

**Comparison of Frontier Efficiency Methods:
An Application to the U.S. Life Insurance Industry**

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Abstract

The objective of this paper is to provide new information on the performance of efficiency estimation methods by applying a wide range of econometric and mathematical programming techniques to a sample of U.S. life insurers. Average efficiencies differ significantly across methods. The efficiency rankings are well-preserved among the econometric methods; but the rankings are less consistent between the econometric and mathematical programming methods and between the data envelopment analysis and free disposal hull techniques. Thus, the choice of estimation method can have a significant effect on the conclusions of an efficiency study. Most of the insurers in the sample display either increasing or decreasing returns to scale, and stock and mutual insurers are found to be equally efficient after controlling for firm size.

Key words. Efficiency estimation, stochastic frontiers, data envelopment analysis, free disposal hull, life insurance industry, organizational form.

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J. David Cummins and Hongmin Zi*

Two primary methodologies have been developed for measuring economic efficiency -- the econometric approach and the mathematical programming approach.¹ Both methodologies involve the estimation of “best practice” frontiers, with the efficiency of specific decision making units (DMUs) measured relative to the frontiers. The econometric approach specifies a functional form for the cost, profit, or production frontier. The methodology is stochastic; firms can be off the frontier because they are inefficient or because of random shocks or measurement errors. Thus, the cost function error term is hypothesized to consist of an inefficiency component and a purely random component. Efficiency is measured by separating the efficiency component from the overall error term. Some variants of the econometric approach require distributional assumptions for the components of the error terms, while others do not require distributional assumptions. By contrast, the mathematical programming approach places less structure on the frontier and is non-stochastic, i.e., any departure from the frontier is measured as inefficiency.²

The choice of estimation methodology has been controversial, with some researchers arguing for the econometric approach (e.g., Berger, 1993) and others for the mathematical programming approach (e.g., Seiford and Thrall, 1990). The econometric approach has been criticized for potentially confounding estimates of efficiency with specification errors. Mathematical programming, on the other hand, is non-parametric and thus less susceptible to specification errors but does not allow DMUs to deviate from the frontier due to purely random shocks. Advocates of the econometric approach disagree about whether distributional assumptions should be imposed on the error term and, if so, which distributions are most appropriate.³ Recent mathematical programming papers proposing the free disposal hull (FDH) methodology have argued that the FDH approach involves less arbitrary assumptions and provides a better fit to the data than data envelopment analysis (DEA) (e.g., Tulkens, 1993, Vanden Eeckaut, Tulkens, and Jamar, 1993).

The primary purpose of this paper is to provide new information on the performance of alternative efficiency estimation methods by applying a variety of estimation techniques to the same data set. The data set consists of 445 U.S. life insurers representing nearly 90 percent of industry assets over the period 1988-1992. We compare the efficiency scores and rank correlations among the methods, measure the consistency of the methods in identifying the DMUs in the most and least efficient quartiles, and correlate the efficiency scores with traditional performance measures such as expense ratios and returns on equity. The methods include DEA, with three returns-to-scale assumptions, FDH, and seven econometric methods.

Few efficiency studies have utilized more than one or two estimation techniques. A review of the literature on the efficiency of financial institutions found only four banking studies and two insurance studies, out of 130 studies surveyed, that compare the econometric and DEA approaches (Berger and Humphrey, 1997).⁴ Our paper is the only one of the six to compare more than one variant of each methodology. A few financial studies have compared two or three econometric methods (e.g., Bauer, Berger, and Humphrey, 1993, Hasan and Hunter, 1996, Berger and Hannan, 1997, and Berger and Mester, 1997) or two or three mathematical programming methods (e.g., DeBorger, Ferrier, and Kerstens, 1995). A more recent bank efficiency study by Bauer, et al. (1998) compares DEA with three econometric techniques. Although we have found no comprehensive surveys of applications to non-financial DMUs, the norm in the papers we have reviewed is similar, i.e., most studies use only one or two estimation methods.⁵ Thus, the present paper is one of the first two financial institutions studies and among the first studies in any industry to compare a wide variety of estimation techniques.

A second objective of this paper is to provide new information on efficiency in the U.S. life insurance industry by investigating two classic topics in industrial organization — economies of scale and the efficiency of alternative organizational forms. Studying efficiency in the life insurance industry is relevant because life insurers are among the most important financial institutions in the U.S. economy, managing about \$2 trillion in assets and employing more than 2 million people. The industry has recently encountered solvency problems, a wave of mergers and acquisitions, conversions of insurers from the

mutual to stock ownership form, and increasing competition from non-traditional sources such as banks, mutual funds, and securities brokers. Thus, additional information on life insurer efficiency should be valuable to regulators, managers, and shareholders.

Relatively few studies have been conducted on the cost efficiency of life insurers. Yuengert (1993) and Gardner and Grace (1993) applied econometric methods to cost efficiency estimation for U.S. life insurers, while Weiss (1986) estimated total factor productivity for one stock and one mutual firm. Each of these studies employed only one of the set of the available estimation methods. Fecher, et al. (1993) measure efficiency in the French life insurance industry using one mathematical programming and one econometric model. Fukuyama (1997) estimated productive efficiency and productivity change in the Japanese life insurance industry using DEA. Ours is the first insurance study to compare a wide variety of methods and also the first to estimate cost efficiency using the DEA and FDH methods.

The rest of this paper is organized as follows. Section II provides an overview of the econometric and mathematical programming methodologies. Section III discusses the measurement of inputs and outputs in life insurance. Section IV presents the empirical results, and section V concludes.

1. Methodology

This section provides an overview of the econometric and mathematical programming methodologies. The reader is referred to Greene (1993), Charnes, et al. (1994), and the other cited references for more details.

1.1. The Econometric Approach

The primary advantage of the econometric approach is its ability to accommodate random noise in efficiency estimation. To separate random error from inefficiency, the cost function is typically specified with two error components:

$$(1) \quad \ln C_i = \ln C(p_i, y_i) + u_i + v_i$$

for $i = 1, \dots, N$, where C_i = observed total costs for firm i , $\ln C(p, y)$ = the log cost function, p_i = a vector of input prices, y_i = a vector of output quantities, u_i = an error term ($u_i \geq 0$) that captures cost inefficiency, and

v_i = a random error term ($-\infty < v_i < \infty$) distributed independently of u_i . An extensive econometric literature exists on the estimation of equation (1), and the most important of the methods are applied in this study. For the econometric methods in this study, we use the standard translog cost function specification.

The general procedure for estimating efficiency using equation (1) is to estimate the cost function and $w_i = u_i + v_i$ and then to calculate efficiency for each observation in the sample as the conditional expectation $E(\exp(-u_i) | w_i)$, providing an estimate of the ratio of frontier costs to actual costs. If distributional assumptions are imposed on the error terms, the approach involves finding the density function $h(w_i)$ of w_i and the joint density $f(u_i, w_i)$ and then obtaining an expression for the conditional mean of $\exp(-u_i)$ based on the distribution $f_u(u_i | w_i)$. The most common distributional assumptions are a normal distribution for v_i and an exponential, truncated normal (usually the half-normal), or gamma distribution for u_i . The estimated w_i and estimated distributional parameters are substituted into the appropriate form of $E(\exp(-u_i) | w_i)$ to obtain an estimate of inefficiency for each observation in the data set.

