Combined Optimization of Portfolio and Risk Exposure of an Insurance Company

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Abstract

This paper presents a model for an insurance company that controls its risk and is allowed to invest in a financial market with just two assets - a risk free asset and a stock. The financial reserve of this company is modelled as an Ito process with positive drift and constant diffusion coefficient. While the diffusion coefficient can be interpreted as the risk exposure, the drift can be understood as the potential profit. The new feature of this paper is to consider that the potential profit of this company depends on the dynamical state of the economy. Thus, in order to take into account the state of the economy, the drift process is modelled as a continuous time Markov chain. The aim is to maximize the reserve of an insurance company whose manager is risk averse. The optimal control problem is formulated and the Hamilton-Jacobi-Bellman equation is solved to yield the solution.

1 Introduction

Since the seminal papers due to Merton [26], [27], the optimal stochastic control methods have been among the most useful recent techniques to deal with problems in economics and finance. This is due to the fact that many economical and financial problems present the necessity of taking decisions based on an objective performance criterion and in the presence of uncertainty. In this context, this paper presents a model for an insurance company that controls its risk and is allowed to invest in a financial market with just two assets as in the Black-Scholes market [4] – a risk free asset and a stock.

Recently, many works have dealt with diffusion models for insurance companies with controllable risk exposure, for instance, [1], [20], [21], [22], [23], [30] and others. This paper, as in the previous papers, considers that the financial reserve of the insurance company is modelled as an Ito process with positive drift and constant diffusion coefficient. While the diffusion coefficient can be interpreted as the risk exposure, the drift can be associated to the potential profit when the number of sold policies is sufficiently large. The new feature of this paper is to consider that the potential profit of the insurance company depends on the dynamical state of the economy. In order to take into account this, the drift process is modelled as a continuous time Markov chain, i.e., the reserve of the company is modelled as a switching diffusion model. This type of model intends to take into account a large class of changes which can affect the potential profit of the insurance company, for instance, legislation changes, population income changes and others. In Brazil, one may point out a good example of a recent change in the federal legislation that had a strong impact on the health insurance companies' potential profit. Until 1998, there was no clear legislation to deal with health insurances in Brazil, but from this date on the situation has completely changed.

Switching diffusions have been used successfully to model a large class of systems with random changes in their structures that may be consequences of abrupt phenomena, for instance, econometric systems [5], manufacturing systems [15], [17] and others. Although most of these works deal with linear models [5], [10], [12], [14], [16], [24], some of these results could be extended to non-linear systems [15] and [17].

Although the idea of modelling by using switching models is not new in the finance literature, most works are set in the context of discrete time models, for instance, [6], [7], [8], [11] and others. In the continuous time setting, few works are available, for instance, [9], [18], [28] and [29]. On the other hand, one may see in this paper that although the control problem is not a linear control problem with quadratic cost, an approach similar to [14] may be conveniently adapted to the present case.

This paper is organized as follows. In section 2, the problem described above is formally stated. In section 3, some properties of the switching diffusions are presented. In section 4, the problem is solved. Finally, section 5 presents some conclusions of this work.

Notation: Stochastic process will be denoted by omitting the argument $\omega \in \Omega$. For instance, $X(t)$ instead of $X(t,\omega)$. The integrals with respect to $dB(t)$ are taken in the sense to Ito. Almost surely is abbreviated a.s.

2 The Model and the Problem Statement

This work addresses the problem of optimizing the wealth of a small investor in the generalized Black-Scholes model [4] where the risk asset price is modelled as a switching diffusion. In this context, one should consider the following statistically mutually independent objects:

a) A two dimensional Brownian motion $B = \{B(t), \mathcal{F}_t^B; s \le t \le T\}$ where $B(t) = \begin{bmatrix} B_R(t) \\ B_{X_1}(t) \end{bmatrix}$ is defined on some probability space $(\Omega^B, \mathcal{F}^B, P^B)$;

b) A homogeneous continuous time Markov chain $\theta = \{\theta(t), \mathcal{F}_t^\theta; s \le t \le T\}$ defined on some probability space $(\Omega^\theta, \mathcal{F}^\theta, P^\theta)$, with right continuous trajectories, and taking values on the finite set $\mathcal{S} = \{1, 2, \ldots, n\}$. One should also assume that $p(t) = \{p_1(t), p_2(t), \ldots, p_n(t)\}$, with $p_i(t) = P^\theta(\theta(t) = i)$, where $i \in \mathcal{S}$, satisfies the following Kolmogorov forward equation $dp/dt = \Lambda p(t)$ where $\Lambda = [\lambda_{ij}]$ is the stationary $n \times n$ transition rate matrix of θ with $\lambda_{ij} \ge 0$ for $i \ne j$, and $\lambda_{ii} = -\sum_{i \ne j} \lambda_{ij}$, i.e., the process is supposed to be conservative.

