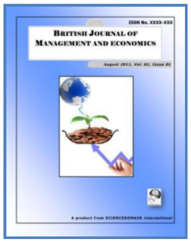

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Optimization Models for Insurance Portfolio Optimization in the Presence of Background Risk

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ABSTRACT

The liability stream of insurance companies often stretches several years into the future. Therefore, there is always the need to determine a portfolio of bonds or other assets whose cash-flows replicate those of the liability stream. Insurance regulatory authorities require that insurance companies must demonstrate solvency. To achieve this, an insurance company needs to determine a fair market value of its liability by finding a replicating portfolio consisting of default-free bonds. This paper presents a class of optimization models that could be employed for portfolio optimization in the presence of background risk.

Keywords: Optimization models; insurance portfolio optimization; background risk;

1. INTRODUCTION

Insurance companies often face a liability stream reaching several years into the future, representing future payouts on insurance products, such as life insurance. Usually, such future liability streams are stochastic, since it is not known today precisely when they will occur. For life insurance, this depends on customers' length of lives and on options, such as cancellation rights. Other examples of future liability streams include home owner mortgages with variable payments and lottery payouts. It is often an interesting problem to determine a

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portfolio of bonds or other assets whose cash-flows replicate those of the liability stream. Regulatory conditions often require that insurance companies need to demonstrate solvency. One way of doing that is to determine a fair market value of their liabilities by finding a replicating portfolio consisting of default-free bonds. This is similar to determining the expected present value (in the case of stochastic cash-flows) of liabilities, using the zero-coupon yield curve.

Insurers constantly look for opportunities to expand their businesses, increase revenues and improve profitability. However, in each line of business and for the firm overall, portfolio growth can lead to increased revenues and an increase in accumulation of risks (Hojgaard and Taskar, 2000). The challenge is to maximize profitability by achieving the optimal risk/reward relationship between loss exposure and expected profit margin. This relationship becomes increasingly important as companies with unmanaged catastrophe accumulations suffer rating pressure and skyrocketing reinsurance costs. While the evaluation of individual risks is important, getting the mix of risks right is equally critical. The stochastic modeling approach to insurance portfolio optimization emphasizes the question of whether the new risk is a good addition to the existing portfolio.

The optimal stochastic control methods have been among the most useful recent techniques developed for dealing with problems in economics and finance. This arises from the fact that many economic and financial problems require taking decisions, based on an objective performance criterion and in a situation of uncertainty (Seppäläinen and Sethuraman, 2003). In this context, there is a need to postulate a model useful for risk optimization and control for insurance industry. Recently, many works including that of Asmussen and Taskar (1997), have dealt with diffusion models for insurance companies with controllable risk exposures. Considering the fact that the financial reserve of insurance companies is modeled as a process with a positive drift and constant diffusion coefficient (risk exposure), the drift can be associated with the potential profit when the number of sold policies is sufficiently large (Cuthbertson, and Nitzsche, 2001).

Switching diffusions have been used successfully to model a large class of system with random changes in their structures that may be consequences of abrupt phenomena, such as in econometric systems (Blair and Sworder, 1975) and manufacturing systems (Ghosh et al, 1993 and 1997). Although most of these works deal with linear models, some of the results could be applied to non-linear systems. The idea of modeling, by using switching models, is not new in finance literature. Most works were set in the context of discrete time models (Dewachter and Veestraeten, 1998).

1.1 Statement of the Research Problem

In an unstable economy, charging rates by businesses tend to be variable. This is because practitioners in such an environment are free to charge arbitrary rates, depending on what they think is required to survive. The result is that anybody, whether efficient or inefficient, can prosper. The insurance business practitioners are not exempted from this circle. The insurance business, like any other business, is concerned with profit/benefit optimization, although vis-a-vis background risk. Going by the general input-output theory, the major inputs of an insurance company are: earned premium, investment income (lagged) and realized capital gain (lagged). The earned premium is made up of: pure risk premium; loading for expenses; and loading for profit.

