides a good balance between time and

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[1]Nevertheless, in the empirical application, we also considered other mother wavelets, such as Paul and Mexican hat. The results obtained are qualitatively similar and available from the authors upon request.

frequency localization. One of the advantages of the Morlet wavelet is its complex nature, which allows for both time-dependent amplitude and phase for different frequencies (see, for example, Adisson (2002) for further details on the Morlet wavelet). Since the wavelength for the Morlet wavelet is given by $\frac{4\pi s}{\omega_0 + \sqrt{2 + \omega_0^2}}$ (see, Torrence and Compo (1998)), then for $\omega_0 = 6$, the wavelet scale s is almost equal to the Fourier period, which eases the interpretation of wavelet analysis. Furthermore, the Morlet wavelet has optimal joint time-frequency concentration (see Teolis, 1998, and Aguiar-Conraria and Soares, 2011c).

As in Fourier analysis, several interesting measures can be defined in the wavelet domain. For instance, one can define the wavelet power spectrum as $|W_x(\tau, s)|^2$. It measures the time series' variance at each time and at each scale. Another measure of interest is the cross-wavelet spectrum, which captures the covariance between two series in the time-frequency space. Given two time series $x(t)$ and $y(t)$, with wavelet transforms $W_x(\tau, s)$ and $W_y(\tau, s)$, one can define the cross-wavelet spectrum as $W_{xy}(\tau, s) = W_x(\tau, s)W_y{}^*(\tau, s)$. As the mother wavelet is complex, the cross-wavelet spectrum is also complex valued and it can be decomposed into real and imaginary parts.

3   Measuring risk with wavelets

The variance has been the most famous moment-based measure of risk in finance ever since the seminal work of Markowitz. The variance is a particularly appropriate measure of risk in segmented markets or if one is interested in a single asset. While variance measures total risk, beta captures the systematic risk. In contrast with variance which treats an asset in isolation, beta reflects the idea that any asset can be viewed as a part of a portfolio. In light of this, the asset's risk can be thought of in terms of the contribution to the variability of the portfolio. According to the CAPM developed by Sharpe (1964), Lintner (1965a, 1965b), and Mossin (1966), it is well known that

$$E[R_i] = R_f + \beta_i [E[R_m] - R_f] \tag{8}$$

where R_i is the return on asset i, R_f is the risk-free asset return and R_m is the market return. Equation (8) states that the expected return on asset i is equal to the risk-free rate (compensating investors for delaying consumption) plus a risk premium (compensating them for taking the risk associated with the investment). The risk premium can be broken into two parts. The term in brackets is the risk premium for the market portfolio, which can be thought of as the risk premium for an average, or representative, asset. To obtain the risk premium for asset i, one has to multiply the risk premium for the average asset by the other term, the risk measure for asset i, that is, the beta. The beta is defined as

$$\beta_i = \frac{Cov(R_i, R_m)}{\sigma^2_{R_m}} \tag{9}$$

where $Cov(R_i, R_m)$ is the covariance between the return on asset i and the return on the market portfolio and $\sigma^2_{R_m}$ is the variance of the portfolio return. For instance, in the context of the world CAPM, a country's beta is defined as the covariance of the country's returns with the world market portfolio divided by the variance of the world market return.

The *rationale* of using beta as a measure of risk is also motivated by the index model developed by Sharpe (1963). Each asset is assumed to respond to the pull of a single factor, which is usually taken to be the market portfolio. The return on asset i can be written as

$$R_{it} = \alpha + \beta_i R_{mt} + \varepsilon_t \tag{10}$$

This model implicitly assumes that two types of events determine the period-to-period variability in the asset's return. On the one hand, events that influence the return on the market portfolio, and through the pull of the market, they induce changes in the return on individual assets. On the

other hand, events that have impact on asset i but no effect on the other assets. Hence, the total risk of asset i can be decomposed as

$$\sigma_{R_i}^2 = \beta_i^2 \sigma_{R_m}^2 + \sigma_{\varepsilon}^2 \tag{11}$$

that is, the variance of the return on asset i can be written as the sum of two terms. The first is called the systematic risk and accounts for that part of the variance that cannot be diversified away, while the second term is called the unsystematic risk and represents the part of the variance that disappears with diversification.

