Probability & Statistics 1 Tutorial Questions

Exercises:

(i) A committee of 4 people is to be chosen from 10 members. How many different committees could be formed?

(ii) John has 8 books but only has space for 3 of them on his shelf. How many arrangements could he make from the 8 books?

(iii) In a game, the contestant rolls the die and whatever number they land on is the amount of money they win. What is the expected value of the winnings?

(iv) A password consists of 1 letter and 4 digits. Calculate the number of possible codes if:

a. The letter must be at the beginning.

b. The letter must be at the beginning and no digit can be repeated.

(v) In a bag there are 5 blue marbles and 3 red marbles. When a marble is picked out, it is put back in.

a. Calculate the probability of picking 3 red marbles in 4 picks?

b. Calculate the probability of picking the 4^{th} red marble on the 8^{th} pick?

c. Calculate the probability of picking at least 1 blue marble in 3 picks?

Past Exam Questions:

Question 1:

Paper 2, 2013, Q1

Question 1 **(25 marks)**

(a) Explain each of the following terms:

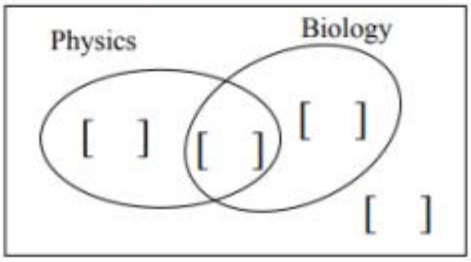
(i) Sample space

(ii) Mutually exclusive events

(iii) Independent events.

(b) In a class of 30 students, 20 study Physics, 6 study Biology and 4 study both Physics and Biology.

(i) Represent the information on the Venn Diagram.



A student is selected at random from this class.
The events E and F are:

E: The student studies Physics

F: The student studies Biology.

(ii) By calculating probabilities, investigate if the events E and F are independent.

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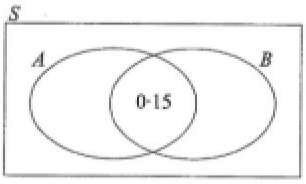
Probability & Statistics 1 Tutorial Questions

Question 2:

[2010 SEC Paper 2, Q1] (25 marks)

Two events are such that P (A) =0.2, P(A ∩ B) =0.15 and P(A' ∩ B)=0.6



i) Find the probability that neither A nor B happens
ii) Find the conditional probability P(A|B)
iii) State whether A and B are independent events and justify your answer

Question 3:

Leaving Cert Paper 2, 2017, Q1

Question 1 **(25 marks)**

When Conor rings Ciara's house, the probability that Ciara answers the phone is $\frac{1}{5}$.

(a) Conor rings Ciara's house once every day for 7 consecutive days. Find the probability that she will answer the phone on the 2nd, 4th, and 6th days but not on the other days.

(b) Find the probability that she will answer the phone for the 4th time on the 7th day.

(c) Conor rings her house once every day for *n* days. Write, in terms of *n*, the probability that Ciara will answer the phone at least once.

(d) Find the minimum value of *n* for which the probability that Ciara will answer the phone at least once is greater than 99%.

Question 4:

Leaving Cert Paper 2, 2018, Q1

In a competition Mary has a probability of 1/20 of winning, a probability of 1/10 of finishing in second place, and a probability of 1/4 of finishing in third place. If she wins the competition she gets €9000. If she comes second she gets €7000 and if she comes third she gets €3000. In all other cases she gets nothing. Each participant in the competition must pay €2000 to enter.

a) Find the expected value of Mary's loss if she enters the competition.

b) Each of the 3 prizes in the competition above is increased by the same amount (€x) but the entry fee is unchanged. For example, if Mary wins the competition now, she would get €(9000 + x). Mary now expects to break even. Find the value of x.