Major outputs of an insurance company include: incurred claims (net of reinsurance recoveries), expenses and dividends to policy holders; and dividends to shareholders. The incurred claims are made up of: incurred claims pertaining to the current accident year, and changes in claim amounts in respect of claims pertaining to previous accident year.

Since the insurance business consists of various classes of business, such as marine, general accident, fire, burglary, life insurance, pension, money – in – transit, workmen compensation and fidelity guarantee, it can be concluded that the inputs and outputs of insurance business originate from either one or a combination of any of the classes of business mentioned above. In an attempt to improve input while minimizing output and risk which, of course, should lead to improved performance, insurance companies tend to spread their risk via re-insurance (statutory proportion), co-insurance (division of risk proportion among insurance companies), and pool insurance (statutory proportion). However, since the environment in which insurance business operates is characterized by risk, uncertainty and volatility which are more pronounced in developing nations, there is need for insurance companies (especially in such environments) to determine, *a priori* of an activity year, the mix of insurance policies (class of businesses) and the proportion of various risks they can optimally carry (both on aggregate and individual business bases), to guarantee an optimum performance for such an activity year.

Considering the inherent risk in insurance business, this article presents models for the determination of optimum portfolio that an insurance company can carry at an acceptable risk level, taking into consideration the volatility and catastrophically unstable nature of the business environment in developing nations.

1.2 Objectives of the Study

The central aim of this study is to design a stochastic model that is capable of predicting the optimum portfolio of insurance business at an acceptable risk exposure level. This, would guarantee the acceptable risk levels for a viable insurance company, evaluate the retention rate of insurance portfolio at a given risk rate, provide good knowledge on the importance of reinsurance on risk adjustment in times of larger claim, and examine the unbearable risk level that would require co-insurance. In particular, the following measures of effectiveness would be observed:

- Minimization of risk, which is measured by the variance, σ^2;
- Maximization of expected return, r, at a specific variance level;
- Maximization of expected returns with risk aversion, ($\lambda a^t x$ - x^t σ^2_x); λ being the risk aversion parameter;
- Minimise risk with respect to benchmark ($[x - x_b]^t$. σ^2 . $[x - x_b]$), with x_b as benchmark portfolio.

1.3 Review of Related Studies

In the early eighties, Leland and Robinste in Acerbi et al. (2001) developed a portfolio insurance (PI) technique, based on the option pricing formula of Black and Scholes (1973). The idea behind it was that a strategy, which would provide protection against market losses while preserving the upward potential, should have considerable appeal to a wide range of investors. Cesari and Cremonini (2003), Do and Faff (2004) provide empirical evidence for the benefits of portfolio insurance in bear markets. The continuing creation of portfolio insurance applications, as well as the mixed research evidence suggest that no consensus has been reached between theory and practice about the effectiveness of portfolio insurance so far.

Recent theoretical works on sampling error in stochastic dominance tests with valid data (Davidson and Duclos, 2000; Barrett and Donald, 2003) have triggered a new wave of stochastic dominance literature. Thanks to the sub-sampling method of Politis and Romano (1994) and insights provided by Linton et al. (2005) it is now possible to handle the sampling error in stochastic dominance tests with valid data. Using these techniques, portfolio insurance strategies can be evaluated more comprehensively by comparing whole return distributions rather than just moments. Moreover, most researches have focused on the 'synthetic put strategy', while little attention has been paid to a serious comparison of this strategy with stop-loss and CPPI strategies. Furthermore, simulation exercises have mostly been limited to Monte Carlo simulation or back-testing. Given the underlying normality assumption of the latter, both approaches fail to correctly assess the performance of the strategies. As opined by Do and Faff, only few studies have so far examined the impact of a different choice of the floor value, rebalancing time and CPPI multiple.

The simplest approach to portfolio protection consists of implementing a stop-loss portfolio insurance strategy. Using this approach, the portfolio is fully invested in risky assets at the start of the insurance programme. As long as the portfolio value exceeds the discounted value of the floor, the portfolio asset allocation remains unchanged. However, the moment the portfolio value drops below the discounted floor value, the whole portfolio is transferred into a risk free investment; hence, this strategy is only subject to a single transaction cost. In the case of an upward market, the portfolio remains fully invested in the risky asset for the entire investment horizon, thereby avoiding any transaction costs. The magnitude of this single transaction cost could be substantial, given the fact that it is computed on the entire portfolio value. This strategy suffers from severe path dependence, which can be explained in terms of its opportunity cost.