We now discuss the wavelet counterpart of the above risk measures[2]. The natural wavelet counterpart of the variance, *i.e.*, total risk, is the wavelet spectrum. As mentioned earlier, the wavelet spectrum for the return on asset i can be obtained as $|W_{R_i}(\tau, s)|^2$ and it measures variance in the time-frequency space. Regarding beta, the wavelet counterpart of (9) is given by

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[2]Another popular measure of risk is what is known as the Value-at-Risk (VaR) (see, for example, Jorion (1997)). The VaR is the minimal potential loss that a portfolio can suffer in the 100α per cent worst cases over a fixed time horizon. Suppose X is a random variable denoting the loss of a given portfolio. The VaR at the $1 - \alpha$ confidence level can be written as

$$VaR_\alpha(X) = \sup\{x \mid P[X \geqslant x] > \alpha\}$$

where $\sup\{x \mid A\}$ is the upper limit of x given event A. Since VaR has several limitations (see, for example, Tasche (2002)), an alternative measure has been proposed , the expected shortfall (also called conditional VaR). The expected shortfall is defined as the conditional expectation of loss when the loss exceeds the VaR level.

$$ES_\alpha(X) = E[X \mid X \geqslant VaR_\alpha(X)]$$

Under the Normal distribution, both the VaR and the expected shortfall are scalar multiples of the standard deviation (see, for example, Yamai and Yoshiba (2005)). Hence, both measures provide essentially the same information as the standard deviation.

$$\beta_i(\tau, s) = \frac{\Re\left(W_{R_i, R_m}(\tau, s)\right)}{\left|W_{R_m}(\tau, s)\right|^2} \tag{12}$$

where $\Re$ denotes the real part of the cross-wavelet spectrum that measures the contemporaneous covariance. Note that the wavelet beta is computed for each frequency around each moment in time and shares the time-frequency localization properties of wavelets. This means that the estimates of beta obtained at low frequencies use more data points than the estimates at high frequencies. In other words. the number of observations used at each time point depends on the frequency.

Additionally, one can assess the importance of systematic risk for explaining total risk of asset i. This can be done by computing the ratio between systematic risk and total risk, $\frac{\beta_i^2 \sigma_{R_m}^2}{\sigma_{R_i}^2}$, which corresponds to the well-known measure of fit , the R^2, for model (10). The wavelet R^2 can be computed as

$$R^2(\tau, s) = \frac{\beta_i(\tau, s)^2 \left|W_{R_m}(\tau, s)\right|^2}{\left|W_{R_i}(\tau, s)\right|^2} \tag{13}$$

In this way, it is possible to quantify the fit of model (10) in the time-frequency space and determine over which periods of time and frequencies the fit is higher. Naturally, $R^2(\tau, s)$ is between 0 and 1, where a value close to 0 can be interpreted as the systematic risk having a small contribution to total risk, while a value close 1 denotes a high importance of systematic risk in determining total variability.

4 The emerging markets case

To illustrate the above suggested measures, we assess the risk faced by an investor in emerging markets over the last twenty years. We use the Morgan Stanley Capital International (MSCI) all country world index and the MSCI emerging markets index taken from Thompson Financial Datastream. The

