An Analysis of Reinsurance Optimisation in Life Insurance
Working Paper

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Abstract

This paper considers optimal reinsurance based on an assessment of the reinsurance arrangements for a large life insurer. The objective is to determine the reinsurance structure, based on actual insurer data, using a modified mean-variance criteria that maximises the retained premiums and minimizes the variance of retained claims while keeping the retained risk exposure constant, assuming a given level of risk appetite. The portfolio of life and disability policies use quota-share, surplus and a combination of both quota-share and surplus reinsurance. Alternative reinsurance arrangements are compared using the modified mean-variance criteria to assess the optimal reinsurance strategy. The analysis takes into account recent claims experience as well as actual premiums paid by insured lives and to the reinsurers. Optimal reinsurance cover depends on many factors including retention levels, premiums and the variance of sum insured values (and therefore claims), as a result an insurer should assess the tradeoff between retained premiums and the variance of retained claims based on its own experience and risk appetite.

Keywords: life insurance, optimal reinsurance, proportional reinsurance, mean-variance criteria.

JEL Classifications: G22, G32, L21

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1 Introduction

In the insurance industry, risk management decisions have important implications for solvency, earnings and tax management and impact directly on the economic value of insurance firms. As a result, determining the optimal extent, type and relative mix of reinsurance and retentions is a key strategic issue for both internal stake-holders, such as policyholders and investors, and external stake-holders, such as reinsurance partners, credit ratings agencies, fiscal authorities, and industry regulators (Krvavych and Sherris, 2006).

Froot *et al.* (1993) provide a theoretical framework for analysis of risk management decisions in terms of market imperfections, including frictional costs, such as taxes, agency costs, and the costs of financial distress and bankruptcy, and the impact of financing policy on investment decisions. They argue that cash flow volatility is costly for shareholders and that by stabilising cash flows following unexpected shock events risk management techniques (such as reinsurance) enhance the market value of (insurance) firms by enabling managers to realise positive net present value (NPV) projects in the investment opportunity set. In addition, Cummins *et al.* (2008) and Shimpi (2002) show that the optimal use of reinsurance can increase shareholder value by substituting for equity thereby reducing the cost of capital and increasing returns from underwriting activities.

A number of recent studies (e.g.,Kaluzska (2001), Verlaak and Beirlant (2003), Lampaert and Walhin (2005), Fu and Khury (2011)) analyse the optimality of reinsurance adopting the classical mean - variance framework, where reinsurance decisions are based on risk-minimisation (e.g., variance-minimisation) and profitability decisions. Lampaert and Walhin (2005) adopt RORAC (return on risk-adjusted capital) maximisation to analyse the optimality of proportional reinsurance in the fire insurance industry. They find that quota-share reinsurance is suboptimal compared to other types of proportional reinsurance covers as it does not reduce the variability of retained claim amounts. However, Lampaert and Walhin (2005) do not consider combinations of these covers which are commonly used in practice.

Verlaak and Beirlant (2003) analyse various combinations of proportional and non-proportional reinsurance protections for a heterogeneous insurance portfolio. They show that the order of application of reinsurance arrangements can change an insurer's risk-profitability trade-off. The paper only allows for variation of the sum insured values in the portfolio but not a range of different types of policies. In practice, insurers write a range of policies with varying risk premiums, claims experience and reinsurance premiums. Reinsurance arrangements can be affected by this variability and the use of multiple retention levels can be required to reflect different reinsurance loading factors.

This paper extends previous research on optimal reinsurance by directly analysing a portfolio of differing policies, allowing for different retention levels and different reinsurance loadings based on actual insurance and reinsurance claims and premiums. Optimal reinsurance structures are assessed using a mean-variance framework. The risk appetite[1] of the insurer is based on their existing reinsurance portfolio and cover-

[1]Risk appetite can be defined as the risk level that insurer is willing to accept and can be measured

age. Different retention levels used to assess optimality will reflect this risk appetite. The objective is to keep the risk appetite constant while assessing different reinsurance structures with multiple retention levels. The reinsurance structure that retains the largest proportion of premium and minimises the variance of retained claims is preferred to other reinsurance structures.

This study contributes to the previous research in two main aspects. First, the current study allows variations in claims experience, levels of premiums for both insurance and reinsurance, and sum insured values, which enables us to produce more realistic insights in determining the optimal reinsurance arrangements. Second, traditional mean-variance approaches either keep risk constant to maximise profits, or keep profits constant to minimise the risk. In the context of reinsurance purchase, this might not be feasible since insurance companies purchase reinsurance (especially proportional types) not only for risk mitigation but also for capital substitution reasons. Therefore, a predetermined risk appetite level[2] is required in the risk analysis.

The remainder of the paper is organised as follows. Section 2 explains the theoretical framework of different reinsurance treaties. Section 3 describes the data used for the numerical assessment. Section 4 presents and discusses the criteria that will be used to determine optimal reinsurance arrangements. Section 5 reports the results and Section 6 concludes.

2 Reinsurance Covers

Consider a life insurer with a portfolio of n_j risks where j defines a different risk cover. Insurance contracts, which include multiple risk covers, are indexed by i and are characterized by the risk exposure (e.g., sum insured value) SI_i, premium P_i and individual loss or claim S_i. A policy can have multiple benefits and hence multiple risks. In a case of no reinsurance the insurer retains all the risk exposure (SI_j), premium (P_j) and it pays the full loss (S_j):

$$SI_j = \sum_{i=1}^{n_j} SI_i, \tag{1}$$

$$P_j = \sum_{i=1}^{n_j} P_i, \tag{2}$$

$$S_j = \sum_{i=1}^{n_j} S_i. \tag{3}$$

as retained risk after applying all reinsurance contracts (Nocco and Stulz, 2006).

[2]Given solvency capital requirements and the loss distribution associated with the risk, the insurer can determine the amount of requited capital to underwrite it's business. Thus, the insurer's capital can restrict the volumes of business underwritten. If the insurer wants to increase underwriting volumes it may increase the premiums, raise more capital or buy appropriate reinsurance. Raising capital can be a lengthy (and complicated) procedure and increasing insurance premiums is limited due to competition in the insurance market. Thus, buying reinsurance is often the most convenient and practical option to increase the insurers capacity (Eden and Kahane, 1988). Insurer's risk appetite is a proxy for the volume of business that can be underwritten holding a predetermined amount of capital. Business volumes above this level need to be reinsured in order to meet solvency capital requirements.

If reinsurance is purchased these amounts are shared between insurer and reinsurer. The way these amounts are shared depends on the type of reinsurance arrangement. Traditionally reinsurance can be split into two categories: proportional and non-proportional. Direct Life Insurance business is generally reinsured under proportional cover; the main characteristics of this arrangement are defined below.

Proportional reinsurance is the simplest way to share an insurance risk. Under this type of cover the insurer and the reinsurer (or reinsurers) agree on a retention ratio (deductable), say $\alpha_i \in [0, 1]$, for each benefit covering risk j in the portfolio. The risk exposure, premium and the loss corresponding to the benefit i are then proportionally shared between the insurer and reinsurer(s), depending on the agreed retention level (α_i). Definition of the retention ratio α_i depends on the type of proportional reinsurance used: quota-share or surplus.

