Retirement Planning in the Light of Changing Demographics

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Hong Wang
Monash University (Australia)

Bonsoo Koo
Monash University (Australia)

Colin O'Hare
Monash University (Australia)

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Hong Wang, Bonsoo Koo and Colin O'Hare[1]

Abstract

With increasing longevity and decreasing fertility rates, governments and policy makers are increasingly engaged in the question of long term retirement planning. In many cases this has included emphasising the need for individuals to take more responsibility for their own retirement planning through tax incentives, compulsion and changes to the age at which state retirement benefits become available. In the case of Australia, as is considered here, long term retirement planning has been focused around the development of a compulsory defined contribution (DC) superannuation system. Here we investigate the interaction between population aging and the sustainability of the superannuation system by modelling a general superannuation scheme to compare the adequacy of retirement funds under a number of alternative scenarios. The model incorporates stochastic longevity forecasts and provides insight into the sufficiency of compulsory retirement saving both now and future. We find that the current pension scheme is more robust to longevity improvements for mid-class individuals however significant gaps arise for low-income individuals as longevity improves. Without addressing these issues, government expenditure is expected to increase substantially.

[1] Department of Econometrics and Business Statistics, Monash University, Melbourne Victoria. 3800 Tel: +61 3 9905 9414 Email: colin.ohare@monash.edu, bonsoo.koo@monash.edu and hong.s.wang@monash.edu.
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1. Introduction

With the increase in longevity and decrease in fertility rates the 21st century has seen unprecedented demographic changes to our populations. These changes are placing pressure on public finances as population's age and the associated retirement and healthcare costs weigh heavily on the economy. Many governments and policy makers are considering the question of long term retirement planning and are developing policies designed to address the increased costs of our aging populations. For a comprehensive survey, see OECD (2013).

The issue of aging populations is an international one. The world ageing report (2013), for example, finds that population ageing is taking place in nearly all parts of the world and projects the proportion of the older population (that is, individuals aged over 60) will increase from 9.7% in 1990 to 21.1% in 2050. The OECD projects a similar trend suggesting that the proportion of the retired population to the working population will move from a current level of 20% to 28% by 2060[2]. When we consider that each additional year of retirement adds around 3% to the capital required to live in retirement[3], it is clear that governments have had to act to address the obvious economic and financial consequences of this.

Whilst the answer seems clear; either we retire later or we save more, governments have to grapple with the political fall out of any decisions that they make, considering that the older population are usually seen as more vunerable in society. In combination with the fact that the older population are more inclined to vote, it is clear why it may prove difficult to make sweeping policy changes to retirement planning. That being said, several options are open to governments and policy makers when considering how to reform retirement planning:

Firstly, governments may encourage individuals to save more of their own funds during their working life to complement any state funded pension provision. This is in line with the life cycle hypothesis, see for example Modigliani and Brumberg (1954) and Ando and Modigliani (1963). Encouraging individuals to save more can be done through tax incentives as is the case in the UK, see Attanasio et al. (2005), or in the extreme through compulsion, as is the case in Australia. One question that must be considered in either case is what is the correct

[2] Source: OECD (2014) assuming a joint male-female retirement age of 65. Note that these are estimated based on the old-age dependence ratio.
[3] This is a consequence of the discounting rate and mortality table and is a well-known rule of thumb for actuaries. See for example the text Actuarial mathematics for Life Contingent Risks.

level of contribution. In this paper we consider this question in the case of the Australian superannuation system.

Secondly, governments could consider increasing the state pension age. Marcin et al (2015) pointed out that addressing the adverse changes in population structure by raising the reitrement age enhances warfare universally. In many countries the process of increasing pension age has already begun[4] (See Farrar et al, 2012) but its implementation is slow owing to the need to provide equity for those approaching retirement as well as managing the expectations of individuals.

Thirdly, and least affordable, the government could raise taxation elsewhere to cover the increasing cost of retirement and welfare although with longevity improvements showing no signs of slowing down this appears, at least of the face of it, to be an unsustainable approach to retirement planning.

In the case of Australia, the government chose to encourage individuals - through compulsion - to save for retirement themselves with the development of the superannuation system in 1992. The superannuation system in Australia is a defined contribution system whereby indiviudals in employment are required to enrol in one of either an industry sponsored or retail superannuation plan (Bateman and Piggott, 2003). See Australian Superannuation Legislation (2015) for more details. In each case the individual's employer is required to contribute into the plan on behalf of the employee. The superannuation guarantee (the minimum contribution required of the employer) started at 3% in 1992 and has gradually increased to 9% of salary by 2002. It is currently 9.5% (as of 2015) and will continue to increase in stages to 12% by 2025. Recognising the ageing population issue the government has also proposed changes to the state pension system, which in time will impact on superannuation. The Treasurer has proposed an increase in the Age Pension age from 65 to 70 by 2035 with the intention that these reforms will ensure the sustainability of the state funded pension. In the coming decades, the Age Pension will be more well-targeted to these in genuine need, and the superannuation system will gradually grow into the primary source of retirement income for the majority of Australians.

[4] For instance, the UK government gradually increases for women from 60 to 65 to match men's and it will plan to increase for both men and women to reach 66 by Oct. 2020. Meanwhile, the australian government increases the Age pension age from 65 to 70 by 2035.

An issue that arises by placing more of the emphasis for retirement funding on the individual is that of longevity risk and whether the funds accumulated will indeed be sufficient to last the individual in retirement. This issue is more acute in the Australian system where post retirement longeivty risk remains with the individual due to the lack of a strong annuity market. Internationally, the question of the economic and welfare effects of longeivty risk being faced by the individual are becoming more prevalent also. For example, even in well developed annuity markets such as the UK, compulsory annuitisation has recently been removed. One of the motivations of this paper, in the context of a significant longevity risk remaining with the individual, is therefore to model the adequacy[5] of retirement funds accumulated within the existing superannuation system. The basic state pension in Australia pays a single individual a pension of $867[6] per fortnight. The Association Superannuation Funds of Australia (ASFA) also define modest and comfortable income in retirement as $23,682, per year and $42,861 per year respectively[7]. In this paper we define adequate to be the level of the basic state pension but note that the results of our analysis would change if we were to use modest or comfortable income as defined by ASFA.

The existing literature on questions around superannuation in the context of longevity risk is relatively scarce although mortality modelling is developed significantly in the past decades. A good suvery paper on the various approaches to modeloing mortality can be found in Booth and Tickle, 2008. A possible reason for the lack of consideration of demographic changes on superannuation, as stated in Bielecki et al (2015), may be that demographic changes are seen to be neutral in the world of superannuation or DC schemes. However, as noted in that paper, changes in demographics will have a profound impact on individuals who will have to make the same amount of funds last them more years in retirement. Failing this, individuals may have to return to employment affecting the labour supply. Several papers consider the post retirement options that are available to individuals. For example Lin et al (2014) use a Monte Carlo simulation to evaluate the impact of various retirement options on early retirement. Further, the ageing population is likely to shift the financial burden from the state pension system to the superannuation system and this further highlights the need for a sustainable superannuation system. Several papers consider questions around pension system in the from

[5] For the case of superannuation in Australia, we define an "adequate" level of retirement income as the government funded state pension rate for low income individuals as of 2015.
[6] See http://www.humanservices.gov.au/customer/enablers/centrelink/age-pension/payment-rates-for-age-pension for details of the state pension levels.
[7] Details on ASFA modest and comfortable retirement incomes can be found at http://www.superannuation.asn.au/resources/retirement-standard

of life-cycle models, see for example Creedy et al (2015), or Koka and Kosempel (2014). In Creedy et al (2015) the authors address questions around household savings and consumption in the event of changes to retirement policy and conclude that the effect on saving rate is modest. Koka and Kosempei (2014), on the other hand, use a life cycle model to consider the welfare implications of a removal of the mandatory pension age, concluding that overall there would be a reduction in the welfare of the individual.

To our knowledge there is very limited work in the economic modelling space considering the adequacy of superannuation systems for individuals. Given the large longevity risk that individuals face, and the corresponding risk that the economy faces through pressure on the state pension system, we feel that questions around the level of contribution and the optimal retirement age are important issues to consider economically. Our paper contributes to the literauture in this area by first modelling the current Australian superannuation system and then considering the impact of changes to the contribution level and / or retirement age on the shortfall in funds required to sustain a adequate retirement income. We stochastically project mortality rates as well as a forecasting of the accumulation of funds invested and address questions around the future sustainability of the current system.

The remainder of the paper is set out as follows. In the next section we discuss the data that we have used to fit and forecast our mortality model. Section 3 discusses the methodology, the mortality forecasting model we use and how we incorporate that with our investment accumulation model. Section 4 presents and discusses our empirical results, considering the current system and various scenarios of different contribution rates and retirement ages. We conclude with some policy recommendations and further research in section 5.

2. The Mortality Model

2.11 The Hyndman-Ullah Method

The forecast accuracy of mortality rates is a key assumption in our superannuation model, we spend some time here discussing the approach before moving onto the superannuation model.

We use the Hyndman-Ullah method (Hyndman and Ullah, 2007) and the product-ratio method (Hyndman et al, 2013) to forecast the Australian mortality rates. There are many mortality forecasting approaches to choose from see for example, Lee and Carter (1992), Renshaw and Haberman (2003 and 2006), Cairns, Blake and Dowd (2006, 2008, 2009), Hyndman and Ullah (2007), Plat (2009) and O'Hare and Li (2012) to name a few. We use the Hyndman-Ullah method as it has been shown to produce accurate forecasts and has been successfully applied to all demographic components including mortality, fertility and migration (the out of sample forecasting performance comparison with Lee-Carter model is provided in the "Empirical Results" section). In conjunction with the product-ratio method, the Hyndman-Ullah method can produce coherence long-run forecasts of sex-specific mortality rates.

