Pension Risk Management in the Enterprise Risk Management Framework

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PENSION RISK MANAGEMENT IN THE ENTERPRISE RISK MANAGEMENT FRAMEWORK

ABSTRACT

This paper presents an enterprise risk management (ERM) model for a firm that is composed of a portfolio of capital investment projects and a defined benefit (DB) plan for its workforce. The firm faces the project, operational and hazard risks from its investment projects as well as the financial and longevity risks from its DB plan. The firm maximizes its capital market value net of pension contributions subject to constraints that control project, operational, hazard, financial and longevity risks as well as an overall risk. The analysis illustrates the importance of integrating pension risk into the firm's ERM program by comparing firm value with and without integrating pension risk with other risks in an ERM program. We also show how pension hedging strategies can impact the firm's net value under the ERM framework. While the existing literature suggests that a longevity swap is less expensive than a pension buy-out because the latter is more capital intensive, this analysis shows that the buy-out is more effective in increasing firm value.

Keywords: defined benefit pension plan, enterprise risk management, conditional value-at-risk, pension de-risking.

1. Introduction

Enterprise risk management (ERM), a new development in the field of risk management, has received unprecedented international attention from both industry and academia in recent years (Lin et al., 2012). ERM represents an integrated risk management method that assesses all enterprise risks and coordinates various risk management strategies in a holistic fashion, as opposed to a silo-based traditional risk management (SRM) approach. In the SRM framework where risk classes are treated in isolation, individual decisions handling idiosyncratic risks can be incompatible with the firm's overall risk appetite and global corporate agenda (Ai et al., 2012). A separate management of individual risk categories can also create inefficiencies due to lack of coordination between various risk management units (Hoyt and Liebenberg, 2011).

Departing from SRM, ERM considers all risk factors, at a holistic level, that ostensibly overcome the limitations of SRM; hence, ERM is likely to create value in multiple dimensions. For example, managing risks in the aggregate facilitates risk control on key drivers of earnings volatility arising from business, operational, credit and market risks (Lam, 2001). Liebenberg and Hoyt (2003) find that firms with greater financial leverage are more likely to establish ERM programs as ERM can mitigate information asymmetry regarding the firm's current and expected risk profile. As noted by Lin et al. (2012), ERM can generate synergies between different risk management activities by coordinating a set of complementary risk management strategies. Furthermore, ERM optimizes the trade-off between risk and return at the enterprise level and thus allows the firm to select investments based on a more accurate risk-adjusted rate (Nocco and Stulz, 2006).

To achieve its proposed benefits and facilitate better operational and strategic decision making, ERM requires firms to encompass all risks that affect firm value. Despite this widely accepted notion, surprisingly, the current ERM practice and literature primarily targets risks that affect the basic balance sheet and disregards the off-balance-sheet items that could impose a significant impact on a firm. Among different off-balance-sheet items, perhaps no other items are more important than corporate pension plans (Shivdasani and Stefanescu, 2010). According to the BrightScope and Investment Company Institute (2014), at the end of the second quarter of 2014, the total value of defined benefit (DB) pension assets held by U.S. firms was about $3.2 trillion, an amount far greater than that of any other off-balance-sheet item. Those firms that offer traditional DB plans to salaried employees are the focus of this paper.

DB pensions introduce significant risks that arise from market downturns, low interest rate environments, new pension accounting standards, and improved life expectancy of retirees. If firms do not control expenses arising from pension risk, they will have to cut costs elsewhere and that will diminish their ability to maintain current operations and invest in new positive net present value (NPV) projects. First, unanticipated improvements in mortality rates increase pension liabilities (Lin and Cox, 2005; Cox et al., 2006; Cox and Lin, 2007; Cox et al., 2010; Milidonis et al., 2011). Second, investment risk constitutes another significant concern for DB plans. The 2007–2009 mortgage and credit crisis and the subsequent drop in discount rates caused double-digit rises in pension funding deficits and notable decreases in the value of many firms. For example, the DB plans of General Motors (GM) were underfunded by $8.7 billion in year 2012. As GM was obligated to infuse cash to cover gaps created by the market downturns, the pension underfunding was one factor decreasing GM's share value even if it operated profitably (Bunkley, 2012).

After the 2008 credit crisis, the profession began moving from an integration of various major business risks to aggregating them with pension risk. As suggested by Kemp and Patel (2011), "For many firms [with DB plans] who already have some form of ERM in place, an initial step might be to extend the governance and risk management function in what may already be an effective framework for decision making in the core business to incorporate the pension risk." Despite this positive direction, with only a few exceptions, the implications of pension risk on firm overall risk have been largely unexplored. The existing literature mainly focuses on a firm's product, investment and/or risk management strategies. Yet little attention has been paid to how pension obligations impact business decisions despite the central role that pension plays in corporate operations. There is also a void in our understanding of how significant it is to incorporate pension risk in an ERM program as no ERM model currently exists to integrate the pension scheme into a firm's decision making processes (Kemp and Patel, 2011). Ai et al. (2012) present an ERM framework to maximize the expected end-of-horizon wealth through investment in real projects and financial assets subject to individual and overall risk constraints. Their ERM model provides a promising starting point for ERM decision making but they do not consider pension risk, a significant risk for firms sponsoring DB plans. In this article, we seek to extend the Ai et al.'s ERM formulation by consolidating pension risk with various business risks. Specifically, we maximize firm value (measured by end-of-horizon operation fund) net of total pension cost subject to separate project, operational, hazard, and pension risk constraints as well as an enterprise-wide overall risk constraint. With this setup, we illustrate the importance of integrating pension risk with other risks in an ERM program. In particular, our numerical example highlights the significant impact that including pension risk in the ERM model can have on firm value. In our example, if we manage pension risk and different business-related risks in a holistic way, it will notably increase firm value by 12.07% relative to SRM.

Over the last decade, firms have sought to de-risk their DB plans, driven by pension deficits due to the latest market downturns and the low interest rate environments. Many firms sponsoring DB plans are confronting an important decision on whether and how to lessen pension obligations. According to a survey from 180 participant responses conducted by Towers Watson in mid-2013, "half of all responding plan sponsors are looking to transfer some or all of their DB plan obligations off their balance sheet" (Towers Watson, 2013). There are generally two major de-risking strategies for companies to offload their pension risks: the ground-up hedging strategy and the excess-risk hedging strategy (Cox et al., 2013). The ground-up hedging strategy, e.g., pension buy-ins or buy-outs, transfers a proportion of the entire pension liability to another party. The excess-risk strategy cedes the longevity risk that exceeds a given level. A prominent example of the excess-risk strategy is the longevity swap. Pension plan sponsors are interested in pension de-risking. In 2013, buy-in and buy-out deals were worth more than £5.5 billion (Hawthorne, 2013). According to Pfeuti (2014), longevity-hedging transactions completed by UK pension funds reached £8.9 billion in 2013, breaking all previous annual records.

While there is a rich literature that explores the rationale and trend of pension de-risking activities, little is known about whether and the extent to which pension risk should be ceded in the ERM framework. To fill the gap, instead of simply comparing different pension hedging tools, as another objective of this paper, we study how much pension risk a plan should transfer given that pension risk and other business risks are managed holistically. In particular, we add a pension hedging decision to our ERM optimization problem. Given this setup, we solve for the plan's optimal pension hedge ratio. Our optimal pension hedge decision ensures that pension de-risking is compatible with global corporate strategic goals. The existing literature suggests that the excess-risk hedging strategy is more attractive than the ground-up strategy as the later is more capital intensive and expensive (Lin and Cox, 2008; Lin et al., 2013). The ground-up strategy covers a proportion of the entire annuity payment so it requires a much higher upfront premium. Our optimization results, however, indicate that subject to enterprise-wide risk constraints, the excess-risk strategy is less effective in improving overall firm performance. This can be explained by the fact that, compared with the ground-up strategy that offloads both pension asset and liability risks, the excess-risk strategy only transfers the high-end longevity risk and retains the entire pension asset risk and other pension liability risk (e.g. interest rate risk). Retaining most of pension risks with excess-risk hedging prevents a firm from investing more in riskier but higher return projects and pension assets and/or requires more project risk hedging, leading to a lower firm value than that with the ground-up strategy.

