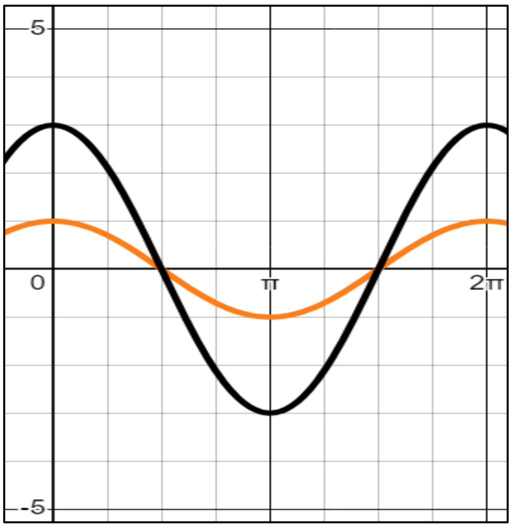
Trigonometry 2 Solutions

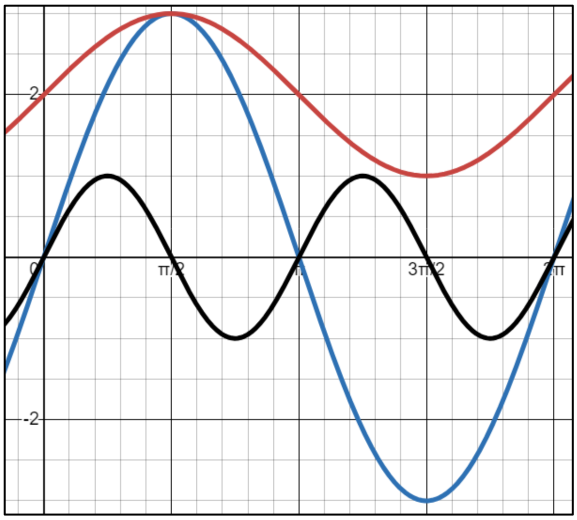
Exercises:

Q1.



Orange line: y = cos(x)
Black line: y = 3cos(x)

Q2.



Blue line: y = 3sin(x)
Black line: y = sin(2x)
Red line: y = 2 + sin(x)

Q3.

(i) Length of arc = r * θ = 5 * $\frac{\pi}{6} = \frac{5\pi}{6}$.

Area = $\frac{1}{2} * r^2 * \theta = \frac{1}{2} * 5^2 * \frac{\pi}{6} = \frac{25\pi}{12}$

(ii) Length of arc = 2 π *r * $\frac{\theta}{360^0}$ = 2 π *6 * $\frac{40}{360} = \frac{4\pi}{3}$.

Area = $\pi * r^2 * \frac{\theta}{360^0} = \pi * 6^2 * \frac{40}{360} = 4\pi$

Q4.

Find the reference angle: Cosθ = $-\frac{\sqrt{3}}{2}$.

⇨ $\theta = \text{Cos}^{-1}(\frac{\sqrt{3}}{2})$

⇨ $\theta = 30^0$

Find the quadrant the angle lies in:

⇨ From our initial equation, Cos is negative.

⇨ Cos is negative in quadrants 2 and 3.

Our reference angle is 30^0 and lies in quadrants 2 and 3.

⇨ For quadrant 2, our angle is 180 – 30 = 150^0

⇨ For quadrant 3, our angle is 180 + 30 = 210^0

General solution is:

$\theta = 150^0 + n*360^0$ or $\theta = 210^0 + n*360^0$

Q5.

Find the reference angle: Tanθ = $\sqrt{3}$.

⇨ $\theta = \text{Tan}^{-1}(\sqrt{3}) = 60^0$

From the initial equation, tan is positive.

⇨ Tan is positive in quadrants 1 and 3.

Our reference angle is 60^0 and lies in quadrants 1 and 3.

⇨ For quadrant 1, our angle is 60^0

⇨ For quadrant 3, our angle is $180^0 + 60^0 = 240^0$.

General solution is:

$\theta = 60^0 + n*360^0$ or $\theta = 240^0 + n*360^0$

We can also write this as:

$\theta = 60^0 + n*180^0$ (period of tan function is 180^0).

Q6.

Find the reference angle: $\text{Sin}3\theta = \frac{1}{2}$.

⇨ $3\theta = \text{Sin}^{-1}(\frac{1}{2}) = 30^0$

From the initial equation, sin is positive:

⇨ Sin is positive in quadrants 1 and 2.

Our reference angle is 30^0 and lies in quadrants 1 and 2.

⇨ For quadrant 1, the angle is 30^0

⇨ For quadrant 2, the angle is $180 – 30 = 150^0$.

Our general solution is:

$3\theta = 30^0 + n*360^0$, $3\theta = 150^0 + n*360^0$.

$\theta = 10^0 + n*120^0$, $\theta = 50^0 + n*120^0$.