The Hyndman-Ullah method follows the functional data analysis (FDA) paradigm, involving the use of non-parametric smoothing techniques and functional Principal Component Analysis (fPCA). It generalises the well-known Lee-Carter method in three aspects: (1) nonparametric smoothing is used whereas the Lee-Carter does not assume smoothness; (2) the use of basis expansion is a significant improvement over the sole use of the first principal component in the Lee-Carter model; (3) more complex time series models are fitted to the principal component coefficients comparison with a random walk with drift model is always used in Lee-Carter model. Hyndman and Ullah (2007) describes the mortality rate (central death rate) in terms of smooth functions and basis expansions. The smoothing process is undertaken in the first equation, and the dynamics are captured in the second equation

$$y_t(x) = s_t(x) + \sigma_t(x)\varepsilon_{t,x},$$

$$s_t(x) = \mu(x) + \sum_{k=1}^{K} \beta_{t,k}\Phi_k(x) + e_t(x),$$

where $y_t(x)$ is the log mortality rate for age x and time t, $\sigma_t(x)$ is the time-varying variance of the log mortality rate for age x and $s_t(x)$ is the underlying smooth function in the first equation. In the second equation, $\mu(x)$ describes the locations of the smooth functions, $\Phi_k(x)$ is a basis function expansion calculated through fPCA and $e_t(x)$ is the model error assumed to be serially uncorrelated. Since the data is observed with error, monotonicity is imposed on high-age groups to reduce the noise in the estimated curves for log mortality rates in the smoothing step (Hyndman and Ullah 2007). Furthermore, Hyndman & Ullah (2007) initially suggested the selection of the number of basis functions (K) by minimizing the mean integrated squared forecast error. In this paper, we use the first six principal components in our mortality forecasting model.

In terms of forecasting, univariate time series models are fitted to the coefficients i.e. $\{\beta_{t,k}\}$ given the basis functions are mutually orthogonal by construction. Let $\hat{\beta}_{T+h|T,k}$ denote the h-step ahead forecast of $\beta_{T,k}$, then the h-step ahead forecast of log mortality rate i.e. $\hat{y}_{T+h|T}$ can be expressed as

$$E\left(y_{T+h|T}\right)=\hat{y}_{T+h|T}=\hat{s}_{T+h|T}=\hat{\mu}(x)+\sum_{k=1}^{K}\Phi_k(x)\hat{\beta}_{T+h|T,k}\,.$$

In this paper, we choose the ARIMA models which give the minimum AIC values.

2.12 The Coherent Mortality Forecasting

Glei and Horiuchi (2007) studied the gender differential in life expectancy across developed countries. They point out that this longevity gap exhibits some stationarity in the long-run. As shown in *Figure 1*, the historical male and female life expectancy at birth (e_0) in Australia is almost always reverting around its mean during the past decades, and this regular pattern suggests that we should apply non-divergent long-run forecasting methods in order to incorporate this regularity into our analysis.

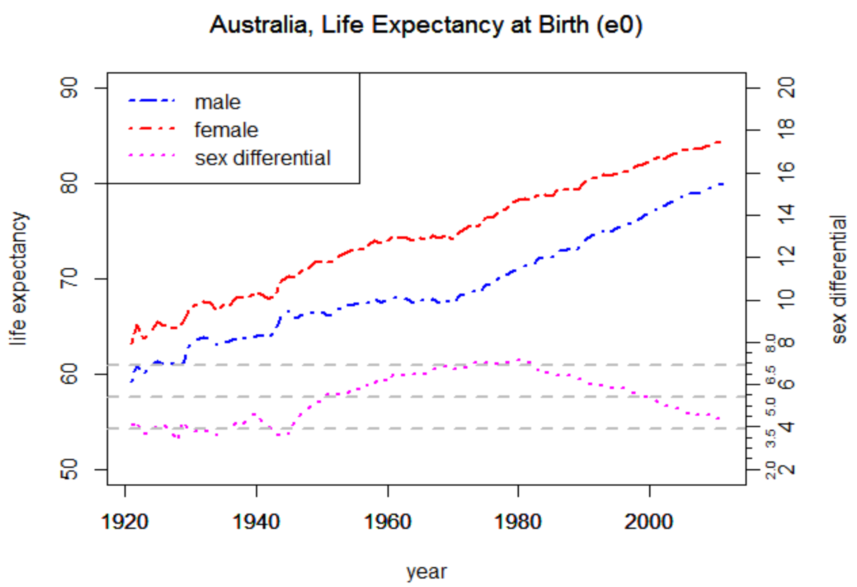

Figure 1: Male and female life expectancy at birth (blue and red) and the sex differential in life expectancy (violet) in Australia from 1921 to 2011. (Data obtained from HMD)

The product-ratio method (Hyndman et al, 2013) addresses this divergence problem in gender-specific mortality forecasts. This longevity gap is preserved in long-run forecasts by constraining the forecast ratio function through the use of stationary time series models. In this paper, the non-divergent (coherent) forecasting is based on the Hyndman-Ullah method in conjunction with the product series and ratio series generated from the smoothed mortality data:

$$p_t(x) = \sqrt{s_{t,M}(x) s_{t,F}(x)} \text{ and } r_t(x) = \sqrt{s_{t,M}(x)/s_{t,F}(x)},$$

where $s_{t,M}(x)$ and $s_{t,F}(x)$ are the smooth functions for male and female mortality rates. The Hyndman and Ullah method is then applied to the log $p_t(x)$ and the log $r_t(x)$

$$\ln[p_t(x)] = \mu_P(x) + \sum_{k=1}^{K} \alpha_{t,k} \Psi_k(x) + \varepsilon_t(x);$$

$$\ln[r_t(x)] = \mu_r(x) + \sum_{k=1}^{K} \gamma_{t,k} \Gamma_k(x) + z_t(x),$$

where $\mu_p(x)$ and $\mu_r(x)$ are the locations of the corresponding smooth functions, $\Psi_k(x)$ and $\Gamma_k(x)$ are the corresponding basis functions, and $\varepsilon_t(x)$ and $z_t(x)$ are the model errors.

In terms of forecasting, the univariate time series models are fitted to $\{\alpha_{t,k}\}$ and $\{\gamma_{t,k}\}$. Let $\hat{p}_{T+h|T}(x)$ and $\hat{r}_{T+h|T}(x)$ denote the h-step ahead forecasts for $p_t(x)$ and $r_t(x)$, the mortality forecasts can be obtained using back-transformations

$$m_{n+h|n,M}(x) = p_{n+h|n}(x) r_{n+h|n}(x),$$

$$m_{n+h|n,F}(x) = p_{n+h|n}(x) / r_{n+h|n}(x),$$

where $p_{n+h|n}(x)$ and $r_{n+h|n}(x)$ are the h-step ahead forecasts of the product and ratio functions and $m_{n+h|n,M}(x)$ and $m_{n+h|n,F}(x)$ are the male and female mortality forecasts respectively. This approach delivers coherent gender-specific mortality forecasts, and the corresponding forecasts of life expectancy at 65 are extrapolated using a standard life-table approach. The prediction interval of life expectancy is explored using the Monte Carlo simulation method, and the prediction intervals are used to calculate the upper/lower boundaries of the optimal contribution rates.

3. The Superannuation Model

For the Australian superannuation system to be fiscally sustainable, there must be a balance between the superannuation savings and retirement income. We measure the deficiency in building a corresponding retirement income by "superannuation saving gap", which is defined as the difference between the accumulated superannuation savings and the value of a lifetime annuity providing certain amount of income throughout the retirement. Thus, the presented model includes two phases: accumulation phase and pension phase. In the accumulation phase, the superannuation balance increases with positive investment returns and new contributions from the employer. In the pension phase, the accumulated superannuation saving is assumed to be converted into a lifetime annuity at retirement. Hence, the deficiency/surplus can be expressed as:

$$S_T = F_T - A_T ,$$

where:

S_T: Deficiency/surplus in the superannuation for people retiring in time T;

A_T: Present value of an adequate lifetime annuity for people retiring in time T;

F_T: Superannuation saving for people retiring in time T.

The overall balance of superannuation must fall into one of the three scenarios: shortfall, surplus and breakeven. We discuss the implications of the resulting superannuation savings according to these scenarios:

(1) When $A_T > F_T$, there is a shortfall in superannuation saving, and the deficiency amount can be calculated by

$$A_T - F_T .$$

In this scenario, people are likely to outlive their superannuation savings and eventually, forced to rely on government funded state pension. To fill the superannuation gap, government would need to increase spending on state pension significantly given the observed adverse changes in population structure. In this paper, we consider two optional government policy changes to address the systematic deficiency in superannuation: increasing the level of compulsory contribution and/or raising the retirement age.

(2) When $A_T < F_T$, there is a surplus in superannuation saving, and the surplus amount can be expressed as

$$F_T - A_T .$$

Superannuation is sufficient to provide adequate retirement incomes and on average funds will outlive the individuals. Typically, high income individuals will fall into this category sine contributions are proportional to the income lvele pre retirement and we have fixed adequacy in terms of the basic state pension level for all individuals. We do not discuss this scenario further in this paper.

(3) When $A_T = F_T$, there is a breakeven, and the superannuation scheme is balanced. In this scenario, incorporating the longevity forecasting interval into our superannuation model, we investigate the corresponding prediction interval of the compulsory contribution rates.

The accumulated superannuation savings and the value of a corresponding lifetime annuity are calculated separately for people who will retire on time horizons between 2014 - 2059. The models for these two components are presented in the following section.

3.1 The Accumulation phase

In the accumulation phase, the employer's contributions are assumed to be paid yearly in advance at 1 July. For people who start working in year t and retire in $T = t + j$, where $j = \{0,1,2,3, \dots, J\}$, we denote F_t to be the initial level of superannuation saving and F_T to be the final superannuation saving. The level of superannuation saving depends on the contribution rate, investment performance, income level and the duration of employment:

$$F_t = \left[F_{t-1} + (C_{t-1} \times I_{t-1})\right] \times (1 + R_{t-1}) ,$$

where:

F_t: Superannuation balance in time t,

C_t : Default contribution rate in time t,

I_t : Annual gross income in time t,

R_t : Investment return in time t.