Remark 2.1. *One may denote a suitable complete probability space* $(\Omega, \mathcal{F}, P^{)}$ *, where* $\mathcal{F} = \mathcal{F}^B \times \mathcal{F}^\theta$ *denotes the* σ*-algebra generated by rectangles* $A^B \in \mathcal{F}^B$ *and* $A^\theta \in \mathcal{F}^\theta$*. Thus, according to the Fubini's theorem, since* P^B *and* P^θ *are* σ*-finite, it follows that* P *is unique and given by* $P(A^B \times A^\theta) = P(A^B)P(A^\theta)$*. For details, see [3].*

Remark 2.2. *The filtration* $\{\mathcal{F}_t\}_{t\in[0,\infty)}$ *may be interpreted as the information available at time* $t \in [0, \infty)$*.*

On the other hand, one may assume that in the case of no risk or investment control, the reserve of the company evolves according to

$$dR(t) = \mu_R dt + \sigma_R dB(t) \tag{2.1}$$

where μ_R is the potential profit and σ_R is the risk exposure of the insurance company. The motivation of this model may be found in [1] and [30]. While σ_R is positive constant, it is assumed that $\mu_R \triangleq \sum_{i=1}^n \delta(i,\theta)\mu_{Ri}$ where, for i, $\theta \in \mathcal{S}$, $\delta(i,\theta)$ is the Kronecker's symbol and the state-wise drift processes μ_{Ri}, for $i = 1, \ldots, n$, are constants.

Additionally, according to the generalized Black-Scholes model [4], one may suppose that the prices X_0 of the risky-free asset and X_1 of the stock are given by

$$dX_0(t) = \rho X_0 dt \tag{2.2}$$

and

$$dX_1(t) = \rho X_1 dt + \sigma X_1 dB_{X_1}(t) \tag{2.3}$$

where ρ, μ and σ are constants.

Remark 2.3. *Because Lipschitz continuity and linear growth conditions are satisfied the equations (2.1), (2.2) and (2.3) have one unique strong solution. The proof follows the same lines of theorem 4.6 on page 128 in [25].*

If one considers that the company controls its risk and is allowed to invest in the financial market, the resulting reserve process (liquid assets of the company or wealth of the company) $(W(t), \theta(t)) \in \mathcal{R} \times \mathcal{S}$ defined on a suitable probability space $(\Omega, \mathcal{F}, P)$ may be described by

$$\begin{aligned} dW(t) &= u_1(\mu_R dt + \sigma_R dB_R(t)) + u_2(\mu W(t)dt + \sigma W(t)dB_{X_1}(t)) \\ &+ (1-u_2)(\rho W(t)dt) \\ &= (u_1\mu_R + u_2\mu W(t) + (1-u_2)\rho W(t))dt + u_1\sigma_R dB_R(t) + u_2\sigma W(t)dB_{X_1} \end{aligned} \tag{2.4}$$

and

$$P(\theta(t+\Delta t) = j/\theta(t) = i) = \lambda_{ij}\Delta t + o(\Delta t) \tag{2.5}$$

where $\lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0$. In this paper, the control is the duple $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$. One should note that u_1, where $0 \leq u_1 \leq 1$, corresponds to the retention level, which is the fraction of all incoming claims that the insuring company will insure by itself. Here, it has been considered that the reinsuring company has the same safety loading as the insuring company, an assumption that is known as "cheap reinsurance". On the other hand, u_2 is the fraction of the agent's wealth that is invested in the risky asset, thereby investing the fraction $(1 - u_2)$ in the safe one, where $0 \leq u_2 \leq 1$.