Under quota-share reinsurance, the retention ratio does not depend on the individual risk characteristics (e.g., sum insured value) and therefore it is constant across all benefits covering the same risk j: $\alpha_i = \alpha_j$. Consequently retained and reinsured risk exposure (SI_j), premium (P_j) and loss (S_j) can be defined as follows:

	Insurer's Share	Reinsurer's Share
Risk Exposure	$SI_j^I = \sum_{i=1}^{n_j} \alpha_j SI_i$	$SI_j^R = \sum_{i=1}^{n_j} (1-\alpha_j) SI_i$
Premium	$P_j^I = \sum_{i=1}^{n_j} \alpha_j P_i$	$P_j^R = \sum_{i=1}^{n_j} (1-\alpha_j) P_i$
Loss	$S_j^I = \sum_{i=1}^{n_j} \alpha_j S_i$	$S_j^R = \sum_{i=1}^{n_j} (1-\alpha_j) S_i$

Quota-share reinsurance reduces the overall amount of claims paid, however it does not reduce the variability of claims in the portfolio [3]:

$$CV_{QS}(S^I) = CV(S). \tag{4}$$

Quota-share reinsurance is an easy and straight forward way to cover an insurance portfolio. Quota-share contracts are simple to administer and there is minimal adverse selection for the reinsurer. A direct insurer has a greater expense than a reinsurer as it has to meet the cost of acquiring the business (e.g., search and screening, policy serving and claims managements costs) (Tiller and Tiller, 2005). For this reason, the reinsurer pays commissions to the direct insurer in order to compensate for a share of its acquisition and administration costs. This commission is usually expressed as a percentage of the original premium and serves as the price for the reinsurance[4] (SwissRe, 2010). For simplicity, we do not separate reinsurance commissions in the analysis; instead this is accounted for in the premium income figures for the reinsurer.

Under surplus reinsurance, the retention rate can be expressed as a function of both the sum insured and the retention chosen by the insurer. If the maximum amount that the insurer wants to pay for a benefit i covering risk j in a case of loss (retention line)

[3] see Lampaert and Walhin (2005).

[4] In our data the life insurer receives around 30 percent in reinsurance commissions; however this amount varies by the type of reinsurance cover.

is denoted as R_j, then the retention ratio, α_i can be expressed as follow:

$$\alpha_i = min(1, \frac{R_j}{SI_i}). \tag{5}$$

Therefore the risk exposure of each benefit for the insurer under a surplus reinsurance cover is:

$$SI_i^I = \begin{cases} min(1, \frac{R_j}{SI_i}) \times SI_i = SI_i, & \text{if } SI_i \leq R_j \\ min(1, \frac{R_j}{SI_i}) \times SI_i = R_j, & \text{if } SI_i > R_j. \end{cases} \tag{6}$$

Equivalently, the insurer retains the loss amount equal to:

$$S_i^I = \begin{cases} min(1, \frac{R_j}{SI_i}) \times S_i = S_i, & \text{if } SI_i \leq R_j \\ min(1, \frac{R_j}{SI_i}) \times S_i = \frac{R_j}{SI_i} S_i, & \text{if } SI_i > R_j. \end{cases} \tag{7}$$

The insurer never pays in excess of the retained amount R_j which can be appealing from the optimality point of view, as surplus reinsurance allows the insurer to retain small losses, therefore reducing the variability of claims in the portfolio (Lampaert and Walhin, 2005).

For each benefit i, where $SI_i > R_j$, the insurer pays the reinsurer a proportion of premium equal to:

$$P_i^R = (1 - \alpha_i) P_i = (1 - \frac{R_j}{SI_i}) P_i. \tag{8}$$

The combination of quota-share and surplus reinsurance can be used (e.g., see Lampaert and Walhin (2005)). The order of application (e.g., quota-share first and then surplus or other way around) will affect retention levels, variance of retained claims and reinsurance premiums. This study focuses only on quota-share and surplus as reinsurance premium data are available only for this type of cover[5]. Under quota-share and surplus arrangement the insurer pays a proportion of incurred losses specified by the retention ratio α_j. However if the sum insured (SI_{ij}) exceeds the retention level R_j, the insurer only pays up to the retention. The risk exposure for the insurer under this arrangement becomes:

$$SI_i^I = \begin{cases} min(1, \frac{R_j}{SI_i}) \times \alpha_j SI_i = \alpha_j SI_i, & \text{if } SI_i \leq R_j \\ min(1, \frac{R_j}{SI_i}) \times \alpha_j SI_i = \alpha_j R_j, & \text{if } SI_i > R_j. \end{cases} \tag{9}$$

This type of arrangement is expected to have an advantage over quota-share reinsurance as it both reduces overall claim amount and the variability of retained claims in the portfolio. If the retention level R_j is the same in both surplus and a combination of quota-share and surplus arrangements the variability of claims is expected to be the same.

[5] see Lampaert and Walhin (2005) for a detailed analysis of surplus and quota-share reinsurance covers.

3 Criteria for Optimal Reinsurance

In order to assess which of the existing reinsurance arrangements, quota-share, surplus or combination of quota-share and surplus, are optimal from the insurer's point of view, both profitability and risk criteria are used. Two measures are employed to assess profitability criteria:

- retained premium-to-retained claims ratio (RPS);
- difference between retained premiums and retained claims.

The first ratio - RPS - shows how much premium income is retained by the insurer in relation to every retained claim dollar. The largest RPS ratio identifies the optimal reinsurance arrangement as it allows the insurer to retain the largest proportion of premium for every retained dollar of claims. RPS can be defined as

$$RPS_j = \frac{\sum_{t=0}^{T_j} \sum_{i=1}^{n_j} (PI_{ti} - PI_{ti}^{R_r})}{\sum_{t=0}^{T_j} \sum_{i=1}^{n_j} (S_{ti} - S_{ti}^{R_r})}, \tag{10}$$

where r represents different reinsurance cover and T_j is a number of time periods[6]. Here claim size (S_i) can be expressed as

$$S_i = x_{jr} SI_i, \tag{11}$$

where x_{jr} is claim severity per unit of risk exposure (e.g., sum insured value):

$$x_{jr} = \frac{\sum_i^{n_{jr}} S_i}{\sum_i^{n_{jr}} SI_i}. \tag{12}$$

Although the first criteria (RPS) describes the profitability in relation to every retained claims dollar it does not show how much of total premium income the insurer retains after paying all claims. Therefore the second profitability measure - the difference between retained premiums and retained claims - is employed, which shows the total amount of retained premium after paying all claims and reinsurance premiums, and represents the underwriting profit (loss) before administrative expenses. The reinsurance arrangement which enables the insurer to maximise retained premiums is theoretically optimal from profitability point of view.

The retained risk is measured by the variance of retained claims and can be expressed as:

$$Var[\sum_{t=0}^{T_j} \sum_{i=1}^{n} (S_{ti} - S_{ti}^{R_r})] \tag{13}$$

The variance of retained claims can impose extra costs for the insurer as large claims variability increases uncertainty and therefore, demands more capital to be held (e.g., see Verlaak and Beirlant (2003)). Therefore, the optimal reinsurance arrangement should minimise the variance of retained claims.

[6]Subscript j allows different risk covers (e.g., income protection, term life, trauma and total and permanent disability (TPD)) to have different time frequencies (e.g., monthly, quarterly, annually) as premiums and claims can differ by the type of insurance cover.

In practice, the mean values of retained claims after applying different reinsurance arrangements can differ. Therefore, the variance of retained claims cannot be compared without the context of the mean of the data. The variance-to-mean ratio is thus estimated in order to make the comparison between different groups. The variance-to-mean ratio is estimated from:

$$\frac{Var[\sum_{t=0}^{T_j}\sum_{i=1}^{n}(S_{ti}-S_{ti}^{R_r})]}{Mean[\sum_{t=0}^{T_j}\sum_{i=1}^{n}(S_{ti}-S_{ti}^{R_r})].} \tag{14}$$

All reinsurance arrangements r are assessed for every group of benefits. This includes the existing (which is generally a combination of all reinsurance arrangements and no reinsurance); three estimated (e.g., quota-share, surplus and combinations of quota-share and surplus); and the case of no reinsurance for which the ratios (specified above) are estimated. This ensures that different benefit characteristics (e.g., claim severity, sum insured values) in each group do not impact the optimality of results. In addition, the study uses actual retention ratios α_{ij} and R_{ij} and so allows these levels to vary among and within every group. The objective is to keep the original amount of retained risk fixed for every group (e.g., $\sum_{i=1}^{n_{jr}}(SI - SI^{R_r})$) while applying different reinsurance arrangements with combinations of different α_{ij} and R_{ij}.