3.2 The Pension phase

To calculate the present value of a lifetime retirement income, equivalently, we calculate the value of a lifetime annuity which pays predetermined cash flow to the beneficiary until death. Specifically, we assume the annuity is paid once per annum in advance, and the first payment is made immediately at the time of retirement (annuity-due) in this paper. Thus, the value of this annuity is obtained by discounting the series of payments back to the time of first payment. The following expression gives the value of annuity $\tilde{A}$

$$\tilde{A} = Pension \times \ddot{a}_{65},$$

where $\ddot{a}_{65}$ is the present value of annuity-due of \$1 p.a. and the present value of an annuity-due is calculated using the standard annuity-due formula given by

$$\ddot{a}_{65} = \sum_t v^t \, {}_t p_{65} \ ,$$

where v is the discount rate[8] and ${}_t p_{65}$ is the probability of the individual still being alive at age $65+t$ in order to receive the next payment. The value of annuity depends on the amount of pension, the discount rate and life expectancy.

The adequacy of retirement income from pension systems is generally evaluated by the replacement ratio (RR). This ratio is given by the amount of fund sustaining a certain level of retirement life divided by the final income (the income at the last year of staying in workplace). In Australia, the government adopted a uniform superannuation scheme for all income groups, and hence the superannuation system always delivers substantially different retirement lump sums for individuals within different income groups. However, the government funded state pension is designed to provide a social safety net such that everyone is promised a minimum level of income at retirement. In this paper we define the adequacy of retirement income by the surplus/shortfall in delivering minimum level of retirement income for all income groups, where that minimum is the state funded Age Pension.

[8] The interest rate is used to discount the cash flows, and we assume the benchmark discount rate equals to the superfund investment return which in turn reflects an international asset mix.

4. Calibrating the model

4.1 Data

In order to forecast future life expectancy we need to fit a suitable model to mortality data in order to produce plausible forecasts. For the purpose of this paper we have used Australian mortality and population data obtained from the Human Mortality Database (HMD) [9]. Specifically, we use the following data: the age-gender-specific central death rates and the start-year (1 January) and mid-year (30 June) populations for age group 0-99 and 100^{+}. The following notation are used:

$D_t(x)$: Deaths in calendar year t of persons of age x,

$P_t(x)$: Population of age x at 1 January of year t,

$E_t(x)$: Population of age x exposed to risk at 30 June of year t,

where $x=0,\ldots,100^{+}$ and $t=1950,\ldots2010$. Here, 100^{+} denotes the open-ended age group for people aged 100 and above. Using the sub-scripts M and F to represent males and females respectively, the age-gender-specific central death rates in calendar year t are estimated from the data as

$$m_{t,M}\left(x\right)=D_{t,M}\left(x\right)/E_{t,M}\left(x\right)$$

$$m_{t,F}\left(x\right)=D_{t,F}\left(x\right)/E_{t,F}\left(x\right).$$

Although mortality data for Australia is available since 1921, only the data from 1950 to 2011 are used in this study. There are two reasons for this; Firstly, there are outliers such as wars and epidemics for the period prior to 1950, and this data may be less reliable when forecasting future rates, and secondly, existing literature in the mortality modelling space tends to rely on data from the mid 1900s onwards, see for example O'Hare and Li (2012) amongst many publications.

To accumulate our superannuation contributions we need to consider an investment vehicle. In order to incorporate all the information about the underlying superannuation portfolio (including asset allocation, international investing, tax issues and etc.) in our analysis, we make

[9] Data consisting of central mortality rates, numbers of deaths and numbers of exposure both by age and by year can be obtained from the website http://www.mortality.org

use of Australian superfunds' investment return series obtained from the SuperRatings database. In particular this will reflect the international mix of assets held by large superannuation funds in Australia. To check the robustness of our analysis, we also consider the investment returns from a simple investment portfolio and present these results in Appendix A. This data is adjusted for inflation and then annualized before use. We denote R_t as the annualized investment return of Australian superfund in year t.

In order to consider the sufficiency of superannuation savings for different income groups, Australian gross incomes for selected percentiles have been retrieved from the Australian Beurau of Statistics, ABS. To remove the effect of inflation, we have transformed this data into 2014 real dollar value by adjusting for the inflation rates. There are two advantages to using income quintile data; firstly, the use of gross income is consistent with the fact that superannuation contributions are calculated from gross incomes, secondly, the use of real dollar valued income makes the superannuation saving shortfall/surplus in the following sections are directly comparable across years. We define I_t Annual average income for Australia in year t and real 2010 Australian dollar in our model.

The default superannuation contribution rates, namely Super Guarantee (SG) charge percentages, are attained from the Australian Taxation Office (ATO). Under the government approved superannuation scheme, the proposed contribution rate path will stay unchanged from 2025 onwards. We denote C_t: the default contribution rate in July of year t.

4.2 The Optimal Superannuation Contribution Rates

For individuals, there are two options for removing any deficiency including increasing the level of superannuation contribution and raising retirement age. In this section, we introduce a simple algorithm to calculate the optimal contribution rates which minimizes the deficiency in superannuation savings. The optimal retirement age is set to the current retirement age in Australia of 65.

In suggesting adjustments to the existing contribution rates we are only considering in this paper chnages to contributions going forward. We also impose some practicalities. Firstly, it is impractical to adjust contribution rates in the past time period, our focus in this paper is the younger generations (2015 - 2059). Therefore, we will only propose contribution rates for the new working generation of 2015. Secondly, we assume that contribution rates should not be increased by more than 0.5% in any one time period. Finally, the prediction interval of the contribution rates is treated as a formula of the corresponding prediction interval of life expectancy.

Now, let $CR = \{CR_{t+1}, CR_{t+2}, \ldots, CR_{t+H}\}$ denote the h-step ahead contribution rates over the prediction horizon and let $S_h\left(CR_{t+1}, \ldots, CR_{t+h}\right)$ be the underlying saving gap for h = 1 ,…, 45. The corresponding superannuation saving can be expressed as

$$S_h\left(CR_{t+1}, \ldots, CR_{t+h}\right) = F_h\left(CR_{t+1}, \ldots, CR_{t+h}\right) - A_h,$$

and the optimal contribution rate $CR^{optimal}$ is obtain by minimizing $CR_{t+h} = \{CR_{t+1}, CR_{t+2}, \ldots, CR_{t+h}\}$ such that $S_h\left(CR_{t+1}, \ldots, CR_{t+h}\right) \geq 0$ subject to the constraint $0 \leq CR_{t+h} - CR_{t+h-1} \leq 0.005$.

The optimal contribution rates can be calculated by increasing the pervious contribution rates $CR = \{CR_{t+1}, \ldots CR_{t+h}\}$ until the expected deficiency in 2059 is removed or the algorithm approaches the end of the prediction horizon. Specifically, let S_H denote the superannuation saving gap at retirement, the contribution rates $CR = \{CR_{t+1}, CR_{t+2}, \ldots, CR_{t+H}\}$ are increased by 0.5% annually and S_H will also be re-evaluated for each increase until

$$S_H \geq 0 \text{ or } CR_{t+h} - CR_{t+h-1} > 0.005.$$

If $CR_{t+h} - CR_{t+h-1} > 0.005$ but the deficiency persists $S_H < 0$, this algorithm will move onto the next set of contribution rates. This process is repeated for $\{CR_{t+2}, CR_{t+3}, \ldots, CR_{t+H}\}$, $\{CR_{t+3}, CR_{t+4}, \ldots, CR_{t+H}\}$, …, until $S_H \geq 0$ or $\boldsymbol{CR = \{\varnothing\}}$.

The randomness in the optimal contribution rate path is driven by the stochastic life expectancy forecasts. Therefore, the contribution rate path calculated using this algorithm which is equivalent to a monotonic transformation of the saving deficiency is also random. Thus, the upper/lower boundaries of the optimal contribution rate path can be calculated by replacing the mean of life expectancy forecasts with its percentile values. This approach of calculating the optimal contribution rate provides probabilistic projection intervals and hence naturally incorporates the longevity risks. For this reason, the accuracy of longevity forecast plays a critical role in calculating the optimal path and the corresponding prediction intervals.

5. Empirical Results

5.1 Australia Mortality Data

Figure 2 shows that mortality in Australia has improved substantially during the past 6 decades. The log male mortality rates and the log female mortality rates exhibit similar shapes except that the male have significantly higher mortality rates at 20s. In addition, both plots indicate more variation within young age groups than that within the old age groups. Overall, females on average have lower mortality rates and hence higher life expectancy than males.

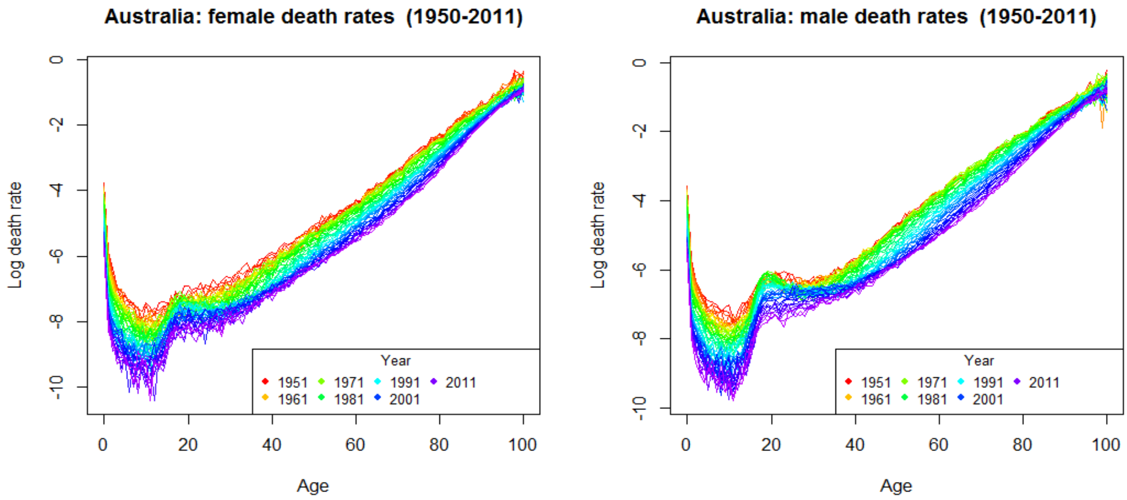

Figure 2: On the left is the female log central death rate (mortality rate) plot, and on the right is the male log central death rate. Both of them uses Australian data form 1950 to 2011.

