



Calculus 1 Solutions

- 1) (i) $4x^3$ (ii) $21x^2$ (iii) $-x^2$ (iv) 1
- 2) (i) $f'(x) = 10x + 7$ (ii) $g'(x) = 6x^2 + 8x + 1$ (iii) $\frac{dy}{dx} = 8$
- 3) $y = 8x + 4$ is the equation of the line. The slope of the line is the derivative.
 $\Rightarrow \frac{dy}{dx} = 8$. 8 is the slope of the line.
- 4) Product Rule: $y = (4 + 3x^2)(6x + 4x^2)$
 Let $u = (4 + 3x^2)$ and $v = (6x + 4x^2)$
 $\frac{dy}{dx} = u * \frac{dv}{dx} + v * \frac{du}{dx}$
 $\frac{du}{dx} = 6x$ $\frac{dv}{dx} = 6 + 8x$
 $\frac{dy}{dx} = (4 + 3x^2)(6 + 8x) + (6x + 4x^2)(6x)$
 $\Rightarrow = 24 + 32x + 18x^2 + 24x^3 + 36x^2 + 24x^3$
 $\Rightarrow = 48x^3 + 54x^2 + 32x + 24$
- 5) Quotient Rule: $y = \frac{(5+6x)}{2x^2}$
 Let $u = (5 + 6x)$ and $v = (2x^2)$
 $\frac{du}{dx} = 6$ $\frac{dv}{dx} = 4x$
 $\frac{dy}{dx} = \frac{(v*du)-(u*dv)}{v^2}$
 $\Rightarrow \frac{dy}{dx} = \frac{(2x^2*6)-((5+6x)*4x)}{(2x^2)^2}$
 $\Rightarrow \frac{dy}{dx} = \frac{12x^2-20x-24x^2}{4x^4}$
 $\Rightarrow \frac{dy}{dx} = \frac{-12x^2-20x}{4x^4}$
 $\Rightarrow \frac{dy}{dx} = \frac{-3x-5}{x^3}$ (Divide above and below by 4x)
- 6) Chain rule: $y = (2 + 3x)^4$ Find $\frac{dy}{dx}$
 $u = v^4$ $v = (2 + 3x)$
 $\frac{du}{dv} = 4v^3$ $\frac{dv}{dx} = 3$
 $\frac{dy}{dx} = \frac{du}{dv} * \frac{dv}{dx} \rightarrow \frac{dy}{dx} = 4v^3 * 3$
 $\frac{dy}{dx} = 12(2+3x)^3$
- 7) Chain rule: $y = \text{Cos}(3x + 5)$. Find $\frac{dy}{dx}$
 $u = \text{Cos}(v)$ $v = (3x + 5)$
 $\frac{du}{dv} = -\text{Sin}(v)$ $\frac{dv}{dx} = 3$
 $\frac{dy}{dx} = \frac{du}{dv} * \frac{dv}{dx} \rightarrow \frac{dy}{dx} = -\text{Sin}(v) * 3$
 $\frac{dy}{dx} = -3\text{Sin}(3x + 5)$



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8) Find the following limits:

$$a) \lim_{x \rightarrow 1} \left(\frac{x^2 + x - 2}{x - 1} \right) = \lim_{x \rightarrow 1} \left(\frac{(x-1)(x+2)}{x-1} \right) = \lim_{x \rightarrow 1} (x + 2) = 1 + 2 = 3$$

$$b) \lim_{n \rightarrow \infty} \left(\frac{2n^2 - 3n + 2}{6n^2 + 5n - 6} \right) = \lim_{n \rightarrow \infty} \left(\frac{2 - \frac{3}{n} + \frac{2}{n^2}}{6 + \frac{5}{n} - \frac{6}{n^2}} \right) = \frac{2}{6} = \frac{1}{3}$$

9) Find the derivative of $f(x) = 5 - 2x$ by first principles.

$$\Rightarrow f(x+h) = 5 - 2(x+h) = 5 - 2x - 2h. \text{ (Subbing in } x+h \text{ for } x)$$

$$\Rightarrow \frac{f(x+h) - f(x)}{h} = \frac{5 - 2x - 2h - (5 - 2x)}{h} = \frac{-2h}{h} = -2$$