2. Firm Risk Overview and Model Assumptions

In this paper, we study a start-up firm that sponsors a DB plan in the US. While we assume the new firm has no cash flow from prior operations, our setup can be readily extended to that of a well-established firm. We also assume this firm is a non-financial firm and does not invest in the financial markets for its main business operation. The firm is composed of two divisions. The first, called the operation division, involves its main business operation. The firm has net revenue from its business operation and is faced with the opportunity to invest in one or more projects that have positive risk-adjusted NPVs. This part of the firm may face project risk, operational risk, insurable hazard risk, and other risks depending on the nature of its business. The second division is called the DB pension division; it is a DB pension plan with assets and liabilities subject to financial, interest rate and longevity risks.

The firm allocates funds to the projects and pension plan to maximize the value of the firm net of total pension cost subject to the constraints in the ERM framework. These constraints relate to the following major risks from the two divisions of the firm considered here.

2.1. **Risks from the Operation Division and Their Assumptions.**

2.1.1. *Project risk.* Project risk is the risk that a project's return is below a minimum acceptable level. Project risk arises, for example, from input and output price changes and changes in customer demands. Some project risk such as price risk can be hedged with derivatives such as futures contracts. Suppose the firm invests in m projects. At the beginning of period t, the firm invests an operation fund F_t^j in project j $(j = 1, 2, \cdots, m)$, of which a proportion ϕ_j is hedged with a hedged rate of return r_h per period.

The unhedged operation fund $\hat{F}_t^j = (1 - \phi_j)F_t^j$ in project j generates a return $\hat{r}_t^j$ that follows a Brownian motion with drift rate $\alpha_{\hat{F}^j} - \frac{1}{2}\sigma_{\hat{F}^j}^2$ and volatility rate $\sigma_{\hat{F}^j}$ as follows:

$$\hat{r}_t^j = \mathrm{d}\left(\log \hat{F}_t^j\right) = \left(\alpha_{\hat{F}^j} - \frac{1}{2}\sigma_{\hat{F}^j}^2\right)\mathrm{d}t + \sigma_{\hat{F}^j}\,\mathrm{d}W_t^j, \quad j = 1, 2, \cdots, m. \tag{1}$$

The Brownian motions W_t^j's are correlated among different projects. They are also correlated with pension valuation rate as well as different pension asset indices to be discussed in Section 2.2.

Then, the after-hedge rate of return of project j, r_t^j, can be modelled as:

$$r_t^j = \hat{r}_t^j(1 - \phi_j) + r_h\phi_j. \tag{2}$$

2.1.2. *Hazard risk.* In addition to the project risk, we also consider the hazard risk in the operation division. Hazard risk is the risk related to safety, fire, theft and natural disasters. A hazard loss will reduce the values of investment projects in the operation division. To measure hazard risk, suppose the unit hazard loss per period of time, h, is a lognormal random variable $h = e^{\mu+\sigma Z}$, where Z is a standard normal. This implies that the expected hazard loss per unit of the operation fund per period equals $\mu_h = e^{\mu+\sigma^2/2}$.

Hazard risk is a pure risk so traditionally it is insurable. Assume at the beginning of each period t, the firm insures a proportion, u (fixed for each period and to be determined by the optimization model), of its total hazard risk. If d is the hazard insurance loading per unit of risk insured, the insurance premium with loading paid at time t ($t = 0, 1, \cdots$) equals $P_t^{hz} = u(1 + d)\mu_h F_t$, where $F_t = \sum_{j=1}^{m} F_t^j$ is the operation fund at time t before purchasing the hazard insurance. The hazard risk retained by the firm after insurance, therefore, decreases to $(1 - u)hF_t$.

2.1.3. *Operational risk.* Operational risk is the risk of unexpected changes in elements related to operations arising, in direct or indirect manner, from people, systems and processes.[1] In this paper, the operational risk excludes the above insurable hazard risk. Compared with project risk and hazard risk, operational risk is more difficult to quantify given that there is not yet a consensus on how to measure operational risk (Ai et al., 2012). Following the Standardized Approach from Basel II and Ai et al. (2012), we assume per dollar project investment, the loss caused by the operational risk from project j in period t, op_{jt}, equals a proportion, $\gamma_p \geq 0$, of project j's total return after project risk hedging, $(1 + r_t^j)$. That is, $op_{jt} = \gamma_p \cdot (1 + r_t^j)$, where r_t^j is the after-hedge return of project j in period t.

2.2. **Risks from the DB Pension Division and Their Assumptions.** A firm sponsoring DB pensions faces three major risks from its pension plan: pension investment risk, interest rate risk and longevity risk. These three risks introduce significant uncertainties. To capture their effects, consider the following extension of the ERM model. Suppose the pension cohort of a firm joins the plan at the age of x_0 at time 0 and retires at the age of x at time T. Following Maurer et al. (2009), we assume before retirement a member who leaves the firm can be immediately replaced by a new member at the same age. That is, the cohort is stable in the entire accumulation phase before time T. We further assume the pension plan forecasts its mortality rates with the Lee and Carter (1992) model.

Given that the retirees will receive a nominal annual survival benefit B after retirement,[2] the present value of the firm's pension liability at time t, PBO_t, equals

$$PBO_t = \begin{cases} \frac{Ba(x(T,t))}{(1+\rho_t)^{T-t}} & t = 1, 2, \cdots, T \\ {}_{t-T}\hat{p}_{x,T} \cdot Ba(y(t)) & y = x+1, x+2, \ldots;\ t = T+1, T+2, \cdots \end{cases} . \quad (3)$$

[1] We do not consider operational risk for the pension division. But it can be added to our model.

[2] The benefit B depends on the retiree's total number of service years and their salaries before retirement.

In the US, the pension valuation rates ρ_t used to calculate private sector sponsors' pension liabilities at time t are usually the high-quality (e.g. AA-rated) bond rates of return (Government Accountability Office, 2014). In this paper, we use the Cox-Ingersoll-Ross (CIR) model (Cox et al., 1985) to illustrate the dynamics of the pension valuation rate ρ_t that satisfies the following process:

$$\mathrm{d}\rho_t = \nu\left(\theta - \rho_t\right)\mathrm{d}t + \sigma_\rho\sqrt{\rho_t}\,\mathrm{d}W_{\rho t}, \tag{4}$$

where ν is the mean-reversion rate, θ and σ_ρ are the long-term mean and instantaneous volatility of the pension valuation rate. The CIR model ensures mean reversion of the pension valuation rate towards the long-term mean θ. With a standard deviation factor $\sigma_\rho\sqrt{\rho_t}$, this model avoids the possibility of negative pension valuation rate as long as ν and θ are positive.

In the first expression of (3), the conditional expected value of life annuity $a(x(T,t))$ as a function of t ($t \leq T$) for age x at retirement T equals $a(x(T,t)) = \sum_{s=1}^{\infty} v_t^s \, {}_s\hat{p}_{x,T}$, where $v_t = 1/(1+\rho_t)$ is the discount factor with the pension valuation rate ρ_t and ${}_s\hat{p}_{x,T}$ is the conditional expected s-year survival rate:

$${}_s\hat{p}_{x,T} = \mathrm{E}\left[{}_s\tilde{p}_{x,T}\,\middle|\tilde{p}_{x,T}, \tilde{p}_{x+1,T+1}, \cdots, \tilde{p}_{x+s-1,T+s-1}\right]. \tag{5}$$

In (5), ${}_s\tilde{p}_{x,T}$ is the probability that a plan member of age x at time T survives to age $x+s$ at the beginning of year $T+s$ given the one-year survival probability $\tilde{p}_{x+i,T+i}$ of age $x+i$ at time $T+i$, $i = 0, 1, \cdots, s-1$. After retirement T, PBO_t ($t > T$) (the second expression in (3)) depends on the life annuity factor $a(y(t))$ for age y ($y > x$):

$$a(y(t)) = \sum_{s=1}^{\infty} v_t^s \, {}_s\hat{p}_{y,t} \qquad y = x+1, \cdots; \; t = T+1, \cdots. \tag{6}$$

The conditional expected s-year survival rate for age y at time t in (6), ${}_s\hat{p}_{y,t}$, is calculated as ${}_s\hat{p}_{y,t} = \mathrm{E}\left[{}_s\tilde{p}_{y,t}\,\middle|\tilde{p}_{y,t}, \tilde{p}_{y+1,t+1}, \cdots, \tilde{p}_{y+s-1,t+s-1}\right]$.