5.2 Out-of-sample forecasting performance comparison

The Lee-Carter (LC) model is widely recognized in mortality modelling literature and often considered as a benchmark method. The strengths of this method are its simplicity and robustness in situations where age-specific log mortality rates have linear trends (Booth et al. 2006). However, only the first principal component and the corresponding score are used to capture the patterns and trends of mortality rates. The Hyndman-Ullah model, by contrast, is an extension of the LC model which utilizes the higher order principal components to capture additional dimension of variability. In this section, using Australian mortality data, we evaluate the forecast accuracy the Lee-Carter model and the Hyndman-Ullah model.

The sample period is split into the training and test sub-groups, we then fit the model using the training set and evaluate the forecast accuracy of fitted model by the test set. We consider 15, 20 and 25 year forecasts and we report the Root Mean Square Forecast Error (RMSFE)

$$RMSFE_T = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left[e_{65,t} - \hat{e}_{65,t}\right]^2}$$

and Mean Absolute Forecast Error (MAE)

$$MAFE_T = \frac{1}{T}\sum_{t=1}^{T}\left|e_{65,t} - \hat{e}_{65,t}\right|$$

in each case in *Table 2*. We plot of 20 years ahead life expectancy forecast against the actual data in *Figure 3*. Both models tend to underestimate the speed of longevity improvements, especially for males. The accurate forecast produced by Hyndman-Ullah model in terms of smaller RMSFE and MAFE is a consequenceof the coherent forecasting of sub-population groups strucutre of the model which addresses any divergence problems in the long-term. Overall, this leads to lower average RMSFE and MAFE for all testing sets. The Hyndman-Ullah model is likely to be more accuarte for Australian data than the Lee-Carter model, and hence the Hyndman-Ullah model is used to calculate the superannuation gap in this paper.

RMSFE		15 years	20 years	25 years
Female	FDM	0.14	0.09	0.10
	LCM	0.14	0.08	0.09
Male	FDM	0.29	0.16	0.17
	LCM	0.36	0.30	0.33
Average	FDM	0.21	0.12	0.14
	LCM	0.25	0.19	0.21
MAE		**15 years**	**20 years**	**25 years**
Female	FDM	0.52	0.34	0.42
	LCM	0.52	0.30	0.35
Male	FDM	1.05	0.59	0.69
	LCM	1.29	1.12	1.36
Average	FDM	0.79	0.47	0.55
	LCM	0.91	0.71	0.86

Table 2: The Root Mean Square Forecast Error (RMSFE) and Mean Absolute Forecast Error (MAE) of the Lee-Carter model (LCM) and the Hyndman-Ullah model (FDM).

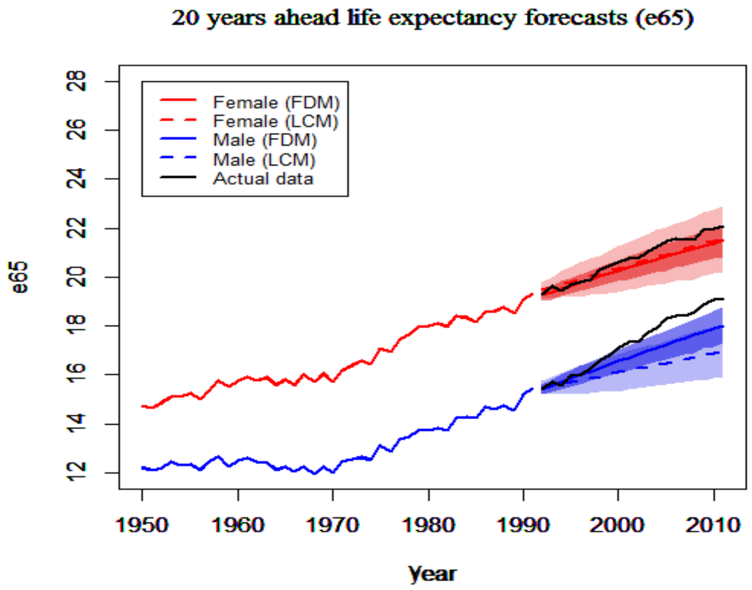

Figure 3: 20 year ahead life expectancy forecasts (out of sample) by the Lee-Carter model (LCM) and the Hyndman-Ullah model (FDM) with comparing to the actual data.

5.3 Mortality and life expectancy forecasts

The product-ratio method involves the use of log product series and log ratio series in the Hyndman-Ullah model. *Figure 4* provides plots of the product function representing the geometric mean of male and female mortality rates. The mean of the log product series captures the general patterns of both male and female log mortality rates. In the product function, the first basis function represents the primary source of variability across all ages, and it has more weighting towards young age groups than older age groups. This is in line with the log mortality data plot where we noted that younger age groups contribute more variation to the mortality data than old age groups. Furthermore, 96.6% of the variability of the product series is absorbed on its own. The coefficients attached to the first basis function have decreased steadily over time, which implies that the historical mortality rates are almost always decreasing. The second basis function positively weights the age group of 20s and negatively weights the groups between age of 0-20 and 30-40. This form of principal component indicates that once we have accounted for overall variability, the next most important source of variability is between the mortality rates for age group of 20s and age groups between ages of 0-20 and 30-40. The second basis function absorbs an extra 1.3% of the variability of the product series.

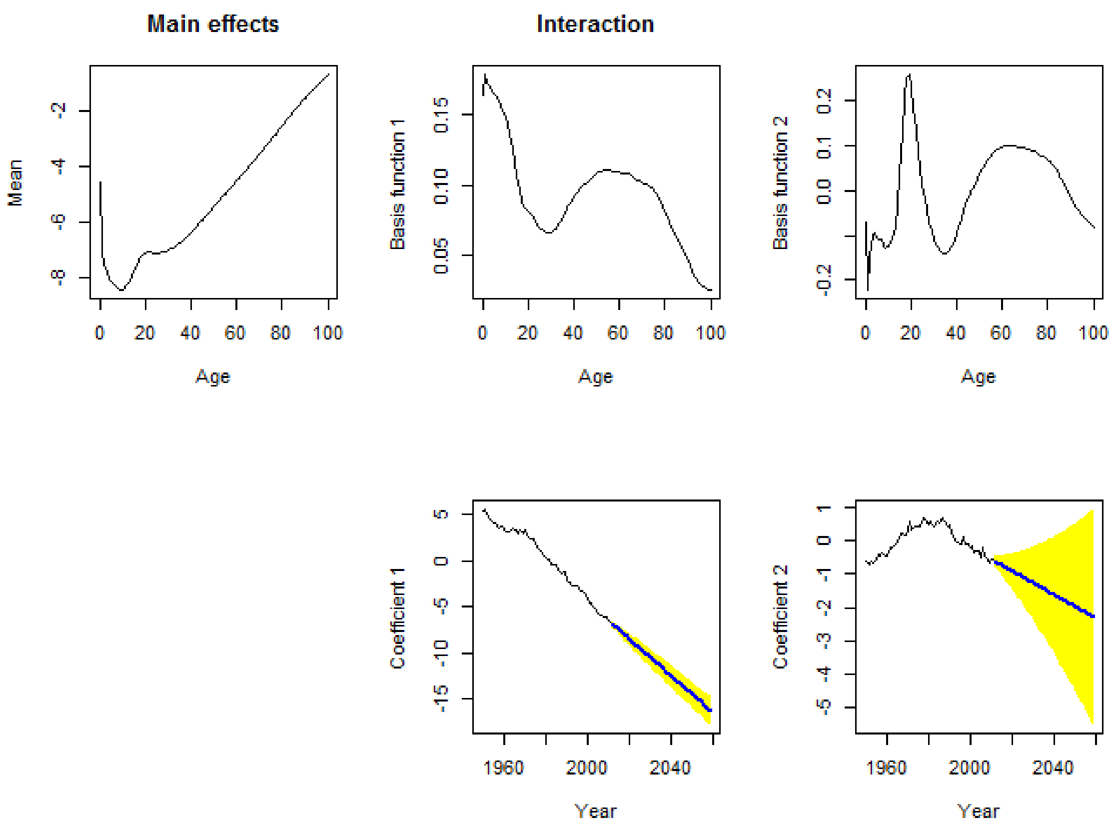

Figure 4: Plots of the product model. The first row is the mean and first two basis functions respectively, and the second row is the corresponding coefficients with forecasted values and 80% prediction intervals in yellow shading from 2011 to 2059.

In *Figure 5* we plots the ratio function**.** Hyndman, Booth & Yasmeen (2013) suggest that the ratio function is simply the square root of the gender ratio (male to female). The mean, first basis function and the second basis function have very similar patterns, except the variability of the plots have increased. This trend implies that the principal components of the ratio series have the same source of variability, which is from the variation of gender specific mortality rates across age groups. The first basis function accounts for 49.3% of the variability and the second basis function absorbs a further 18.9% of the total variability.