MSCI emerging markets index is a free float-adjusted market capitalization index and consists of the following 23 emerging market country indices: Argentina, Brazil, Chile, China, Colombia, Czech Republic, Egypt, Hungary, India, Indonesia, Israel, Korea, Malaysia, Mexico, Morocco, Peru, Philippines, Poland, Russia, South Africa, Taiwan, Thailand, and Turkey. The MSCI all country index is also a free float-adjusted market capitalization weighted index and consists of 46 country indices comprising the above 23 emerging market countries and 23 developed country indices. The developed market country indices included are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, the United Kingdom, and the United States. The data sample ranges from January 1988 to December 2008, whereas monthly returns are computed as the percentage change of the stock price indices considering end of month figures. The returns on the MSCI stock indices are expressed in US dollars, that is, we consider the case of an American investor investing in emerging markets. In Figures 1 and 2, we plot the monthly stock indices and the monthly returns, respectively. In Table 1, we report some descriptive statistics for both the world and emerging markets indices as well as for the individual emerging market countries. The results are in line with well-known facts about emerging markets, namely that average returns are higher, as well as volatility.

First, we focus on total risk as measured by the variance. In Figure 3, we present the wavelet spectrum for the return on the emerging markets index. Results on a country-by-country basis are presented in Figure 6[3] The results

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[3]In order to save space, we present the results for only the most important emerging market countries, namely, Brazil, China, India, Korea, Mexico, Russia, South Africa, and Taiwan. We also include the Turkish case, as it is particularly relevant for the analysis of the overall results. The weight of these 9 countries is around 75 per cent in the emerging markets index. The results for the remaining countries are available from the authors upon request.

are presented through a contour plot as there are three dimensions involved. The x-axis refers to time and the y-axis to frequency. To ease interpretation, the frequency is converted to time units (years). The contour plot uses a gray scale where darker areas correspond to higher values of the wavelet power spectrum. From the analysis of the wavelet spectrum several findings emerge. First, one can see that the volatility of monthly stock returns is concentrated at high frequencies, that is, the short-term fluctuations dictate the variance of the series. In fact, frequencies associated with movements longer than one year are almost negligible in terms of contribution to total variance. Besides the varying feature across frequencies, one can see that variance has also changed over time. In particular, one can clearly detect periods of higher volatility, namely around 1990, 1998, 2001, and 2008. The volatility at the end of the 1980s and beginning of the 1990s is related with the crisis involving several Asian countries, such as Taiwan and Indonesia. The highest volatility period is around 1998, when several crises occurred, starting with the East Asian crisis during 1997 and 1998, when Thailand, Malaysia, the Philippines, Indonesia, and Korea underwent severe financial and currency crises, and the Russian default in August 1998. The volatility in 2001 reflects the Turkish crisis, while the more recent episode is related with the subprime mortgage crisis and the subsequent liquidity squeeze in the US, which started in mid-2007 and sent shock waves around the world. Despite these episodes of changing variance over time, there is no evidence of a persistent upward or downward trend in the volatility in the emerging markets (see also Bekaert and Harvey (1997, 2003)). Indeed, as discussed by Bekaert and Harvey (2003), it is not clear from finance theory that volatility should increase or decrease when markets are liberalized. In fact, if on the one hand equity market liberalization may lead to higher volatility due to an increase of the importance of short-term fluctuations, on the other hand, one may expect lower volatility coming from long-term swings. However, our results suggest that no trend is present for either high or low frequencies.

To assess the systematic risk of stock market returns in emerging countries, we present in Figure 4 the wavelet beta for the emerging markets index considering as a proxy for the market portfolio the world index. The solid (dashed) line delimits the region where the wavelet beta is statistically higher (lower) than the overall beta, estimated to be 1.17, with a significance level of 5 per cent.[4]. For the sample as a whole, the beta is slightly higher than 1 (see Table 1) in line with the findings of Estrada (2000). However, one can find noteworthy variation in the results across frequencies and over time. First, the beta coefficient seems to be more stable over time at low frequencies and more time-varying at high frequencies. Second, at low frequencies the beta is around 1, while at high frequencies one can identify regions in the time-frequency space where the beta is near 3. The periods where the beta is highest include the Mexican crisis in 1994, the year 1998 when crisis hit several emerging markets, late 2005 through 2006 encompassing the Turkish crisis in the Spring of 2006, and the most recent period since mid-2007 with the awakening of a global financial crisis.