Table 1: Income Protection Benefit Summary Statistics

Benefit Class		Variable (000, A$)*	Mean	Std	Min	Max	Skew	Sum	n
Retail Life	Short & Long Term Benefits	SI	5.850	3.971	0.001	37.745	2.302	143,351.3	24,504
		SI^R	2.622	3.050	0.000	32.338	3.351	64,247.6	24,504
		PI	2.799	3.272	0.000	79.022	4.770	68,576.9	24,504
		x	0.161	0.066	0.000	0.431	0.818	-	24,504
		S	0.958	0.849	0.000	12.570	3.186	23,463.8	24,504
	Business Expenses	SI	7.956	6.787	0.400	60.000	1.866	5,084.1	639
		SI^R	2.999	3.851	0.000	48.000	4.338	1,916.2	639
		PI	2.359	2.836	0.029	28.028	3.710	1,507.3	639
		x	0.018	0.009	0.000	0.034	-0.788	-	639
		S	0.137	0.180	0.000	1.220	2.411	87.8	639
Direct Life		SI	2.664	1.187	0.001	5.788	0.449	40,063.1	15,036
		SI^R	0.733	0.811	0.000	2.894	2.403	11,017.7	15,036
		PI	1.119	0.919	0.000	7.472	1.075	16,821.8	15,036
		x	0.055	0.030	0.011	0.154	0.725	-	15,036
		S	0.135	0.079	0.000	0.293	0.170	2,027.6	15,036

* for all variables except x. SI - sum insured value (for IP benefits sum insured value is a monthly payment value); SI^R - sum reinsured value; PI - total premium income received up to date; x - claim severity ratio, S - paid and reported but not paid claim amount up to date.

4 Data Summary

The original data-set consists of approximately 450,000 life insurance benefits, 100,000 Retail Life and 350 thousand Direct Life[7], commencing between January, 2008 and

[7]Retail Life is traditional life insurance and is provided via dealers and advisers. Direct Life provides the insurance over the telephone or internet, marketed using detailed customer analysis. Risk exposure,

January, 2011[8] for a large Australian life insurer. All benefits are split into four groups ($j = 4$), depending on their risk cover: income protection (IP), term life, total and permanent disability (TPD) and trauma and further four sub-groups depending on the original reinsurance arrangements ($r = 4$): no reinsurance, quota-share, surplus, combination of quota-share and surplus. 100 percent reinsured benefits are excluded from the samples as the risk and profitability rations give the same results for all reinsurance types. Sub-groups of benefits (jr) with no claims made in a period of analysis are also eliminated as claims data are required for risk and profitability ratio estimation. Finally, the sub-groups (rj) consisting of less that 1 percent of total number of benefits are excluded from the sample as they give bias results when compared with the larger ones. This gives the final sample of 425,000 life insurance benefits: 95,000 Retail Life and 330,000 Direct Life.

Table 2: Total and Permanent Dissability Benefit Summary Statistics

Variable (000, A$)*	Mean	Std	Min	Max	Skew	Sum	n
SI	592.405	496.965	2.625	6400.000	2.352	8,059,079.9	13,604
SI^R	180.337	366.883	0.000	5400.000	3.491	2,453,305.4	13,604
PI	1.469	2.707	0.002	71.691	7.544	19,986.1	13,604
x	0.001	0.003	0.000	0.010	3.314		13,604
S	0.748	2.527	0.000	66.253	8.903	10,181.6	13,604

* for all variables except x. SI - sum insured value; SI^R - sum reinsured value; PI - total premium received up to date; x - claim severity ratio, S - paid and reported but not paid claim amount up to date.

Table 1 reports Retail[9] and Direct Life benefit summary statistics covering IP risk. Over 40,000 benefits are opened between January, 2008 and January, 2011 with 60 percent of them covering Retail Life business. The average sum insured value varies depending on the benefit class from the lowest of A$2,600 in Direct Life to the largest of A$8,000 in Business Expenses group. The lowest reinsurance coverage is in Direct Life business, the insurer cedes less than a quarter of total IP risk. The most heavily reinsured are Retail Life short-term and long-term benefits; here reinsurance covers just under a half of total risk. Claims severity (x) varies from the smallest of 0.02 for Business Expenses to the largest of 0.16 for short and long term benefits. Total IP premium income (PI) exceeds A$85 million with 81 percent of this sum generated in Retail Life group. The average amount of premium income received from one benefit is around A$2,500 for Retail Life and A$1,000 for Direct Life.

Table 2 reports TPD Retail Life benefit summary statistics[10]. Over 13,500 benefits were opened in the period of analysis. The average sum insured value (SI) is just under

premiums and claim severity differ between these types of insurance businesses. Therefore, the optimal reinsurance analysis is required separately for these groups.

[8]The insurer experienced a number of mergers over the last couple of decades, therefore, complete and reliable data were available only for a period of January, 2008 and January, 2011 at the time the study was carried out.

[9]Retail Life benefits are split into two groups: short-term and long-term Retail Life benefits and Business Expenses. As the later has higher average sum insured values and less claims severity it results in different reinsurance terms and premiums.

[10]Direct Life Business is not offered for TPD cover.

Table 3: Term Life Benefit Summary Statistics

Benefit Class		Variable (000, A$)*	Mean	Std	Min	Max	Skew	Sum	n
Retail Life		SI	690.473	700.835	0.00110	22580.250	5.818	28,784,444.7	41,688
		SI^R	138.824	479.654	0.00000	21530.246	11.564	5,787,307.4	41,688
		PI	2.250	5.417	0.00000	434.935	19.799	93,782.7	41,688
		x	0.000	0.001	0.00000	0.017	14.055		41,688
		S	0.181	0.630	0.00000	30.259	20.143	7,538.8	41,688
Direct Life	Funeral Plan	SI	9.265	4.031	0.00000	34.730	0.533	2,528,456.3	272,891
		SI^R	1.955	2.599	0.00000	16.538	1.080	533,588.4	272,891
		PI	0.527	0.665	-0.00002	11.307	2.022	143,926.1	272,891
		x	0.000	0.000	0.00003	0.000	-0.868		272,891
		S	0.001	0.001	0.00000	0.007	0.563	381.1	272,891
	Other Term Life	SI	241.967	169.690	6.00000	1050.000	1.212	10,013,307.5	41,383
		SI^R	65.188	79.936	0.00000	289.407	1.187	2,697,680.9	41,383
		PI	1.146	5.262	0.00000	419.578	18.917	47,437.7	41,383
		x	0.002	0.001	0.00056	0.003	-0.801		41,383
		S	0.450	0.386	0.00346	1.677	1.125	18,608.2	41,383

* for all variables except x. SI - sum insured value; SI^R - sum reinsured value; PI - total premium received up to date; x - claim severity ratio, S - paid and reported but not paid claim amount up to date.

A$600,000 with the smallest of A$2,600 and the largest of A$6.4 million. Total TPD risk exposure exceeds A$8 billion with a third of this risk being reinsured. The claims severity ratio (x) is just 0.1 percent making the average claims for every benefit just under A$750. The small claim severity ratio is explained by the short time span of analysis.

Term Life benefit summary statistics are reported in Table 3. All benefits are split into Retail and Direct Life groups with Direct Life being further divided into Funeral Plan and other term life benefits[11]. There are 356 thousand benefits opened in the period of analysis with the majority of them covering Funeral Plan. Retail Life benefits have larger sum insured values (e.g., A$690,000) with a large variation. Direct Life benefit average sum insured values are smaller (e.g., over A$9,000 in Funeral Plan and just under A$250,000 in other term life) with significantly smaller variations. The possibility of a Term Life claim is significantly larger in Direct Life business compared to Retail Life (e.g., claim severity is 0.2 percent for Direct Life and 0.03 - for Retail Life). Therefore, the average premium income relative to average sum insured value is larger for Direct Life compared to Retail Life. As with other covers, the most heavily reinsured are Retail Life benefits (e.g., 20 percent of total risk is reinsured).

Table 4 reports summary statistics for Retail Life Trauma benefits[12]. Over 16,000 benefits were opened between January, 2008 and January, 2011. The average sum insured value exceeds A$250,000 however there is a large variation in these values. The insurer cedes around one third of total Trauma risk. Average premium income for each benefit

[11]Funeral Plan sum insured values are significantly lower compared with other term life benefits. In order to asses the outcome of different reinsurance covers this group of benefits is separated.