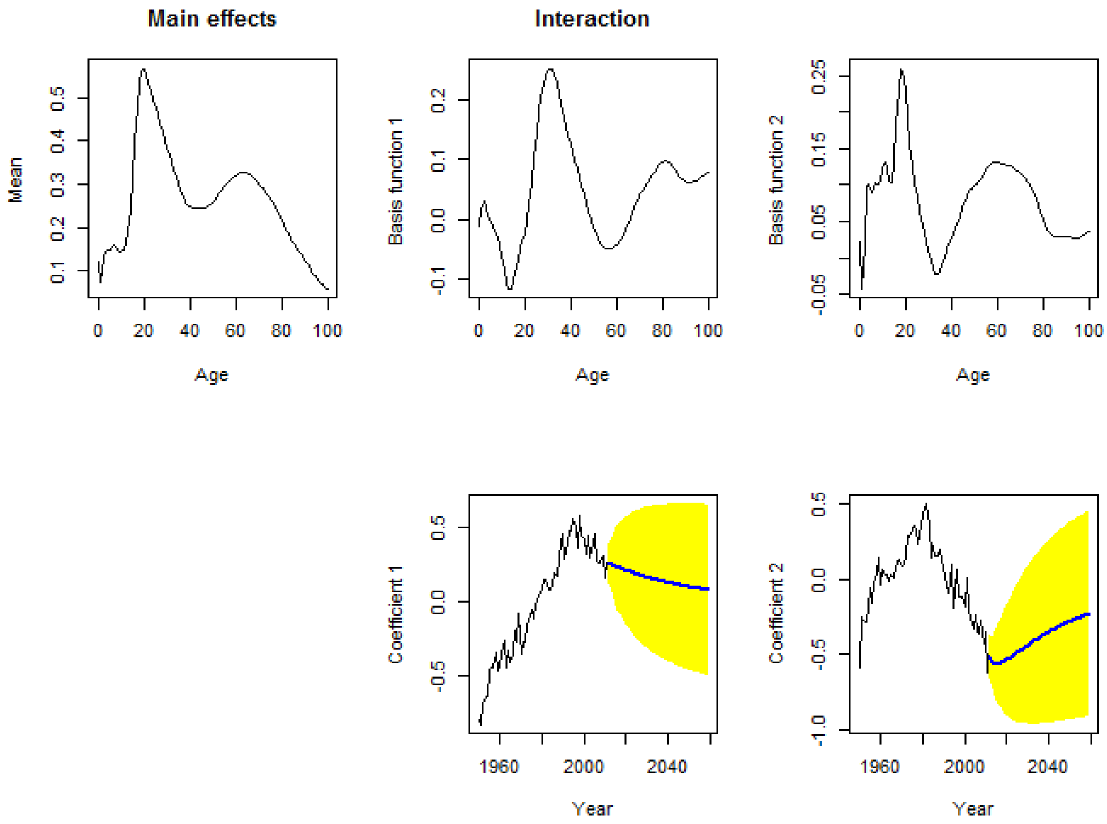

Figure 5: Plots of the ratio model. The first row is the mean and first two basis functions respectively, and the second row is the corresponding coefficients with forecasted values and 80% prediction intervals in yellow shading from 2011 to 2059.

We extrapolate the life expectancy forecasts using the standard life table approach, exploring uncertainties using Monte Carlo simulation methods. *Figure 6* shows the gender-specific life expectancy at 65 from 1950 to 2059. Note that the regularity of gender differentials in life expectancy is maintained over the prediction horizon and the male life expectancy forecasts and female life expectancy forecasts are non-divergent. The estimated life expectation at 65 is 22 for female and 19 for male in 2010, and these are expected to be increase to 27 for female and 24 for male before 2059. The improving longevity is an unavoidable result of technological advances in medical care, and this trend reveals that ignoring the dynamic of longevity will lead to overestimating the sustainability of superannuation system.

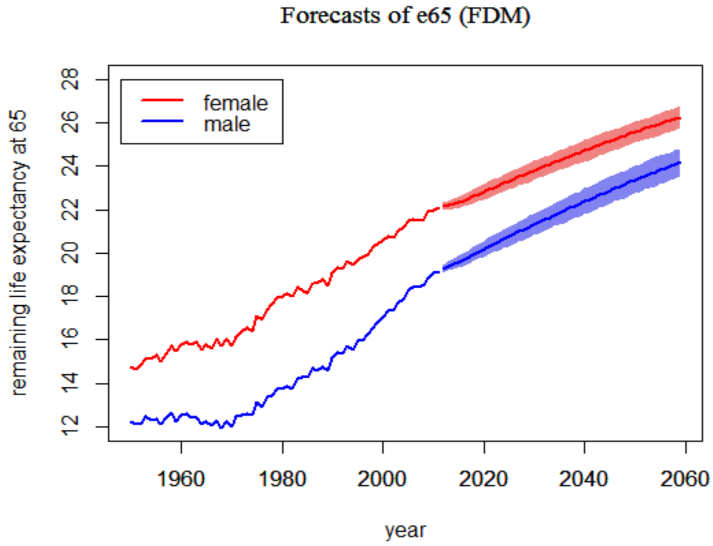

Figure 6: Sex-specific life expectancy at age 65 for Australia. Forecasts are from 1950 to 2011, and forecasted values are from 2011 to 2059 with 80% prediction intervals shown as shading.

For comparison purposes, the life expectancy forecasts produced by Hyndman-Ullah model (FDM) and Lee-Carter Model (LCM) are ploted in the same graph in *Figure 7*. There are two remarkable distinctions between these forecasts by two methods; Firstly, FDM produces non-diverge long-term forecasts for male and female, in contrast, the forecasts by LCM is diverge in the long-term. It is seen that in the Australian data, the non-diverge long-term life expectancy forecasting for male and female produces less forecasting error than the normal one. In detail, the mean of forecasts by FDM is closer to those by LCM for female than for male. Secondly, the prediction interval for the FDM is narrower and hence more informative when using these forecasts to predict the optimal contribution rate path. The basis functions and their scores for LCM are reported in Appendix B.

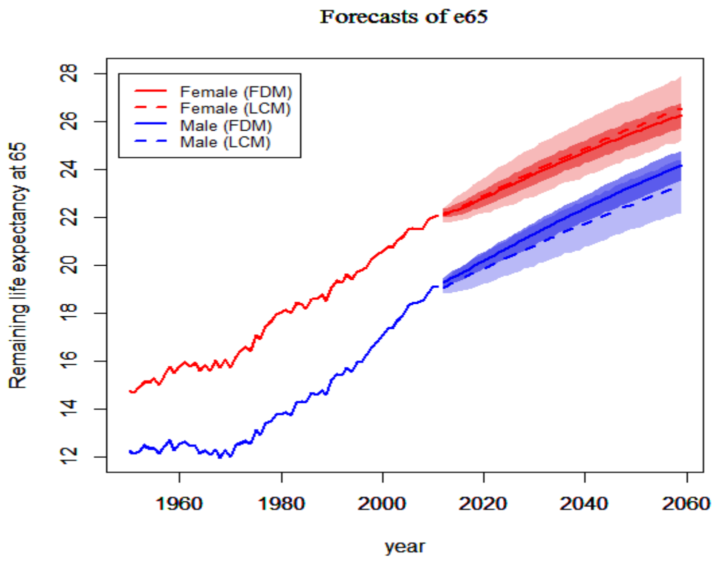

Figure 7: Sex-specific life expectancy at age 65 for Australia. The 80% prediction intervals for the Hyndman-Ullah model (FDM) is in darker colour and the prediction intervals for the Lee-Carter model (LCM) is in lighter colour.

5.4 Investigating the Superannuation Gaps

In this section, we assess the consequences of superannuation gap for selected income percentile groups and evaluate the impact of government co-contribution on the low-income groups. In order to calculate the superannuation gap for the projection period of 2014-2059, the real incomes (in 2014 dollar value) are forecasted using Autoregressive integrated moving average (ARIMA) models and the investment returns are forecasted using Autoregressive fractional integrated moving average (ARFIMA) model for the same projection period, where the models are selected using the Akaike Information Criterion (AIC). Further, we assume that the pension payments are equal at the time of retirement to the adequate level of retirement income and the annuity amount is given by pension multiplied by the appropriate life annuity. In addition, the government co-contribution is available for individuals who earn less than $35,454 and make voluntary after-tax contributions to a superfund in 2014. Although some low-income individuals are facing the difficulty of making voluntary after-tax contributions in reality, in this paper we assume that people who are eligible for government co-contribution will make their voluntary after-tax contributions in order to boost their superannuation savings.

We plot the shortfall or surplus in superannuation savings on the vertical axis, and on the horizontal axis we plot the time horizon. The method for superannuation drawdown varies from individual to individual due to the personal savings, incomes and etc. In this paper, we consider a simple method of calculating the present value of annuity i.e. Pension $\ddot{a}_t$ where the payments are level over time. We then compare the accumulated superannuation savings with the present value of a life annuity at the time of retirement.

Figure 8 illustrates the superannuation gap for the 10 and 20 income percentiles in real 2014-dollar values. For individuals within these two income groups, the government will make superannuation co-contribution which has a match-up rate of 50% with a maximum amount of \$500 if these individuals make voluntary after-tax contributions of \$1,000 to a superfund. This co-contribution mechanism has been incorporated into our analysis. For the 10 and 20 income percentile groups, the superannuation gap is stable during the entire projection period with females requiring more superannuation in gerneral due to their longer life expectancy. The deficiencies in superannuation saving decrease almost linearly from 2015-2036 since for people retiring during that period, the years of participating in superannuation system are increasing over time. Everything else being equal, individuals who participated in SG longer will always end up with more superannuation saving at retirement. After 2036, when maximum participation is reached, the superanuation gap levels off and there is a deficiency of around \$18,000 for female and \$14,000 for male in the 10 income percentile group and \$16,000 for female and \$12,000 for male in the 20 income percentile group even if these individuals have fully participated SG and enjoyed government co-contributions. For a comparsion purpose, *Figure 9* plots the superannuation gap for 10 and 20 income percentiles without government co-contributions. As expected, the superannuation gaps become even wider and there is a deficiency of around \$29,000 for female and \$25,000 for male in the 10 income percentile group and \$23,000 for female and \$28,000 for male in the 20 income percentile group.