Assumption 2.1. *$\mathcal{U}$ is to be the set of admissible policies. Such a process $u : \mathcal{R}^2 \times \mathcal{S} \to \mathcal{U}$ is called an admissible policy if, for $l = 1, 2$, $u_l \in [0, 1]$ and u_l satisfies the following conditions:*

a) Restriction on growth condition

$$u_l^2(x(t), \varepsilon(t)) \leq L_1 \int_0^t (1 + x^2(s))dK(s) + L_2(1 + \varepsilon^2(t) + x^2(t)) \tag{2.6}$$

b) Lipshitz condition

$$|u_l(x(t), \varepsilon(t)) - u_l(y(t), \varepsilon(t))|^2 \leq L_1 \int_0^t (x(t) - y(t))^2 dK(s) + L_2(x(t) - y(t))^2 \tag{2.7}$$

where L_1 and L_2 are positive constants, $K(\cdot)$ is a non-decreasing right continuous function, $0 \leq K(\cdot) \leq 1$, $x(\cdot)$ and $y(\cdot)$ are continuous measurable functions, $\varepsilon(t) \in \mathcal{S}$ and $s \leq t \leq T$.

Remark 2.4. *From assumption 2.1, the equation (2.4) has one unique strong solution. The proof follows the same lines of theorem 4.6 on page 128 and theorem 4.9 on page 142 in [25].*

Assumption 2.2. *In this work, the jump sizes are considered predictable in $\theta = \{\theta(t), \mathcal{F}_t^\theta; s \leq t \leq T\}$, i.e. one does not know if a jump will occur, but if it does, its intensity is known.*

Remark 2.5. *One may see that assumption 2.2 is not too restrictive, since $\mu_R \triangleq \sum_{i=1}^n \delta(i, \theta)\mu_{Ri}$ is the potential profit of the insurance company and this state may be accessed with some precision by the manager of this company.*

One may define a function $\Phi : [s, T] \times \mathcal{R} \times \mathcal{S} \to \mathcal{R}$ defined on a vector space endowed with the product topology. Thus, again according to the Fubini's theorem and assumption 2.6, one may get $E_W[\Phi] = E_{B\times\theta}[\Phi] = \int_{\Omega^B} \int_{\Omega^\theta} \phi dP^B dP^\theta$. For details, see [3].

Problem 2.1. *Suppose that, starting with the initial reserve $W(s) = w$ at time $t = s$, the manager of an insurance company wants to maximize the total expected profit given by $E_W[\int_s^T \exp(-\gamma t)U(W(t))dt / W(s) = w, \theta(s) = i]$, where $T = inf\{t > s : W(t) = 0\}$. In this work, one should assume that the utility function U of the company manager is increasing* *and concave (the manager is risk averse). Thus, the problem is to find the value function* $\Phi(s,w,i)$ *and a Markov control* $u^*(t,W,i) = \begin{bmatrix} u_1^*(t,W,i) \\ u_2^*(t,W,i) \end{bmatrix}$ *such that* $\Phi(s,w,i)$ *is given by*

$$\Phi(s,w,i) = \sup_{0 \le u_l \le 1} J(s,w,i,u) \quad \text{for } l = 1,2 \tag{2.8}$$

where

$$\begin{aligned} J &= E_W\left[\int_s^T \exp(-\gamma t)U(W(t))dt/W(s) = w, \theta(s) = i\right] \\ &= E_{B\times\theta}\left[\int_s^T \exp(-\gamma t)U(W(t))dt/W(s) = w, \theta(s) = i\right] \end{aligned} \tag{2.9}$$

Remark 2.6. *The assumption that the manager of the insurance company is risk averse is a significant difference between this work and [30] and many references in it. Moreover, if one assumes that the utility function is a convenient concave function, it is possible to employ similar tools to that ones presented by [26] and [27]. If the utility function were linear as in [30], the problem that has been proposed here would become too hard and it would likely have no closed solution. This would happen since the resultant Hamilton-Jacobi-Bellman partial differential equation wouldn't have separable variables as in [30], as well as many references in it and also in [26] and [27].*

3 Switching Diffusions: Basic Properties

This section intends to review some basic properties of switching diffusions . It follows the same steps of [14].

Proposition 3.1. *The process* $(W(t), \theta(t))$ *is a Markov process.*

Sketch of proof: Consider the following statements

a) $\theta(t)$ is a Markov process according to its definition. For details, see [19];

b) $B(t)$ is also a Markov Process, since its increments are independent;

c) The sources of uncertainty in $W(t)$ are $B(t)$ and $\theta(t)$ and they are independent. On the other hand, the process $(W(t), \theta(t))$ is defined on a σ-algebra $\mathcal{F} = \mathcal{F}^B \times \mathcal{F}^\theta$, generated by rectangles $A^B \in \mathcal{F}^B$ and $A^\theta \in \mathcal{F}^\theta$.