[12]Direct Life benefits account for 15 percent of the total number of trauma benefits. As no claims are made for Direct Life benefits in a period of analysis, they are eliminated from the sample.

Table 4: Trauma Benefit Summary Statistics

Variable (000, A$)*	Mean	Std	Min	Max	Skew	Sum	n
SI	251.034	283.865	0.001	6450.000	4.415	4,070,007.3	16,213
SI^R	77.037	218.727	0.000	5700.000	6.349	1,248,999.7	16,213
PI	1.922	3.821	0.000	122.781	8.595	31,159.7	16,213
x	0.002	0.003	0.000	0.014	2.919		16,213
S	0.555	1.455	0.000	33.369	6.276	8,992.1	16,213

* for all variables except x. SI - sum insured value; SIR - sum reinsured value; PI - total premium received up to date; x - claim severity ratio, S - paid and reported but not paid claim amount up to date.

is just under A$2,000 and the average claim is A$450.

5 Mean Variance Analysis

The mean-variance criterion are applied to each type of risk cover: IP, term life, TPD and trauma.

5.1 Income Protection

Table 5 summarises estimation results for IP benefits. Table 5 is divided into three

Table 5: Income Protection Mean-Variance Estimation Results

Benefit Class		Reinsurance Structure	RPS	Retained Premium - Retained Claim (A$ 000)	Retained Claim (A$ 000)				n
					Mean	Variance	Variance-to-Mean Ratio	Sum	
Retail Life	Short & Long Term Benefits	No Reinsurance	2.92	45,113.11	0.958	0.721	0.753	23,463.81	24,504
		Quota-Share	3.56	34,235.75	0.546	0.280	0.513	13,389.59	24,504
		Quota-Share&Surplus	3.79	34,442.60	0.538	0.173	0.321	13,192.96	24,504
		Surplus	3.23	29,395.17	0.539	0.214	0.398	13,205.91	24,504
		Original Reinsurance Structure	3.53	33,155.78	0.535	0.233	0.436	13,099.29	24,504
	Business Expenses	No Reinsurance	17.16	1,419.52	0.137	0.033	0.138	87.82	639
		Quota-Share	19.63	1,051.17	0.088	0.016	0.179	56.41	639
		Quota-Share&Surplus	19.98	1,088.51	0.090	0.014	0.151	57.36	639
		Surplus	17.69	986.80	0.093	0.011	0.114	59.12	639
		Original Reinsurance Structure	19.74	1,074.28	0.090	0.013	0.140	57.34	639
Direct Life		No Reinsurance	8.30	14,794.25	0.135	0.006	0.047	2,027.59	15,036
		Quota-Share	10.93	12,662.56	0.085	0.002	0.022	1,275.19	15,036
		Quota-Share&Surplus	9.18	12,108.37	0.098	0.003	0.033	1,480.31	15,036
		Surplus	8.19	11,103.82	0.103	0.003	0.033	1,544.92	15,036
		Original Reinsurance Structure	13.00	15,299.44	0.085	0.002	0.022	1,275.19	15,036

parts: the first two parts report the results for Retail Life benefits (e.g., short and long

term benefits[13] and Business Expenses[14]), and the third reports results for Direct Life benefits[15]. Each part gives the results for no reinsurance, quota-share, quota-share and surplus, surplus and original reinsurance structure, which usually combines specified proportions of the others.

The largest amount of premiums after paying all claims for short and long term benefits is retained under no reinsurance purchase as the expected value of reinsurance premiums exceeds the expected value of the reinsurance risk (e.g., reinsured claim amount)[16]. However the variance of total claims is significantly larger compared to the variances of the retained claims after any type of reinsurance is taken (see Figure 1). On average, the largest RPS ratio and the greatest amount of retained premiums from all reinsurance arrangements are obtained under a combination of quota-share and surplus (e.g., $RPS = 3.8$ and retained premium after claims $= A\$34.4$ million.) making this cover optimal under profitability criteria. The smallest RPS ratio and retained premiums after claims are obtained under surplus reinsurance (e.g., $RPS = 3.2$ and retained premiums after claims $= A\$29.40$ million.). This result is not surprising as the later cover is significantly more expensive compared with quota-share or quota-share and surplus covers (e.g., 90 percent of original premium for every A$1 reinsured under surplus compared to 70 percent of original premium for every A$1 reinsured under the other two arrangements). The smallest retained claim variance-to-mean ratio for short and long-term benefits is again obtained under a quota-share and surplus arrangement making this cover the optimal under the retained risk minimisation criteria. Benefits reinsured under quota-share reinsurance experience the largest retained claim variance compared with the other two reinsurance structures. This result confirms the

[13]Quota-share reinsurance retention for short and long term benefits α vary between 55 and 65 percent depending on the original insurer's risk appetite ($\sum_{i=1}^{n}(SI - SI^{R_r})$). The ratio of 0.7 is used to estimate reinsurance premium income, that is, reinsurance premium income is equal to 70 percent of total insurance premium income for every A$1 reinsured (e.g., representing 30 percent of reinsurance commission). The quota-share and surplus reinsurance retention percentage α is equal to 50 or 65 and retention line R varies between A$6,000 and A$10,000 depending on the original benefit retention level. The ratio of quota-share and surplus reinsurance premium income varies depending on the retention levels with an average of 0.7. The surplus reinsurance retentions R vary between A$1 thousand and A$5 per month. Here the average reinsurance premium is 90 percent of the total insurance premium income for every A$1 reinsured. However it varies depending on retention level R and IP type of risk cover.

[14]Retention percentage α for Business Expenses benefits varies between 55 and 65 percent for quota-share reinsurance with an average reinsurance premium income equal to 70 percent of total premium income for every A$1 reinsured. Two retention levels R are used for quota-share and surplus cover - A$12,000 and A$18,500 with retention percentage α equal to 65 percent for both levels. The ratio of reinsurance premium income varies by retention levels with an average of 0.7. Retention levels R of A$5,000 and A$12,000 are used for surplus reinsurance with an average reinsurance premium income ratio of 0.9.

[15]Originally, Direct Life benefits are either reinsured under quota-share arrangement or not-reinsured. A retention α of 50 percent is used for all Direct Life IP benefits. The average quota-share reinsurance premium income is just 6 percent of total premium income for every A$1 reinsured. The low amount of reinsurance premium income in Direct Life business results from the different reinsurance arrangements. That is, the first year marketing allowance for the reinsurer of 100 percent of the gross reinsurance premium leads to a twenty month zero net reinsurance premium and therefore significantly lowers the total amount of reinsurance premium received. Retention percentages α of 75 percent and a retention level R of A$4,200 are used for quota-share and surplus arrangements. Quota-share and surplus reinsurance premiums are 70 percent of total insurance premiums for every A$1 reinsured. In addition, two retention levels R are used for surplus cover: A$2,500 and A$5,000 with a reinsurance premium income ratio of 0.9.

[16]See Venter (1992) for premium calculation principles.