The use of mode 0 distributions such as the exponential and half-normal to model inefficiency has been criticized for imposing the restriction that most firms are clustered near full efficiency, with higher degrees of inefficiency being increasingly unlikely. However, it is not necessarily true that the half normal or exponential distributions place the majority of firms “near” the mode. In the half normal, the amount of probability mass in a fixed interval to the right of 0 is decreasing in σ_u and can be quite small for reasonable values of the interval size defined as “near full efficiency” and σ_u .⁶ Thus, although the monotonicity of the half normal and exponential may be a limitation for some data sets, using one of these distributions does not arbitrarily confine a high proportion of the firms to near full efficiency. We provide information on this issue by analyzing the appropriateness of monotonicity for our sample.

We estimate equation (1) using maximum likelihood estimation (MLE) under both the normal/half-normal and normal/exponential distributional assumptions.⁷ Two versions are estimated for each set of distributional assumptions -- equation (1) is estimated separately for each year of the sample period; and a panel version of equation (1) is estimated where the cost function parameters are constrained to be constant

over the sample period. Both formulations have merits. Allowing the parameters to vary by year captures any changes of cost technology over time, while the panel version provides more degrees of freedom.

To test the sensitivity of the results to the monotonicity of the half normal and exponential assumptions, we also estimate equation (1) using the more general gamma distribution for u_i , retaining the assumption of normality for v_i . The disadvantage of the normal/gamma model is that a closed form for the likelihood function of the composed error term, $u_i + v_i$, is not available unless the shape parameter of the gamma has an integer value (an Erlang form). To avoid arbitrarily restricting the shape parameter, we estimate this model using the modified ordinary least squares method (MOLS) suggested by Greene (1990).

With panel data, it is possible to avoid imposing distributional assumptions on the error components.⁸ In the ‘distribution free’ method of Schmidt and Sickles (1984) and Berger (1993), the inefficiency error term is assumed to be constant over time, i.e., the following specification is used:

$$(2) \quad \ln C_{it} = \ln C(p_{it}, y_{it}) + u_i + v_{it}$$

No distributional assumptions are imposed on u_i or v_{it} . Rather, an estimate of the efficiency is obtained by averaging the estimated overall error, $w_{it} = u_i + v_{it}$, over the sample period on the assumption that the random error v_{it} will average out over time. We estimate equation (2) using generalized least squares (GLS), as in Schmidt and Sickles (1984), and also estimate an ordinary least squares (OLS) version of equation (2) where the parameter vector is allowed to vary over the sample period, as in Berger (1993). For the Schmidt and Sickles and Berger methods, cost efficiency is estimated for each firm as:

$$(3) \quad E[u_i | w_{i1} \dots w_{iTN}] = \exp(\min_i(\bar{w}_i) - \bar{w}_i)$$

where \bar{w}_i denotes the average over the sample period of the residuals w_{it} for firm i , and $\min_i(\bar{w}_i)$ is the minimum average error term for the firms in the sample.

1.2. The Mathematical Programming Approach

Efficiency measurement also has been formulated in a mathematical programming framework, usually called data envelopment analysis (DEA), first by Charnes, Cooper, and Rhodes (1978), and subsequently

modified by Banker, Charnes, and Cooper (1984), and Byrnes, Färe, and Grosskopf (1984), among others. Because DEA focuses primarily on the technological aspects of production correspondences, it can be used to estimate technical and scale efficiency without requiring estimates of input and output prices. Thus, this approach has been used extensively in the regulated sector (e.g., Banker, Conrad, and Strauss, 1986) and the non-profit sector (e.g., Lewin, Morey, and Cook, 1982). If estimates of input prices are available, cost efficiency also can be measured using DEA (e.g., Aly, et al., 1990, and Ferrier and Lovell, 1990).

This study applies the three conventional DEA models — the constant (CRS), variable (VRS), and non-increasing returns to scale (NIRS) models — to estimate cost efficiency. We also use DEA to estimate technical efficiency and use the CRS, VRS, and NIRS estimates of technical efficiency to measure scale economies for each firm in the sample (Aly, et al., 1990). Finally, we compute efficiency using the recently developed free-disposal hull (FDH) model (Tulkens, 1993), which relaxes the convexity assumption of the VRS frontier model.

2. Outputs, Inputs, and Input Prices

Defining outputs of an insurance firm has been a challenging task. Most of extant life insurance cost studies use premiums as proxies for outputs (e.g., Grace and Timme, 1992, Gardner and Grace, 1993, and Fecher, et al., 1993). However, premiums do not represent output quantity but rather are equal to revenue (price times quantity) (Yuengert, 1993).

The outputs of life insurers may be measured by the services they provide to customers. In general, life insurers provides two principal services: risk bearing/risk pooling services and intermediation services. Life insurers collect premiums and annuity considerations from customers and redistribute most of the funds to those policyholders who sustain losses (the risk bearing/risk pooling service). Funds are collected in advance of paying benefits and invested in assets until claims are paid (the intermediation service).

Incurred benefit payments are used here to proxy for the risk bearing/pooling services of a life insurance firm because benefit payments represent the delivery of contingent dollars to policyholders (Cummins and Weiss, 1993). Incurred benefits are further categorized into ordinary life insurance (Y_1), group

life insurance (Y_2), individual annuities (Y_3), group annuities (Y_4), and accident and health insurance (Y_5) to allow for the different characteristics of the major product categories in the life insurance industry. The intermediation service of an insurance firm is proxied by additions-to-reserves (Y_6), which denote reserves set up for new business, new deposit funds, and new reserves set up for existing policies due to premium payments and investment returns.⁹ All outputs are deflated by the CPI to the base year 1988.

Three inputs are used in this study: labor (X_1), financial capital (X_2), and materials (X_3). The price for labor input (P_1) is obtained by the Divisia index based on U.S. Department of Labor data on average weekly wages for employees and agents working in the life insurance sector (SIC 6311). A premium weighted index is used, with average weekly wages by state for SIC 6311 weighted by the proportion of premiums written by the insurer in each state. Financial capital is included as an input for the risk-pooling/risk-bearing function because insurers hold capital to back their promise to pay benefits if losses are larger than expected or investment returns fall below expectations.¹⁰ The price of capital (P_2) is measured as a three-year moving average of the ratio of net income to equity capital. The price of the materials input (P_3) is calculated by the Divisia index of the deflators for its components, representing the major non-labor inputs purchased by insurers.¹¹ Thus, all three input prices vary over time and by insurer.

The insurance financial data were obtained from the regulatory annual statements filed by insurers as reported on the National Association of Insurance Commissioners (NAIC) life insurance data tapes for 1988-1992. Groups of affiliated insurers under common ownership were treated as decision making units, along with unaffiliated single insurers. Recognizing the group as the DMU minimizes distortions arising from intra-group transactions. In order to use methodologies such as the distribution free approach that require balanced pooled samples, insurers were included in the sample if data are available in every year during the period 1988-1992. Very small firms (assets < \$ 10 million in 1990) and firms specializing in reinsurance are excluded from the sample. The final sample consists of 445 firms each year, accounting for nearly 90 percent of industry assets. Descriptive statistics for the sample are presented in Table 1.