Thus,$(W(t), \theta(t))$ is a Markov process.

Proposition 3.2. *The process* $\{(W(t), \theta(t)); s \le t \le T\}$ *has sample paths that are continuous from the right.*

Proof: It is obvious from equation (2.4) and the definition of $\theta(t)$.

Proposition 3.3. *The process $\{(W(t), \theta(t)); s \leq t \leq T\}$ has a stochastically continuous transition probability. Therefore, is uniquely defined by its infinitesimal generator.*

Proof: It follows from proposition 3.1, proposition 3.2 and the Dynkin's formula.

Definition 3.1. *Let T_h be the operator defined on the space of $\mathcal{B}(\mathcal{R} \times \mathcal{R} \times \mathcal{S})$ of bounded measurable scalar functions Φ defined on $\mathcal{R} \times \mathcal{R} \times \mathcal{S} = \mathcal{X}$ and equipped with the norm $\|\Phi\| = \sup_{x \in \mathcal{X}} |\Phi(x)|$ as follows*

$$T_h\Phi(s, w, i) = E_W[\Phi(s + h, W(s + h), \theta(s + h))/W(s) = w, \theta(s) = i] \tag{3.1}$$

Thus, one may define the infinitesimal generator L of a family of transition probabilities of the Markov process $\{(W(t), \theta(t)); s \leq t \leq T\}$ as

$$L\Phi(s, W(s), i) = \lim_{h \to 0} \frac{T_h\Phi(s, W(s), i) - T_0\Phi(s, W(s), i)}{h} \tag{3.2}$$

where the limit is the uniform limit in $\mathcal{B}(\mathcal{R} \times \mathcal{R} \times \mathcal{S})$. The domain of definition $\mathcal{D}_L \subset \mathcal{B}(\mathcal{R} \times \mathcal{R} \times \mathcal{S})$ consists of all functions for which limit in (3.2) exists. For details and examples, see page 36 in [2].

Remark 3.1. *$L\Phi$ can be interpreted as the infinitesimal "average" change of the function Φ.*

Remark 3.2. *If $\Phi \in \mathcal{D}_L$ then $\lim_{h\to 0} T_h\Phi(t, W(t), \theta(t)) = \Phi(t, W(t), \theta(t))$.*

Now, one should notice that $\Phi \in \mathcal{D}_L$ is the class of functions that continuous derivatives of first order in t on $[0, T]$ and first and second orders in $W(t)$ exists almost everywhere.

Proposition 3.4. *The infinitesimal generator of $\{(W(t), \theta(t)); s \leq t \leq T\}$, with $\{(W(t); s \leq t \leq T\}$ that satisfies the equation (2.4) and $\{\theta(t); s \leq t \leq T\}$ that satisfies assumption 2.2 and $u \in \mathcal{U}$ is given by*

$$\begin{aligned} L^u\Phi(t, W, i) &= \frac{\partial \Phi}{\partial t} + (u_1\mu_{Ri} + u_2\mu W + (1 - u_2)\rho W)\frac{\partial \Phi}{\partial W} \\ &+ \frac{1}{2}(u_1^2\sigma_R^2 + u_2^2\sigma^2 W^2)\frac{\partial^2\Phi}{\partial W^2} + \sum_{j=1}^{n} \lambda_{ij}\Phi(t, W, j) \end{aligned} \tag{3.3}$$

Proof: From equation 3.2, one may write

$$\begin{aligned} L^u\Phi(t, W, i) &= \lim_{h\to 0} \frac{T_h\Phi(s, W(s), i) - T_0\Phi(s, W(s), i)}{h} \\ &= \lim_{h\to 0} \frac{1}{h}\{E_{B\times\theta}[\Phi(s + h, W(s + h), \theta(s + h))/W(s) = w, \theta(s) = i] \\ &- T_0\Phi(s, W(s), i)\} \end{aligned}$$