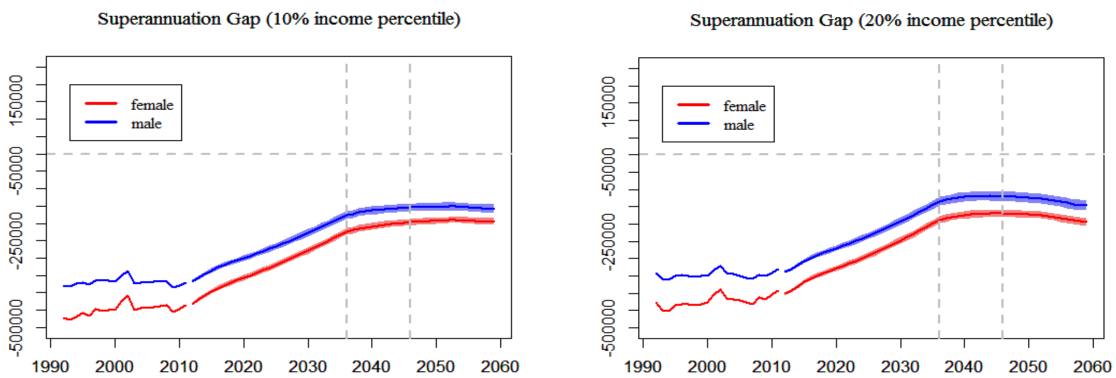

Figure 8: Superannuation Gap for 10 income percentile (left) and 20 income percentile (right), both with government co-contribution full amount.

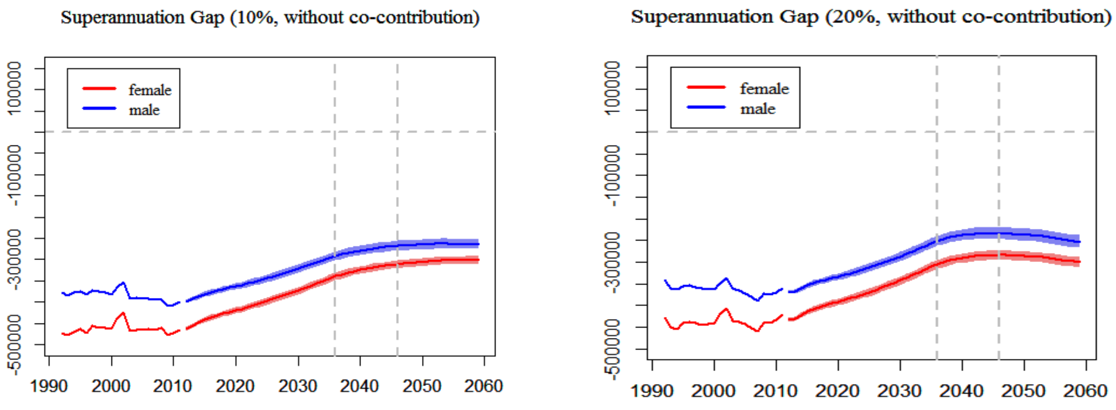

Figure 9: Superannuation Gap for 10% income percentile (left) and 20% income percentile (right), both without government co-contribution.

Figure 10 and Figure 11 show the superannuation gap for 30%, 50%, 70% and 90% income percentiles respectively. Individuals within these income groups are not eligible for the governement co-contribution full amount and hence the superannuation savings are purely sourced from the complusary contributions and investment returns. With the government approved superannuation scheme, individuals within the 30 income percentile group face a shortfall in superannuation saving and have to access the Age Pension benefits as a supplement. For the 50 income percentile group, the superannuation gap for male is deminished after 2045 when the first generation who fully participated in SG with a contribution rate at least 9% retire. Although there is a deficiency for female, this gap can be easily filled with Age Pension benefits or voluntary savings as income increases.

For the 70 and 90 income percentile individuals, the current superannuation system will deliver huge amount of surplus on the top of maintaining a basic lifestyle, especially for the 90 income percentile groups. This indicates that a uniform superannuation scheme is probably not suitable for a certain income percentile groups.

At one end of the income distribution, under the government approved superannuation scheme, the low income groups will face superannuation deficiencies in retirement and are forced to choose Age Pension as their main retirement income given that the Age Pension is still generous in Australia. At the other end of the income distribution, the high income groups are expected to produce huge amount of superannuation surpluses, which are locked up until their retirement. However, these high income individuals are least likely to access Age Pension at retirement. Hence, the current superannuation scheme faces a dilemma of effectively delivering retirement incomes to different income percentile groups.

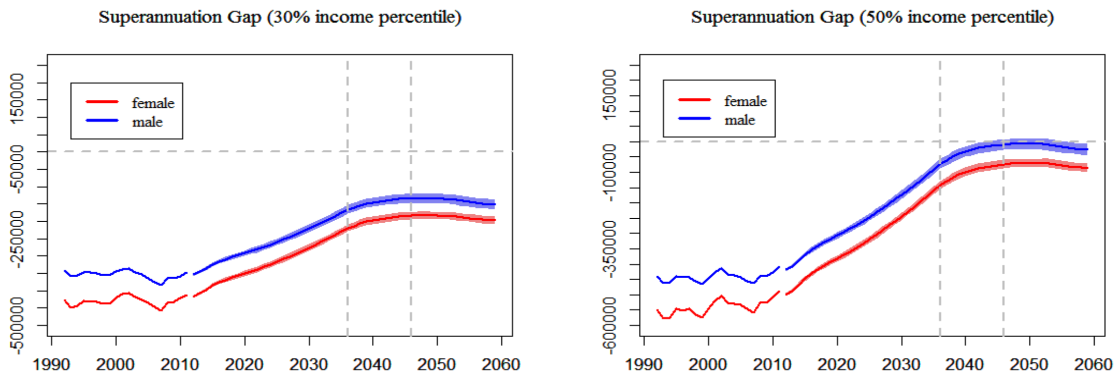

Figure 10: Superannuation Gap for 30% income percentile (left) and 50% income percentile (right).

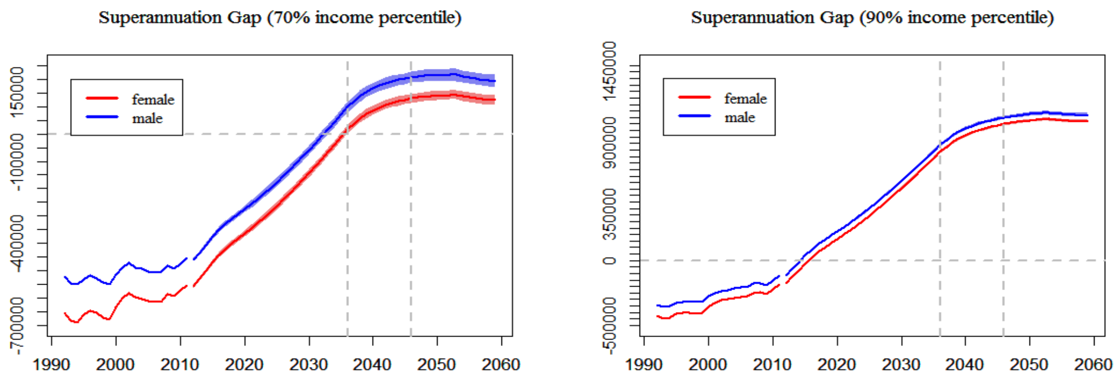

Figure 11: Superannuation Gap for 70% income percentile (left) and 90% income percentile (right).

6. The Optimal Contribution Rates

Given the demographic changes occurring in Australia, superannuation savings, instead of Age Pension benefits, are expected to become the main source of retirement incomes for most Australians. Hence, the new generation of workers will be expected to rely more on self-funded retirement income. In order the address the policy dilemma faced by the government, we propose a combination of two plausible superannuation reform policies. On one hand, for the new entrants to the workplace, we propose an optimal superannuation scheme, which can boost superannuation saving level and hence diminish the superannuation gaps for low-income percentile groups. On the other hand, we suggest the adoption of higher government co-contribution match-up rates for these low-income groups.

The current government co-contribution match-up rate is 50% with a maximum amount of $500, and to be eligible for the co-contribution, low income individuals (within lowest two income percentile groups) need to make voluntary after-tax contribution. Since the lower income groups generally end up with more superannuation saving shortfalls and it is hard to expect these individuals to make more voluntary contributions, increasing government co-contribution match-up rates is a more sensible approach.

In the following sections, we analyse the low-income groups separately due to the distinct longevity characteristics for male and female and different superannuation saving capability.

6.1 *10%* Income Percentile Group

Since the contribution rates are restricted to increasing by 0.5% each year and need to be kept low, increasing the co-contribution match-up rate provides a more sensible way to boost the superannuation saving for low-income groups. Our analysis suggests that the government need to increase the co-contribution match-up rate from 0.5 to 2.5 with a maximum co-contribution amount of $2,500 in order to remove the superannuation gap for 0%-10% income percentile group whilst keeping their contribution rates less than 20%. *Tables 3* and *4* report the forecasted optimal contribution rates for female and male respectively.

Mean and 80% boundary of target contribution rates (%) for female (2.5 government match-up rate)															
Year	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029
Upper	9.5	9.5	9.5	9.5	9.5	9.5	10	10.5	11	11.5	12	12.5	13	13.5	14
Mean	9.5	9.5	9.5	9.5	9.5	9.5	10	10.5	11	11.5	12	12.5	13	13.5	14
Lower	9.5	9.5	9.5	9.5	9.5	9.5	10	10.5	11	11.5	12	12.5	13	13.5	14
Year	2030	2031	2032	2033	2034	2035	2036	2037	2038	2039	2040	2041	2042	2043	2044
Upper	14.5	15	15.5	16	16.5	17	17.5	18	18.5	19	19.5	20	20.5	21	21.5
Mean	14.5	15	15.5	16	16.5	17	17.5	18	18.5	19	19.5	20	20.5	20.5	20.5
Lower	14.5	15	15.5	16	16.5	17	17.5	18	18.5	18.5	18.5	18.5	18.5	18.5	18.5
Year	2045	2046	2047	2048	2049	2050	2051	2052	2053	2054	2055	2056	2057	2058	2059
Upper	22	22.5	23	23.5	23.5	23.5	23.5	23.5	23.5	23.5	23.5	23.5	23.5	23.5	23.5
Mean	20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5
Lower	18.5	18.5	18.5	18.5	18.5	18.5	18.5	18.5	18.5	18.5	18.5	18.5	18.5	18.5	18.5

Table 3: Mean and 80% boundary of optimal contribution rates for female with 2.5 government co-contribution match-up rate (10% income percentile group).