The objective of the synthetic put strategy is to implement a dynamic strategy which continuously revises the portfolio mix, while the same payoff can be achieved, as obtained by the purchase of a put option on the whole portfolio. Portfolio protection can also be achieved by implementing a 'constant proportion portfolio insurance (CPPI) strategy 6', introduced by Black and Jones (1987) and Perold (1986). The strategy is based on the assumption that a portfolio is composed of a risk free asset on the one hand, and a risky asset on the other hand, and subtracting the floor value from the portfolio value yields the cushion. In contrast to the synthetic put approach, this strategy has the advantage that it does not depend on the time to maturity, as the CPPI strategy does not have an expiration date.

The risk of the portfolio insurance strategies can be expressed in terms of the standard deviation of returns. In this context, a high risk would indicate that extreme returns (positive as well as negative) are likely to occur. However, since the aim of portfolio insurance is to limit downside losses, an asymmetric risk measure yields a more appropriate risk indication. The VaR is such an asymmetric risk measure which focuses on the downward tail of the return distribution. The VaR measure denotes the maximum loss at a certain confidence level. Duffie and Pan (1997) remark that the VaR should mainly be used as a relative benchmark, that is, to compare the risk of portfolios *ceteris paribus* (given the same time horizon and confidence level). Christoffersen et al. (2001) point out that the VaR computation suffers from a major drawback in that no comprehensive discussion of the value-at-risk measure reference is available.

2. THEORETICAL FRAMEWORK

Going by the input-output portfolio theory, there are certain inputs and outputs peculiar to insurance business. These include:

- reinsurance inputs: written premium (net of premium for outward reinsurance);
- returns on investment/capital income; realized capital gain; unrealized capital gain; and
- outputs: incurred claim (net of reinsurance recoveries); expenses; dividends to policy holders; dividends to shareholders.

The assumption here is that the underwriting year is the same as the insurance company financial year. However, a written premium can be split into three different components, namely: pure risk premium; loading for expenses; and loading for profit. On the other hand, incurred claim can be subdivided into three components, namely: incurred claim for the accident year; changes in claims amount in respect of claims pertaining to previous accident years; and reserve for claims to be due for payment in years other than the accident year. These variables may be denoted as follows:

P - written premium (gross)
P_{-ro} – premium net of reinsurance outwards
R_o - return on investment
R_c - realised capital gain/capital economic value at the beginning of the year.
$\check{R}$ – unrealised capital gain
C - incurred claim
C_{-Rr} – net claim after reinsurance recovers
e_x – expenses
D_p - dividend paid to policy holders
D_s - dividend paid to shareholders
ΔRc – increase in capital economic value during the financial year.

It is important to note that P_{-ro} is a composite function of: pure risk premium, loading for expenses and loading for profit. In other words, P_{-ro} may be expressed as:

$$P_{-ro} = P_r + P_{ex} + P_\pi + U \tag{2.1}$$

With P_r as pure risk premium, P_{ex} as loading for expenses, P_π as loading for profit, while U is any other special features related to premium loading.

Also, incurred claim, C, could be seen as a function of C_t (claim for current year), C_{t-1} (claim for previous year payable this year), C_{t+1} (reserve made for claim payable in future, but pertaining to the current accident year), and $(C_{t+1} – C_{t-1})$ (difference between reserve and actual claim). Thus, C can be expressed as:

$$C = \{C_t + C_{t-1} + C_{t+1} + (C_{t+1} – C_{t-1})\} \tag{2.2}$$

2.1 Assumption

For the purpose of this study, only the model assumption is employed (although we sometimes have both simplified and model assumptions).