Additionally, from equation (2.5), one may get

$$\begin{aligned} L^u\Phi(s,W,i) &= \lim_{h\to 0}\frac{1}{h}\{\sum_{j=1}^{n} E_B[\Phi(s+h,W(s+h),j)/W(s)=w]\cdot P^{\theta}[\theta(s+h)=j/\theta(s)=i] \\ &- T_0\Phi(s,W(s),i)\} \\ &= \lim_{h\to 0}\frac{1}{h}\{\sum_{\substack{j=1\\ i\neq j}}^{n} E_B[\Phi(s+h,W(s+h),j)/W(s)=w]\cdot(\lambda_{ij}h+o(h)) \\ &+ E_B[\Phi(s+h,W(s+h),i)/W(s)=w]\cdot(1+\lambda_{ii}h+o(h)) \\ &- T_0\Phi(s,W(s),i)\} \\ &= \lim_{h\to 0}\frac{1}{h}\{E_B[\Phi(s+h,W(s+h),i)/W(s)=w]-T_0\Phi(s,W(s),i)\} \\ &+ \lim_{h\to 0}\{\sum_{j=1}^{n} E_B[\Phi(s+h,W(s+h),j)/W(s)=w]\lambda_{ij}\} \end{aligned}$$

And, from definition 3.1 and remark 3.2

$$\begin{aligned} L^u\Phi(s,W,i) &= \lim_{h\to 0}\frac{1}{h}\{E_B[\Phi(s+h,W(s+h),i)/W(s)=w]-\Phi(s,W(s),i)\} \\ &+ \sum_{j=1}^{n}\lambda_{ij}\Phi(s,W(s),j) \end{aligned}$$

Finally, from equation (2.4) and equations on pages 41 and 42 in [2], one can see that

$$\lim_{h\to 0}\frac{1}{h}\{E_B[\Phi(s+h,W(s+h),i)/W(s)=w]-\Phi(s,W(s),i)\} = \\ \frac{\partial\Phi}{\partial s}+(u_1\mu_{Ri}+u_2\mu W+(1-u_2)\rho W)\frac{\partial\Phi}{\partial W}+\frac{1}{2}(u_1^2\sigma_R^2+u_2^2\sigma^2W^2)\frac{\partial^2\Phi}{\partial W^2}$$

Thus, the proof is complete.

Theorem 3.1. *The Hamilton-Jacobi-Bellman equation associated to this problem is given by*

$$\sup_u\{L^u\Phi(t,W,i)+exp(-\gamma t)U(W)\}=0 \tag{3.4}$$

with boundary conditions $\Phi(T,W,i)=U(W)$.

Proof: It follows from the Dynkin's formula and the Bellman's optimality principle.

Theorem 3.2. *(Dynamic Programming Verification Theorem) Let* Φ *be the solution of the dynamic programming equation* $\sup_u\{L^u\Phi(t,W,i)+exp(-\gamma t)U(W)\}=0$ *with boundary conditions* $\Phi(t,W,i)=U(W)$. *Then:*

a) $J(t,W,i,u)\leq\Phi(t,W,i)$ *for any admissible feedback control* u *and any initial data;*

b) *If* u^* *is an admissible feedback control such that* $L^{u^*}\Phi(t,W,i)+exp(-\gamma t)U(W) = \sup_u\{L^u\Phi(t,W,i)+exp(-\gamma t)U(W)\}=0$. *Thus* u^* *is optimal;*

Proof: This proof follows the same lines of [13].

4 Problem Solution

The results presented in section 3 will be used to solve problem 2.1. In general, it is difficult to find the explicit solution of the problem 2.1. However, one may circumvent this problem when the utility function $U(W)$ is given by a power function $U(W) = W^r$, $0 < r < 1$.

Theorem 4.1. *If $\frac{\partial^2 \Phi}{\partial W^2} < 0$, $\frac{\partial \Phi}{\partial W} > 0$, in equation (3.3), and $U(W) = W^r$, $0 < r < 1$, then the optimal control u^* that solves the problem 2.1 is given by*

$$u^*(W,i) = \begin{bmatrix} u_1^* \\ u_2^* \end{bmatrix} \tag{4.1}$$

where

$$u_1^*(W,i) = min\left(\frac{\mu_{Ri} W}{\sigma_R^2(1-r)},\ 1\right) \quad \text{for } \theta(t) = i \tag{4.2}$$

and

$$u_2^*(W,i) = min\left(\frac{\mu - \rho}{\sigma^2(1-r)},\ 1\right) \quad \text{for } \theta(t) = i \tag{4.3}$$

and the value function is given by $\Phi(t, W, i) = f(t, i)W^r$ where $f(t, i)$ are the unique solutions of the system of ordinary differential equations given by

$$\frac{df(t,i)}{dt} + \left(\rho r - \frac{\mu_{Ri}^2 r}{2\sigma_R^2(r-1)} - \frac{(\mu-\rho)^2 r}{2\sigma^2(r-1)}\right) f(t,i) + \sum_{j=1}^{n} \lambda_{ij} f(t,j) + \exp(-\gamma t) = 0 \tag{4.4}$$

with boundary conditions $f(T, i) = 1$, for $i = 1, \cdots, n$.

