Coordinate Geometry: The Line – Hints & Tips

General Hints and Tips

1 **Always draw diagrams**. This is useful in every question, but it is particularly helpful with questions relating to the circle or more difficult questions.

2 Make sure you **know which formulae are in the tables,** and where in the tables they are. Formulae in the tables:

- **Slope of a line**
  $\frac{(y2 - y1)}{(x2 - x1)}$
- **Distance between 2 points**
  $\sqrt{(x2 - x1)^2 + (y2 - y1)^2}$
- **Midpoint formula**
  $\left(\frac{x1 + x2}{2}, \frac{y1 + y2}{2}\right)$
- **Equation of a line (2 different formats)**
  $(y - y1) = m\,(x - x1)$
- **Area of a triangle with one point at the origin**
  $(\frac{1}{2}|x1y2 - x2y1|)$
- **Point dividing a line segment in the ratio a:b**
  $a : b = \left(\frac{bx1 + ax2}{a + b}, \frac{by1 + ay2}{a + b}\right)$
- **To find the angle between 2 lines:**
  $Tan\theta = \pm(m1 - m2)/(1 + m1.m2)$
- **Perpendicular distance from a point to a line**
  $\frac{|ax1+by1+c|}{\sqrt{a^2+b^2}}$

3 **Learn other formulae** off by heart.

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The Line

1 To get an **equation of a line** you always need 2 things:

- **A point**
- **A slope**

Once you have these, use the formula $y - y_1 = m(x - x_1)$

Slopes:

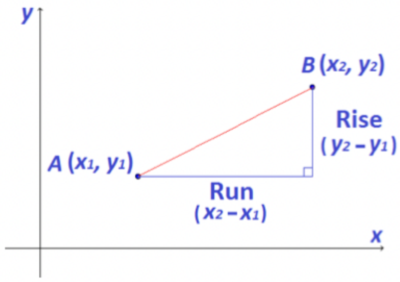
The **slope** or gradient of a line is a value that describes both the direction and the steepness of the line.

The given diagram shows the points $A(x_1, y_1)$ and B (x_2, y_2)

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}}$$

The slope, m, of the line passing through the points (x_1, y_1) and (x_2, y_2) is

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$



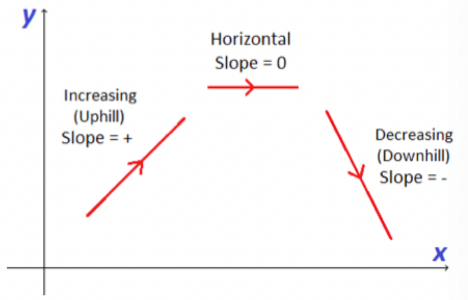
Positive and Negative Slopes

Graphs are read from left to right.

If a line is increasing (going uphill), it has a positive slope.

A horizontal line has a slope of zero.

If a line is decreasing (going downhill), it has a negative slope.



2 To check if a point is on a line, substitute it into the equation.
If the answer = 0, then the point is on the line, otherwise it is not.

3 To plot a line, you need two points on the line.
An easy way to find points on a line is:
Let x = 0, solve for y. This will give you a point (0,y)
Let y = 0, solve for x. This will give you a point (x,0)
Use these two points to plot the line.

4 If a line **intersects the x-axis**, then **y = 0** at that point.
If a line **intersects the y-axis**, then **x = 0** at that point.

5 Use simultaneous equations to find the point of intersection between 2 lines.

6 If lines are **parallel**, their **slopes are equal**.

Parallel Lines

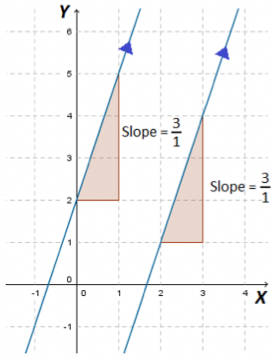
If two lines are parallel if their slopes are equal.

If $l_1 || l_2$ then $m_1 = m_2$

In the given diagram, the two lines are parallel, therefore their slopes are equal:

$$\frac{3}{1} = \frac{3}{1}$$

If two lines are parallel, their slopes are equal.



If lines are **perpendicular**, then multiplying their slopes together equals -1 (**$m_1.m_2$ = -1**)
An example - you want the slope of a line and are told it is perpendicular to another line with slope 2/3
Turn it upside down and change the sign of it. So in this case, the slope of the line you want is -3/2

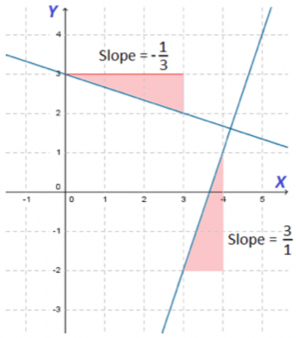
Perpendicular Lines

If two lines are perpendicular, when we multiply their slopes we get -1.

If $l_1 \perp l_2$ then $(m_1)(m_2) = -1$

In the given diagram, the two lines are perpendicular, therefore the product of the slopes is:

$$-\frac{1}{3} \times \frac{3}{1} = -\frac{3}{3} = -1$$



If two lines are perpendicular, the product of their slopes is -1.

1. **Note:** If we know the slope of a line and we need to find the slope of a line perpendicular to it, we turn the given slope upside down and change the sign.

7 To use the area of a triangle formula (½$|x_1y_2 - x_2y_1|$) one of the points needs to be (0,0).
If you are looking for the area of a triangle, where no points are at the origin (0,0), use translations to bring one of the points to (0,0) and then use the formula as normal.
Alternatively, you can use the area = ½ base x perpendicular height formula.

8 If 3 or more points lie on the same line, they are said to be collinear.
To check if 3 points (e.g. a, b, c) are collinear, see what the slopes of |ab| and |bc| are.
If they are the same, then the points are collinear, otherwise they are not.
An alternative way of doing this is to calculate the area of the triangle using the 3 points.
If the area = 0, then the points are collinear, otherwise they are not.