Now we need to list each angle where $0 \leq \theta \leq 360^0$:

N=0: $\theta = 10^0 + 0*120^0$, $\theta = 50^0 + 0*120^0$

$\theta = 10^0$, $\theta = 50^0$

N=1: $\theta = 10^0 + 1*120^0$, $\theta = 50^0 + 1*120^0$

$\theta = 130^0$, $\theta = 170^0$

N=2: $\theta = 10^0 + 2*120^0$, $\theta = 50^0 + 2*120^0$

$\theta = 250^0$, $\theta = 290^0$

N=3: $\theta = 10^0 + 3*120^0$, $\theta = 50^0 + 3*120^0$

$\theta = 370^0$, $\theta = 410^0$. These are greater than 360^0 so are not included.

Final list: $\theta = 10^0, 50^0, 130^0, 170^0, 250^0, 290^0$.

Exam Questions:

Q1. 2021 Paper 2 Question 4b

Q4	Model Solution – 30 Marks
(b)	$\tan(\text{angle}) = -\sqrt{3}$, so reference angle $= 60°$
	$\mathbf{150° \leq B + 150° \leq 510°}$
	In Quad's 2 or 4, so angles are $\mathbf{300°}$ or $\mathbf{480°}$
	$B + 150 = 300$ **or** $B + 150 = 480$
	So $B = 150°$ **or** $B = 330°$
	OR
	$\tan(B + 150) = \frac{\tan B + \tan 150}{1 - \tan B \tan 150}$
	$= \frac{\tan B - \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)\tan B} = -\sqrt{3}$
	So $\tan B - \frac{1}{\sqrt{3}} = -\sqrt{3} - \tan B$
	So $2\tan B = -\frac{2}{\sqrt{3}}$, i.e. $\tan B = -\frac{1}{\sqrt{3}}$
	Reference angle $= 30°$
	In Quad's 2 or 4, so $B = 150°$ **or** $B = 330°$

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Q2. 2020 Paper 2 Question 4

Q4	Model Solution – 25 Marks
(a)	Reference angle: $\frac{\pi}{6}$ 2^{nd} Quadrant: $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$ $\frac{\theta}{2} = \frac{5\pi}{6} + 2n\pi$ $\theta = \frac{5\pi}{3} + 4n\pi$ $n = 0 \Rightarrow \theta = \frac{5\pi}{3} = 300°$ 4^{nd} Quadrant: $2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ $\frac{\theta}{2} = \frac{11\pi}{6} + 2n\pi$ $\theta = \frac{11\pi}{3} + 4n\pi$ $n = 0 \Rightarrow \theta = \frac{11\pi}{3} = 660°$
(b)	Area of ΔCOA = Area of Sector − 21 $= \frac{1}{2}r^2\theta - 21 = 8{\cdot}4$ Area of ΔCOA: $\frac{1}{2}\|CO\|\|7\| \sin 1{\cdot}2 = 8{\cdot}4$ $\|CO\| = \frac{8{\cdot}4}{3{\cdot}5 \sin 1{\cdot}2} = 2{\cdot}57$ $\|BC\| = 7 - 2{\cdot}6 = 4{\cdot}4$ cm

Q3. 2019 Paper 2 Question Q4b

Trigonometry 2 Solutions

(b)

Let length of side be x

Diagonal of any face $= \sqrt{x^2 + x^2} = \sqrt{2}x$

Internal diagonal $= x^2 + \left(\sqrt{2}x\right)^2 = \sqrt{3}x$

By cosine rule:

$$x^2 = \left(\frac{\sqrt{3}x}{2}\right)^2 + \left(\frac{\sqrt{3}x}{2}\right)^2 - 2\frac{\sqrt{3}x}{2}\frac{\sqrt{3}x}{2}\cos A$$

$$\cos A = \frac{\left(\frac{\sqrt{3}x}{2}\right)^2 + \left(\frac{\sqrt{3}x}{2}\right)^2 - x^2}{2\left(\frac{\sqrt{3}x}{2}\right)\left(\frac{\sqrt{3}x}{2}\right)}$$

$$\cos A = \frac{1}{3}$$

Or

Drop perpendicular from intersecting diagonals to side of cube, thereby creating angle $A/2$ at vertex in a right-angled triangle.

$$\sin\frac{A}{2} = \frac{\frac{x}{2}}{\frac{\sqrt{3}x}{2}} = \frac{1}{\sqrt{3}}$$

$$\therefore \cos\frac{A}{2} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\cos A = 2\cos^2\frac{A}{2} - 1 = 2\left(\frac{2}{3}\right) - 1 = \frac{1}{3}$$