The importance of the systematic risk in explaining total risk in emerging markets can be assessed through Figure 5, where we plot the wavelet R^2. Once again, the time-frequency analysis can provide valuable insights. For instance, although the R^2 of equation (10) for the sample period as a whole is close to 0.5, meaning that the systematic part is as important as the unsystematic one for total variance (see Table 1), one can see that the value of R^2 changes considerably across frequencies and over time. A finding is that the importance of the systematic risk in emerging markets is relatively high and stable over time at low frequencies. The proportion of the total variance explained by the systematic component is around 80 per cent at frequencies associated with fluctuations that last longer than four years. In contrast, for higher frequencies we observe a time-varying influence of the systematic part. In particular, we have values around 30 per cent up to the

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[4]The critical values were obtained through a Monte Carlo simulation exercise.

14

mid-1990s followed by a relatively steady increase thereafter, though clearly interrupted during 2004 and at the beginning of 2007, attaining values near 80 per cent at the end of the sample. This finding supports the idea that the increase of the correlation between the emerging markets and world indices at the end of the 1990s highlighted by Bekaert and Harvey (2003) may be of a permanent nature.

5   Conclusions

Although most textbook models assume volatilities and covariances to be constant, it has long been acknowledged among both finance academics and practitioners that market risk varies over time. Besides taking into account such time-varying feature, the risk profile of an investor, in terms of investment horizon, makes it also crucial to assess risk at the frequency level. Naturally, a short-term investor is more interested in the risk associated with high frequencies whereas a long-term investor focuses on lower frequencies. This paper provides a new look into market risk measurement by resorting to wavelet analysis, as it allows one to evaluate the time and frequency-varying features within a unified framework. In particular, we derive the wavelet counterpart of well-known measures of market risk. We consider total risk, as measured by the variance of returns, the systematic risk, captured by the beta coefficient, and we provide the tools to assess the importance of systematic risk on total risk in the time-frequency space.

To illustrate the method, we consider the emerging markets case, which has received a great deal of attention in the literature over the last twenty years. As those countries have experienced a changing economic environment, it is particularly interesting to see how market risk has changed across frequencies and over time. We find that the variance of monthly returns is determined essentially by short-run fluctuations and that the volatility has changed over time. In particular, the periods of higher volatility are associated with several economic crises that hit the emerging markets. Regarding

the systematic risk, we find that the beta coefficient is relatively stable at low frequencies, presenting a value of around 1. In contrast, at higher frequencies, the beta coefficient varies considerably, attaining values as high as 3 in some economic episodes. Additionally, we assessed the importance of the systematic risk in explaining total risk in emerging markets. Again, we find noteworthy variation in the results across frequencies and over time. We conclude that the importance of systematic risk in emerging markets is relatively high and stable over time at low frequencies. At higher frequencies, the influence of the systematic part was relatively low before the mid-1990s, but increased gradually thereafter, attaining values also relatively high at the end of the sample. All of these results highlight the importance of considering time and frequency-varying features in risk assessment. Hence, wavelet analysis can be a valuable tool for obtaining additional insights that may influence risk-taking decisions.

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Table 1 - Descriptive statistics of monthly stock returns
(in percentage)