Mean and 80% boundary of target contribution rates (%) for male (2.5 government match-up rate)															
Year	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029
Upper	9.5	9.5	9.5	9.5	9.5	9.5	10	10.5	11	11.5	12	12.5	13	13.5	14
Mean	9.5	9.5	9.5	9.5	9.5	9.5	10	10.5	11	11.5	12	12.5	13	13.5	14
Lower	9.5	9.5	9.5	9.5	9.5	9.5	10	10.5	11	11.5	12	12.5	13	13	13
Year	2030	2031	2032	2033	2034	2035	2036	2037	2038	2039	2040	2041	2042	2043	2044
Upper	14.5	15	15.5	16	16	16	16	16	16	16	16	16	16	16	16
Mean	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5
Lower	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13
Year	2045	2046	2047	2048	2049	2050	2051	2052	2053	2054	2055	2056	2057	2058	2059
Upper	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
Mean	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5
Lower	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13

Table 4: Mean and 80% boundary of optimal contribution rates for male with 2.5 government co-contribution match-up rate (10% income percentile group).

In general, females require higher contribution rates than males due to their longer life expectancy. This is reasonable from a demographic perspective. In addition. It also worth pointing out that the policy suggestion made here are only one amongst a selection of reasonable proposals of a similar kind which can be calculated using the model proposed in this paper.

Figure 12 visualizes the superannuation gap for 10 income percentile group using the mean of the optimal contribution rates. With the proposed policy reform, the entrants to the workplace would no longer face systematic superannuation deficiency.

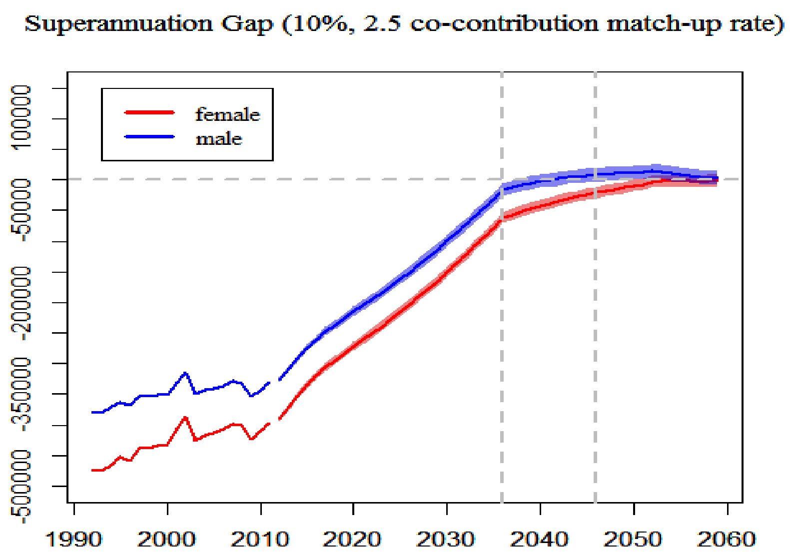

Figure 12: Superannuation Gap for 10 income percentile groups with 2.5 government co-contribution match-up rate. (Calculated using the means of optimal contribution rates)

6.2 *20%* income percentile group

The 10%-20% income percentile group has a smaller superannuation gap. We suggest that, for this group, the government need to increase the co-contribution match-up rate from 0.5 to 1.8 with a maximum co-contribution amount of $1,800 such that the contribution rates are not increased to an unaffordable high level. *Tables 5* and *6* summarize the optimal contribution rate paths to minimize the superannuation gap under 1.8 co-contribution match-up rates for female and male respectively. The contribution rates we suggested here have a similar path to that for the 0-10% income percentile group under the lower match-up rate.

Mean and 80% boundary of target contribution rates (%) for female (1.8 government match-up rate)															
Year	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029
Upper	9.5	9.5	9.5	9.5	9.5	9.5	10	10.5	11	11.5	12	12.5	13	13.5	14
Mean	9.5	9.5	9.5	9.5	9.5	9.5	10	10.5	11	11.5	12	12.5	13	13.5	14
Lower	9.5	9.5	9.5	9.5	9.5	9.5	10	10.5	11	11.5	12	12.5	13	13.5	14
Year	2030	2031	2032	2033	2034	2035	2036	2037	2038	2039	2040	2041	2042	2043	2044
Upper	14.5	15	15.5	16	16.5	17	17.5	18	18.5	19	19.5	20	20.5	21	21.5
Mean	14.5	15	15.5	16	16.5	17	17.5	18	18.5	19	19.5	20	20.5	20.5	20.5
Lower	14.5	15	15.5	16	16.5	17	17.5	18	18.5	19	19	19	19	19	19
Year	2045	2046	2047	2048	2049	2050	2051	2052	2053	2054	2055	2056	2057	2058	2059
Upper	22	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5
Mean	20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5
Lower	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19

Table 5: Mean and 80% boundary of optimal contribution rates for female with 2.5 government co-contribution match-up rate (20% income percentile group).

Mean and 80% boundary of target contribution rates (%) for male (1.8 government match-up rate)															
Year	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029
Upper	9.5	9.5	9.5	9.5	9.5	9.5	10	10.5	11	11.5	12	12.5	13	13.5	14
Mean	9.5	9.5	9.5	9.5	9.5	9.5	10	10.5	11	11.5	12	12.5	13	13.5	14
Lower	9.5	9.5	9.5	9.5	9.5	9.5	10	10.5	11	11.5	12	12.5	13	13.5	14
Year	2030	2031	2032	2033	2034	2035	2036	2037	2038	2039	2040	2041	2042	2043	2044
Upper	14.5	15	15.5	16	16.5	17	17	17	17	17	17	17	17	17	17
Mean	14.5	15	15.5	16	16	16	16	16	16	16	16	16	16	16	16
Lower	14.5	15	15	15	15	15	15	15	15	15	15	15	15	15	15
Year	2045	2046	2047	2048	2049	2050	2051	2052	2053	2054	2055	2056	2057	2058	2059
Upper	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17
Mean	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
Lower	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15

Table 6: Mean and 80% boundary of optimal contribution rates for male with 2.5 government co-contribution match-up rate (20% income percentile group).

Figure 13 shows the superannuation gap for 10%-20% income percentile group using the corresponding mean of the optimal contribution rates. The superannuation gap for male is almost diminished at 2036, when the first generation who fully participated in SG retires. The superannuation gap for both females and males levels off to zero in 2059 when the new entrants to the workplace retire under the proposed policy reforms.

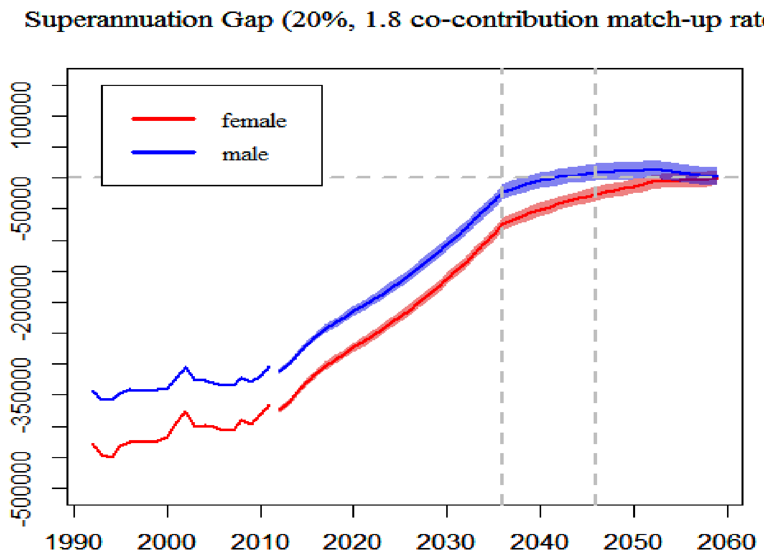

Figure 13: Superannuation Gap for 20 income percentile groups with 1.8 government co-contribution match-up rates. (Calculated use the mean of optimal contribution rate)

7. Concluding Remarks

Driven by the demographic changes, pension pressure on the government funded state pension is progressively being transferred to the superannuation system in Australia. This paper provides insights of the sufficiency of the superannuation scheme, both today and in the future, to governments and policy makers in terms of longevity risks. We explicitly incorporate the uncertainty of future longevity improvements in the superannuation model and find that when factoring in the predicted improvements in mortality rates the superannuation system become less sustainable. In the context of retirement planning, our analysis indicates that low-income individuals require substantially more government support at retirement. The resulting additional government expenditure would gradually place greater financial burden on the government during the coming decades. In this paper, we make use of stochastic mortality forecasting and demonstrate a plausible path of future contributions to the superannuation system that would remove any deficiencies in retirement savings.

Although mortality modelling techniques have been developed enormously over the past decades, to our knowledge, longevity risks are often seen to be neutral in the world of superannuation or defined contribution schemes in the literature. Most attention has been given to state pension systems and the macroeconomic implications of policy changes. In this paper we connect stochastic mortality modelling to retirement planning and this idea can also be easily extend to the problem of optimal retirement ages and the old age dependence ratio for example.

The focus of this paper has been on the sufficiency of retirement saving at an individual level. Hence, the analysis undertaken in this paper does not account for the variation of non-demographic factors including the income and substitution effects of implementing these fiscal policies etc. However, our model can be easily extended to account for these factors in conjunction with some appropriate assumptions. We leave these issues to future studies.