10)

(a) Differentiate the function $2x^2 - 3x - 6$ with respect to x from first principles.

$$f(x) = 2x^2 - 3x - 6$$

$$f(x+h) = 2(x+h)^2 - 3(x+h) - 6 = 2x^2 + 4xh + 2h^2 - 3x - 3h - 6$$

$$f(x+h) - f(x) = 4xh + 2h^2 - 3h$$

$$\lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right) = \lim_{h \rightarrow 0} \left(\frac{4xh + 2h^2 - 3h}{h} \right) = 4x - 3$$

(b)

$$f(x) = \frac{2x}{x+2}$$

$$\text{Let } u(x) = 2x \Rightarrow u'(x) = 2 \text{ and } v(x) = x+2 \Rightarrow v'(x) = 1$$

$$f'(x) = \frac{(x+2)(2) - 2x(1)}{(x+2)^2} = \frac{4}{(x+2)^2}$$

$$f'(x) = \frac{1}{4} \Rightarrow \frac{4}{(x+2)^2} = \frac{1}{4}$$

$$\Rightarrow 16 = (x+2)^2$$

$$\Rightarrow x+2 = 4 \text{ or } x+2 = -4$$

$$\Rightarrow x = 2 \text{ or } x = -6$$

$$\text{or } x^2 + 4x - 12 = 0$$

$$(x-2)(x+6) = 0$$

$$\Rightarrow x-2 = 0 \text{ or } x+6 = -0$$

$$\Rightarrow x = 2 \text{ or } x = -6$$

$$f(-6) = \frac{-12}{-6+2} = 3 \text{ and } f(2) = \frac{4}{2+2} = 1$$

Points $(-6, 3)$ and $(2, 1)$



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11)

<p>(a)</p>	$f(x+h) - f(x) = (2x+2h+4)^2 - (2x+4)^2$ $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} =$ $\lim_{h \rightarrow 0} \frac{(2x+2h+4)^2 - (2x+4)^2}{h}$ $= \lim_{h \rightarrow 0} \left(\frac{[(4x^2 + 8hx + 4h^2 + 16x + 16h + 16)] - (4x^2 + 16x + 16)}{h} \right)$ $= \lim_{h \rightarrow 0} \frac{8hx + 4h^2 + 16h}{h}$ $= 8x + 16$	<p>Scale 10D (0, 2, 5, 8, 10)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> any $f(x+h)$ <p><i>Mid Partial Credit</i></p> <ul style="list-style-type: none"> limit of $\frac{f(x+h)-f(x)}{h}$ <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> limit of $\frac{(2x+2h+4)^2 - (2x+4)^2}{h}$ <p>Notes:</p> <ul style="list-style-type: none"> omission of limit sign penalised once only answer not from 1st Principles merits 0 marks
<p>(b) (i)+ (ii)</p>	$y = x \cdot \sin \frac{1}{x}$ $\frac{dy}{dx} = \sin \frac{1}{x} + x \left(\cos \frac{1}{x} \right) \left(-\frac{1}{x^2} \right)$ $\frac{dy}{dx} = \sin \frac{1}{x} - \frac{1}{x} \cos \frac{1}{x}$ $\frac{dy}{dx} = \sin \frac{\pi}{4} - \frac{\pi}{4} \cos \frac{\pi}{4}$ $= 0.15$	<p>Scale 15D (0, 4, 7, 11, 15)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> any correct differentiation <p><i>Mid Partial Credit</i></p> <ul style="list-style-type: none"> product rule applied <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> correct differentiation <p>Note: one penalty for calculator in wrong mode</p>



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12)