	Sample period	Mean	Standard deviation	Minimum	Maximum	Beta	R^2
World	Feb-88 up to Dec-08	0.41	4.30	-19.91	11.11	-	-
Emerging markets	Feb-88 up to Dec-08	0.90	6.90	-29.29	18.11	1.17	0.53
Argentina	Feb-88 up to Dec-08	2.21	16.30	-41.68	95.36	0.91	0.06
Brazil	Feb-88 up to Dec-08	2.29	15.71	-66.93	81.25	1.64	0.20
Chile	Feb-88 up to Dec-08	1.21	7.18	-29.11	21.55	0.75	0.20
China	Jan-93 up to Dec-08	0.13	11.09	-27.67	46.50	1.23	0.23
Colombia	Jan-93 up to Dec-08	1.24	9.55	-28.55	30.31	0.73	0.11
Czech Republic	Jan-95 up to Dec-08	1.27	8.44	-29.44	30.08	0.85	0.20
Egypt	Jan-95 up to Dec-08	1.51	9.64	-32.62	42.02	0.79	0.13
Hungary	Jan-95 up to Dec-08	1.45	10.59	-43.35	46.16	1.44	0.36
India	Jan-93 up to Dec-08	0.83	8.80	-28.56	22.00	0.93	0.21
Indonesia	Feb-88 up to Dec-08	1.46	15.17	-40.83	93.93	1.08	0.09
Israel	Jan-93 up to Dec-08	0.58	7.26	-18.89	26.95	0.97	0.33
Korea	Feb-88 up to Dec-08	0.78	11.23	-31.26	70.59	1.29	0.25
Malaysia	Feb-88 up to Dec-08	0.69	8.79	-30.31	49.95	0.90	0.20
Mexico	Feb-88 up to Dec-08	1.78	9.28	-34.26	28.93	1.15	0.29
Morocco	Jan-95 up to Dec-08	1.06	5.66	-15.20	23.93	0.26	0.04
Peru	Jan-93 up to Dec-08	1.49	9.41	-36.04	35.58	0.99	0.21
Philippines	Feb-88 up to Dec-08	0.65	9.48	-29.29	43.35	0.94	0.18
Poland	Jan-93 up to Dec-08	1.92	14.74	-34.94	118.29	1.60	0.22
Russia	Jan-95 up to Dec-08	2.33	17.23	-59.23	61.13	2.00	0.26
South Africa	Jan-93 up to Dec-08	0.92	8.07	-30.84	21.26	1.20	0.40
Taiwan	Feb-88 up to Dec-08	0.67	10.94	-33.67	46.44	0.96	0.14
Thailand	Feb-88 up to Dec-08	0.72	11.49	-34.05	43.18	1.32	0.25
Turkey	Feb-88 up to Dec-08	1.78	17.40	-41.24	72.30	1.38	0.12

