Revision Tutorial November 2024

Question 1.

(a) In a triangle ABC, the lengths of the sides are a, b and c.
Using a formula for the area of a triangle, or otherwise,
prove that

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}.$$

(b) In a triangle XYZ, $|XY| = 5$ cm, $|XZ| = 3$ cm and $|\angle XYZ| = 27°$.

(i) Find the two possible values of $|\angle XZY|$. Give your answers correct to the nearest degree.

(ii) Draw a sketch of the triangle XYZ, showing the two possible positions of the point Z.

(c) In the case that $|\angle XZY| < 90°$, write down $|\angle ZXY|$, and hence find the area of the triangle XYZ, correct to the nearest integer.

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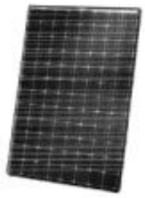
Revision Tutorial November 2024

Question 2.

A stand is being used to prop up a portable solar panel. It consists of a support that is hinged to the panel near the top, and an adjustable strap joining the panel to the support near the bottom.

By adjusting the length of the strap, the angle between the panel and the ground can be changed.



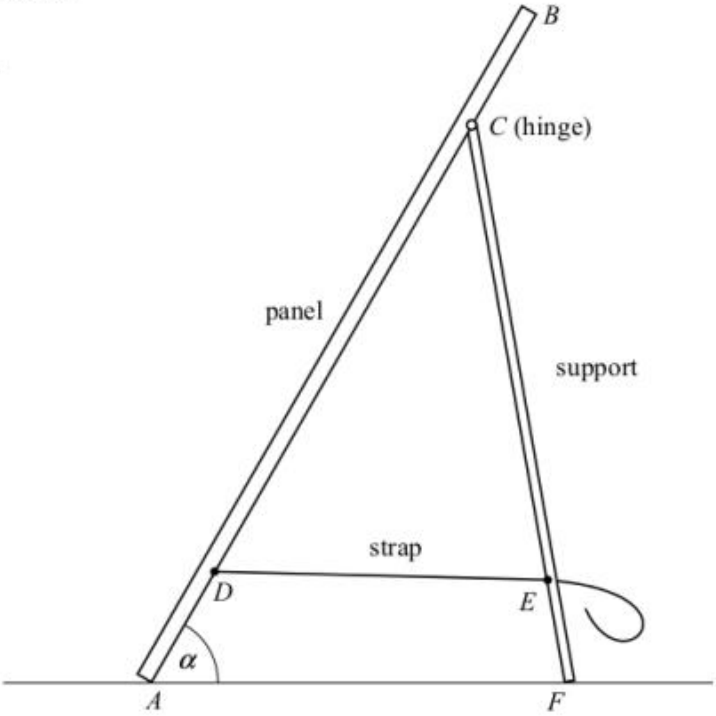
The dimensions are as follows:

$|AB| = 30$ cm

$|AD| = |CB| = 5$ cm

$|CF| = 22$ cm

$|EF| = 4$ cm.



(a) Find the length of the strap $|DE|$ such that the angle α between the panel and the ground is $60°$.

(b) Find the maximum possible value of α, correct to the nearest degree.

Revision Tutorial November 2024

Question 3.

Find all the values of x for which $\sin(3x) = \frac{\sqrt{3}}{2}$, $0 \le x \le 360$, x in degrees.

Question 4.

Prove that $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$.

Revision Tutorial November 2024

Question 5. (Question 2, Paper 1, 2000)

(a) Solve for x, y, z

$$3x - y + 3z = 1$$
$$x + 2y - 2z = -1$$
$$4x - y + 5z = 4$$

(b) Solve $x^2 - 2x - 24 = 0$

Hence, find the values of x for which

$$\left(x + \frac{4}{x}\right)^2 - 2\left(x + \frac{4}{x}\right) - 24 = 0 \qquad x \in R, \quad x \neq 0$$

(c) (i) Express $a^4 - b^4$ as a product of three factors

(ii) Factorise $a^5 - a^4b - ab^4 + b^5$

Use your results from (i) and (ii) to show that

$a^5 + b^5 > a^4b + ab^4$

where a and b are positive unequal real numbers.

Revision Tutorial November 2024

Question 6. (Question 1, Paper 2, 2000)

(a) The equation of a circle is $x^2 + y^2 = 130$.

Find the slope of the tangent to the circle at the point $(-7, 9)$.

(b) $x^2 + y^2 - 6x + 4y - 12 = 0$ is the equation of a circle.

Write down the coordinates of its centre and the length of its radius.

$x^2 + y^2 + 12x - 20y + k = 0$ is another circle, where $k \in R$.

The two circles touch externally. Find the value of k.

(c) A circle intersects a line at the points $a(-3, 0)$ and $b(5, -4)$.

The midpoint of [ab] is m. Find the coordinates of m.

The distance from the centre of the circle to m is $\sqrt{5}$.

Find the equations of the two circles that satisfy these conditions.

Question 7. (Question 1, Paper 2, 2001)

(a) A circle with centre (-3, 7) passes through the point (5, -8)

Find the equation of the circle.

(b) The equation of a circle is $(x+1)^2 + (y-8)^2 = 160$

The line $x-3y+25 = 0$ intersects the circle at the points p and q.

(i) Find the co-ordinates of p and the co-ordinates of q.
(ii) Investigate if [pq] is a diameter of the circle.

(c) The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ passes through the points (3,3) and (4, 1)
The line $3x - y - 6 = 0$ is a tangent to the circle at (3,3)

(i) Find the real numbers g, f and c.
(ii) Find the co-ordinates of the point on the circle at which the tangent parallel to $3x-y-6=0$ touches the circle.