3. Empirical Results

This section first reports the overall efficiency results based on the econometric and mathematical programming methodologies and then presents our tests of economies of scale and the relative efficiency of stock and mutual insurers. The section concludes with an analysis of the appropriateness of assuming a monotonically distributed inefficiency error term for the firms in our sample.

3.1. Comparison of Efficiency Estimates

3.1.1. Average Efficiencies. The average cost efficiency estimates using the econometric methods are presented in the first seven columns of Table 2.¹² The first five columns show the results based on the econometric models which impose distributional assumptions on the error terms. There is no noticeable difference between the average cost efficiency estimates from the year-by-year and panel versions of the normal/half-normal MLE models (labeled Half and Half-P, respectively, in the table) nor between the year-by-year and panel estimates from the normal/exponential models (Exp and Exp-P in the table). The average efficiencies are lower with the normal/half-normal models (averaging 0.61) than with the normal/exponential models (which average 0.71). Still higher efficiencies are obtained with the gamma model, averaging 0.85 for the sample period.

The distribution free models, which constrain efficiency to be equal over the sample period, provide considerably lower efficiency estimates than the econometric models which impose distributional assumptions on the error terms. The DFA estimates average 0.44 and 0.46, respectively, for the methods of Schmidt and Sickles (1984) (labeled DFA-SS) and Berger (1993) (labeled DFA-B). This suggests that it may not be appropriate to constrain efficiency to equality over the sample period for this data set. Another possible explanation for the low average efficiencies of the DFA methods in comparison with the econometric models that use distributional assumptions for the error terms is that the time period we use in averaging the residuals is relatively short (5 years). Thus, our DFA efficiency estimates may contain a significant amount of random error. An overall conclusion is that the choice of distributional assumptions and estimation technique, particularly the decision to allow inefficiencies to vary by year, has a significant effect on the average

efficiency estimates using the econometric approach.

Our results with the normal/half-normal and normal/exponential models are consistent with those of Yuengert's (1993) normal/half-normal model, which also ranged from 0.6 to about 0.7, depending on firm size class (he did not estimate the normal/exponential model). Our efficiency estimates from the normal/gamma model are higher than Yuengert's normal/gamma estimates which ranged from approximately 0.50 to 0.65. However, he arbitrarily set the gamma shape parameter to 2, whereas we use Greene's MOLS method to avoid constraining this parameter.¹³ The results are also consistent with those of Gardner and Grace (1993), who find average efficiency of 0.47 using DFA-B over the period 1985-1990.

Of the results using the four different mathematical programming models (DEA-CRS, DEA-NIRS, DEA-VRS, and FDH), we report the efficiency measures for DEA-VRS and FDH, because the underlying assumptions for these models are the least stringent.¹⁴ These results are presented in the last two columns of Table 2. The DEA-VRS and FDH models yield very different results. The average levels of cost efficiency for FDH are the highest (0.90-0.92) of all methods used in this study, including the econometric models. On the other hand, the DEA-VRS efficiency scores are much lower than most of the other models (0.46 on average) and about the same as the distribution free econometric estimates (DFA-B and DFA-SS).

The low level of cost efficiency for DEA is mainly due to technical inefficiency rather than allocative inefficiency.¹⁵ FDH leads to the opposite finding, but the FDH technical and allocative efficiency measures are fairly close to each other.¹⁶ The large difference between the DEA-VRS and FDH estimates is easily explained by the fact that in the FDH model, each firm is compared only to actual observations, leading to many "self-efficient" firms, while in the DEA-VRS model, each firm is compared to a convex combination of efficient firms and thus has a much greater chance of being dominated by sets of efficient firms.¹⁷

Our finding that FDH yields substantially higher efficiency estimates than DEA is consistent with prior research on other industries (e.g., Vanden Eeckaut, Tulkens, and Jamar, 1993, DeBorger and Kerstens, 1996). Thus, it is increasingly clear that researchers face an important choice not only between the econometric and mathematical programming approaches but also as to whether convexity should be imposed when using

mathematical programming.

It is not surprising that the DEA-VRS estimates are lower than the normal/half-normal, normal/exponential, and normal/gamma estimates because the latter models allow insurers to depart from the frontier due to random error as well as inefficiency, whereas DEA measures any departure from the frontier as inefficiency. On the other hand, it is somewhat surprising that the cost efficiency measures for DEA-VRS are very close to those for the distribution-free econometric models. This again suggests that our DFA estimates may contain a significant amount of random error.

3.1.2. Consistency of Scores Across Methods. The results in Table 2 show that the choice of methodology has an important impact on the estimated average efficiency scores. However, for many purposes, such as public policy or managerial decision making, it is not so much the absolute values of the scores that matter but rather the ranking of DMUs in terms of efficiency. For example, to evaluate the potential effect of mergers and acquisitions or to rate the effectiveness of alternative underwriting systems, human resource policies, etc., it is useful to know the efficiency of DMUs relative to their peers.

We explore the consistency of the models in ranking insurers by presenting pairwise Spearman rank correlation coefficients of the 5 year average cost efficiencies for the insurers in the sample. The correlation coefficients, presented in Table 3, show that the econometric models are highly consistent in ranking insurers according to their efficiency, with pairwise correlation coefficients no lower than 96 percent. Thus, even though the distribution-free methods (DFA-B and DFA-SS) produce lower efficiency scores than the econometric models that use distributional assumptions, they produce comparable efficiency rankings. This finding is consistent with Berger and Mester (1997), who find correlations of about 0.99 between the normal/half-normal and DFA estimates for a sample of banks.

The rank correlations between the econometric models and the mathematical programming models are much lower, in the range of 50 to 60 percent. Thus, the choice between the econometric and mathematical programming approaches is important if one is interested in ranking DMUs. The rank correlation coefficient *between* the two different programming models is also relatively low (67 percent),

mainly due to the fact that many firms in the FDH model are self-efficient, again emphasizing the importance of the convexity assumption.

Our ranking results are consistent with those of DeBorger and Kerstens' (1996) study of Belgian municipalities. They report rank correlations between the normal/half normal and DEA efficiency scores of about 0.83 and rank correlations between the normal/half normal and FDH scores of about 0.60. The rank correlation between DEA and FDH in their study is 0.66, similar to our value of 0.67.¹⁸ In Hjalmarsson, Kumbhakar, and Heshmati's (1996) study of Colombian cement plants, the rank correlations between efficiency scores based on their normal/half-normal model most directly comparable with ours and their DEA scores are also similar to our correlations, ranging from 0.65 to 0.73. On balance, our results as well as the cited studies suggest that alternative econometric models produce efficiency rankings that are highly correlated; but correlations tend to be significantly lower between econometric and mathematical programming results and between DEA and FDH results.

Another consistency measure is the degree to which the methods to identify the same firms as being in the highest and lowest efficiency groups. Categorizing firms as "most" and "least" efficient is important for identifying best and worst managerial practices in an industry (Bauer, et al., 1998). We measure the consistency of the methods in identifying the most and least efficient firms by analyzing the pairwise agreement between methods in placing firms in the highest and lowest efficiency quartiles based on the five-year average efficiency scores. The results, presented in Table 4, show that the econometric methods are reasonably consistent in identifying the most efficient firms, with pairwise agreement statistics mostly in the 85 to 90 percent range. The degrees of agreement between the econometric methods and DEA are much lower, between 50 and 52 percent. FDH is not informative in this comparison because 235 of the 445 DMUs are measured as self efficient over the entire sample period. The methods in general are somewhat more consistent in identifying the least efficient firms than they are in identifying the most efficient firms. Again, the degree of agreement is relatively high among the econometric methods and considerably lower between the econometric methods and the mathematical programming methods and also low between DEA and FDH. Thus, the choice of method can be important in identifying best and worst practices in an industry.