2.1.1 Model assumption

The model assumptions are that:

- all random variables in the model have a finite second order moment;
- payments pertaining to a given period are made at the end of the period;
- there is no deferred premium;
- the pure risk premium is the present value of expected loss payment;
- loss reserve is equal to the present value of expected future loss;
- discount factors, used to assess the pure risk premium and the loss reserve, is based on the yielding curve, as defined by the bond market;
- assets of insurance companies are valued at market value;

From the model assumption, we express the increase in capital economic value during the financial year as:

$$\Delta Rc = E(C) + P - C - \Delta C_{t-1} + \Delta A \qquad (2.3)$$

where the symbol Δ stands for change/increase, E is expectation and A is the invested income plus capital gain.

We also have the following expressions:

$$\text{Underwriting risk} = C - E(C) \qquad (2.4)$$

$$\text{Loss reserve risk} = \Delta C_{t-1} - E(\Delta C_{t-1}) \qquad (2.5)$$

$$\text{Asset risk} = \Delta A - E(\Delta A) \qquad (2.6)$$

$$\text{Total company risk} = \Delta Rc - E(C) = \sigma \qquad (2.7)$$

From the above, one can deduce a simplified underwriting risk, whereby the asset of an insurance company is split into two, namely:

(i) Liability fund = A_L
(ii) Capital fund = A_U

Thus, underwriting risk in expression (3.4) is transformed to:

$$A = A_L + A_U \qquad (2.8)$$

This implies that some of the liability assets (A_L) are earmarked to cover the liabilities of the company and the rest of the asset (capital fund - A_U) to match the equity of the company. This is only feasible where there is no loss reserve risk and when the following associated assumptions also hold:

- Time of payment in respect of outstanding losses is perfectly known to the company;
- Those assets, which cover liability, perfectly match the amounts and maturity of the liability;
- We can discount liability with a factor corresponding to the liability fund;
- Any change in yield curve will have a perfectly offsetting effect on ΔC_{t-1} and $\Delta A_{L;}$

- Capital fund is invested at the risk free rate of return: $\Delta Au = P_oR_c$

The increase in capital (profit) will now be:

$$\Delta Rc = E(C) + P\pi - C - \Delta C_{t-1} + \Delta A_L + \Delta Au \quad (2.9)$$

and

$$\Delta Rc = E(C) + P_\pi - C + P_oR_c \quad (2.10)$$

where:

$P_oR_c =$ Risk free rate of return on capital assets

From expression (3.10) above, we obtain:

$$P_o = \frac{\Delta R_c}{R_c} - \frac{E(C)}{R_c} - \frac{P\pi}{R_c} + \frac{C}{R_c} \quad (2.11)$$

Using equation (3.7) in equation (3.11) yields:

$$P_o = \frac{\sigma}{R_c} - P + r \quad (2.12)$$

with $r = \frac{P\pi}{\sigma C}$ (2.13)

The trade-off between risk (σ) and excess return (P-P_o) is thus linear and the slope of the line is equal to the ratio of underwriting return (Pπ) and the underwriting risk (σ C). The singular objective here would be to maximize the underwriting risk return ratio (r). Therefore, the resultant effect of the objective function would serve as the efficient boarder of the set of all risk return pairs (σ, P), which can be achieved, if only (σ, P) is on a straight line. Thus, an increase in return becomes achievable by the company through increase in risk. The choice of a specific point (σ*, P*) on the efficient boarder is equivalent to the choice of capital level of the company. Indeed, if (σ*, P*) is given, then by the very definition of σ and P, we have:

$$R_c = \frac{\sigma(\Delta R_c)}{\sigma^*} = \frac{P\pi}{P^* - P_o} \quad (2.14)$$