<p>(b)</p> $\frac{d(fg(x))}{dx} = \frac{1}{(3(x+5)^2 + 2)}(6(x+5))$ $\frac{d(fg(\frac{1}{4}))}{dx} = \frac{6(\frac{21}{4})}{3(\frac{21}{4})^2 + 2} = \frac{504}{1355}$ $= 0.372$ <p style="text-align: center;">OR</p> $f(x) = \ln(3x^2 + 2)$ $g(x) = (x + 5)$ $f[g(x)] = \ln[3(x+5)^2 + 2]$ $= \ln(3x^2 + 30x + 77)$ $f'(x) = \frac{6x + 30}{3x^2 + 30x + 77}$ $x = \frac{1}{4}: f'(x) = \frac{31.5}{84.6875} = 0.3719$ $= 0.372$	<p>Scale 5C (0, 3, 4, 5)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> Any correct differentiation $fg(x)$ formulated <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> $\frac{d(fg(x))}{dx}$ found <p>Note: Work with $f(x) \times g(x)$ merits low partial credit at most</p>
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13)

<p>(a) (i)</p> $h(10)$ $= 0.001(10)^3 - 0.12(10)^2 + p(10) + 5$ $= 30$ $10p = 36$ $p = 3.6$	<p>Scale 5C(0, 2, 3, 5)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> $h(10)$ with some relevant substitution <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> Equation in p
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<p>(a) (ii)</p> <table border="1" style="margin-bottom: 10px;"> <tr><td>x</td><td>0</td><td>10</td><td>20</td><td>30</td><td>40</td></tr> <tr><td>$h(x)$</td><td>5</td><td>30</td><td>37</td><td>32</td><td>21</td></tr> </table> <table border="1" style="margin-bottom: 10px;"> <tr><td>x</td><td>50</td><td>60</td><td>70</td><td>75</td></tr> <tr><td>$h(x)$</td><td>10</td><td>5</td><td>12</td><td>21.875</td></tr> </table>	x	0	10	20	30	40	$h(x)$	5	30	37	32	21	x	50	60	70	75	$h(x)$	10	5	12	21.875	<p>Scale 15D (0, 4, 8, 12, 15)</p> <p>14 items are required: 5 table entries and 9 plots (which also need to be joined appropriately for <i>Full Credit</i>)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> Any one item correct <p><i>Mid Partial Credit:</i></p> <ul style="list-style-type: none"> any 7 items correct <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> any 11 items correct <p><i>Full Credit –1:</i></p> <ul style="list-style-type: none"> All items correct but points not joined or joined incorrectly All items but one correct, and points appropriately joined
x	0	10	20	30	40																		
$h(x)$	5	30	37	32	21																		
x	50	60	70	75																			
$h(x)$	10	5	12	21.875																			



Calculus 1 Solutions

<p>(b) (i)</p>	$h'(x) = 0.003x^2 - 0.24x + 3.6$	<p>Scale 5C (0, 2, 3, 5)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> • correct differentiation of 1 term <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> • correct differentiation of 2 terms
<p>(b) (ii)</p>	$h'(x) = 0.003x^2 - 0.24x + 3.6$ $h'(20) = 0.003(20)^2 - 0.24(20) + 3.6$ $= 0, \text{ so [local] max/min at } x = 20$ $h''(20) = 0.006(20) - 0.24 < 0,$ <p>so local max</p> <p>Also $h(20) > h(0)$ and $h(20) > h(75)$</p> <p style="text-align: center;">OR</p> $h'(x) = 0.003x^2 - 0.24x + 3.6$ $h'(20) = 0.003(20)^2 - 0.24(20) + 3.6$ $= 0, \text{ so [local] max/min at } x = 20$ <p>From graph, turning point at $x = 20$ is a [local] max, and it is above the two endpoints [0 and 75]</p>	<p>Scale 10C (0, 3, 7, 10)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> • $0.003x^2 - 0.24x + 3.6$ • States $h'(x) = 0$, or similar <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> • Shows that $h'(20) = 0$, but no further justification that it is the max in the range [0, 75].
<p>(b) (iii)</p>	$h''(x) = 0.006x - 0.24 = 0$ $0.006x = 0.24$ $x = 40$ $h(40) = 0.001(40)^3 - 0.12(40)^2$ $+ 3.6(40) + 5$ $= 21 \text{ [m]}$	<p>Scale 5C (0, 2, 3, 5)</p> <p><i>Note:</i> work presented in (b)(iii) must involve calculus, or be based on calculus from (b)(ii), to be awarded any credit.</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> • Some correct differentiation of $h'(x)$ • $h''(x)$ indicated <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> • $x = 40$