Figure 1 - Monthly stock price indices
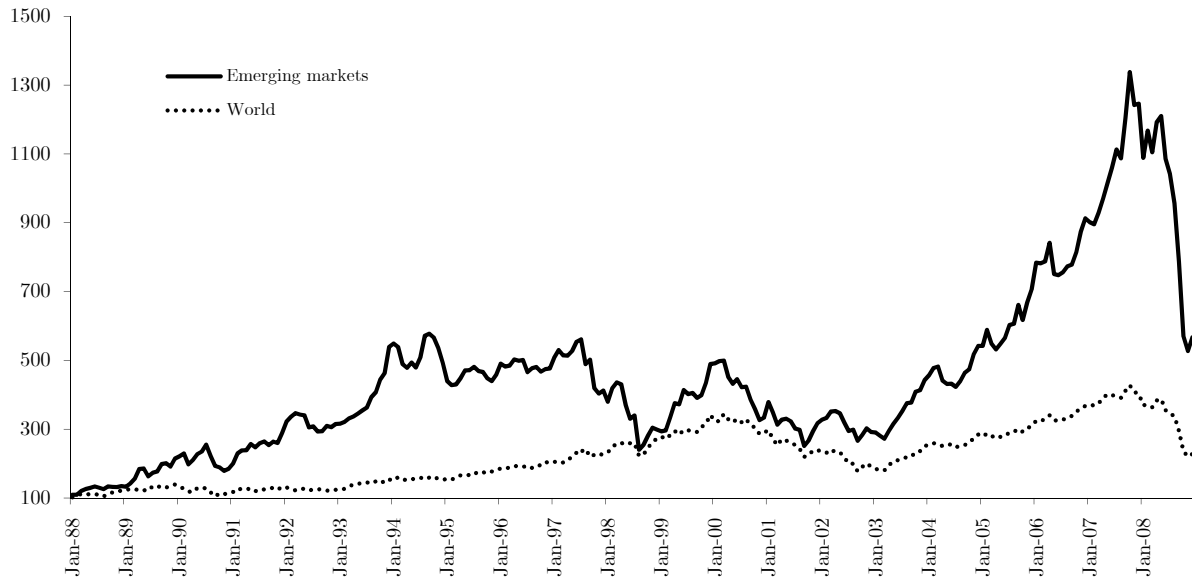
Figure 2 - Monthly stock returns
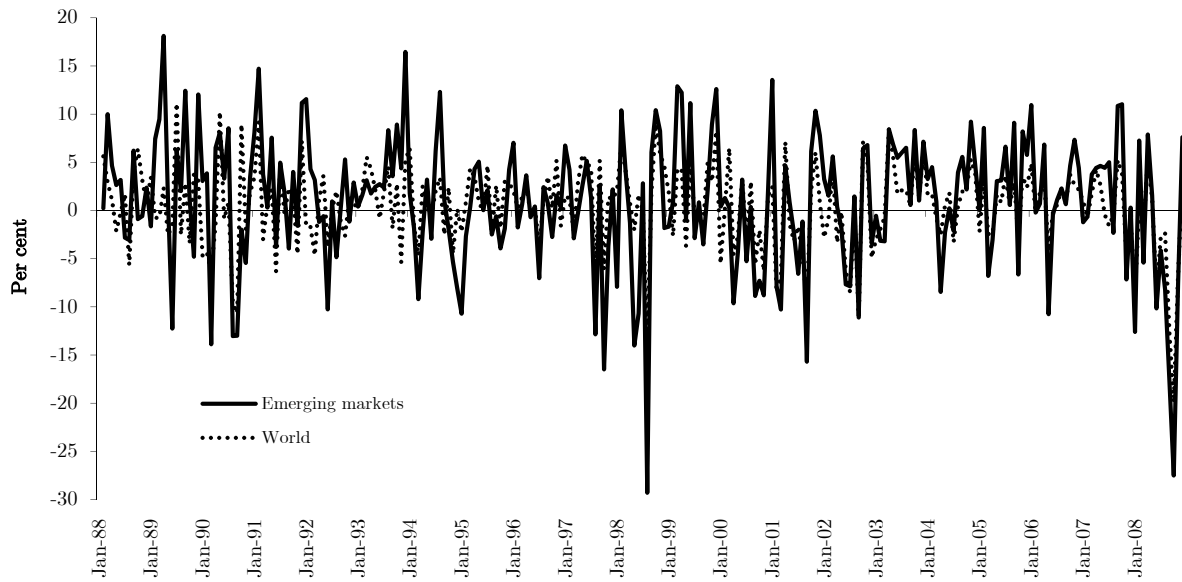
Figure 3 - Wavelet spectrum for the emerging markets returns
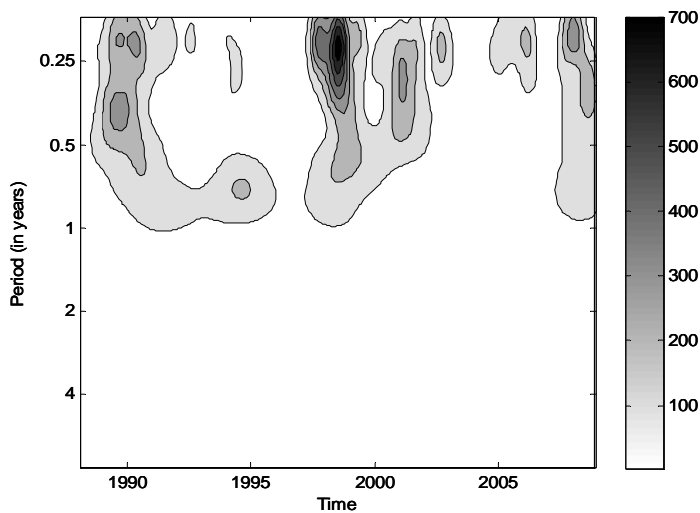
Note: Values of the wavelet spectrum presented in a gray scale.