Proof: Firstly, it is necessary to find $u(t, W, i)$ that maximizes the Hamilton-Jacobi-Bellman equation given by (3.4). If $\frac{\partial^2 \Phi}{\partial W^2} < 0$ and $\frac{\partial \Phi}{\partial W} > 0$, since equation (3.4) is a second degree polynomial in u_1 and u_2, then

$$u_1^*(W,i) = min\left(-\frac{\mu_{Ri}\frac{\partial \Phi}{\partial W}}{\sigma_R^2 \frac{\partial^2 \Phi}{\partial W^2}},\ 1\right) \tag{4.5}$$

and

$$u_2^*(W,i) = min\left(-\frac{(\mu-\rho)\frac{\partial \Phi}{\partial W}}{\sigma^2 W \frac{\partial^2 \Phi}{\partial W^2}},\ 1\right) \tag{4.6}$$

Thus, if one takes into account (3.3) and substitutes (4.5) and (4.6) into (3.4), then the solution of (3.4) is given by $\Phi(t, W, i) = f(t, i)W^r$, for $i = 1, \cdots, n$, where $f(t, i)$ is the solution of the system of n ordinary differential equations given by (4.4) where the final conditions calculated from the boundary condition of the partial differential equation are $f(T, i) = 1$ for $i = 1, \cdots, n$. Substituting $\Phi(t, W, i) = f(t, i)W^r$ in (4.5) and (4.6), one may arrive to equation (4.2) and (4.3).

Remark 4.1. *It is easy to prove that the system (4.4) is a real positive one. For details, see theorem 4.1 on page 62 in [31].*

Remark 4.2. *One may see that the control law given by equation (4.1) requires explicit knowledge of the state of* $\theta(t)$*.*

Remark 4.3. *It is interesting to note that the system of* n *ordinary differential equations given by equation (4.4) is similar to the set of interconnected Riccati equations that arises in continuous time markovian jump linear quadratic control, for instance, see [14].*

Remark 4.4. *It is very interesting to interpret equation (4.2). One may see that there are two different periods in the life of an insurance company. When the insurance company is small as compared to the potential claims, i.e.* $W < \frac{(1-r)\sigma_R^2}{\mu_{Ri}}$*, the manager of this insurance company should think about the possibility of reducing the risk which the company is faced and his or her decision depends on the dynamical state of the economy. On the other hand, when the insurance company grows, sharing the risk, i.e. reinsuring, is not an interesting procedure since the risk may not be a threat of bankruptcy anymore and it also means sharing the profit.*

Remark 4.5. *The interpretation of equation (4.3) is the same as that one given by [26] and [27]. The choice of the optimum portfolio depends on the risk premium* $(\mu - \rho)/\sigma$*.*

5 Conclusions

This paper has presented a model for an insurance company that controls its risk and is allowed to invest in a financial market with just two assets as in the Black-Scholes market [4] – a risk free asset and a stock. The interesting new feature of this model paper is to consider that the potential profit of the insurance company depends on the dynamical state of the economy. Based on this statement, the proposed problem has been completely solved by means of dynamic programming arguments when the complete observation case is considered. This paper can also be seen as an extension of some works that deal with linear stochastic equations with jump parameters, for instance, see [14].

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References

[1] S. Asmussen and M. Taksar, "Controlled diffusion models for optimal dividend pay-out," *Insurance: Mathematics and Economics* 20, 1997, 1–15.

[2] L. Arnold, *Stochastic differential equations: theory and applications*, John Wiley and Sons, New York, 1974.

[3] R. G. Bartle, *The elements of integration*, John Wiley and Sons, New York, 1966.

[4] F. Black and M. Scholes, "The pricing of options and corporate liabilities," *Journal of Political Economy* 81, 1973, 637–659.

[5] W. Blair Jr. and D. Sworder, "Continuous time regulation of a class of econometric models," *IEEE Transactions on Systems, Man and Cybernetics* 5, 1975, 341–346.