On the other hand, if R_c is given, we have:

$$\sigma^* = \frac{\sigma(\Delta R_c)}{Rc} \quad (2.15)$$

and

$$P^* = P_o + R\pi/R_c \quad (2.16)$$

The above models can be used in verifying whether or not (σ*, P*) is on the efficient boarder. It is important to note that the choice of a specific point on the efficient boarder could be arbitrary. It all depends on the balance between the investors' hunger for profit and aversion to risk. It can equally be assumed that the owners of insurance companies or the managers, acting on behalf of the owners, have a quadratic utility function

$$V(P) = a + bP - cP^2 \quad (2.17)$$

where b, c, > 0. The utility function is only meaningful for $P \leq P_{max} = b/2c$ since above P_{max}, the function decreases. However, if

$$\text{Prob.}\ (P > P_{max}) \approx a + bP - cP^2 - C\,\sigma^2 \quad (2.18)$$

this defines a set of indifference curves. All points (P, σ), which yield the same value of V = E(VP), are on the same inductance curve. Assuming that the efficient boarder is a straight line $P - P_o = R_o$, it is easily seen that the risk return pair, which maximizes the utility function of the company, is:

$$\sigma^* = \frac{(P_{max} - P_o)\, r}{1 + r^2} \qquad (2.19)$$

where:

$$P^* = P_o + r\sigma^*$$

Therefore, the corresponding capital amount is:

$$R_c = \frac{\sigma(\Delta R_c)}{\sigma^*} \qquad (2.20)$$

3. OPTIMIZATION MODELS

Among the various models that could be used when resolving optimization problems in the insurance industry are combined portfolio reinsurance risk model and portfolio optimization model.

3.1 Combined Portfolio Re-insurance Risk Model

During underwriting risk return ratio maximization, when loadings of individual risks are given, the issue of portfolio reinsurance comes into play. The first of this type of portfolio heterogenity is as follows:

Assuming that $x_i, x_{2,} \ldots , x_n$ are uncorrelated risks of a portfolio, then

$$\sum_{i=1}^{n} C = x_j \qquad (3.1)$$

If *m* denotes the loading for risk, the company keeps a share α_i for its own account and cedes a share $(1 - \alpha_i)$ to re-insurers. Under the above assumption, the choice of $\alpha_I, \alpha_2 \ldots \alpha_n$ which maximizes the net underwriting risk return ratio is:

$$r_{net} = \frac{\sum_{j=1}^{n} \alpha_i m_j}{\left\{ \sum_{j=1}^{n} \alpha_i^2 \sigma_j^2 \right\}^{1/2}} \qquad (3.2)$$

However;

$$\alpha_i = q \frac{m_j}{\sigma_j^2} \qquad (3.3)$$

where q is some norming constant which must be chosen in such a way that $0 \le \alpha_i \le 1$

For all i, with the so defined set of retentions, we have:

$$r_{net} = \left\{ \sum_{j=1}^{n} \frac{m_j^2}{\sigma_i^2} \right\}^{1/2} \qquad (3.4)$$

Meanwhile, because of the peculiarities of the insurance businesses, in some cases, the retention of each risk might be the net monetary amount retained for all risks. In this case, we express

$$\begin{cases} L_i & \text{with probability } P = \text{Retention} \\ & \text{with probability } 1-P = \text{Ceding} \end{cases}$$

and

$$m_j = E(x_i)\lambda = PL_i\,\lambda \tag{3.5}$$

We can now combine (4.4) and (4.5) to have:

$$\mathrm{Var}\,(x_i) = P\,(1\text{-}P)\,L_i^2 \approx PL_i^2 \tag{3.6}$$

This holds for all P<1 and at this level, the optimal retention becomes

$$\alpha_i = \frac{qm_i}{\sigma^2_i} \approx q\,\frac{PL_i\,\lambda}{PL_i^2} = \frac{1}{L_i}\,q\,\lambda \tag{3.7}$$
$$= \alpha_i L_i = q\lambda$$

It is important to note that the re-insurance engagement that maximizes underwriting risk return ratio is a surplus treaty where the retention is equal to the smallest sum insured. On a gross basis, the risk return ratio is:

$$r = \frac{\sum_{j=1}^{n} L_i\,P\,\lambda}{\left\{\sum_{j=1}^{n} L_i^2 P\right\}^{½}} = \frac{\lambda\,.\sqrt{P}\ \sum_{i=1}^{n} L_i}{\left\{\sum_{j=1}^{n} L_i^2\right\}^{½}} \tag{3.8}$$

On a net basis, the risk return ratio is

$$r_{net} = \left\{\sum_{j=1}^{n} \frac{m_j}{\sigma_j^2}\right\}^{½} = \lambda\sqrt{P} \tag{3.9}$$

This means that $r_{net} \geq r$, the inequality, is expected to be strict, unless all L_i^2 are equal.