Figure 4 - Wavelet beta for the emerging markets
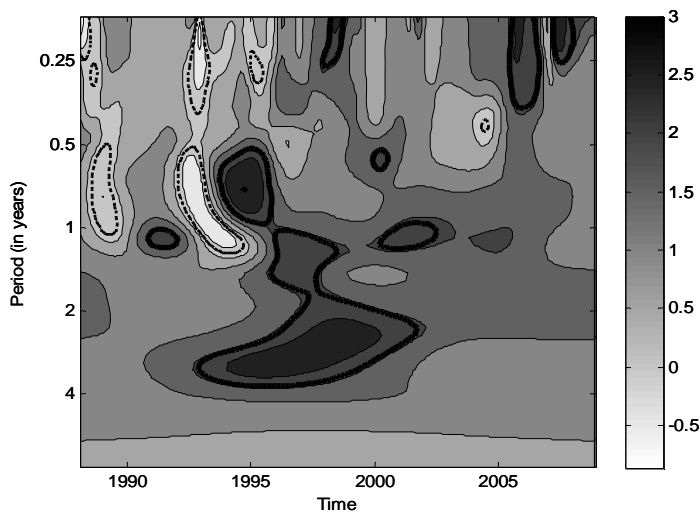
Note: Values of the wavelet beta presented in a gray scale. The solid (dashed) line delimits the region where the beta is statistically higher (lower) than the overall beta with a significance level of 5 per cent.

Figure 5 - Wavelet R^2 for the emerging markets
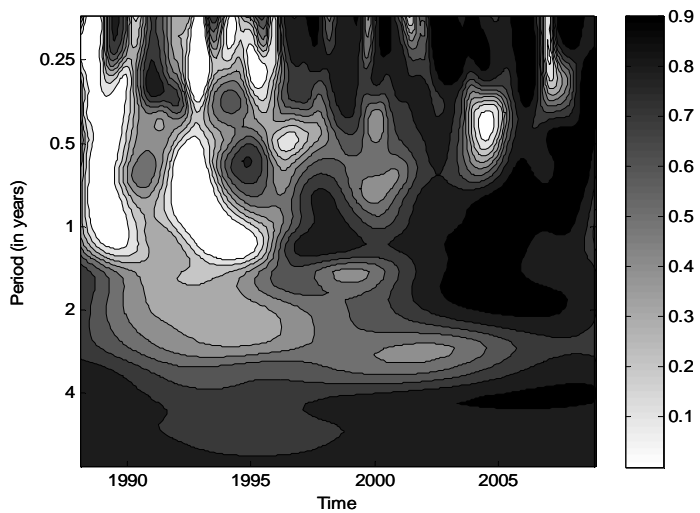
Note: Values of the R^2 (ranging between 0 and 1) presented in a gray scale.

Figure 6 - Results for several emerging market countries

Note: Values of the wavelet spectrum presented in a gray scale.

Note: Values of the wavelet beta presented in a gray scale. The solid (dashed) line delimits the region where the beta is statistically higher (lower) than the overall beta with a significance level of 5 per cent.

Note: Values of the R² (ranging between 0 and 1) presented in a gray scale.

Figure 6 - Results for several emerging market countries (continued)

Note: Values of the wavelet spectrum presented in a gray scale.

Note: Values of the wavelet beta presented in a gray scale. The solid (dashed) line delimits the region where the beta is statistically higher (lower) than the overall beta with a significance level of 5 per cent.

Note: Values of the R^2 (ranging between 0 and 1) presented in a gray scale.

Figure 6 - Results for several emerging market countries (continued)

Wavelet spectrum

Wavelet beta

Wavelet R^2

Note: Values of the wavelet spectrum presented in a gray scale.

Note: Values of the wavelet beta presented in a gray scale. The solid (dashed) line delimits the region where the beta is statistically higher (lower) than the overall beta with a significance level of 5 per cent.

Note: Values of the R^2 (ranging between 0 and 1) presented in a gray scale.

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