[6] N. Bollen, S. Gray and R. Whaley, "Regime switching in foreign exchange rates: evidence from currency option prices," *Journal of Econometrics* 94, 2000, 239–276.

[7] H. Dewachter and D. Veestraeten, "Expectation revisions and jumps in asset prices," *Economic Letters* 59, 1998, 367–372.

[8] H. Dewachter, "Can Markov switching models replicate chartist profits in the foreign exchange market?," *Journal of International Money and Finance* 20, 2001, 25–41.

[9] G. Di Masi, Y. Kabanov and W. Runggaldier, "Mean variance hedging of options on stocks with markov volatilities," *Theory of probability and Applications* 39, 1995, 172–182.

[10] F. Dufour and R. Elliott, "Adaptive control of linear systems with Markov perturbations," *IEEE Transactions on Automatic Control* 43, 1998, 351–372.

[11] M. Evans and K. Lewis, "Do long term swings in the dollar affect estimates of the risk premia?," *The review of financial studies* 8, 1995, 709–742.

[12] M. Everdij and H. Blom, "Embedding adaptive JLQG into LQ martingale control with a completely observable stochastic control matrix," *IEEE Transactions on Automatic Control* 41, 1996, 424–430.

[13] W. Fleming and R. Rishel, *Deterministic and stochastic optimal control*, Springer Verlag, New York, 1975.

[14] M. Fragoso and E. Hemerly, "Optimal control for a class of noisy linear systems with Markovian jumping parameters and quadratic cost," *International Journal of Systems Science* 22, 1991, 2553–2561.

[15] M. Ghosh, A. Arapostatathis and S. Marcus, "Optimal control of switching diffusions with application to flexible manufacturing systems," *SIAM Journal on Control and Optimization* 31, 1993, 1183–1204.

[16] M. Ghosh, A. Arapostatathis and S. Marcus, "A note on an LQG regulator with Markovian switching and pathwise average cost," *IEEE Transactions on Automatic Control* 40, 1995, 1919–1921.

[17] M. Ghosh, A. Arapostatathis and S. Marcus, "Ergodic control of switching diffusions," *SIAM Journal on Control and Optimization* 35, 1997, 1952–1988.

[18] X. Guo, "Information and option prices," *Quantitative Finance* 1, 2001, 38–44.

[19] D. Heyman and and M. Sobel, *Stochastic processes in operations research: Volume I*, McGraw Hill Book Company, New York, 1982.

[20] B. Hϕjgaard and M. Taksar, "Optimal proportional reinsurance policies for diffusion models," *Scandinavian Actuarial Journal* 2, 1998a, 166–180.

[21] B. Hϕjgaard and M. Taksar, "Optimal proportional reinsurance policies for diffusion models with transactions costs," *Insurance: Mathematics and Economics* 22, 1998b, 41–51.

[22] B. Hϕjgaard, "Controling risk exposure and dividend payout schemes: insurance company example," *Mathematical Finance* 9, 1999, 153–182.

[23] B. Hϕjgaard and M. Taksar, "Optimal dynamic portfolio selection for a corporation with controllable risk and dividend distribution policy," *Working Paper — Department of Mathematics, Aalborg University*, 2000.

[24] Y. Ji and H. Chizeck, "Jump linear quadratic Gaussian control in continuous time," *IEEE Transactions on Automatic Control* 37, 1992, 1884–1892.

[25] R. Liptser and A. Shiryayev, *Statistics of random process I: general theory*, Springer-Verlag, Berlin, 1977.

[26] R. Merton, "Lifetime portfolio selection under uncertainty: the continuous case," *The Review of Economics and Statistics* 51, 1969, 247–257.

[27] R. Merton, "Optimum consumption and portfolio rules in a continuous time model," *Journal of Economic Theory* 3, 1971, 373–413.

[28] V. Naik, "Option valuation and hedging strategies with jumps in the volatility of asset returns," *Journal of Finance* 48, 1993, 1969–1984.

[29] R. Norberg, "A time continuous Markov chain model with applications to insurance," *Journal of Applied Stochastic Models and Data Analysis* 11, 1995, 245–256.

[30] M. Taksar, "Optimal risk/dividend distribution control methods – Applications to insurance," *Mathematical Methods of Operation Research* 1, 2000, 1–42.

[31] J. Zabczyk, *Mathematical control theory: An introduction*, Birkhauser, Boston, 1995.