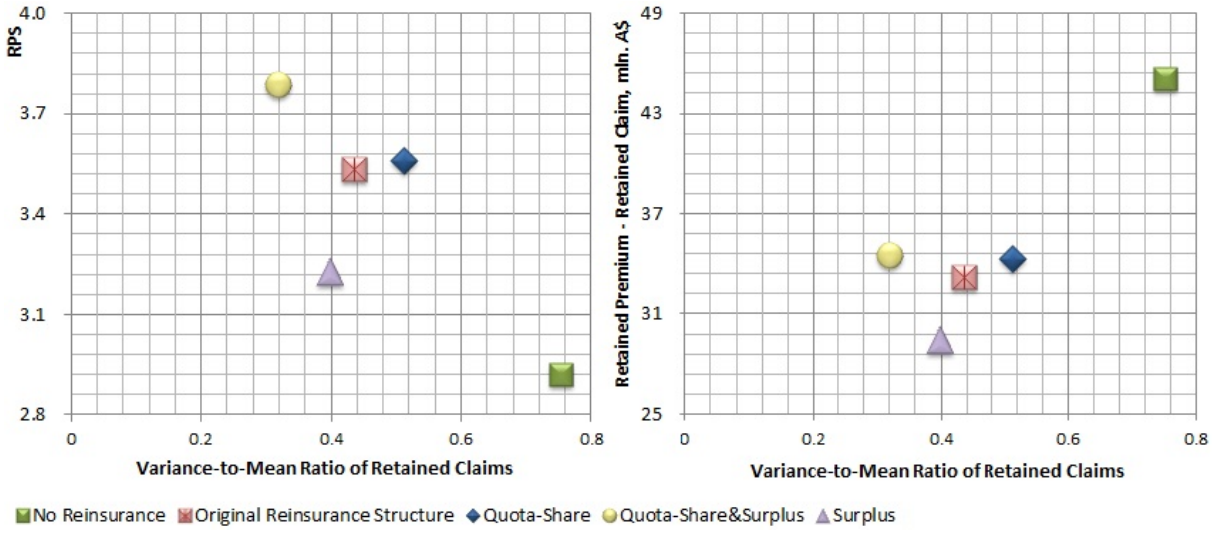

Figure 1: IP - Retail Life Short and Long Term Benefit RPS Ratio and Retained Premium after Claims Plot

theoretical reinsurance result that quota-share reinsurance is not as efficient in minimising underwriting risk compared with surplus or a combination of quota-share and surplus. Under the mean-variance criterion the original reinsurance structure, which combines all three reinsurance arrangements with some benefits not-reinsured, does not perform optimally as it does not lead to the minimal variance-to-mean ratio and does not maximise RPS or retained premiums after paying claims.

The RPS ratio and retained premiums after claims for Business Expenses are similar to those for IP short and long term benefits (see Figure 2). Again, the largest RPS ratio and retained premium income after paying claims are obtained in the group reinsured under quota-share and surplus (e.g., $RPS = 20.0$, retained premiums after claims $= A\$1.09$ million) making this cover the optimal under the profitability criteria. However, as total claim variance is relatively small ($Var = 0.033$), quota-share and surplus reinsurance does not optimally reduce it. The smallest variance-to-mean ratio is obtained under surplus reinsurance; but this cover enables the insurer to retain the smallest premium income after paying claims. As none of the reinsurance covers optimises the mean-variance criteria, a combination of them gives the best results. Therefore, the original reinsurance arrangement (which combines all reinsurance covers with some benefits not-reinsured) is the optimal as it results in one of the largest RPS and retained premiums after claims and one of the smallest variance-to-mean ratio. If the assumption of predetermined (e.g., constant) risk appetite is not taken into consideration, the arrangement of no reinsurance purchase could be the optimal as the variance of total claims is relatively low and it also enables the insurer to retain the largest amount of premium income (1.4 million.).

As mentioned in Footnote 15, the insurer does not pay any quota-share reinsurance premium for the first 20 months in Direct Life business making the original reinsurance cover much cheaper compared to other types of reinsurance arrangement. Therefore, the ratio, referred to as 'Quota-Share', is reported where the percentage of reinsurance premium income is estimated after eliminating the benefits with zero or negative premium income figures (an increase to 27 percent of total premium income for

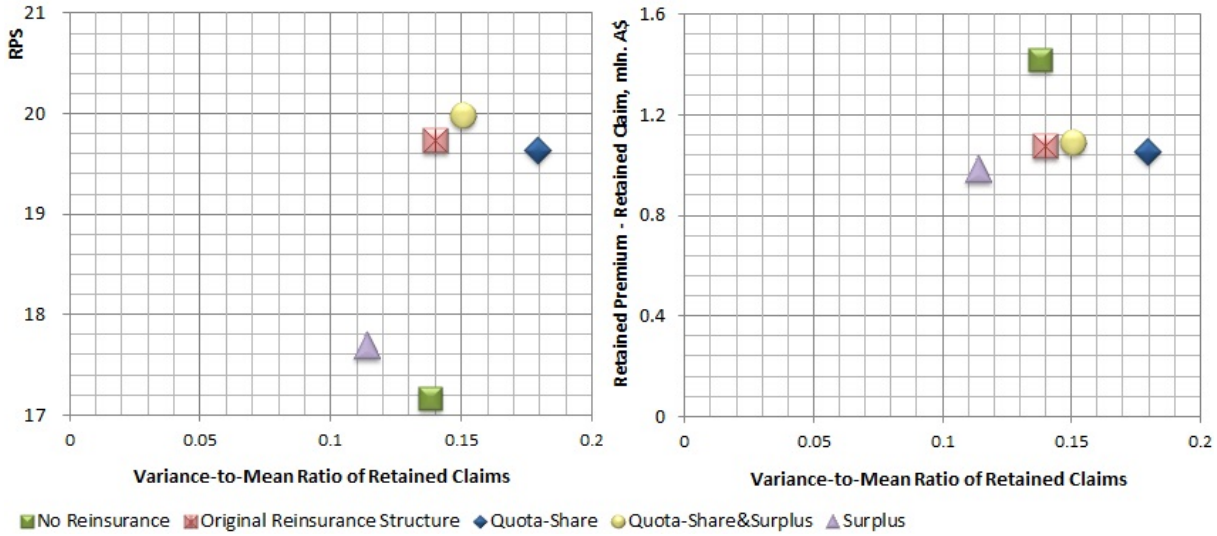

Figure 2: IP - Retail Life Business Expenses RPS Ratio and Retained Premium after Claims Plot

every A$1 reinsured). Figure 3 shows that the largest RPS ratio, the largest amount of retained premiums after claims and the smallest mean-to-variance ratio of retained claims are obtained under quota-share reinsurance arrangement (e.g., $RPS = 10.9$, retained premiums after claims $= A\$12.66$ million. and variance-to-mean ratio $= 0.02$) making this cover the optimal compared with the other two reinsurance covers. These figures are derived from significantly lower quota-share reinsurance premium income and a relatively small variance of total claims (e.g., the average claim for every IP Direct Life benefit is equal to $A\$135$ with the variance of just $A\$6$). Again, if the assumption of constant risk appetite is not taken into consideration, the no reinsurance purchase is optimal as it enables the insurer to retain the largest amount of premium income (e.g., $A\$14.8$ million).