3.1.3. Efficiency and Firm Size. To investigate the relationship between efficiency and firm size, we present the average cost efficiency measures classified by asset size in Table 5 as well as the correlation coefficients between the efficiency scores and assets.¹⁹ There is a positive relationship between size and efficiency for all of the methodologies. The correlation coefficients between size and efficiency are statistically significant at the 10 percent level or better for all methods except the gamma model. The highest correlation is for DEA-VRS, which has a correlation coefficient with size more than three times larger than any other method. Thus, conclusions about the size-efficiency relationship also can be affected by methodological choice.²⁰

3.1.4. Efficiency and Conventional Performance Measures. To provide additional information on the reasonableness of the methods, we also correlate the five-year average efficiency scores with two conventional performance measures used in the insurance industry -- the ratio of expenses to premiums and the ratio of net income plus benefits to equity. The latter ratio is used because benefits incurred, an important output for insurers, is subtracted from revenues to obtain net income. The results, shown in Table 6, indicate that nearly all of the methods produce efficiency scores that are significantly correlated with the conventional performance measures. The only exceptions are the two distribution-free methods, which are not significantly correlated with the expense to premium ratio. The highest correlation with the expense to premium ratio is provided by the FDH estimates. VRS has the highest and FDH the second-highest correlations with the returns plus benefits to equity ratio, and the exponential model performs relatively well among the econometric methods.

3.2. Economies of Scale

As mentioned above, the life insurance industry is currently experiencing a wave of mergers and acquisitions, including acquisitions of medium size regional companies by foreign and domestic financial services conglomerates as well as “mega-mergers” among industry giants. Because mergers and acquisitions require the approval of state insurance commissioners and are often rationalized on efficiency grounds, the issue of economies of scale is of significant relevance.

We estimate scale economies using the DEA approach (see Aly, et al., 1990). The scale economy results, presented in Table 7, reveal that the vast majority of firms in the industry are operating at either increasing or decreasing returns to scale.²¹ Only about 6 percent of the 445 firms in our sample are attaining the economic ideal of operating at constant returns to scale. About 63 percent of firms, most with assets of less than \$300 million, are operating in the range of increasing returns to scale. Thus, in general, mergers of firms with less than \$300 million of assets have the potential to reduce production costs in the industry. Mergers of firms with more than \$1 billion in assets appear to be much more difficult to justify on cost grounds because most firms in this range are operating at decreasing returns to scale. Stock insurers are more likely to be operating at increasing returns to scale than mutuals, mainly due to the size skewness within the two organizational forms — stocks are predominantly small firms and mutuals are predominantly large.²² Stocks and mutuals are discussed in more detail in the following section.

3.3. Stocks and Mutuals

Many studies have investigated the hypothesis that utility-maximizing managers who are not owners of the firm may pursue their own interests and engage in "expense preference behavior," especially in regulated industries where different organizational forms, such as stocks and mutuals, coexist (see, for example, Akella and Greenbaum, 1988, and Mester, 1989). The expense preference hypothesis predicts that mutuals will have higher costs than stocks because the mutual form of ownership affords owners less effective mechanisms for controlling and disciplining managers than the stock ownership form.

A somewhat more sophisticated agency theoretic hypothesis is that firms with alternative organizational forms are sorted into market segments where they have comparative advantages in dealing with various types of principal-agent problems (see Mayers and Smith, 1981). For example, the stock form of ownership is expected to be more effective than the mutual form in controlling owner-manager conflicts because the stock ownership form provides more effective mechanisms for controlling management. On the other hand, the mutual form of ownership is expected to be more effective in controlling owner-customer conflicts because the ownership and customer functions are merged in the mutual ownership form. This

analysis suggests that the alternative organizational forms may be equally efficient in the market segments where they have respective advantages. We refer to this hypothesis as the “efficient sorting” hypothesis.

The insurance industry provides a particularly interesting environment for studying organizational form because stock and mutual insurers have coexisted in the industry for many decades. We conduct empirical tests to provide evidence on the expense preference and efficient sorting hypotheses in the life insurance industry. Recall that the expense preference hypothesis predicts that mutuals will be less efficient than stocks, whereas the efficient sorting hypothesis implies that mutuals and stocks are likely to be equally efficient.

The average cost efficiencies of stock and mutual insurers are presented in Table 8. The top panel of the table, based on the full sample, shows that mutuals have efficiency scores that are higher than those of stocks under every estimation methodology. Although this seems to contradict the expense preference hypothesis, the result may be due to size effects because mutuals are larger on average than stock insurers. For example, the mean assets for stocks in our sample is \$1.78 billion and the median is \$119.5 million, whereas the mean and median assets for mutuals are \$6.91 billion and \$472 million, respectively.

To control for the effects of size, we choose a size-stratified random sample of stock insurers that matches the size distribution of mutuals, where size is based on average total assets.²³ This procedure yields matched samples of 95 stock insurers and 95 mutuals. The mean and median efficiencies for the insurers in the matched samples are shown in the middle and lower panels of Table 8. Although mutual firms still show slightly higher efficiencies, the differences in cost efficiencies seem negligible between the matched samples. We test for efficiency differences between stocks and mutuals by conducting an analysis of variance (ANOVA) and three non-parametric tests -- the Wilcoxon, Van der Waerden, and Savage tests.²⁴ On the basis of the tests, we generally cannot reject the hypothesis that the cost efficiency of stock and mutual insurers have the same mean (for ANOVA) or the same distribution (for the other three tests) at the 5 percent significance level.²⁵ Moreover, all of the Z-statistics from the non-parametric tests (except the Wilcoxon test on DEA-VRS) exhibit negative signs, suggesting that stock insurer efficiency is no greater than that of

mutuals. The results therefore are not consistent with the expense preference hypothesis but rather are consistent with the efficient sorting hypothesis.

The hypothesis test results are stronger for the mathematical programming methods than for the econometric methods. For example, whereas the results as a whole imply a clear failure to reject the hypothesis that mutuals and stocks are equally efficient after controlling for size, the results might appear ambiguous if one were to rely solely on the normal/half normal panel (Half-P) model. Thus, our results suggest that more than one methodology should be used in analyzing efficiency, unless there is a strong theoretical rationale for preferring a particular method or class of methods for a given data set.

3.4. Monotonicity of the Inefficiency Error Distribution

As suggested above, the normal/half normal and normal/exponential models have been criticized because of the possibility that they may inappropriately impose monotonicity on the inefficiency component of the error term in equation (1). Following the approach in Berger (1993), we investigate the appropriateness of the monotonicity assumption by examining the five-year average residuals from the DFA-B model.²⁶ The empirical probability density function of the five-year average residuals is shown in Figure 1 along with a fitted gamma density function obtained by maximum likelihood estimation.²⁷

The non-zero mode of the distribution of average residuals shown in Figure 1 does **not** necessarily imply that monotonicity is inappropriate for our data set. The reason is that the plotted DFA residuals are obtained as the five-year averages of the cost function residuals for each firm and thus (up to division by a constant) represent the convolution of the five individual-year inefficiency error terms (plus whatever component of the white noise term is not eliminated by averaging). Under the assumptions that the probability distribution of the inefficiency error is stationary over time and that the random error averages out over the sample period, the inefficiency term for each year would be gamma distributed with shape parameter equal to $4.1/5$ or 0.81 and scale parameter of $4.8/5 = 0.97$.²⁸ A gamma distribution with a shape parameter of 0.81 has a mode at zero and a shape similar to that of the exponential distribution, which of course is gamma distributed with shape parameter equal to 1 . Thus, neither the monotonicity assumption nor the exponential

distributional assumption seems to be patently inappropriate for our data set.