Suppose the firm allocates PA_0 to fund its pension plan at time 0. If the pension fund is invested in n asset indices, the accumulated pension fund of the plan at time t, $PA_t,\ t = 1, 2, \cdots$, equals

$$PA_t = \sum_{i=1}^{n} A_{i,t-1}(1 + r_{i,t}) \quad t = 1, 2, \cdots, \tag{7}$$

where $A_{i,t-1}$ is the amount invested in asset index i at time $t-1$ and $r_{i,t}$ is the log-return of asset index i in period t. We model $A_{i,t}, i = 1, 2, \cdots, n; t = 1, 2, \cdots$, as a geometric Brownian motion:

$$\frac{\mathrm{d}A_{i,t}}{A_{i,t}} = \alpha_{A_i}\,\mathrm{d}t + \sigma_{A_i}\,\mathrm{d}W_{it}, \tag{8}$$

where α_{A_i} is the drift and σ_{A_i} is the instantaneous volatility of asset index i. We further assume the Brownian motions W_{it}'s are correlated among different asset indices. They are also correlated with the Brownian motions W_t^j's of different projects in (1) and $W_{\rho t}$ of the pension valuation rate in (4). We use the correlated log-normal model (8) as it has great advantages in tractability and easiness in calibrating parameters.

For each period, the following balance equation holds:

$$\sum_{i=1}^{n} A_{i,t} = \begin{cases} PA_t + NC + k_t \cdot UL_t & t = 1, 2, \cdots, T \\ PA_t + k_t \cdot UL_t - B \cdot {}_{t-T}\hat{p}_{x,T} & t = T+1, T+2, \cdots \end{cases}, \tag{9}$$

where NC is a constant annual normal contribution (to be determined by optimization in the later sections) and UL_t represents the plan's underfunding at time t, calculated as:

$$UL_t = \begin{cases} PBO_t - PA_t - NC & t = 1, 2, \cdots, T \\ PBO_t - PA_t + B \cdot {}_{t-T}\hat{p}_{x,T} & t = T+1, T+2, \cdots \end{cases}. \tag{10}$$

In (9), $k_t = 1/\sum_{i=0}^{q-1}(1+\rho_t)^{-i}$ is the pension amortization factor at time t, where the plan amortizes its unfunded liability over $q > 1$ periods at the plan's periodic discount rate ρ_t.

Suppose that the plan sponsor invests a proportion w_i of the initial accumulated fund PA_0 in asset i, $i = 1, 2, \ldots, n$. In our basic framework, w_i is set at time 0 and will be determined by optimization. Therefore, from equations (7), (9), and (10), $A_{i,t}$ is calculated as

$$A_{i,t} = \begin{cases} (1-k_t)A_{i,t-1}(1+r_{i,t}) + (1-k_t)NC \cdot w_i + k_t \cdot PBO_t \cdot w_i, & t = 1, 2, \cdots, T \\ (1-k_t)A_{i,t-1}(1+r_{i,t}) - (1-k_t)B \cdot {}_{t-T}\hat{p}_{x,T} \cdot w_i + k_t \cdot PBO_t \cdot w_i, & t = T+1, \cdots \end{cases}, \tag{11}$$

where $A_{i,0} = w_i \cdot PA_0$ for $i = 1, 2, \cdots, n$.

The pension cost PC_t at time t is determined by the normal contribution (NC) and the underfunding or overfunding penalty as follows:

$$PC_t = \begin{cases} NC + SC_t(1+\psi_1) - W_t(1-\psi_2) & t = 1, 2, \cdots, T \\ SC_t(1+\psi_1) - W_t(1-\psi_2) & t = T+1, \cdots \end{cases}, \tag{12}$$

where the supplementary contribution SC_t and the withdrawal W_t are calculated as follows:

$$SC_t = \max\{k_t \cdot UL_t, 0\},$$

$$W_t = \max\{-k_t \cdot UL_t, 0\}.$$

Note that either SC_t or W_t is positive depending on whether the plan has underfunding ($UL_t \geq 0$) or overfunding ($UL_t \leq 0$). The penalty factor ψ_1 represents the unit opportunity cost arising from unexpected mandatory supplementary contributions that force the firm to forgo positive NPV projects while ψ_2 accounts for the excise tax imposed on pension fund early withdrawals and the loss of tax benefits when the firm reduces its pension contributions.

Assume the maximal possible age of the cohort is $x_0 + \tau$ where $\tau > T$. In the paper, we set the investigation horizon of the firm at τ. Following Maurer et al. (2009), Cox et al. (2013), Lin et al. (2014), and Lin et al. (2015), we define total pension cost TPC as the present value of all pension contributions PC_t in the time horizon τ:

$$TPC = \sum_{t=1}^{\tau} \frac{PC_t}{(1+\rho_t)^t} = \sum_{t=1}^{T} \frac{NC}{(1+\rho_t)^t} + \sum_{t=1}^{\tau} \frac{SC_t(1+\psi_1) - W_t(1-\psi_2)}{(1+\rho_t)^t}. \tag{13}$$

Moreover, following Ngwira and Gerrard (2007), we calculate the total underfunded liability over τ years, TUL, as the present value of all future underfundings $UL_t, t = 1, 2, \ldots, \tau$. That is,

$$TUL = \sum_{t=1}^{\tau} \frac{UL_t}{(1+\rho_t)^t}.$$

3. Basic Enterprise Risk Management Optimization

3.1. **Basic Optimization Problem.** Suppose this new start-up firm raises a fund of M_0 allocated to m investment projects and a DB plan at time 0. A proportion w_{jp} of M_0 is invested in project $j, j = 1, 2, \cdots, m$. After financing m projects at time 0, the remaining amount, $PA_0 = M_0 \left(1 - \sum_{j=1}^{m} w_{jp}\right)$, goes to the pension plan. As noted earlier, PA_0 is invested in n assets with the weights of $w_1, w_2, \cdots, w_n$, where $\sum_{i=1}^{n} w_i = 1$.

For simplicity, we assume the firm switches to a defined contribution (DC) plan for new hires after the DB pension cohort reaches the retirement age x at time T. In a DC plan, after a firm pays fixed contributions into an individual account in the accumulation phase, it assumes no more obligations. Instead, the entire investment risk and longevity risk are taken by employees/retirees. As the DC plan in general does not increase a firm's risk, we assume zero DC contributions after time T. This assumption will not change our conclusions but allow us to focus on the DB pension effect for a given cohort. If a firm wants to investigate the effects of different DB pension cohorts for a given horizon of interest, our model can be flexibly modified to meet this need.

We further assume in each period, the pension cost PC_t is proportionately allocated to project j according to the following weight:

$$Nw_j = \frac{w_{jp}}{\sum_{l=1}^{m} w_{lp}} = \frac{F_0^j}{\sum_{l=1}^{m} F_0^l} \qquad j = 1, 2, \cdots, m, \tag{14}$$

where $F_0^j = w_{jp} \cdot M_0$ is the fund invested in project j at time 0 before project risk hedge. That is, the labor per dollar project investment is the same for each project. For simplicity, the proportion of PC_t allocated to project j stays at Nw_j throughout τ years. The following recursive equation holds:

$$F_t^j = F_{t-1}^{j*}(1-\gamma_p)(1+r_t^j) - Nw_j \cdot PC_t - (1-u)hF_{t-1}^j, \ t = 1, 2, \cdots \tau; j = 1, 2, \cdots, m,$$

where $F_{t-1}^{j*} = F_{t-1}^j(1 - u(1+d)\mu_h)$ is the fund of project j after the hazard insurance premium at time $t-1$. The last term on the right hand side of the equation, $(1-u)hF_{t-1}^j$, represents the retained hazard loss from project j. Then, the aggregate cash flow or the total fund in the operation section F_t after project risk hedge from all projects at time t ($t = 1, 2, \cdots, \tau$) equals

$$F_t = \sum_{j=1}^{m} F_{t-1}^j[(1 - u(1+d)\mu_h)(1-\gamma_p)(1+r_t^j) - (1-u)h] - PC_t. \tag{15}$$

That is, the after-hedge fund in the operation section F_t at time t equals the gross return from m projects $\sum_{j=1}^{m} F_{t-1}^{j*}(1+r_t^j)$ net of the cost of operational risk $\sum_{j=1}^{m} F_{t-1}^{j*}(1+r_t^j)\gamma_p$, the pension cost PC_t and any retained hazard loss $(1-u)h\sum_{j=1}^{m} F_{t-1}^j$.

At time τ when the pension cohort reaches its terminal age $x_0 + \tau$, the DB pension section may have some undistributed funds (an overfunding) or require an additional contribution to cover a shortfall (an underfunding). After we credit/debit a possible overfunding/underfunding that is neither amortized nor recognized in PC_τ, the adjusted operation fund or firm value F'_τ at the end-of-horizon τ equals:[3]

$$F'_\tau = F_\tau - (1 - k_\tau)\left[\max\{UL_\tau, 0\}(1 + \psi_1) - \max\{-UL_\tau, 0\}(1 - \psi_2)\right],$$

where F_τ is the operation fund before adjusted for the full pension effect at time τ.