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Appendixes

A. Superannuation gaps calculated using weighted returns from different asset classes

Calibrating the investment return data

To check the robustness of our analysis, we need to consider other investment assets. In this section we consider a portfolio invested in 2 asset classes, stocks and bonds. Although superannuation funds in Australia will invest globally, the purpose of our paper is to develop a model of accumulation and subsequent deccumulation in retirement. We use domestic investment data in our model but of course the model could be easily calibrated to a different investment data set. The following data are used: the mid-year ASX/S&P 200 total return index during 1992-2014 and the mid-year 10-year Australian government bond yield during 1992-2014. The ASX/S&P 200 total return index is a benchmark in share indices with high market capitalization and liquidity (trading volume) in Australia, and the 10-year Australian government bond is the AAA rated fixed income benchmark. The following notations are used:

$P_{s,t}$ = ASX/S&P 200 total return index at 30 June of calendar year t;

$P_{b,t}$ = 10-year Australian government bond yield at 30 June of calendar year t.

These data are adjusted for inflation, and we denote the log-returns for the stock and bond as:

$$r_{s,t} = \ln(P_{s,t+1}) - \ln(P_{s,t}) \text{ and}$$

$$r_{b,t} = \ln(1 + P_{b,t}),$$

where $r_{s,t}$ is the log-return for ASX/S&P 200 total return index, and $r_{b,t}$ is the log-return for the 10-year Australian government bond.

The current superannuation reform requires the authorized MySuper[10] product to provide a single investment option from 2014. We use the portfolio proportion rate i.e. w to construct different portfolio mixes. For an annually rebalanced portfolio, the percentage return i.e. R_t is the weighted sum of the stock return and the bond return for the same year, i.e.

$$R_t = \left[(1-w)\times\exp(r_{s,t}) + w\times\exp(r_{b,t})\right] - 1 \ .$$

[10] MySuper is part of the Stronger Super reforms. It requires the employers must only pay default superannuation contributions to an authorized MySuper product.

Then, we forecast the portfolio return using the ARIMA models and we choose the models according to the AIC.

The superannuation gap

We illustrate three investment portfolios: growth, balanced and defensive. *Figure 14* shows the superannuation gap with investing in a growth portfolio (portfolio mixture proportion of 0.3). These superannuation gaps are similar to that using the returns for superfunds. *Figure 15* illustrates the superannuation gaps with the investment returns from a balanced portfolio. This yields the smallest superannuation gaps among all the three portfolios we considered here. Lastly, the superannuation gaps produced by investing in the defensive portfolio are as in *Figure 16*, and these gaps are the widest among the three.

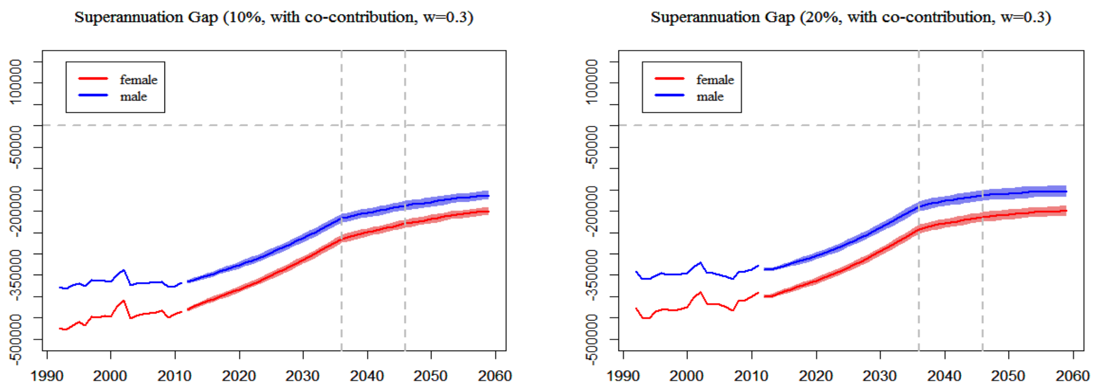

Figure 14: Superannuation Gap for 10% income percentile (left) and 20% income percentile (right) with the investment portfolio of w = 0.3.

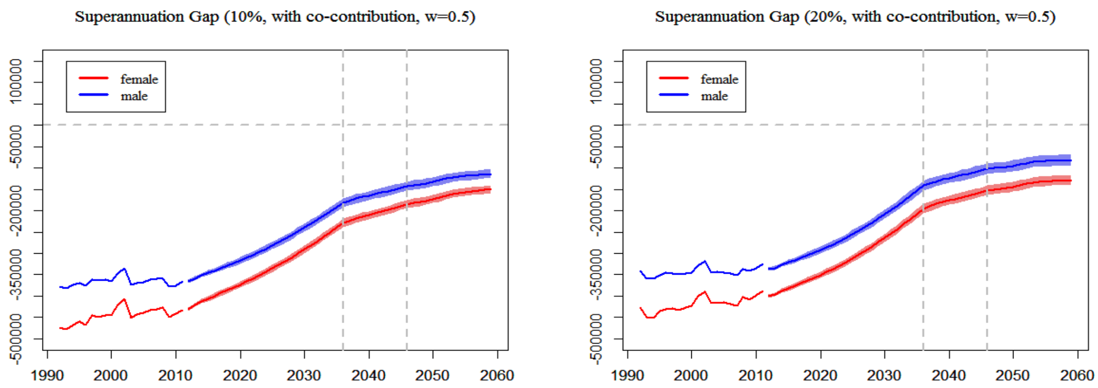

Figure 15: Superannuation Gap for 10% income percentile (left) and 20% income percentile (right) with the investment portfolio of w = 0.5.

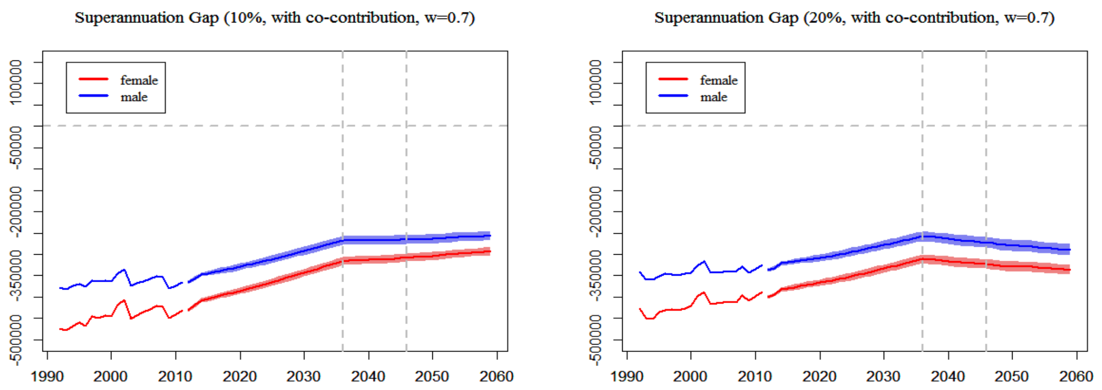

Figure 16: Superannuation Gap for 10% income percentile (left) and 20% income percentile (right) with the investment portfolio of $w = 0.7$.

To conclude, these superannuation gaps produced by investing in different portfolios are similar to the gaps in the section of "investigating the superannuation gaps".

B. Basis functions and corresponding scores for LCM

Figure 17 and *Figure 18* show the basis functions and their scores for Australian sex-specific log mortality rate forecasts from 2012 to 2059 by the Lee-Carter model. The LCM only uses the first principal component. The first basis function compares the variability in the mortality for the young age groups to that of old age groups, and the their scores decrease over time. These first principal components explains 90.4% variability for female mortality and 90.9% variability for male mortality respectively.

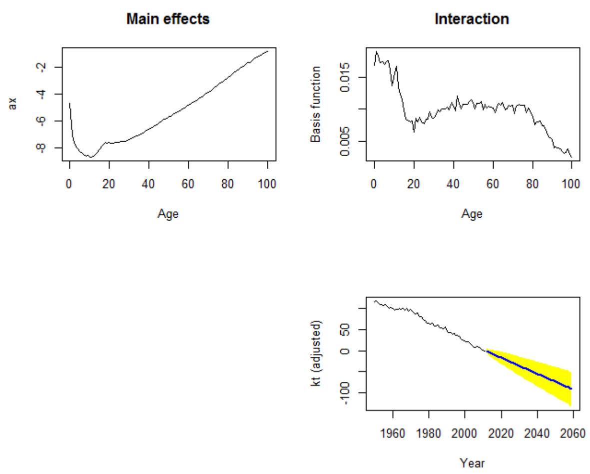

Figure 17: Basis function and its score for log female mortality rates forecasts from 2012 to 2059 by LCM.

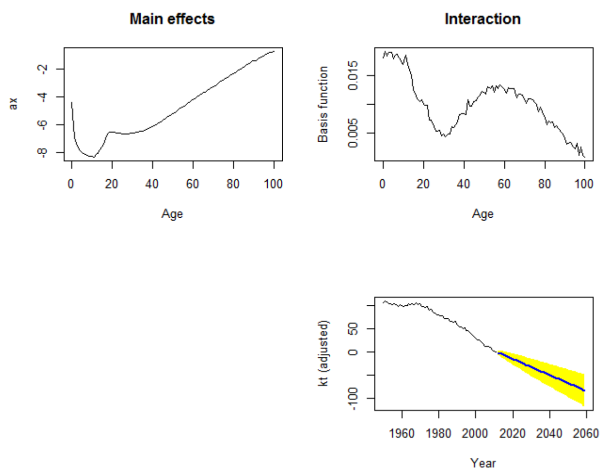

Figure 18: Basis function and its score for log male mortality rate forecasts from 2012 to 2059 by LCM.

List of Abbreviation

ABS: Australian Bureau of Statistics

AIC: Akaike Information Criterion

ARFIMA: autoregressive fractional integrated moving average

ASX: Australian Securities Exchange

ATO: Australian Taxation Office

FDA: Functional Data Analysis

fPCA: functional Principal Component Analysis

GFC: Global Financial Crisis

GDP: Gross domestic product

HMD: Human Mortality Database

OECD: The Organisation for Economic Co-operation and Development

PCA: Principal Component Analysis

PC: principal component

SG: Superannuation Guarantee

S&P: Standard & Poor’s

SVD: Singular Value Decomposition

TFR: total fertility rate (babies per woman)