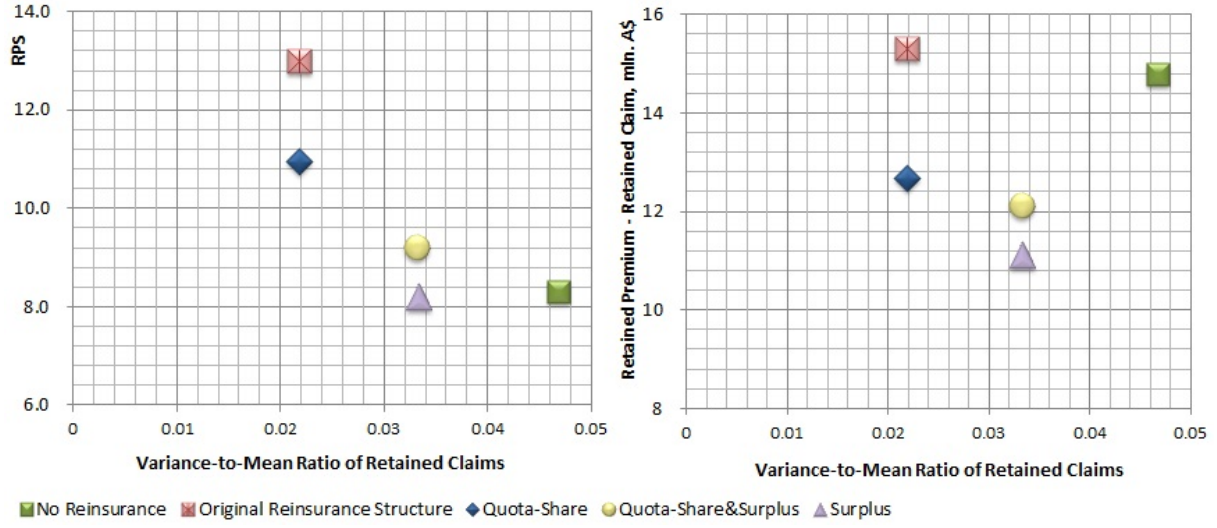

Figure 3: IP - Direct Life RPS Ratio and Retained Premium after Claims Plot

5.2 Total and Permanent Disability

Table 6 reports mean-variance estimation results for TPD risk cover[17]. The optimal reinsurance structure for TPD benefits is surplus reinsurance as it results in the largest RPS ratio, the largest amount of retained premium income after claims and the smallest variance-to-mean ratio. This result could be affected by the large variability in claim values and relatively few surplus retention levels. Surplus enables the insurer to retain the smallest claim amounts without paying any reinsurance premiums leading to the largest RPS ($RPS = 2.54$) and the largest amount of retained premiums after claims (e.g., 9.6 million.). As the majority of the sum insured values are retained at A$500,000 and A$750,000, surplus reinsurance reduces the variance of the retained claims leading to the smallest variance-to-mean ratio. As the majority of TPD benefits (e.g., 80 percent) are originally reinsured under surplus reinsurance, the ratios for the original reinsurance structure are very close to the surplus ratios (see Figure 4).

Table 6: TPD Mean-Variance Estimation Results

Reinsurance Structure	RPS	Retained Premium - Retained Claim (A$ 000)	Retained Claim (A$ 000)				n
			Mean	Variance	Variance-to-Mean Ratio	Sum	
No Reinsurance	1.96	9,804.48	0.748	6.388	8.535	10,181.64	13,604
Quota-Share	2.15	8,534.13	0.546	3.603	6.600	7,426.17	13,604
Quota-Share&Surplus	2.23	9,180.75	0.548	3.363	6.134	7,458.79	13,604
Surplus	2.54	9,606.62	0.459	1.488	3.245	6,239.23	13,604
Original Reinsurance Structure	2.50	9,349.99	0.459	1.488	3.243	6,241.80	13,604

[17]Quota-share retention α varies between 45 and 75 percent with the largest retention used for the group of benefits originally reinsured under surplus arrangement. Again, the ratio of 0.7 is used to estimate the quota-share reinsurance premium income. A number of different combinations of retention levels R and retention percentages α are used for quota-share and surplus reinsurance with the most common $R = A\$400,000, A\$1million., A\$2.5million.$ and $\alpha = 0.75, 0.65, 0.8$ percent respectively. On average the insurer paid 60 percent of total insurance premium income for every A$1 reinsured. Surplus retention levels R vary between A$50 thousand and A$1.5 million with the most common retentions of A$500,000 and A$750,000, however for benefits originally reinsured under quota-share and surplus this retention level is much smaller. Surplus reinsurance premium income varies depending on the retentions and the type of risk cover with an average premium of 75 percent of total premium for every A$1 reinsured.

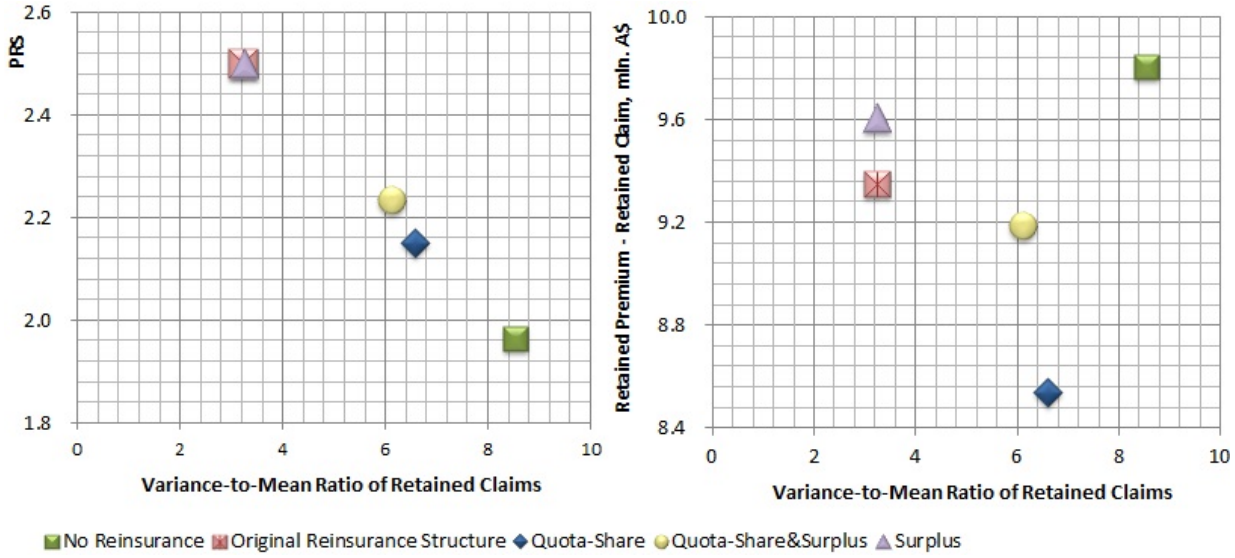

Figure 4: RPS Ratio and Retained Premium after Claims Plot for TPD benefits

5.3 Term Life

Table 7 reports mean-variance estimation results for Term Life benefits. The table is split into three parts: one reports Retail Life[18] results and the other two - Direct Life[19] (e.g., Funeral Plan and Direct Term Life). Results shown in Table 7 do not clearly distinguish which reinsurance structure is optimal for the Retail Life business. The variance-to-mean ratio of the retained claims is smallest under surplus reinsurance (e.g., 0.21). However, this cover leads to the smallest retained premium after claims amount (A74 million.) and one of the smallest RPS ratios (e.g., $RPS = 17.6$). The largest amount of retained premiums after claims and the largest RPS ratio is obtained under combination of quota-share and surplus reinsurance (e.g., retained premiums after claims $= 80.2$ mln. and $RPS = 19.0$). Nevertheless, the variance of retained claims under this cover is larger than under surplus reinsurance (Variance-to-Mean ratio $= 0.27$). Figure 5 shows that the ratios for all reinsurance covers are quite close to each other (located on the left side of the plot). This figure could be as a result of significant number of

[18]Quota-share retention α for Retail Life benefits vary between 15 and 90 percent depending on the original insurer's risk appetite. The average quota-share reinsurance premium income equal to 70 percent of the total premium income for every A1 reinsured. Retention line R for surplus reinsurance vary between A50 thousand and A3 million with the most common retention levels being A$500,000, A$750,000, A$1 million and A$1.5 million. Reinsurance premium income varies depending on these retention levels with the average of 78 percent of total premium income for every A1 reinsured. A number different combinations of retention percentages α and retention levels R are used for quota-share and surplus arrangement depending on the original risk appetite. The reinsurance premium percentage also varies from 45 to 85 percent with an average of 60 percent.

[19]Quota-share retention ratio α is 50 percent for both Direct Life groups. Again quota-share reinsurance premium income is estimated using both the original data (with percentage of 20 and 1.5 for Funeral Plan and other term life benefits respectively) and the data without zero or negative premiums (the percentage increases to approximately 40). Quota-share and surplus retention level R of A$9,300 with retention percentage α of 65 is used for Funeral Plan benefits and $R = A\$300,000$ with $\alpha = 60$ percent - for other term life benefits. Reinsurance premium income for quota-share and surplus is estimated using a ratio of 0.6. Surplus retention levels R of A$8,700 and A$250,000 are used for Funeral Plan and other term life benefits respectively. For these reinsurance premium income is equal to 78 percent of total premium income for every A$1 reinsured.