The primary goal of a corporation is to maximize its value. In our setup, it is equivalent to maximize the value of all invested projects net of pension effects. Accordingly, we propose a model in an integrated ERM framework to solve for the optimal project risk hedge ratio $\phi = [\phi_1, \phi_2, \cdots, \phi_m]$, hazard insurance ratio u, project investment proportions $w_p = [w_{1p}, w_{2p}, \cdots, w_{mp}]$, pension asset weights $w = [w_1, w_2, \cdots, w_n]$, and pension normal contribution NC, so as to maximize the expected value of the adjusted operation fund or firm value at time τ:

$$\underset{\phi, u, w_p, w, NC}{\text{Maximize}} \; \mathrm{E}[F'_\tau], \tag{16}$$

subject to the following constraints:

Constraint 1: Project risk (at time τ) As discussed in Section 2.1.1, project risk is the risk of potential losses due to unsatisfactory performance of a firm's real project operations. The projects here include all real business related activities. One major reason to include a project risk constraint in our optimization model is to ensure that the retained project risk after hedging is within the firm's risk appetite and thus the firm can meet its credit rating target (Ai et al., 2012). Riskier positive NPV projects generally offer higher expected returns but they also introduce higher downside risk. Given a hedge on project risks, the gross return per unit of capital invested across all projects over τ periods equals

$$R(w_p, r, \phi) = \sum_{j=1}^{m} w_{jp} \prod_{t=1}^{\tau} \left(1 + r_t^j\right) = \sum_{j=1}^{m} w_{jp} \prod_{t=1}^{\tau} \left((1 + \hat{r}_t^j)(1 - \phi_j) + (1 + r_h)\phi_j\right).$$

[3]While the horizon of interest here ends at time τ, the firm is expected to continue operating afterward, in our setup, with DC plans.

Given the risk appetite parameter α_1, the left-tail α_1-level VaR of the accumulated gross return is defined as:

$$\text{VaR}_{\alpha_1}(R(w_p, r, \phi)) = \min\{\beta_1 | \Pr[R(w_p, r, \phi) \leq \beta_1] \geq \alpha_1\}.$$

Conditional Value-at-risk (CVaR) is a commonly used downside risk measure. To control the project downside risk, we specify the CVaR-type project risk constraint across all projects as follows:

$$\text{E}[R(w_p, r, \phi) | R(w_p, r, \phi) \leq \text{VaR}_{\alpha_1}(R(w_p, r, \phi))] \geq \sum_{j=1}^{m} w_{jp}(1 + r_{p0}^j)^\tau. \tag{17}$$

In (17), r_{p0}^j is the minimal acceptable return of project j with a project hedge ratio ϕ_j:

$$r_{p0}^j = \hat{r}_{p0}(1 - \phi_j) + r_h \phi_j, \tag{18}$$

where $\hat{r}_{p0}$ is the minimal acceptable rate of return without hedge. This constraint requires that the left-tail α_1-level CVaR of the accumulated gross return across m projects over τ periods should be greater than or equal to the minimal acceptable level.

Constraint 2: Operational risk (at time τ) Denote the total operational losses across all projects over τ periods as

$$OP(w_p, r, \phi) = \gamma_p \sum_{j=1}^{m} \sum_{t=1}^{\tau} F_{t-1}^j (1 - u(1 + d)\mu_h)(1 + r_t^j)(1 + r_{p0}^j)^{\tau - t}.$$

Given the risk appetite parameter α_2, the right-tail α_2-level VaR of the overall operational losses equals

$$\text{VaR}_{\alpha_2}(OP(w_p, r, \phi)) = \min\{\beta_2 | \Pr[OP(w_p, r, \phi) \geq \beta_2] \leq \alpha_2\}.$$

Suppose in each period the firm specifies its operational risk limit for project j equals to a proportion, l_{op}, of the expected available fund based on the after-hedge minimal acceptable periodic net return r_{p0}^j ($j = 1, 2, \cdots, m$). To manage the operational risk across all projects in a holistic way, we aggregate operational losses and define an overall operational risk limit across all projects over τ periods as follows:

$$\zeta_{op} = l_{op} \cdot \text{E}[\sum_{j=1}^{m} \sum_{t=1}^{\tau} F_{t-1}^{r_{p0}^j} (1 - u(1 + d)\mu_h)(1 + r_{p0}^j)^{\tau - t + 1}]. \tag{19}$$

That is, (19) defines a cap on losses due to operational risk. The minimum operation fund for project j ($j = 1, \cdots, m$) at time $t-1$ given r_{p0}^j, $F_{t-1}^{r_{p0}^j}$, in (19) is calculated following the recursive formula:

$$F_{t-1}^{r_{p0}^j} = F_{t-2}^{r_{p0}^j}[(1-u(1+d)\mu_h)(1-\gamma_p)(1+r_{p0}^j) - (1-u)h] - Nw_j \cdot PC_{t-1}, \tag{20}$$

where $F_0^{r_{p0}^j} = F_0^j = w_{jp} \cdot M_0$.

The CVaR-type operational risk constraint requires that the expected value of the highest α_2-level total operational losses should be less than or equal to the firm's maximal acceptable operational loss ζ_{op} defined in (19). That is,

$$\mathrm{E}[OP(w_p, r, \phi) | OP(w_p, r, \phi) \geq \mathrm{VaR}_{\alpha_2}(OP(w_p, r, \phi))] \leq \zeta_{op}. \tag{21}$$

Constraint 3: Hazard risk (at time τ) After the firm insures a proportion u of its overall hazard risk, the retained hazard losses over τ periods equal

$$HZ(u) = (1-u)h \sum_{t=1}^{\tau} \sum_{j=1}^{m} F_{t-1}^j (1 + r_{p0}^j)^{\tau - t}.$$

Assume in each period the firm is willing to retain a hazard loss caused by one or more hazard events up to l_h per unit of the expected operation fund, subject to a risk appetite α_3, where the risk limit l_h and the risk appetite α_3 are managerial inputs. The CVaR-type hazard risk constraint is defined as

$$\mathrm{E}[HZ(u) | HZ(u) \geq \mathrm{VaR}_{\alpha_3}(HZ(u))] \leq \mathrm{E}[l_h \sum_{t=1}^{\tau} \sum_{j=1}^{m} F_{t-1}^j (1 + r_{p0}^j)^{\tau - t}]],$$

where $\mathrm{VaR}_{\alpha_3}(HZ(u))$, the right-tail α_3-level VaR of total hazard losses, is defined as:

$$\mathrm{VaR}_{\alpha_3}(HZ(u)) = \min\{\beta_3 | \Pr[HZ(u) \geq \beta_3] \leq \alpha_3\}.$$

Constraint 4: Pension risk I (at time 0) Following the literature (e.g. Cox et al. (2013)), we require the expected present value of total unfunded liability TUL at time 0 equal to zero. That is,

$$\mathrm{E}(TUL) = 0.$$

Constraint 5: Pension risk II (at time 0**)** Given the firm's pension risk appetite α_4, we require the expected value of the top α_4-level total unfunded liabilities to be not greater than some predetermined upper limit ζ_{TUL}. That is,

$$\mathrm{E}[TUL|TUL \geq \mathrm{VaR}_{\alpha_4}(TUL)] \leq \zeta_{TUL},$$

where $\mathrm{VaR}_{\alpha_4}(TUL)$ is the right-tail α_4-level VaR of total unfunded liabilities:

$$\mathrm{VaR}_{\alpha_4}(TUL) = \min\{\beta_4 | \Pr[TUL \geq \beta_4] \leq \alpha_4\}.$$

Constraint 6: Overall risk (at time τ**)** The overall risk is the risk that a firm's total available funds are insufficient to meet its debt and pension obligations so that it has to be liquidated. Due to diversification effects, risk integration allows natural hedges among different risks from real projects, pension assets or a broader range of business units. ERM takes advantage of these natural hedge opportunities by allowing different operation and pension elements to interact through a dependence structure. To reflect this benefit, our overall risk constraint considers different risks at a holistic level. It requires that the total value of all projects net of costs of operational risk, pension contributions and retained hazard losses should be sufficient to cover the entire financial obligations. We assume the total financial obligations at time τ equal a proportion c of the future value of the initial operation fund $M_0 \cdot \left(\sum_{j=1}^m w_{jp}(1+\hat{r}_{p_0})^\tau\right)$ evaluated at the minimal acceptable periodic return $\hat{r}_{p_0}$. Then the CVaR-type overall risk constraint is formulated as:

$$\mathrm{E}[F_\tau'^* | F_\tau'^* \leq \mathrm{VaR}_{\alpha_5}(F_\tau'^*)] \geq \left[cM_0 \cdot \left(\sum_{j=1}^m w_{jp}(1+\hat{r}_{p_0})^\tau \right) \right],$$

where $F_\tau'^*$ is the adjusted operation fund after the hazard insurance at time τ, i.e., $F_\tau'^* = F_\tau' - P_\tau^{hz} = (1-u(1+d)\mu_h)F_\tau'$. Here, the left-tail α_5-level VaR of the firm value net of pension costs equals

$$\mathrm{VaR}_{\alpha_5}(F_\tau'^*) = \min\{\beta_5 | \Pr[F_\tau'^* \leq \beta_5] \geq \alpha_5\}.$$

We can view this overall risk constraint as an insolvency constraint at the firm level after considering the pension effect. It requires that the left-tail α_5-level CVaR of the firm value net of pension costs to be not lower than the promised payment on the debt.