Checking whether the five year average residuals would have a shape similar to the empirical density function in Figure 1 if the inefficiency residual were distributed as a half-normal is more difficult because the convolution of half-normals does not have a closed form expression. To test the reasonableness of the half-normal, we simulate the convolution of an error term consisting of normal and half-normal components. We simulate one million sequences of five draws from the normal and half normal, summing and averaging the results in each case to simulate the results of applying DFA.²⁹ The resulting distribution has a shape similar to that of the empirical probability density function of the averaged residuals from the DFA methodology.³⁰ (Even though the half-normal is monotonic with a mode at zero, sums of half normals have a non-zero mode and approach symmetry as the number of half normals in the sum increases.) Therefore, at least for life insurers, the monotonicity assumption does not appear to be inappropriate. Nor does the monotonicity assumption confine a high proportion of firms to near full efficiency. For example, only 2.2 percent of firms are more than 90 percent efficient based on our normal/half-normal panel (Half-P) model.

4. Summary and Conclusions

The principal objective of this study is to provide new information on the effects of estimation method on frontier cost efficiency estimates by measuring cost efficiency for the same data set using a variety of econometric and mathematical programming techniques. A secondary objective is to analyze some classic industrial organization issues in the life insurance industry. We evaluate the estimation methods using four primary criteria — average efficiency scores, rank correlations of efficiency scores, the consistency of methods in identifying best and worst practice firms, and the correlation of efficiency scores with conventional performance measures. Two classic industrial organization issues are investigated — economies of scale and the relative efficiency of the stock and mutual insurers. Finally, we provide evidence on the appropriateness of imposing monotonicity on the inefficiency error term in econometric models.

The findings indicate that the choice of efficiency estimation method can make a significant difference on several important dimensions. Average efficiencies are significantly higher for the econometric models

that use distributional assumptions than for the distribution-free econometric models or DEA. DEA estimates are expected to be lower in general than econometric estimates because DEA measures all departures from the frontier as inefficiency, whereas the econometric techniques separate inefficiency from random noise. The comparability of our DEA and DFA estimates may be due to the relatively short time period we use in obtaining the DFA results. The highest average efficiency estimates are provided by the FDH technique, because it identifies many firms as self-efficient.

The efficiency rankings for the DMUs in the sample are well-preserved within the set of econometric methods. The rankings are less well-preserved between the econometric and mathematical programming methods and likewise between the DEA and FDH methods. The performance of methods in identifying firms in the highest and lowest efficiency quartiles leads to similar conclusions. Thus, the choice of method also matters if one is interested in ranking DMUs or identifying best and worst practices.

The distributional assumption imposed on the error term in the econometric models makes little difference with our data. Of much greater importance is the choice between econometric and mathematical programming methods, on the one hand, and the decision of whether to impose the convexity assumption in mathematical programming, on the other. Both the econometric and mathematical programming efficiency scores are significantly correlated with conventional performance measures, but the correlations tend to be somewhat higher for the mathematical programming methods.

The economies of scale estimates indicate that firms with less than \$300 million in assets predominantly exhibit increasing returns to scale, while firms with more than \$1 billion in assets tend to exhibit decreasing returns to scale. Thus, consolidation among smaller firms has the potential to reduce costs in the industry, but mega-mergers appear difficult to justify on cost grounds. We find no evidence that mutual insurers are less efficient than stock insurers, contrary to the expense preference hypothesis but consistent with the hypothesis that alternative organizational forms are sorted into market segments where they have comparative advantages. Finally, we conclude that the monotonicity assumption appears to be appropriate for the inefficiency component of our cost function error terms.

The results suggest that the controversy regarding distributional error term assumptions in the econometric approach may be a bit of a red herring, but this finding needs to be tested for robustness using other data sets. Our analysis confirms prior findings that the data envelopment analysis (DEA) and the free disposal hull (FDH) mathematical programming methodologies tend to give significantly different results. Thus, the appropriateness of the convexity assumption in mathematical programming needs further investigation. Although our tests of economic hypotheses are generally robust to the choice of estimation methodology, it is probably advisable in most cases to use more than one methodology when analyzing efficiency to ensure that the findings are not being driven by specification errors.

NOTES

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¹A review of the two approaches appears in Lovell (1993). More details on the econometric approach appear in Greene (1993) and DEA is discussed further in Charnes, et al. (1994).

²Stochastic versions of mathematical programming efficiency models have been developed (for a discussion, see Lovell, 1993). These models, which are used much less frequently than the non-stochastic models, are not considered in this paper.

³Alternative distributional assumptions are discussed in Greene (1993). A “distribution free” approach is developed in Schmidt and Sickles (1984) and Berger (1993).

⁴The studies, other than this paper, surveyed in Berger and Humphrey (1997) that compare the econometric and DEA approaches are Ferrier and Lovell (1990), Giokas (1991), Fecher, et al. (1993), Eisenbeis, Ferrier, and Kwan (1997), and Resti (1997).

⁵Studies that compare more than two methods include DeBorger and Kerstens (1996) and Hjalmarsson, Kumbhakar, and Heshmati (1996).

⁶Likewise, in the exponential, the amount of probability mass near the origin can be large or small depending upon the exponential parameter value.

⁷Linear homogeneity in input prices and the standard symmetry conditions are imposed in estimating the cost function.. The likelihood function for (1) is found in Greene (1993). For an application of this model to the insurance industry, see Cummins and Weiss (1993).

⁸A method which avoids distributional assumptions for cross-sectional data is the ‘thick frontier’ approach (TFA) (see, for example, Bauer, Berger, and Humphrey, 1993). We do not use this method because our sample is not large enough to permit us to obtain comparable point estimates of efficiency for individual DMUs. For a banking application of TFA that provides point estimates see Bauer, et al. (1998).

⁹Yuengert (1993) was the first to use additions-to-reserves as an output measure for life insurers.

¹⁰The value of physical capital held by insurers is small relative to the other input categories. Consequently, physical capital is included in the materials category.

¹¹Eleven component indices were used to calculate the business services price: GNP implicit price deflator for communications (for advertising), the CPI for reading material (for books, printing, and stationery), the GNP deflator for wholesale trade (bureau and association fees), the CPI for legal services (legal fees), the implicit price deflator for capital stock (capital equipment such as computers), the GNP deflator for business services (accounting fees, claims settlement fees, etc.), the CPI index for medical care (medical exam fees), the fixed weight price index for insurance (insurance purchased), the CPI for food consumed away from home (travel expenses), the GNP deflator for tenant rental of nonfarm dwellings (rental expenses), and the overall CPI (for miscellaneous expenses).