Table 7: Term Life Mean-Variance Estimation Results

Benefit Class		Reinsurance Structure	RPS	Retained Premium - Retained Claim (A$ 000)	Retained Claim (A$ 000)				n
					Mean	Variance	Variance-to-Mean Ratio	Sum	
Retail Life		No Reinsurance	12.44	86,243.89	0.1808	0.396	2.192	7,538.79	41,688
		Quota-Share	18.06	79,317.44	0.1115	0.044	0.392	4,649.95	41,688
		Quota-Share&Surplus	19.00	80,228.29	0.1069	0.029	0.272	4,456.48	41,688
		Surplus	17.59	74,040.10	0.1070	0.023	0.213	4,462.13	41,688
		Original Reinsurance Structure	17.79	74,798.73	0.1068	0.024	0.226	4,453.86	41,688
Direct Life	Funeral Plan	No Reinsurance	377.66	143,545.00	0.0014	9.64E-07	6.90E-04	381.10	272,891
		Quota-Share	491.95	130,787.35	0.0010	5.15E-07	5.16E-04	266.40	272,891
		Quota-Share&Surplus	465.00	126,441.60	0.0010	4.66E-07	4.67E-04	272.52	272,879
		Surplus	424.80	128,372.62	0.0011	4.06E-07	3.65E-04	302.91	272,879
		Original Reinsurance Structure	517.29	137,537.49	0.0010	5.15E-07	5.16E-04	266.40	272,891
	Other Term Life	No Reinsurance	2.55	28,829.56	0.4497	0.149	0.331	18,608.17	41,383
		Quota-Share	3.34	27,261.75	0.2811	0.058	0.207	11,634.58	41,383
		Quota-Share&Surplus	3.21	25,740.05	0.2815	0.048	0.171	11,651.27	41,383
		Surplus	2.86	26,135.68	0.3396	0.052	0.154	14,054.97	41,383
		Original Reinsurance Structure	4.05	35,520.79	0.2811	0.058	0.207	11,634.58	41,383

different retentions used under all reinsurance arrangements. A large number of retention levels for surplus and combination of quota-share and surplus increases the variability of overall retained claims and therefore brings the results closer to quota-share arrangement. As all reinsurance arrangements significantly reduce the variance of retained claims the cover that allows to retain largest amount of retained premiums and leads to the largest RPS ratio should be preferable. The original reinsurance arrangement which combines all reinsurance covers with the majority of benefits being reinsured under surplus leads to one of the smallest variance-to-mean ratio (0.23) and a larger RPS and retained premiums after claims compared to surplus alone.

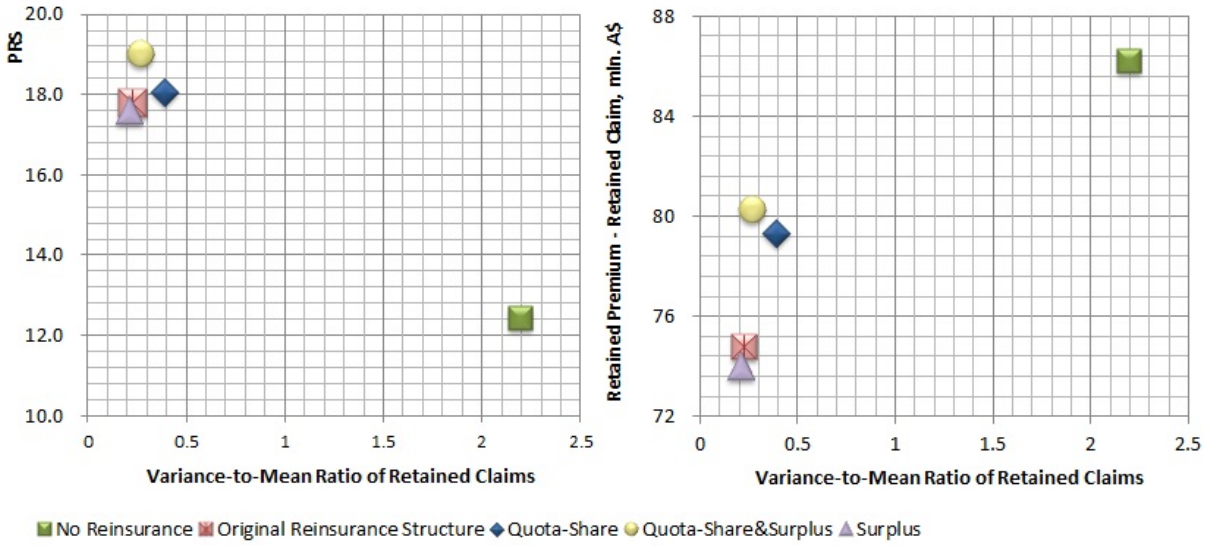

Figure 5: Retail Term Life RPS Ratio and Retained Premium after Claims Plot

Figure 6 shows that the largest RPS ratio and amount of retained premiums after claims

are obtained under quota-share reinsurance for both Direct Life groups. This result is affected by significantly lower quota-share reinsurance premiums. The variance-to-mean ratio of retained claims is smallest under surplus cover. However as the variability of total claim amounts is small in both groups, the reduction of this variability after applying different reinsurance arrangements is marginal. Therefore quota-share is the optimal cover for both Funeral Plan and other term life benefits covering Direct Life business as it allows the insurer to retain the largest RPS ratios and the largest amount of retained premiums after claims compared with other two reinsurance covers. If a predetermined risk appetite assumption is not taken into account the no reinsurance purchase case can be optimal for Funeral Plan as it retains a significantly larger amount of premiums after paying claims (e.g., *A*$143.5 million.). The variance of other term life benefit claims is significantly larger compared with Funeral Plan claims; here the decision whether no reinsurance purchase or quota-share arrangement is optimal depends on the cost of capital that needs to be held in order to mitigate higher underwriting risk without reinsurance.