Constraint 7: Budget constraint (at time 0) The following budget constraint holds for the pension fund:

$$w_1 + w_2 + \cdots + w_n = 1.$$

Constraint 8: Strategic constraint (at time 0) To maximize firm value, keep market presence and have sufficient funds to support all pension obligations, it is desirable to have a minimum amount of capital invested in real projects. Thus, a minimum proportion γ_{rp} of the firm's total capital M_0 is required to invest in real projects at time 0:

$$\gamma_{rp} \leq w_{1p} + w_{2p} + \cdots + w_{mp} \leq 1.$$

Constraint 9: Range constraints (at time 0)

$$\begin{aligned} &0 \leq \phi_j, u, w_{jp}, w_i \leq 1, \qquad i = 1, 2, \cdots, n; j = 1, 2, \cdots, m \\ &NC \geq 0. \end{aligned} \tag{22}$$

3.2. **Numerical Results.** Here we use an example to illustrate how to apply our ERM optimization model considering pension effects. Suppose the firm invests a proportion w_{sp} of the total raised capital $M_0 = 200$ in a 5-year short-term project and a proportion w_{lp} in a 15-year long-term project at time 0. The unhedged operation funds invested in the short-term and long-term projects, $\hat{F}_t^{sp}$ and $\hat{F}_t^{lp}$, generate returns according to the geometric Brownian motion described in (1). The fund in the 5-year short-term (15-year long-term) project account will be used to fund another identical 5-year short-term (15-year long-term) project at the end of each 5-year (15-year) period. The assumed drifts and instantaneous volatilities of the short-term project (SP) and the long-term project (LP) are shown in Panel A of Table 1.

We further assume the firm invests the pension funds $PA_0 = M_0(1 - w_{sp} - w_{lp})$ in $n = 3$ asset indices at time 0: S&P500 index, Merrill Lynch corporate bond index and 3-month T-bill with the weights of w_1, w_2 and w_3, respectively. Following the log-normal model (8), we estimate the drifts and instantaneous volatilities of the S&P500 index, Merrill Lynch corporate bond index, and the 3-month T-bill based on the monthly data from 1995 to 2010[4]. We convert the monthly estimates to the annual parameters and present them in Panel A of Table 1.

[4]The data for the S&P500 total return index, the Merrill Lynch corporate bond total return index and the 3-month T-bill total return index are obtained from DataStream.

TABLE 1. Assessment of Projects, Pension Assets, and Pension Valuation Rate

Panel A: Drifts and Instantaneous Volatilities of Geometric Brownian Motion						
	SP	LP	S&P500	Corp. Bond	T-Bill	
α	0.1000	0.1200	0.0964	0.0715	0.0348	
σ	0.1000	0.1200	0.1696	0.0566	0.0061	
Panel B: Correlations						
	SP	LP	S&P500	Corp. Bond	T-Bill	IRS Bond
SP	1					
LP	0.5000	1				
S&P500	0.0500	0.0500	1			
Corp. Bond	-0.0250	-0.0250	0.2534	1		
T-Bill	-0.0500	-0.0500	0.0466	0.0381	1	
IRS Bond	-0.0250	-0.0250	-0.2241	0.1222	0.1398	1

TABLE 2. Maximum Likelihood Parameter Estimates of Pension Valuation Rates

Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
ν	0.1821	θ	0.0569	σ_ρ	0.0035

Our estimation of the CIR model follows two steps. Using the IRS composite corporate bond rates from January 2001 to December 2010,[5] we first obtain the maximum likelihood estimates of the CIR model (4) for the pension valuation rate ρ_t. Our estimation results are shown in Table 2. Then, with the estimated values of $\mathrm{d}W_{\rho t}$, we calibrate the correlation between the Brownian motion of the pension valuation rate ρ_t in (4) and the Brownian motions of pension assets $A_{i,t}$ ($i = 1, 2, 3$) in (8) based on the monthly data from January 2001 to December 2010. The correlations among the three pension assets are also estimated based on the data of 2001-2010. We report the estimated correlations in Panel B of Table 1. In addition, Table 1 shows the assumed correlations between the projects and the pension assets as well as the pension valuation rate.

We focus our study on a cohort with all members who join the plan at age $x_0 = 50$ in time 0 and retire at age $x = 65$ after $T = 15$ years. We further assume the pension cohort has the same mortality experience as that of the US male population and the plan forecasts its mortality rates following the Lee and Carter (1992) procedure with the mortality data from 1933 to 2010 in the Human Mortality Database[6]. The terminal age of the pension participants is set at 110. Lee and Carter (1992)'s model

[5]Data source: `www.irs.gov/Retirement-Plans/Composite-Corporate-Bond-Rate-Table` (data downloaded on November 13, 2015).

[6]Available at `www.mortality.org` or `www.humanmortality.de` (data downloaded on June 21, 2013).

accounts for both age-specific mortality variations and a general time trend of mortality evolution for all ages.[7] The plan offers an annual survival benefit of $B = 15$ and it amortizes its unfunded liability over $q = 7$ years. Moreover, the firm forecasts its pension valuation rates and pension asset returns based on (4) and (8) with the data at the end of 2010 as the starting values.

TABLE 3. Parameter Values

Risk Appetite					Risk Limits			
α_1	α_2	α_3	α_4	α_5	$\hat{r}_{p0}$	l_{op}	l_h	ζ_{TUL}
0.025	0.025	0.025	0.025	0.01	5.00%	0.35	0.015	250
Hazard Risk					Op Risk Factor	Strategic Factor	Borrowing Factor	Project Factor
μ	σ	μ_h	d		γ_p	γ_{rp}	c	r_h
-6.913	2.148	0.01	0.2		0.02	0.8	0.3	4.00%
Pension Risk								
ψ_1	ψ_2	B	q	ρ_0				
0.2	0.5	15	7	6.14%				

The proposed integrated optimization framework maximizes the expected firm value $\mathrm{E}[F'_\tau]$ at time $\tau = \omega - x_0 = 60$, subject to the project risk, pension risk, hazard risk, operational risk and overall risk constraints. The parameters for the optimization are shown in Table 3. All of these assumptions can be easily relaxed to fit a firm's specific needs. The optimal holistic decisions for the project investment, project risk hedging, hazard insurance, and pension asset investment based on Problem (16) and Constraints 1–9 are presented in Table 4.

TABLE 4. Optimal Investment, Project Risk Hedge, Insurance and Pension Decisions Based on ERM Optimization Model (16)

w_{sp}	w_{lp}	u	w_1	w_2	w_3	NC	ϕ_{sp}	ϕ_{lp}	$\mathrm{E}[F'_\tau]$	$\mathrm{E}[TPC]$
60.06%	19.94%	90.07%	4.29%	70.25%	25.46%	2.77	0.00%	10.12%	8727.74	46.57

The optimal solution in Table 4 suggests that the plan should invest $w_{sp} = 60.06\%$ of the initial total capital in the short-term project and $w_{lp} = 19.94\%$ in the long-term project to achieve the highest $\mathrm{E}[F'_\tau] = 8727.74$. The long-term project has a higher return than the short-term project, but it also has a higher volatility. To satisfy the business and pension risk constraints as well as the overall risk constraint, the firm has to invest more in the short-term project and hedge a proportion $\phi_{lp} = 10.12\%$ of

[7]To conserve space, the model estimation and mortality projections for our numerical illustrations are not reported here. Instead, they are provided in the Appendix published on the internet.

the long-term project risk. While project risk hedge reduces the firm's risk, it lowers the expected firm value $\mathrm{E}[F'_\tau]$ because the hedged portion of the project earns a lower hedged rate of return $r_h = 4\%$.