¹²We initially estimated the models that incorporate the half normal assumption using the more general truncated normal model, which allows a mode greater than zero. However, using likelihood ratio tests, we were unable to reject the hypothesis that the value of the mode parameter was equal to zero. Thus, the analysis presented here uses the half normal rather than the truncated normal model. Hjalmarsson, Kumbhakar, and Heshmati (1996) also failed to reject the half-normal hypothesis.

¹³Our estimated gamma shape parameters vary by year and are equal to 1.400, 0.790, 2.675, 1.224, 0.457 for 1988-1992, respectively. Yuengert's (1993) sample was for 1989, and he assumed a shape parameter of 2.0 for that year.

¹⁴The efficiency measures for the DEA-CRS and DEA-NIRS models are slightly less than those for the DEA-VRS model.

¹⁵The technical and allocative efficiency estimates are not shown in the table but are available from the authors on request. For the DEA-VRS method, technical efficiency averaged 0.58 for the period as a whole and allocative efficiency averaged 0.82.

¹⁶For the FDH approach, technical efficiency averaged 0.98 and allocative efficiency averaged 0.93.

¹⁷The numbers of self-efficient firms in the DEA-VRS estimates are 25, 32, 35, 35, and 41 for the years 1988 through 1992, respectively; while the number of FDH self efficient firms for these years are 329, 313, 312, 336,

and 340.

¹⁸They estimated the DEA frontier under the assumption of variable returns to scale (VRS).

¹⁹The value for assets are based on the year 1990.

²⁰The conclusion that efficiency is positively related to size in the industry is consistent with the findings of Gardner and Grace (1993). However, Yuengert (1993) found that efficiency and size were statistically unrelated.

²¹The results presented in Table 7 are for 1990. The results for other years are very similar and, therefore, are not shown.

²²This is due to the fact that it is much easier to capitalize a new stock firm than a new mutual, so that most new entrants during the past several decades have adopted the stock ownership form.

²³All stock insurers with assets greater than \$13 billion are included in the sample (there are 11 stock insurers and 10 mutuals in this size category). The remaining firms are then placed in \$100 million size strata and random samples of stocks are selected from each stratum to exactly match the number of mutuals in that stratum.

²⁴ANOVA assumes that the underlying distribution is normal and tests for differences in means across groups. The non-parametric tests do not require a distributional assumption and compare the entire structure of the distribution of efficiency scores not just the central tendency.

²⁵The test statistics are available from the authors on request.

²⁶The residuals from the DFA-SS model are very similar and lead to the same conclusions.

²⁷The Bowman-Shelton statistic easily rejects the normal distribution as a model for the empirical distribution, with a test statistic of 70.4. On the other hand, the Kolmogorov-Smirnov statistic for the gamma is 0.04, implying that we cannot reject the gamma distribution hypothesis. Our parameterization of the gamma is:

$$f(x) = \frac{\lambda^r}{\Gamma(r)} (\lambda x)^{r-1} e^{-\lambda x}$$

The estimated scale parameter (λ) = 4.8608 and the estimated shape parameter (r) = 4.0518. The Bowman-Shelton statistic is defined as: $B = (\text{sample size}) * [\text{skewness}^2/6 + (\text{kurtosis}-3)^2/24]$, which is distributed as χ^2 with 2 degrees of freedom.

²⁸The shape parameter result is due to the stationarity assumption plus the fact that the convolution of independent gamma distributions with the same scale parameter is also gamma distributed with a shape parameter equal to the sum of the shape parameters of the distributions included in the convolution. The scale parameter result is due to dividing by 5 to obtain the five-year average residuals.

²⁹That is we simulate $r_i = \sum_j (u_{ij} + v_{ij})/5$, where u_{ij} = simulated random draw from a half-normal distribution, v_{ij} = simulated random draw from a standard normal distribution, $i = 1, \dots, 1,000,000$, and $j = 1, 2, \dots, 5$. The parameters of the half-normal and normal distributions are obtained from the half-normal panel model (Half-P).

³⁰The graph of the distribution of simulated residuals is available from the authors on request.

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Table 1
Descriptive Statistics: 445 U.S. Life Insurers
Averages 1988-1992

Variable	Mean	StdDev	Maximum	Minimum*
Outputs (mil. \$)				
Y ₁	34.5	116.0	1,494.6	0.001
Y ₂	20.8	120.9	3,073.0	0.001
Y ₃	18.8	60.4	818.4	0.001
Y ₄	34.7	202.6	2,671.2	0.001
Y ₅	79.2	295.4	4,539.2	0.001
Y ₆	492.9	1,667.4	25,228.0	0.001
Inputs				
X ₁ (thous.)	159.4	383.6	4,974.3	2.02
X ₂ (mil.)	2,419.3	8,299.6	155,216.2	2.83
X ₃ (mil.)	20.0	56.4	914.6	0.51
Input Prices				
P ₁	499.1	33.29	628.5	363.1
P ₂	0.12	0.10	1.02	0.01
P ₃	1.09	0.07	1.26	0.85
Costs (mil.\$)	313.4	867.3	13,015.6	1.79
Assets (mil.\$)	2,617.1	9,873.5	137,281.0	8.5

Note: Y₁ = individual life insurance benefit payments
Y₂ = group life insurance benefit payments
Y₃ = individual annuities benefit payment
Y₄ = group annuities benefit payments
Y₅ = accident and health insurance benefit payments
Y₆ = additions to reserves
X₁ = quantity of labor input
X₂ = quantity of financial capital input
X₃ = quantity of materials input
p₁ = price of labor input
p₂ = price of financial capital input
p₃ = price of materials input

All dollar valued quantities are expressed in constant 1988 dollars.

*Some DMUs in the sample did not produce all outputs. The value of 0.001 was inserted in the translog function when zero outputs were encountered.

Table 2. - Cost Efficiency Measures: Econometric Approach

	Half	Exponential	Half-P	Exp-P	Gamma	DFA-B	DFA-SS	DEA-VRS	FDH
(1988)									
Mean	0.58	0.70	0.61	0.71	0.86	0.47	0.44	0.42	0.92
Std. Dev.	0.22	0.17	0.18	0.16	0.08	0.04	0.04	0.24	0.18
Max	0.94	0.92	0.92	0.92	0.90	1.00	1.00	1.00	1.00
Min	0.05	0.07	0.07	0.07	0.23	0.04	0.04	0.02	0.06
(1989)									
Mean	0.63	0.73	0.61	0.70	0.86	0.47	0.44	0.46	0.91
Std. Dev.	0.17	0.15	0.18	0.16	0.07	0.04	0.04	0.25	0.19
Max	0.92	0.92	0.93	0.93	0.93	1.00	1.00	1.00	1.00
Min	0.09	0.08	0.08	0.08	0.21	0.04	0.04	0.04	0.07
(1990)									
Mean	0.61	0.71	0.61	0.71	0.79	0.47	0.44	0.45	0.90
Std. Dev.	0.18	0.16	0.18	0.16	0.10	0.04	0.04	0.24	0.19
Max	0.93	0.93	0.94	0.93	0.90	1.00	1.00	1.00	1.00
Min	0.09	0.10	0.09	0.10	0.32	0.04	0.04	0.04	0.09
(1991)									
Mean	0.61	0.69	0.61	0.71	0.86	0.47	0.44	0.48	0.92
Std. Dev.	0.20	0.19	0.18	0.16	0.08	0.04	0.04	0.24	0.18
Max	0.94	0.94	0.91	0.92	0.94	1.00	1.00	1.00	1.00
Min	0.05	0.06	0.07	0.07	0.20	0.04	0.04	0.04	0.08
(1992)									
Mean	0.63	0.73	0.61	0.71	0.86	0.47	0.44	0.50	0.92
Std. Dev.	0.18	0.16	0.17	0.15	0.07	0.04	0.04	0.25	0.18
Max	0.93	0.93	0.91	0.91	0.90	1.00	1.00	1.00	1.00
Min	0.06	0.06	0.06	0.06	0.12	0.04	0.04	0.03	0.04
(5-Year Average)									
Mean	0.61	0.71	0.61	0.71	0.85	0.47	0.44	0.46	0.91
Std. Dev.	0.17	0.15	0.16	0.14	0.07	0.18	0.18	0.22	0.16
Max	0.90	0.90	0.87	0.89	0.91	1.00	1.00	1.00	1.00
Min	0.07	0.07	0.07	0.07	0.22	0.04	0.04	0.04	0.07
Median	0.63	0.75	0.63	0.75	0.87	0.45	0.42	0.40	1.00