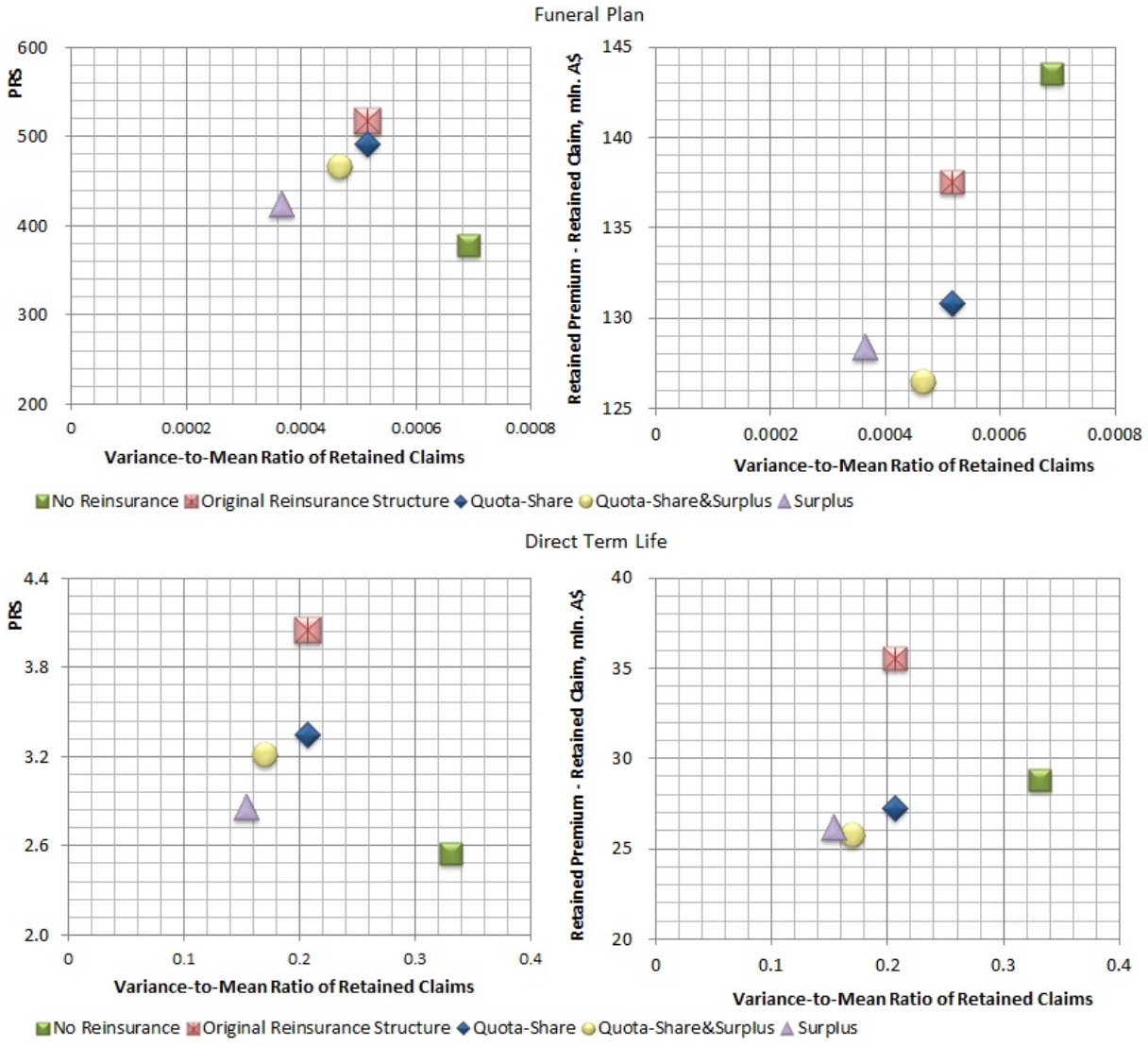

Figure 6: Direct Term Life RPS Ratio and Retained Premium after Claims Plot

5.4 Trauma

Mean-variance estimation results for Retail Life trauma benefits[20] are reported in Table 8. The results show that the largest RPS ratio and the largest amount of retained premi-

Table 8: Trauma Mean-Variance Estimation Results

Reinsurance Structure	RPS	Retained Premium - Retained Claim	Retained Claim (A$ 000)				n
			Mean	Variance	Variance-to-Mean Ratio	Sum	
No Reinsurance	3.47	22,167.58	0.555	2.116	3.815	8,992.14	16,213
Quota-Share	4.91	20,865.51	0.329	0.511	1.552	5,339.10	16,213
Quota-Share&Surplus	4.67	19,788.79	0.333	0.377	1.133	5,396.80	16,213
Surplus	4.37	18,251.86	0.334	0.307	0.920	5,412.84	16,213
Original Reinsurance Structure	4.40	18,385.91	0.334	0.307	0.921	5,412.55	16,213

ums after claims are obtained under quota-share reinsurance ($RPS = 4.9$ and retained premiums after claims $= 20.9$ million). The variance of retained claims is the smallest under surplus cover (Variance-to-Mean ratio $= 1.55$) (see Figure 7). These are con-

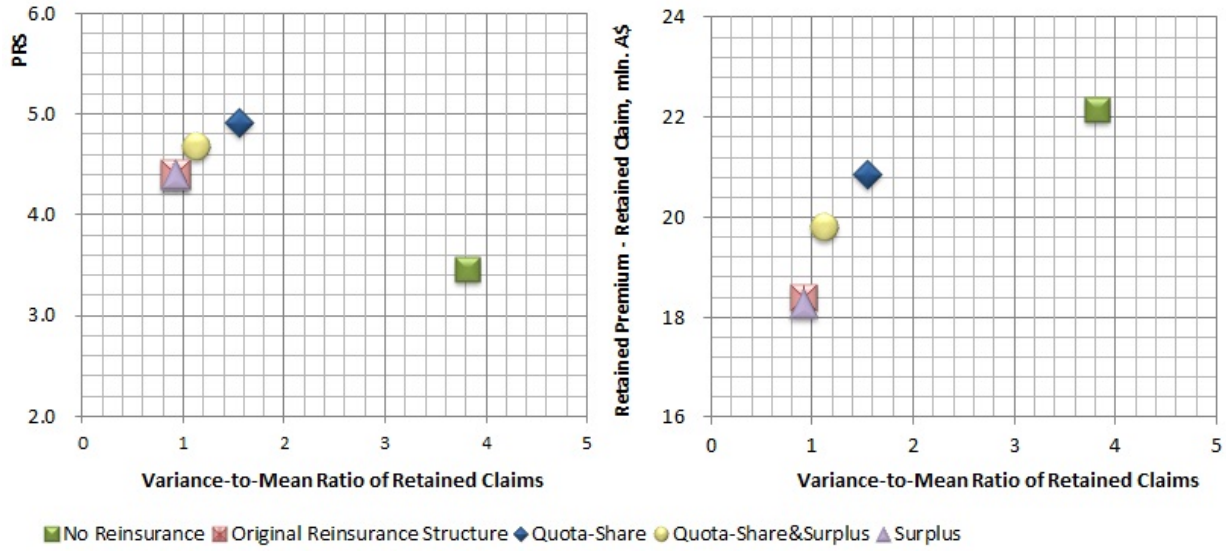

Figure 7: Trauma RPS Ratio and Retained Premium after Claims Plot

sistent with the previous results: quota-share reinsurance (or no reinsurance) can be optimal if the variance of total claims is relatively small. When the variance of these

[20]Retention percentage α varies between 35 and 80 for quota-share reinsurance with the premium income of 50 percent of original insurance premium income for every A$1 reinsured. Surplus reinsurance retention line R varies between A$50,000 and A$1 million with the most common retained amounts of A$250,000 and A$375,000. Surplus reinsurance premium income varies depending on the retention level R and the cover type with the average of 72 percent of total premium income for every A$1 reinsured. Several different combinations of retention percentages α and retention lines R are used for quota-share and surplus reinsurance depending on the original insurer's risk appetite. The most common combination of $\alpha = 75$ percent and $R = A\$400,000$ is used. The average reinsurance premium income for this type of cover is equal to 62 percent of original premium for every A$1 reinsured.

amounts is relatively large, quota-share (or no reinsurance) does not perform optimally. Therefore, a combination of quota-share and surplus or surplus cover should be chosen. The choice of the cover depends on the tradeoff between reinsurance premiums and the cost of capital needed to mitigate underwriting risk. The original reinsurance structure, where the majority of benefits are reinsured under surplus reinsurance, results in one of the smallest variance-to-mean ratio. It also results in one of the smallest RPS and retained premiums after claims.

6 Conclusions

This paper has analysed the optimal reinsurance structures for a heterogeneous life insurance portfolio based on actual policy, claim and reinsurance data for a large Australian life insurer. Three different proportional reinsurance structures (quota-share, surplus and combination of quota-share and surplus) as well as the no-reinsurance purchase case were assessed and compared with the original reinsurance structure that is used by the insurer for term life, TPD, income protection and trauma benefits.

A modified "mean-variance" framework was adopted in order to assess which of the reinsurance structures has the most desirable properties. A trade-off between three criteria, two for retained premiums and one for the variance of retained claims, were used to assess reinsurance optimality:

- retained premium-to-retained claims ratio (RPS) and total retained premiums after claims,
- variance of retained claims (variance-to-mean ratio).

The paper found that the type of optimal reinsurance arrangement depends on a combination of different benefit characteristics. The number of retention levels can influence the results, that is, when the portfolio is homogenous (e.g., only one or two different retention levels are used), surplus or a combination of quota-share and surplus reinsurance are optimal as these covers minimises the variance of retained claims[21].

Claim variance has an influence on the choice of optimal reinsurance cover. When the variance of total claim amounts is relatively small, quota-share reinsurance tend to be the optimal cover as the price of this cover is almost always the lowest. However, when the variance is relatively large, quota-share does not perform optimally and either surplus or quota-share and surplus should be chosen. In most cases, surplus reinsurance results in the smallest retained claim variance, however it also leads to the smallest retained premium amounts.

Finally, if the assumption of a predetermined risk appetite and therefore a predetermined amount of assets available is not taken into consideration, the no reinsurance purchase case can be optimal for the lines of business where the variance of total claims is relatively small as it always allows the insurer to retain the largest amount of premium income after paying claims.

[21] The choice between these two arrangements depends on the price of reinsurance.

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