While there is a positive loading on the insurance, hazard risk transfer can lower firm risk. It allows the firm to invest more in the long-term project and thus increases firm value. In this example, the firm should transfer $u = 90.07\%$ of its entire hazard risk to an insurance company. The optimal annual pension normal contribution before retirement is $NC = 2.77$. In this case, the expected total pension cost equals $\mathrm{E}[TPC] = 46.57$. Table 4 also shows that $w_1 = 4.29\%$ of the pension fund should be invested in the S&P 500 index, $w_2 = 70.25\%$ in the the Merrill Lynch corporate bond index, and $w_3 = 25.46\%$ in the 3-month T-bill. As the risky pension assets introduce significant risk, the firm invests a notable proportion of its pension funds in the 3-month T-bill to satisfy the average and downside risk constraints on its pension total underfunded liability.

3.3. **What if Pension Risk is not Integrated?** We have shown how to integrate pension risk in an ERM framework, but is it important to consider pension risk in our ERM formulation? To answer this question, here we examine the case when the pension risk is not recognized or modeled in the ERM optimization. That is, the firm separately assesses its pension risk and decides how to mitigate it on its own. After allocating a proportion of the initial capital to the pension plan, the firm invests the remaining fund in real projects and deducts the total pension cost at the end of the horizon τ. To make the cases comparable, we keep the upper/lower bounds of Constraints 1–9 for the following pension only and project only cases the same as those in the ERM example of Section 3.2.

Suppose at time 0, the firm allocates 40 to the pension fund, the same amount as the optimal PA_0 $(= M_0(1 - w_{sp} - w_{lp}) = 40)$ in the ERM case. Consistent with Lin et al. (2015), in a silo pension risk management setup, we solve for the optimal pension asset weights $w_i^{silo} \in [0,1]$ for $i = 1, 2, \cdots, n$ ($n = 3$ in our example) and normal contribution $NC^{silo} \geq 0$ to minimize the expected total pension cost

$$\underset{w^{silo}, NC^{silo}}{\text{Minimize}} \qquad \mathrm{E}\left[TPC^{silo}\right] \tag{23}$$

subject to the pension risk constraints (Constraints 4 and 5), the budget constraint (Constraint 7), and the range constraints on w_i^{silo} and NC^{silo}. With this setup, in our example, the optimal solution of Problem (23) for the weights in S&P500 (w_1^{silo}), Merrill Lynch corporate bond index (w_2^{silo}), T-Bill (w_3^{silo}), as well as the normal contribution (NC^{silo}) leads to the same expected total pension cost $\mathrm{E}\left[TPC^{silo}\right] = 46.57$

as that with ERM in Table 4. This is not a surprising result. DB pensions impose cost to a firm and reduce firm value. In the ERM framework considering pension risk, if we can reduce total pension cost (determined by pension asset allocation and normal contribution), we will increase firm value. In the SRM framework, following the pension literature, our optimization problem is to minimize total pension cost. As pension contributions are mandatory and independent of real projects performance, SRM and ERM have the same objective with respect to total pension cost. Thus, all else equal, given the same initial pension fund PA_0 in both cases, SRM and ERM are expected to have very close (if not the same) expected total pension cost.

As noted earlier, the minimum expected total pension costs of SRM and ERM turn out to be the same in our example. As the ERM optimization problem has exactly the same pension constraints as those in SRM, in this case, we have confirmed that the optimal solution for pension asset allocation and normal contribution with ERM is also an optimal solution for SRM. It is worth noting that SRM can have more than one optimal solution for pension asset allocation and normal contribution, all of which achieve the same minimum expected total pension cost; the optimal solution that is the same as that with ERM is just one of them.

Given the available fund of 160 $(= M_0 - PA_0 = 200 - 40)$ at time 0 for the real project investment and $w_{sp}^{silo} + w_{lp}^{silo} = 80\%$, we maximize the expected value of the operation fund $\mathrm{E}[F_\tau^{silo}]$ for the firm at time τ with respect to the weights invested in the short-term and long-term projects $w_p^{silo} = [w_{sp}^{silo}, w_{lp}^{silo}]$, the project risk hedge ratios $\phi^{silo} = [\phi_{sp}^{silo}, \phi_{lp}^{silo}]$, and the hazard insurance ratio u^{silo}. That is,

$$\underset{w_p^{silo},\phi^{silo},u^{silo}}{\text{Maximize}} \ \mathrm{E}[F_\tau^{silo}], \tag{24}$$

where F_τ^{silo} is determined recursively as $F_t^{silo} = \sum_{j=1}^m F_{t-1}^{silo,j}[(1 - u^{silo}(1+d)\mu_h)(1-\gamma_p)(1+r_t^{silo,j}) - (1-u^{silo})h]$, given $F_0^{silo,j} = M_0 \cdot w_{jp}^{silo}$. This optimization problem is subject to the project risk constraint (Constraint 1), the operational risk constraint (Constraint 2), the hazard risk constraint (Constraint 3), the overall risk constraint (Constraint 6), the strategic constraint (Constraint 8), the range constraints on w_{jp}^{silo}, ϕ_j^{silo} and u^{silo}, but without the pension-related constraints (Constraints 4, 5 and 7). These constraints are similar to those for the ERM model (16) except that firm values and choice variables have a superscript $silo$.

Using the IRS composite corporate bond rate in December 2010 as the pension valuation rate $\rho_0 = 6.14\%$ at time 0, the expected total pension cost $\mathrm{E}[TPC^{silo}] = 46.57$ at time 0 is equivalent to a future value of $1662.76 = 46.57 \times (1 + 6.14\%)^{\tau}$ at time $\tau = 60$. As this amount is paid by the operation fund, the expected firm value with SRM net of pension costs at time τ, $\mathrm{E}[F_{\tau}'^{silo}]$, is calculated as $\mathrm{E}[F_{\tau}'^{silo}] = \mathrm{E}[F_{\tau}^{silo}] - \mathrm{E}[TPC^{silo}] \cdot (1 + \rho_0)^{\tau}$.

TABLE 5. Optimal Project Investment, Project Risk Hedge, and Insurance Decisions with Silo Project Risk Management Strategy

w_{sp}^{silo}	w_{lp}^{silo}	u^{silo}	ϕ_{sp}^{silo}	ϕ_{lp}^{silo}	$\mathrm{E}[F_{\tau}'^{silo}]$	$\mathrm{E}[TPC^{silo}]$
23.67%	56.33%	80.42%	2.89%	33.08%	7674.58	46.57

The results are shown in Table 5. In this scenario when the real projects and the pension plan are managed separately, the expected firm value is notably reduced to $\mathrm{E}[F_{\tau}'^{silo}] = 7674.58$, a 12.07% drop from the previous ERM optimum considering pension effects $\mathrm{E}[F_{\tau}'] = 8727.74$. Without aggregating the risks of real projects and the pension plan, the firm is subject to a higher risk because it forgoes diversification benefits. To reduce risk within its risk appetite, the firm has to hedge 24.15% $(= (23.67\% \times 2.89\% + 56.33\% \times 33.08\%)/(23.67\% + 56.33\%))$ of its operation fund 160 at time 0, much higher than the hedge ratio 2.52% with ERM. As the hedged operation fund has a much lower rate of return, a higher project risk hedge ratio with SRM leads to a lower expected firm value. This finding highlights the importance of integrating pension risk with a firm's business strategic planning.[8]

4. PENSION DE-RISKING IN THE ERM FRAMEWORK

Operating a DB plan increases volatilities of corporate earnings, balance sheets, and free cash flows. As a result, many companies have considered or have taken steps to reduce the risk associated with their DB plans via pension de-risking. The pension ground-up de-risking strategy, including buy-ins and buy-outs, transfers a proportion of the entire pension liabilities to a third party. The excess-risk de-risking strategy such as longevity insurance and swaps, instead, only cedes a proportion of the high-end longevity risk embedded in a pension plan to a risk taker. The ground-up strategy allows a firm to remove interest rate, inflation, asset and longevity risks from its pension account while the excess-risk strategy

[8]As a robust check, we also investigate whether integrating pension risk into an ERM program is important when a pension ground-up hedging strategy or a pension excess-risk hedging strategy is implemented. The results from this analysis further confirm that the expected firm value with SRM is notably lower than that with ERM. To conserve space, we do not report these results. They are shown in the Appendix published on the internet.

only transfers longevity risk. As the ground-up strategy involves more risk transfer, it is more expensive and entails a much higher cash outflow upfront than the excess-risk strategy. In this section, we compare the costs and benefits of these two pension de-risking approaches and assess their effectiveness in the ERM framework.