Revision Tutorial November 2024

Question 8. (Question 4, Paper 2, 2001)

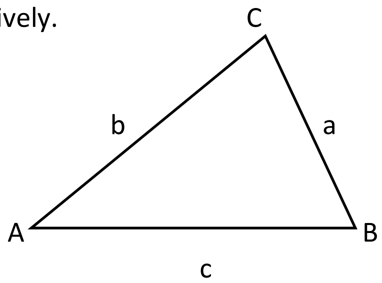
(a) The length of an arc of a circle is 10 cm. The radius of the circle is 4 cm. The measure of the angle at the centre of the circle subtended by the arc is . θ

(i) Find in radians. θ
(ii) Find in degrees, correct to the nearest degree. θ

(b)
(i) Write cos2x in terms of sin x
(ii) Hence, find all the solutions of the equation $\cos 2x - \sin x = 1$

in the domain $0° \leq x \leq 360°$

(c) A triangle has sides a, b and c.
The angles opposite a, b and c are A, B and C, respectively.



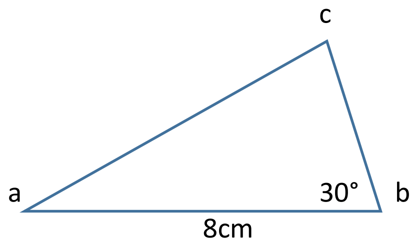
(i) Prove that $a^2 = b^2 + c^2 - 2bc \cos A$

(iii) Show that $c\,(b \cos A – a \cos B) = b^2 - a^2$

Revision Tutorial November 2024

Question 9. (Question 5 , Paper 2, 2002)

(a) The area of triangle abc is 12 cm2. |ab | = 8 cm and |∠abc | = 30° Find |bc| .



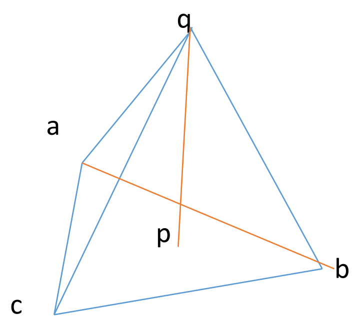
(b) (i) Prove that

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

(ii) Hence, or otherwise, prove that tan 22.5° = √2 -1

(c) A vertical radio mast [pq] stands on flat horizontal ground. It is supported by three cables that join the top of the mast, q, to the points a, b and c on the ground. The foot of the mast, p, lies inside the triangle abc. Each cable is 52 m long and the mast is 48 m high.

(i) Find the (common) distance from p to each of the points a, b and c.
(ii) Given that |ac| = 38 m and |ab| = 34 m, find |bc| correct to one decimal place.



Revision Tutorial November 2024

Question 10. (Question 4, Paper 2, 2003)

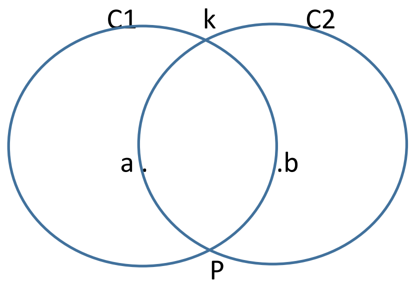
(a) The circumference of a circle is 30 π cm. The area of a sector of the circle is 75 cm^2 .
Find, in radians, the angle in this sector.

(b) Find all the solutions of the equation sin 2x + sin x = 0 in the domain $0° \leq x \leq 360°$.

(c) C1 is a circle with centre a and radius r.
C2 is a circle with centre b and radius r.
C1and C2 intersect at k and p. $a \in C2$ and $b \in C1$.

(i) Find, in radians, the measure of angle kap.

(iii) Calculate the area of the intersection region. Give your answer in terms of r and π .



Revision Tutorial November 2024

Question 11. (Question 2, Paper 1, 2024)

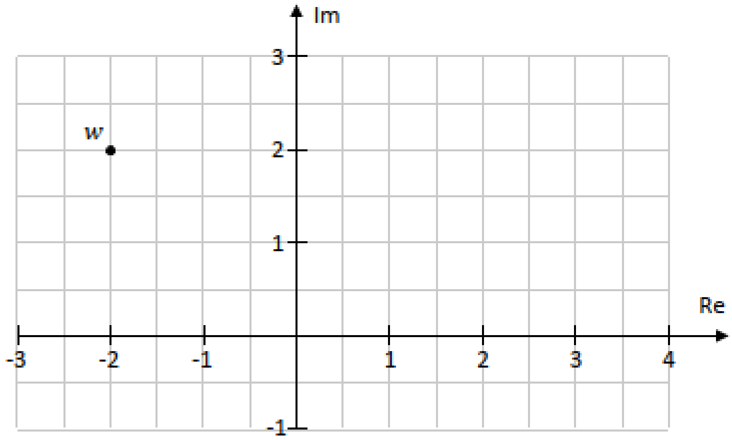
In this question, $i^2 = -1$.

(a) Find the two solutions of the following equation in z, where z is a complex number.
Give each answer in the form $a + bi$, where $a, b \in \mathbb{R}$.

$$z^2 + 12z + 261 = 0$$

(b) Use **de Moivre's theorem** to write $(1 - \sqrt{3}\, i)^9$ in the form $a + bi$, where $a, b \in \mathbb{R}$

(c) The point $w = -2 + 2i$ is shown in the Argand diagram below.



(i) **Using the diagram above, plot** and **label** the complex number
u= $4\left(Cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ accurately as possible

(ii) The complex number o is $0 + 0i$. Find the size of the angle $\angle wou$ in **radians**.