NOTE: Half = half-normal distribution for inefficiency error term, estimated year-by-year;
Exponential = exponential distribution for inefficiency error term, estimated year-by-year;
Half-P = half-normal inefficiency assumption, estimated using entire panel;
Exp-P = exponential inefficiency assumption, estimated using entire panel;

Gamma = gamma distribution for inefficiency error term, estimated year-by-year (Greene, 1990);
DFA-B = distribution free method of Berger (1993);
DFA-SS = distribution free method of Schmidt and Sickles (1984);
DEA-VRS = data envelopment analysis with the variable returns to scale constraint imposed;
FDH = free disposal hull.

Table 3. - Spearman Correlation Coefficients Among Alternative Efficiency Measures
Five-Year Average (1988-1992)

	Half	Exp	Half-P	Exp-P	Gamma	DFA-B	DFA-SS	DEA-VRS	FDH
Half	1.00								
Exp	0.96	1.00							
Half-P	0.98	0.98	1.00						
Exp-P	0.97	0.98	0.99	1.00					
Gamma	0.97	0.98	0.97	0.97	1.00				
DFA-B	0.98	0.98	0.98	0.97	0.99	1.00			
DFA-SS	0.96	0.96	0.98	0.98	0.97	0.98	1.00		
DEA-VRS	0.60	0.59	0.58	0.57	0.56	0.59	0.57	1.00	
FDH	0.55	0.56	0.55	0.56	0.52	0.51	0.52	0.67	1.00

NOTE: Half = half-normal distribution for inefficiency error term, estimated year-by-year;
 Exp = exponential distribution for inefficiency error term, estimated year-by-year;
 Half-P = half-normal inefficiency assumption, estimated using entire panel;
 Exp-P = exponential inefficiency assumption, estimated using entire panel;
 Gamma = gamma distribution for inefficiency error term, estimated year-by-year;
 DFA-B = distribution free method of Berger (1993);
 DFA-SS = distribution free method of Schmidt and Sickles (1984);
 DEA-VRS = DEA estimate with variable returns to scale constraint;
 FDH = free disposal hull.

Table 4. - Pairwise Agreement on Firms in Most and Least Efficient Quartiles
Based on Five-Year Average Efficiencies (1988-1992)

	Half	Exp	Half-P	Exp-P	Gamma	DFA-B	DFA-SS	DEA-VRS	FDH
Most Efficient Quartile									
Half	1.00								
Exp	0.96	1.00							
Half-P	0.88	0.88	1.00						
Exp-P	0.89	0.90	0.98	1.00					
Gamma	0.87	0.86	0.87	0.87	1.00				
DFA-B	0.92	0.88	0.88	0.88	0.92	1.00			
DFA-SS	0.81	0.80	0.87	0.87	0.86	0.86	1.00		
DEA-VRS	0.52	0.52	0.51	0.50	0.51	0.51	0.50	1.00	
FDH*	NA	NA	NA	NA	NA	NA	NA	NA	NA
Least Efficient Quartile									
Half	1.00								
Exp	0.96	1.00							
Half-P	0.92	0.90	1.00						
Exp-P	0.90	0.90	0.97	1.00					
Gamma	0.90	0.90	0.87	0.88	1.00				
DFA-B	0.95	0.93	0.90	0.88	0.92	1.00			
DFA-SS	0.92	0.90	0.94	0.92	0.91	0.92	1.00		
DEA-VRS	0.65	0.64	0.66	0.64	0.61	0.64	0.63	1.00	
FDH	0.58	0.60	0.57	0.59	0.57	0.57	0.56	0.66	1.00

NOTE: Half = half-normal distribution for inefficiency error term, estimated year-by-year;

Exp = exponential distribution for inefficiency error term, estimated year-by-year;

Half-P = half-normal inefficiency assumption, estimated using entire panel;

Exp-P = exponential inefficiency assumption, estimated using entire panel;

Gamma = gamma distribution for inefficiency error term, estimated year-by-year;

DFA-B = distribution free method of Berger (1993);

DFA-SS = distribution free method of Schmidt and Sickles (1984);

REM = random effects model estimate using maximum likelihood;

DEA-VRS = DEA estimate with variable returns to scale constraint;

FDH = free disposal hull.

*The top quartile comparisons are not relevant for FDH because 235 of 445 DMUs are measured as self efficient over the sample period.

Table 5. - Efficiency Measures by Asset Size Class:
Five Year Averages (1988-1992)

Asset Size Class	Half	Exp	Half-P	Exp-P	Gamma	DFA-B	DFA-SS	DEA-VRS	FDH
10M-30M	0.63	0.71	0.63	0.72	0.86	0.53	0.51	0.47	0.84
30M-50M	0.64	0.73	0.64	0.73	0.86	0.51	0.49	0.41	0.88
50M-100M	0.63	0.73	0.64	0.73	0.86	0.51	0.49	0.43	0.90
100M-300M	0.59	0.69	0.59	0.69	0.83	0.43	0.40	0.34	0.90
300M-1B	0.59	0.70	0.59	0.70	0.84	0.43	0.40	0.41	0.95
1.0B-2.5B	0.59	0.70	0.59	0.70	0.84	0.43	0.39	0.49	0.97
2.5B-5.0B	0.56	0.66	0.56	0.66	0.81	0.40	0.36	0.49	0.92
5.0B-10.0B	0.62	0.72	0.62	0.72	0.85	0.46	0.41	0.69	1.00
≥ 10.0B	0.66	0.76	0.65	0.75	0.86	0.47	0.44	0.84	1.00
Correlation *	0.106	0.083	0.094	0.078	0.039	0.092	0.099	0.389	0.108
P-Value **	0.025	0.080	0.047	0.096	0.405	0.051	0.035	0.000	0.022

*Pearson correlation coefficient between efficiency score and assets.

**Probability value for test of the null hypothesis that correlation coefficient is equal to zero.

NOTE: M = millions of dollars; B = billions of dollars.