3.2 Portfolio Optimization Model

Since a portfolio is optimal, if and only if the corresponding risk return ratio ($\underline{\sigma}$, $\underline{P}$, $\underline{A}$) is maximum, then there is a need to revisit:-

$$\Delta R_{c\,=}\ [(E(C_c) + P\pi - C) + (m_i - \Delta L_i)] - RL\,.\,L + \Delta A] \tag{3.10}$$

The first two terms are insurance risks (underwriting risk and loss reserve development risk), while the last two terms are financial risks (yield curve risk and asset risk). This assumes that there are N different categories of assets R_j. If a random variable denotes the amount invested in asset category, we can then have:

$$\Delta A = \sum_{i=1}^{n} R_i\,.A_j \tag{3.11}$$

If P_o is the return of the risk free assets, we can obtain the representation below for excess profit of each company.

$$\Delta R_c - P_o R_c = \{E(C) + P\pi - C) + (m_i - \Delta L_i) - (R_L - P_o) . L + \sum_{j=1}^{n}(R_j - P_o) . A_j \quad (3.12)$$

This is realizable at a point where the sum of liabilities equals the sum of the assets of a company. However, if the objective is to maximize the risk return ratio:

$$r = \frac{P(R_j) - Po}{\sigma(R_c)} = \frac{E(\Delta R_c) - P_o . R_c}{\sigma(\Delta R_c)} \quad (3.13)$$

Then since we have to minimize the risk return ratio of the underwriting and the loss reserve sub-portfolio through reinsurance buying, the excess profit of a company thus becomes

$$\Delta R_c - P_o R_c = \sum_{j=1}^{n}\alpha_j\{E(C) + P\pi_j - x_i)\} + \sum_{j=1}^{n}\beta_i(P\pi_j - x_i) - (R_L - P_o).L + \sum_{j=1}^{n}(R_j - P_o) . A_j \quad (3.14)$$

This leads to a more homogenously and less catastrophe exposed portfolio and, hence, to a higher risk return ratio of sub-portfolios. This constitutes the crux and hub of the present study.

4. EXPERIMENTAL ANALYSIS

Information used in this experimented analysis was obtained from annual reports and accounts of insurance companies in Nigeria.

4.1 Risk Return analysis

From the insurance point of view, there are two types of risks involved: one on possibility of return on investment and the other on possible profit on ordinary business (underwriting). Denoting the two risks by X_1 and X_2, respectively, the analysis yielded the following values:

$$X_1 = \begin{cases} 1 & \text{probability} = 10^{-5} \end{cases}$$

and

$$X_2 = \begin{cases} 100 & \text{probability} = 10^{-3} \\ 0 & \text{probability} = 0.872 \end{cases}$$

Source: Field survey (2010)

Since $\sigma = 10^{-5}$ risks of the first type and $\sigma = 10^{-3}$ risk of the second type, with the profit loading $P_\pi = 0.25\%$ of the pure premium risk, the computational outcome becomes:

$$\sigma(S) = \sqrt{10^{-3}\,(10^5+10^7)} = 100.5$$

where, $\sigma(S)$ is the rate at which the aggregate risk of each subclass of portfolio would yield returns. However the result of the analysis revealed a risk level of 0.060 and income level of 6.0. According to the above position, the re-insurance arrangement, which maximizes the underwriting risk return ratio, is a surplus treaty with a retention of 1 on the net basis, (that is, net income). Thus, r = 0.301, Ic= 3.03

From this result, the net underwriting risk return ratio is much higher than the gross. The implication of this is that, since insurance companies are expected to have reduced their risk retention after reinsurance and treaty, their risk level is expected to have dropped, considering the reduction in the volume or percentage of the risk carried.