4.1. **Pension Ground-up Hedging Strategy.**

4.1.1. *Optimization Setup.* With the pension ground-up de-risking strategy, the firm transfers a proportion h^G of the pension liability $Ba(x(T,0))$ to a hedge provider at time $t = 0$ where $0 \leq h^G \leq 1$. To implement this strategy, the firm needs to pay a hedge price equals to

$$HP^G = \frac{h^G(1+\delta^G)B\bar{a}(x(T,0))}{(1+\rho_0)^T},$$

where $\bar{a}(x(T,0)) = \mathrm{E}[a(x(T,0))]$. The unit hedge cost δ^G accounts for a premium of taking pension risk as well as transaction costs. Later at time $T+1, T+2, \cdots$, the firm will receive a proportion h^G of benefits due to retirees from the hedge provider.

Given the ground-up strategy, the ERM optimization problem of the firm with respect to the project risk hedge ratios $\phi^G = [\phi_1^G, \phi_2^G, \cdots, \phi_m^G]$, the insurance ratio u^G, the project weights $w_p^G = [w_{1p}^G, w_{2p}^G, \cdots, w_{mp}^G]$, the pension asset weights $w^G = [w_1^G, w_2^G, \cdots, w_n^G]$, the normal contribution NC^G, and the ground-up hedge ratio h^G is expressed as:

$$\underset{\phi^G, u^G, w_p^G, w^G, NC^G, h^G}{\text{Maximize}} \ \mathrm{E}[F_\tau'^G], \tag{25}$$

where the objective function $\mathrm{E}[F_\tau'^G]$ is constructed similar to $\mathrm{E}[F_\tau']$ in Section 3.1 but based on the variables and parameters with a superscript G explained in this section and the Appendix published on the internet. Optimization problem (25) is subject to a set of constraints similar to Constraints 1–9 without pension de-risking in Section 3.1. It is worth noting that, compared with the no-hedge case, the ground-up strategy has an additional range constraint in Constraint 9, $HP^G \leq PA_0^G$, to ensure the hedge price does not exceed the fund allocated to the pension plan at $t = 0$.[9] That is, the plan is not allowed to borrow money to pay for the cost of the ground-up strategy. The detailed ground-up hedging formula derivations and constraints are explicitly given in the Appendix published on the internet.

[9]This range constraint can be modified or removed if a bond is floated to cover the costs of the firm in transferring the pension risks.

4.1.2. *Numerical Example.* Here we continue the example in Section 3.2, but assume that the plan implements a ground-up hedging strategy by transferring a proportion h^G of its total pension obligations to an insurer at time 0.

TABLE 6. Optimal Ground-up Hedging Strategies with Different Hedge Cost Parameters δ^G

δ^G	w_{sp}^G	w_{lp}^G	u^G	w_1^G	w_2^G	w_3^G	NC^G	h^G	ϕ_{sp}^G	ϕ_{lp}^G	$\mathrm{E}[F_\tau'^G]$	$\mathrm{E}[TPC^G]$
0.00	58.82%	21.18%	83.05%	31.08%	68.92%	0.00%	1.19	41.28%	0.00%	8.41%	10193.68	32.98
0.05	58.38%	21.62%	83.17%	25.64%	74.36%	0.00%	1.36	37.26%	0.00%	8.81%	10003.83	36.45
0.10	59.34%	20.66%	83.32%	20.14%	79.86%	0.00%	1.51	32.92%	0.00%	8.44%	9835.16	39.34
0.15	59.73%	20.27%	83.45%	14.54%	85.46%	0.00%	1.66	28.19%	0.00%	8.40%	9686.63	41.59
0.20	59.99%	20.01%	83.56%	10.39%	89.61%	0.00%	1.79	24.57%	0.00%	8.42%	9557.24	43.46
0.25	59.84%	20.16%	83.66%	9.28%	90.72%	0.00%	1.89	23.69%	0.00%	8.62%	9439.66	45.51

The results based on (25) are shown in Table 6. Table 6 indicates that as long as the firm hedges some of its pension risk with a hedge ratio $h^G > 0$, the firm can achieve a firm value $\mathrm{E}[F_\tau'^G]$ at time τ higher than $\mathrm{E}[F_\tau'] = 8727.74$, the value when the firm does not hedge. For example, at zero hedge cost ($\delta^G = 0$), the ground-up de-risking strategy notably increases $\mathrm{E}[F_\tau'^G]$ to 10193.94, a 16.80% rise compared to $\mathrm{E}[F_\tau'] = 8727.74$ without hedge. Even when the hedge cost is high at $\delta^G = 0.25$, $\mathrm{E}[F_\tau'^G]$ is still 8.16% higher than $\mathrm{E}[F_\tau']$. The ground-up strategy reduces the pension risk as well as the firm's overall risk. Thus, it allows the firm to hedge less and invest more in the riskier but higher-return long-term project within its risk appetite. Moreover, it invests all pension fund in risky pension assets (*i.e.*, $w_1 + w_2 = 1$ and $w_3 = 0$) and thus achieves a significant reduction in the normal contribution and the expected total pension cost. As a result, the hedged firm value exceeds the unhedged. Overall, our findings suggest that the ground-up strategy is effective in improving firm performance based on our ERM framework with pension de-risking.

Consistent with Cox et al. (2013), Lin et al. (2014) and Lin et al. (2015), Table 6 shows that h^G and $\mathrm{E}[F_\tau'^G]$ decrease and $\mathrm{E}[TPC^G]$ increases with the hedge cost. An increase in the hedge cost parameter δ^G from 0 to 0.25 causes a drop in the hedge ratio from $h^G = 41.74\%$ to $h^G = 23.69\%$. This result highlights the adverse effect of hedge cost on the hedge level. Moreover, when the hedge cost parameter δ^G increases from 0 to 0.25, the proportion invested in S&P index drops significantly and the investment in the less risky corporate bond index increases accordingly (w_1 drops from 31.69% to 9.28% and w_2 rises from 68.31% to 90.72%). As the firm transfers less pension risk due to a higher hedge cost, to satisfy the pension-related Constraints 4 and 5, it has to invest more in the safer asset. Interestingly, the weight of 3-month T-bill w_3 stays at 0 when the hedge cost increases. It is because the expected annual return of the 3-month T-bill 3.48% is much lower than the long-term mean of the pension valuation rate $\theta = 5.69\%$, making the 3-month T-bill unattractive when the ground-up pension hedging is adopted.

4.2. **Pension Excess-Risk Hedging Strategy.**

4.2.1. *Optimization Setup.* Suppose, at time 0, the firm implements an excess-risk strategy to hedge the risk that the s-year survival rate of the retirees of age x at time T exceeds its expectation ${}_s\bar{p}_{x,T} = \mathrm{E}[{}_s\hat{p}_{x,T}]$ at time $T+s$. That is, the firm purchases a set of European call options written at time 0 and exercised at $T+1, T+2, \cdots$ with payoffs equal to $\max\left[B_s\hat{p}_{x,T} - B_s\bar{p}_{x,T}, 0\right]$ $(s = 1, 2, \cdots)$. With a hedge ratio of h^E, the firm needs to pay a hedge price equals to

$$HP^E = \frac{h^E(1+\delta^E)\mathrm{E}\left[\sum_{s=1}^{\tau} v_0^s \max\left[B_s\hat{p}_{x,T} - B_s\bar{p}_{x,T}, 0\right]\right]}{(1+\rho_0)^T},$$

where δ^E is the hedge cost per dollar hedged in the excess-risk hedging strategy and the discount factor equals $v_0 = 1/(1+\rho_0)$. With a full hedge against the risk above the expectation, the expression $\mathrm{E}\left[\sum_{s=1}^{\tau} v_0^s \max\left[B_s\hat{p}_{x,T} - B_s\bar{p}_{x,T}, 0\right]\right]$ represents the expected payment from the risk taker at retirement T. After hedging pension longevity risk, the total fund available at time 0 reduces to $PA_0 - HP^E$.