Also: $\sin\frac{A}{2} = \frac{1}{\sqrt{3}} \rightarrow \frac{A}{2} = 35{\cdot}2643896^\circ$

$$A = 70{\cdot}5287792^\circ$$

$$\cos A = 0{\cdot}33236$$

Q4. 2018 Paper 2 Question 4

Q4 — **Model Solution – 25 Marks**

(a)

$2x = 150 + 360n$ or $2x = 210 + 360n$

$x = 75 + 180n$ $\quad$ $x = 105 + 180n$

$n = 0 \Rightarrow x = 75^\circ$ $\quad$ $n = 0 \Rightarrow x = 105^\circ$

$n = 1 \Rightarrow x = 255^\circ$ $\quad$ $n = 1 \Rightarrow x = 285^\circ$

(b)

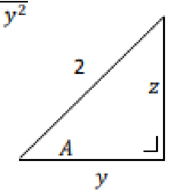
$2^2 = y^2 + z^2$

$$z = \sqrt{4 - y^2}$$

$\sin 2A = 2\sin A\cos A$

$$2\left(\frac{\sqrt{4 - y^2}}{2}\right)\left(\frac{y}{2}\right)$$

$$= \frac{y\sqrt{4-y^2}}{2}$$



Or

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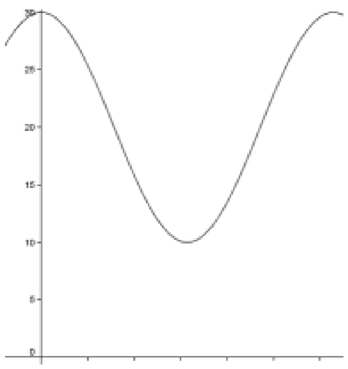
Q5. 2018 Paper 2 Question 9

Q9	Model Solution – 40 Marks
(a)	$\frac{10}{\sin 15} = \frac{30}{\sin x}$ $\sin x = \frac{30 \sin 15}{10}$ $\sin x = 0{\cdot}77645$ $x = 51°$
(b) (i)	period = 2π Range = [10, 30]

(b) (ii)

α	$0°$	$90°$	$180°$	$270°$	$360°$
$f(\alpha)$ (cm)	30	18·28	10	18·28	30

(b) (iii)



(c)

$$r^2 = 36^2 + (31 + r)^2 - 2(36)(31 + r)\cos 10°$$
$$r^2 = 1296 + 961 + 62r + r^2 - (2232\cos 10° - 72r\cos 10°)$$
$$8{\cdot}906r = 58{\cdot}91$$
$$r = 6{\cdot}62$$
$$r = 7$$

Q6. 2016 Paper 2 Question 7

Trigonometry 2 Solutions

Q7	Model Solution – 55 Marks
(a) (i)	$\lvert EC\rvert^2 = 3^2 + 2{\cdot}5^2 = 15{\cdot}25$ $\lvert EC\rvert = \sqrt{15{\cdot}25}$ $\lvert EC\rvert = 3{\cdot}905$ $\Rightarrow \lvert AC\rvert = 1{\cdot}9525$ $= 1{\cdot}95$
(a) (ii)	$\tan 50^\circ = \frac{\lvert AB\rvert}{1{\cdot}95}$ $\lvert AB\rvert = 1{\cdot}95(1{\cdot}19175) = 2{\cdot}23239$ $\lvert AB\rvert = 2{\cdot}3$
(a) (iii)	$\lvert BC\rvert^2 = 1{\cdot}95^2 + 2{\cdot}3^2$ $\lvert BC\rvert = 3\cdot015377$ $\lvert BC\rvert = 3$ Also: $\sin 40^\circ = \frac{1{\cdot}95}{\lvert BC\rvert}$ or $\cos 40^\circ = \frac{2{\cdot}3}{\lvert BC\rvert}$ or $\cos 50^\circ = \frac{1{\cdot}95}{\lvert BC\rvert}$ or $\sin 50^\circ = \frac{2{\cdot}3}{\lvert BC\rvert}$
(a) (iv)	$3^2 = 3^2 + 2{\cdot}5^2 - 2(3)(2{\cdot}5)\cos\propto$ $15\cos\propto = 6{\cdot}25$ $\propto = 65^\circ$ or $\cos\propto = \frac{1{\cdot}25}{3}$ $\propto = 65^\circ$

(a) (v)	$A = 2\times$ isosceles triangle $+ 2\times$ equilateral triangle $= 2\times\left[\frac{1}{2}(2{\cdot}5)(3)\sin 65^\circ\right] +$ $2\times\left[\frac{1}{2}(3)(3)\sin 60^\circ\right]$ $= 14{\cdot}59$ $A=15$

(b)	$\tan 60^\circ = \frac{3}{\lvert CA\rvert}$ $\Rightarrow \lvert CA\rvert = \sqrt{3}$ $\lvert CE\rvert = 2\sqrt{3}$ $x^2 + x^2 = (2\sqrt{3})^2$ $x = \sqrt{6}$