4.2 Catastrophe Exposure Analysis

From the earlier risk values in the risk return ratio analysis, using the catastrophe exposure model,

$$\mathrm{Cov}(x_1, x_2) = \mathrm{Cov}\,(x_1 + x_2 \ldots\ldots X_n)$$
$$= \sigma_1, \sigma_2 + \sigma_c^{\,2}$$

and

$$\mathrm{Var} = n\,\sigma^2 + n^2\,\sigma_c^{\,2}$$

where,

n is the number of portfolio/class of business,
σ is the risk level of normal business
σ_c is the risk of catastrophic business and
x_i is the individual class of business

However, in an attempt to unravel the level of catastrophic explosive, pure risk premium was denoted by υ_o and ordinary risk of a catastrophic risk by υ_c. Therefore, the risk level thus becomes

$$r = \frac{P\pi}{\sigma(s)} = \frac{n(\upsilon_o + \lambda_o + \upsilon_c\lambda_c)}{(n\sigma^2 + n^2\sigma_c^{\,2})}$$
$$= \frac{n(\upsilon_o\lambda_o + \upsilon_c\lambda_c)}{(\sigma^2/n + \sigma_c^{\,2})^{1/2}}$$

From the analyzed result, the following values were obtained:-

υ_o = 0.1; υ_c = 0.05
σ_o = 3.16; σ_c = 0.5
λ_o = 5%; λ_c= 12%
n = 10^7(considering 7 classes of business)

where, υ_o is the ordinary risk; υ_c is the catastrophe risk; σ_o is the yield of ordinary risk; σ_c is the yield of catastrophe risk; λ_o is the loading for ordinary risk; λ_c is the loading for catastrophe risk, and n is the number of classes of business.

The net corresponding qualities for the gross portfolio are:

$\sigma^* = 0.0060$, $P^* - P_o = 0.0002$ and $\upsilon = 8.343$

From these values, it is seen that it would be totally uninteresting to insure the gross portfolio without being able to reinsure a sizeable part of the catastrophe risks. Since portfolios are insured separately and insurance companies optimize their capital allocation according to the indifference curve analysis, the following results were obtained through the analysis of the combined portfolio and portfolio with optimal risk return ratio.

In Table 1, portfolio number 4 (Engineering) is the combined portfolio and portfolio number 6 (Good-in-transit) is the optimum portfolio. This result reveals that combining portfolios results in substantial improvement of the risk-return ratio.

Also, when portfolios are combined in a non-optimal way, there is a tendency for gross subsidization between portfolios. The fair loading computations were $P\pi_1 = 0.14$, $P\pi_2 = 0.44$, $P\pi_3 = 0.02$, $P\pi_4 = 0.68$, $P\pi_5 = 0.72$ and $P\pi_6 = 0.30$, $P\pi_7 = 0.62$; whereas the actual loadings were: 0.2, 0.6, 0.8, 0.4, 0.14, 0.56 and 0.6, respectively.

Table 1: Combined portfolio and portfolio with optimal risk return ratio

Portfolio No.	r	σ	P^*-P_o	U
1	0.200	3.85%	0.77%	26.0
2	0.300	5.50%	1.65%	36.3
3	0.400	6.90%	2.76%	65.2
4	0.509	8.09%	4.12%	63.2
5	0.518	8.17%	4.23%	43.7
6	0.421	7.3%	4.04%	41.6
7	0.384	6.9%	3.19%	40.2

5. CONCLUSION

From the premise of this research work, it is concluded that:

- Since insurance is a game of chance, where probability plays a significant role, absolute values may not give valid and reliable information. This explains the need to revert to stochastic modeling, which canvases the use of risk, variances and expected values for mathematical computation.
- In the three states of nature, the most important of them all is the state of risk. Therefore, the importance of stochastic modeling in analysing portfolios and risk in the insurance business cannot be overemphasized.

One useful stochastic tool, as adopted in this study, was the generalization of the application of the “Markowitz portfolio optimization method to finance and insurance risks”. This was adopted to allow for allowance of symmetrical treatment of insurance and financial risk, as well as simultaneous optimization of portfolios.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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