The optimization problem of the firm under the excess-risk strategy is as follows:

$$\underset{\phi^E, u^E, w_p^E, w^E, NC^E, h^E}{\text{Maximize}} \ \mathrm{E}[F_\tau'^E]. \tag{26}$$

The objective function $\mathrm{E}[F_\tau'^E]$ has a similar expression to that of the no-hedge strategy $\mathrm{E}[F_\tau']$ but with a superscript E. The constraints for (26) are similar to Constraints 1–9 without pension de-risking in Section 3.1 except a superscript E as well as an additional range constraint in Constraint 9, $HP^E \leq PA_0^E$, to ensure the hedge price does not exceed the fund allocated to the pension plan at $t = 0$. The detailed excess-risk hedging formula derivations and constraints are explicitly given in the Appendix published on the internet.

4.2.2. *Numerical Illustrations.* We continue the example in Section 3.2 with the same parameters as those used to obtain the optimal solution in Table 4. Suppose the firm implements the excess-risk de-risking strategy to hedge a proportion h^E of its high-end longevity risk from its pension cohort aged $x_0 = 50$ at time 0. The strike level at time $T+s$ is specified at the expected s-year survival rate, ${}_s\bar{p}_{x,T}$,

$s = 1, 2, \ldots$ where $T = 15$ and $x = x_0 + T = 65$. Table 7 summarizes the results at different unit hedge costs δ^E with the excess-risk hedging strategy.

TABLE 7. Optimal Excess-Risk Hedging Strategies with Different Hedge Cost Parameters δ^E

δ^E	w_{sp}^E	w_{lp}^E	u^E	w_1^E	w_2^E	w_3^E	NC^E	h^E	ϕ_{sp}^E	ϕ_{lp}^E	$\mathrm{E}[F_\tau'^E]$	$\mathrm{E}[TPC^E]$
0.00	57.97%	22.03%	85.00%	3.13%	75.71%	21.16%	2.57	100.00%	0.00%	11.00%	8956.35	44.88
0.05	57.72%	22.28%	84.99%	3.12%	75.72%	21.16%	2.58	100.00%	0.00%	11.13%	8948.60	45.03
0.10	57.47%	22.53%	84.98%	3.11%	75.74%	21.15%	2.58	100.00%	0.00%	11.26%	8940.82	45.17
0.15	57.43%	22.57%	85.01%	3.10%	75.75%	21.15%	2.59	100.00%	0.00%	11.29%	8932.99	45.32
0.20	57.39%	22.61%	85.04%	3.10%	75.76%	21.14%	2.60	100.00%	0.00%	11.31%	8925.16	45.46
0.25	56.76%	23.24%	84.97%	3.09%	75.77%	21.14%	2.60	100.00%	0.00%	11.61%	8917.35	45.61

Table 7 shows that to achieve the highest $\mathrm{E}[F_\tau'^E]$, when $\delta^E \leq 0.25$, the firm should transfer the entire risk $h^E = 100\%$ above the strike level, ${}_s\bar{p}_{65,15}$, $s = 1, 2, \ldots$. Different from those in the ground-up hedging strategy in Table 6, the hedge ratio h^E and the pension asset allocation of the excess-risk strategy are not sensitive to the hedge cost in all scenarios of interest. While the excess-risk hedging has an increase in the risky pension asset investments than the no-hedge case (for example, compare $w_1 + w_2 = 74.54\%$ without hedge in Table 4 and $w_1^E + w_2^E = 78.84\%$ with the excess-risk strategy given $\delta^E = 0$ in Table 7), this increase is much smaller than that of the ground-up hedging (for example, compare $w_1 + w_2 = 74.54\%$ without hedge in Table 4 and $w_1^G + w_2^G = 100.00\%$ with the ground-up strategy given $\delta^G = 0$ in Table 6). This difference can be explained by the fact that the excess-risk strategy only transfers the high-end longevity risk but the ground-up strategy removes the interest rate risk, inflation risk, asset risk and longevity risk. As a result, the excess-risk hedging has a lower risk reduction and thus has less leeway in increasing the firm's investment in the riskier but higher-return pension assets within risk tolerance. This leads to a lower end-of-horizon firm value than the ground-up strategy. For example, when $\delta^E = 0$, the excess-risk strategy has an expected firm value of $\mathrm{E}[F_\tau'^E] = 8956.35$ at time τ, only 2.62% higher than that without hedge, $\mathrm{E}[F_\tau'] = 8727.74$. In contrast, the ground-up strategy improves the firm value at time τ by 16.80% when $\delta^G = 0$. Even if δ^G is higher, for example, at $\delta^G = 0.25$, the firm still has $\mathrm{E}[F_\tau'^G] = 9439.66$, higher than $\mathrm{E}[F_\tau'^E] = 8956.35$ when δ^E is zero. Given this, we conclude that, subject to the enterprise-wide risk constraints, the ground-up de-risking strategy is more effective in improving the overall firm performance than the excess-risk strategy.

Notice that the expected total pension cost of the excess-risk hedging, $\mathrm{E}[TPC^E]$ in Table 7, is much higher than that of the ground-up hedging, $\mathrm{E}[TPC^G]$ in Table 6, especially when the ground-up hedge ratio h^G is high. The total pension cost in pension de-risking equals the sum of pension hedge cost and the present value of all future pension normal and supplementary contributions associated with the *retained* pension liabilities after pension de-risking. While there is a hedge cost δ^G per dollar pension liabilities transferred, paying a lump-sum premium HP^G to implement the ground-up hedging at time 0 reduces the total pension cost as the firm does not need to make any contributions to serve the ceded liabilities. Moreover, it lowers high pension underfunding or overfunding costs that are a function of the penalty factors ψ_1 and ψ_2 in (12). In contrast, the price HP^E of excess-risk hedging paid at time 0 is low even with a 100% hedge ratio because this strategy only transfers the high-end longevity risk. As the firm has to retain most of pension liabilities and underfunding/overfunding costs, its total pension cost is just somewhat lower than that in the no-hedge case but much higher than that of the ground-up hedging.

5. Sensitivity Analyses

Now we explore how parameter choices affect our optimal solutions.[10] In general, the expected firm value at time τ, $\mathrm{E}[F'_\tau]$, decreases as the annual survival benefit B increases or the mortality improves more than our expectation. This is consistent with the intuition that a higher survival benefit or a more significant mortality improvement tends to increase firm pension liabilities and thus reduce the fund for business operations as well as the firm value. We also investigate how a firm will change its strategies when the operational risk and overall risk constraints become more stringent. We find a lower risk appetite results in a higher project risk hedge and/or a reduced investment in the long-term project that is more profitable but riskier than the short-term project. That is, the firm has to sacrifice the total return on its operation fund to meet a lower risk tolerance target, thus leading to a lower firm value. In addition, we study the impact of the risk reward ratios of the pension asset indices. We observe that an increase (decrease) in the average returns of the three asset indices raises (reduces) the firm value and reduces (increases) the total pension cost. Finally, we examine the impact of the project correlation parameter $\rho_{sp,lp}$. Our results indicate that the firm hedges less (more) project risk when $\rho_{sp,lp}$ is lower (higher). Due to diversification benefits, a lower correlation between the projects allows the DB plan sponsor to hedge

[10]To conserve space, we do not report these sensitivity results. They are reported in the Appendix published on the internet.

less and thus achieve a higher firm value. Overall, the analyses illustrate that our formulation leads to intuitive results. These help demonstrate the reliability of our ERM optimization models.

6. Conclusion

In this paper, we integrate the pension and business risks in the ERM framework and show how strategic decisions can be made in a holistic way considering both sets of risks. We construct an ERM optimization model that maximizes the expected firm value subject to different pension and business risk constraints as well as an overall risk constraint. Our analysis indicates that the performance of a firm will suffer if the firm ignores pension risk in its strategic planning.

Sponsoring a DB pension plan has become an increasingly difficult business problem (Lin et al., 2015). It has led many firms to transfer their pension risk to another party via a ground-up or an excess-risk pension de-risking strategy. Thus, this paper brings a pension hedging component to the study of the ERM optimization model, motivated by the proliferation of recent pension de-risking activities. Our model has important implications for pension plans as it describes how a de-risking strategy can increase a firm's value, whereas most of the earlier research on ERM does not numerically analyze pension risk transfer. Moreover, our results suggest that, while the pension excess-risk de-risking strategy is less capital intensive, it underperforms the ground-up strategy in terms of value creation. The extended analysis in this paper adds to the pension and longevity securitization literatures as it examines value effect and offers a more balanced view on each pension de-risking strategy.

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