Half = half-normal distribution for inefficiency error term, estimated year-by-year;

Exp = exponential distribution for inefficiency error term, estimated year-by-year;

Half-P = half-normal inefficiency assumption, estimated using entire panel;

Exp-P = exponential inefficiency assumption, estimated using entire panel;

Gamma = gamma distribution for inefficiency error term, estimated year-by-year;

DFA-B = distribution free method of Berger (1993);

DFA-SS = distribution free method of Schmidt and Sickles (1984);

DEA-VRS = DEA estimate with variable returns to scale constraint;

FDH = free disposal hull.

Table 6
Pearson Correlation Coefficients Between Efficiencies And Other Performance Measures

Ratio	Half	Exp	Half-P	Exp-P	Gamma	DFA-B	DFA-SS	DEA-VRS	FDH
Expenses/Premiums	-0.17	-0.22	-0.14	-0.19	-0.18	-0.07	-0.04	-0.17	-0.36
	2.84e-04	2.26e-06	2.89e-03	4.78e-05	1.19e-04	1.38e-01	3.97e-01	2.84e-04	1.55e-15
RBOE	0.14	0.15	0.12	0.12	0.13	0.13	0.14	0.35	0.24
	2.89e-03	1.40e-03	1.08e-02	1.08e-02	5.71e-03	5.71e-03	2.89e-03	1.11e-14	2.28e-07

NOTE: For each ratio, the top number is the correlation coefficient with efficiency and the bottom number is the probability of obtaining the correlation coefficient under the null hypothesis that the correlation = 0.

Expense Ratio = the ratio of expenses to premiums

RBOE = the ratio of net income plus benefits to equity

Half = half-normal distribution for inefficiency error term, estimated year-by-year;

Exp = exponential distribution for inefficiency error term, estimated year-by-year;

Half_P = half-normal inefficiency assumption, estimated using entire panel;

Exp_P = exponential inefficiency assumption, estimated using entire panel;

Gamma = gamma distribution for inefficiency error term, estimated year-by-year;

DFA-B = distribution free method of Berger (1993);

DFA-SS = distribution free method of Schmidt and Sickles (1984);

DEA-VRS = DEA estimate with variable returns to scale constraint;

FDH = free disposal hull.

Table 7
Economies of Scale in the U.S. Life Insurance Industry: 1990

Asset Size Class	Returns to Scale					
	Increasing		Constant		Decreasing	
	Stocks	Mutuals	Stocks	Mutuals	Stocks	Mutuals
10M - 30M	65	6	3	1	0	0
30M - 50M	44	4	0	2	1	0
50M - 100M	40	6	2	1	5	0
100M - 300M	55	14	2	0	6	5
300M - 1B	21	14	6	0	15	8
1B - 2.5B	5	2	3	0	26	6
2.5B - 5B	2	1	2	0	17	5
5B - 10 B	1	0	1	1	14	7
> 10B	0	0	3	1	11	11
Total: Number	233	47	22	6	95	42
Percent: Stocks	78.5%		7.4%		32.0%	
Percent: Mutuals		49.5%		6.3%		44.2%
Percent: Overall		62.9%		6.3%		30.8%

Note: M = millions of dollars, B = billions of dollars.

Table 8. - Efficiency Comparison: Stocks vs. Mutuals

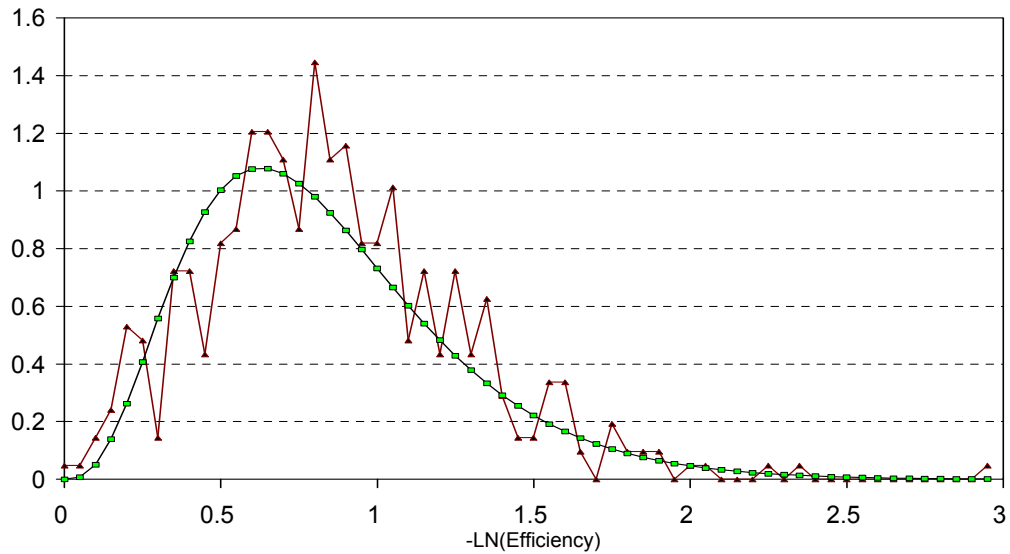
Means From Entire Sample										
	N	Half	Exp	Half-P	Exp-P	Gamma	DFA-B	DFA-SS	DEA-VRS	FDH
Stock	350	0.60	0.70	0.60	0.70	0.84	0.46	0.43	0.44	0.90
Mutual	95	0.67	0.76	0.66	0.75	0.87	0.51	0.48	0.54	0.97

Means From Matched Samples										
	N	Half	Exp	Half-P	Exp-P	Gamma	DFA-B	DFA-SS	DEA-VRS	FDH
Stock	95	0.63	0.74	0.63	0.73	0.86	0.48	0.47	0.53	0.96
Mutual	95	0.67	0.76	0.66	0.75	0.87	0.51	0.48	0.54	0.97

Medians From Matched Samples										
	N	Half	Exp	Half-P	Exp-P	Gamma	DFA-B	DFA-SS	DEA-VRS	FDH
Stock	95	0.64	0.76	0.68	0.75	0.87	0.46	0.43	0.48	1.00
Mutual	95	0.71	0.80	0.69	0.80	0.89	0.51	0.49	0.48	1.00

NOTE: Half = half-normal distribution for inefficiency error term, estimated year-by-year;
 Exp = exponential distribution for inefficiency error term, estimated year-by-year;
 Half-P = half-normal inefficiency assumption, estimated using entire panel;
 Exp-P = exponential inefficiency assumption, estimated using entire panel;
 Gamma = gamma distribution for inefficiency error term, estimated year-by-year;
 DFA-B = distribution free method of Berger (1993);
 DFA-SS = distribution free method of Schmidt and Sickles (1984);
 DEA-VRS = DEA estimate with variable returns to scale constraint;
 FDH = free disposal hull.

Figure 1
Empirical and Fitted Distributions of DFA Average Residuals



Note: The graph is based on individual firm average residuals obtained by averaging the cost function residuals for each firm across the sample period 1988-1992. The empirical density function is defined at $n(i)/N$, where $n(i)$ = number of firms with average residuals r in the range $p(i-1) < r \leq p(i)$, where $p(0) = 0$, $p(1) = 0.05, \dots$, normalized so that the sum under the empirical density sums approximately to 1.0. Recall that the estimate of efficiency under the DFA method is cost efficiency = $\exp(-r)$, so the horizontal axis in the chart plots the DFA estimates of the inefficiency component of the cost function residual. The fitted gamma distribution has scale parameter = 4.8608 and shape parameter = 4.0518.