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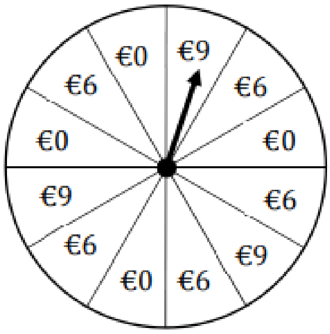
Question 5:

Leaving Cert Paper 2, 2023, Q1

A circular spinner has 12 sectors, as follows:

- 5 sectors are labelled €6
- 3 sectors are labelled €9
- The rest are labelled €0.



In a game, the spinner is spun once.
The spinner is equally likely to land on each sector.
The player gets the amount of money shown on the sector that the spinner lands on.

(a) Fiona plays the game a number of times.

Work out the probability that Fiona gets €6, then €9, then €6 the first three times she plays. Give your answer correct to 4 decimal places.

(b) Rohan also plays the game a number of times. Find the probability that Rohan gets €9 for the 3rd time, on the 8th time that he plays the game. Give your answer correct to 4 decimal places.

(c) Olga plays the game 2 times. Find the probability that Olga gets less than €16 in total from playing the game. Give your answer correct to 4 decimal places.

Question 6:

Leaving Cert Paper 2, 2022, Q1

(a) The table below gives some details on the number of different types of students in a university. There are 22 714 students in the university in total.

	Age (years)		**Total**
	23 or younger	**24 or older**	
Undergraduate	12 785	2922	**15 707**
Postgraduate	1353		
Total		**8576**	**22 714**

(i) Fill in the three missing values to complete the table above.

(ii) One student is picked at random from the students in the university. Let O be the event that the student is 24 years old, or older. Let U be the event that the student is an undergraduate. Are the events O and U independent? Justify your answer.

(b) Three people are picked at random from a class. Find the probability that all three were born on the same day of the week. Assume that the probability of being born on each day is the same.

(c) There are b boys and g girls in a class. $\frac{3}{5}$ of the students in the class are girls. 4 boys and 4 girls join the class. One student is then picked at random from the whole class. The probability that this student is a girl is now $\frac{4}{7}$. Find the value of b and the value of g.

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Probability & Statistics 1 Tutorial Questions

Question 7:

Leaving Cert Paper 2, 2015, Q8

In basketball, players often have to take free throws. When Michael takes his first free throw in any game, the probability that he is successful is 0·7.
For all subsequent free throws in the game, the probability that he is successful is:

- 0·8 if he has been successful on the previous throw
- 0·6 if he has been unsuccessful on the previous throw.

(a) Find the probability that Michael is successful (S) with all three of his first three free throws in a game.

P(S, S, S) =

(b) Find the probability that Michael is unsuccessful (U) with his first two free throws and successful with the third.

P(U, U, S) =

(c) List all the ways that Michael could be successful with his third free throw in a game and hence find the probability that Michael is successful with his third free throw.

(d) (i) Let p_n be the probability that Michael is successful with his n^{th} free throw in the game (and hence $(1-p_n)$ is the probability that Michael is unsuccessful with his n^{th} free throw). Show that $p_{n+1} = 0 \cdot 6 + 0 \cdot 2 p_n$.

(ii) Assume that p is Michael's success rate in the long run; that is, for large values of n, we have $p_{n+1} \approx p_n \approx p$.
Using the result from part **(d) (i)** above, or otherwise, show that $p = 0 \cdot 75$.

(e) For all positive integers n, let $a_n = p - p_n$, where $p = 0 \cdot 75$ as above.

(i) Use the ratio $\frac{a_{n+1}}{a_n}$ to show that a_n is a geometric sequence with common ratio $\frac{1}{5}$.

(ii) Find the smallest value of n for which $p - p_n < 0 \cdot 00001$.

(f) You arrive at a game in which Michael is playing. You know that he has already taken many free throws, but you do not know what pattern of success he has had.

(i) Based on this knowledge, what is your estimate of the probability that Michael will be successful with his next free throw in the game?

Answer: _______________

(ii) Why would it **not** be appropriate to consider Michael's subsequent free throws in the game as a sequence of Bernoulli trials?

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