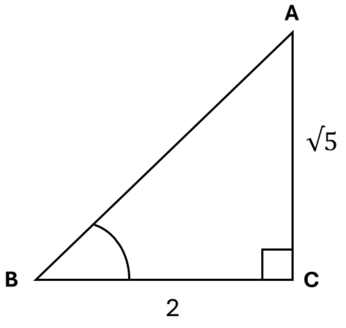
Trigonometry 1 Solutions

<u>Exercises:</u>

Q1

Draw a diagram:



Using Pythagoras $|AB|^2 = \sqrt{5}^2 + 2^2$

$|AB|^2 = 9 \Rightarrow |AB| = 3$

- ⇨ Sin B = $\frac{\sqrt{5}}{3}$
- ⇨ Cos B = $\frac{2}{3}$

Q2

(i)

$$\frac{8}{\sin(27)} = \frac{x}{\sin(89)}$$

- ⇨ X = $\frac{8*sin(89)}{\sin(27)}$ = 17.61cm

(ii)

$$\frac{5}{\sin(53)} = \frac{x}{\sin(25)}$$

- ⇨ X = $\frac{5*sin(25)}{\sin(53)}$ = 2.65cm

(iii)

$$\frac{10}{\sin(55)} = \frac{x}{\sin(65)}$$

- ⇨ X = $\frac{10*sin(65)}{\sin(55)} = 11.06$cm

Q3

< BAC = 180 – 90 – 20 = 70^0

|AC|:

$$\frac{32}{\sin(90)} = \frac{|AC|}{\sin(20)}$$

|AC| = $\frac{32*\sin(20)}{\sin(90)}$ = 10.94

|BC|:

Using Pythagoras, $32^2 = 10.94^2 + |BC|^2$

$|BC|^2 = 904.3164 \Rightarrow |BC| = 30.07$

Q4

(i)

$$\frac{10}{\sin(45)} = \frac{|SQ|}{\sin(30)}$$

$|SQ| = \frac{10*\sin(30)}{\sin(45)} = 5\sqrt{2}$m

(ii)

Using Pythagoras: $|SR|^2 + 3^2 = (5\sqrt{2})^2$

$|SR|^2 = 41 \Rightarrow |SR| = \sqrt{41}$m

(iii)

$$\frac{10}{\sin(45)} = \frac{|PS|}{\sin(115)}$$

$|PS| = \frac{10*\sin(105)}{\sin(45)} = 5+5\sqrt{3}$m

Q5

Using Cosine Rule: $|BC|^2 = 8^2 + 13^2 – 2*8*13*\text{Cos}(85)$

$|BC|^2 = 214.8716 \Rightarrow |BC| = 14.66$cm

For angle B, using Cosine Rule: $13^2 = 8^2 + 14.66^2 – 2*8*14.66*\text{Cos}(B)$

-109.9156 = -234.56*Cos(B)

$B = \text{Cos}^{-1}(\frac{-109.9156}{-234.56}) = 62.06^0$

Angle C = 180 – 85 – 62.06 = 32.94^0

Q6

Using formula: $\frac{1}{2}*7*18*\text{Sin}(65)$

= 57.097cm^2

Q7

All angles are 60^0 and all sides are the same length.

Let length of triangle = x.

Using formula: $\frac{1}{2}*x*x*\text{Sin}(60) = 9\sqrt{3}$

- ⇨ $X^2 = \frac{2}{Sin(60)} * 9\sqrt{3}$
- ⇨ $X^2 = 36 \Rightarrow X = 6$

Q8

(i) π radians = 180^0

$\frac{\pi}{5}$ radians = $\frac{180^0}{5}$ = 36^0

(ii) $180^o = \pi$ radians

$1^0 = \frac{\pi}{180}$ radians

$210^0 = \frac{210\pi}{180}$ radians = $\frac{7\pi}{6}$ radians

Exam Questions:

Q1 - 2022 Paper 2 Question 4b

Q4	Model Solution – 30 Marks
(b)	*Sine Rule*: $\frac{180-45}{2} = 67{\cdot}5°$ $\frac{x}{\sin 67{\cdot}5} = \frac{10\sqrt{2-\sqrt{2}}}{\sin 45}$ $x = \frac{10\sqrt{2-\sqrt{2}}\ \sin 67{\cdot}5}{\sin 45} = 10$ **OR** *Cosine Rule:* $\left(10\sqrt{2-\sqrt{2}}\right)^2 = x^2 + x^2 - 2(x)(x)\cos 45°$ $(2-\sqrt{2})x^2 = 100(2-\sqrt{2})$ $x = 10$ [as $x > 0$]

Q2 - 2022 Paper 2 Question 9

Q9	Model Solution – 50 Marks
(a)	Area Field 1 $= \frac{1}{2}(35)(30)\sin 50$ $= 402{\cdot}1 \ldots = 402$ [m²] [nearest m²] Area Field 2 $= \frac{1}{3} \times$ Area Field 1 [*since both have common perp. height*] $= \frac{402{\cdot}1\ldots}{3} = 134$ [m²] [nearest m²] **OR** Total area $= \frac{1}{2}(35)(40)\sin 50 = 536{\cdot}2 \ldots$ So, area Field 2 $= 536{\cdot}2 \ldots - 402{\cdot}1 \ldots$ $= 134$ [m²] [nearest m²]
(b)	$\|CB\|^2 = 35^2 + 30^2 - 2(35)(30)\cos 50$ $= 775{\cdot}14 \ldots$ $\|CB\| = \sqrt{775{\cdot}14\ldots} = 27.8 \ldots = 28$ [m] [$\in \mathbb{N}$] Perimeter $= 35 + 30 + 28 = 93$ [m] [$\in \mathbb{N}$]

Q3 – 2020 Paper 2 Question 3

Trigonometry 1 Solutions

Q3	Model Solution – 25 Marks
(a)	$\frac{6}{\sin 17°} = \frac{\lvert HF\rvert}{\sin 35°}$ $\lvert HF\rvert = \frac{6\sin 35°}{\sin 17°} = 11{\cdot}77$ $\frac{11{\cdot}77}{\sin 95°} = \frac{x}{\sin 33°}$ $x = \frac{11{\cdot}77(\sin 33°)}{\sin 95°}$ $x = 6{\cdot}43$ m

Q4 – 2019 Paper 2 Question 9

Q9	Model Solution – 55 Marks
(a)	$\lvert SG\rvert^2 = 30^2 + 58^2 - 2(30)(58)(\cos 68)$ $= 2960{\cdot}369$ $\lvert SG\rvert = 54{\cdot}409$ m $\lvert SG\rvert = 54{\cdot}4$
(b)	$\frac{54{\cdot}4}{\sin 68} = \frac{30}{\sin \angle HSG}$ $\sin \angle HSG = 0{\cdot}51131$ $\lvert \angle HSG\rvert = 30{\cdot}75$ Or $\cos \angle HSG = \frac{54{\cdot}4^2 + 58^2 - 30^2}{2(54{\cdot}4)(58)}$ $= 0 \cdot 859432$ $\lvert \angle HSG\rvert = 30{\cdot}747^{\circ} = 30{\cdot}75$
(c)	Area $\Delta GSH = \frac{1}{2}(30)(58)\sin 68 = 806{\cdot}65$ Also Area ΔGSH: $\frac{1}{2}(54{\cdot}4)(58)\sin 30{\cdot}75$ and $\frac{1}{2}(54{\cdot}4)(30)\sin 81{\cdot}25$

Q5 - 2023 Paper 2 Question 7b

(b)	$\lvert\angle POR\rvert = 5^\circ$ $\dfrac{\lvert RO\rvert}{\sin 87} = \dfrac{20}{\sin 5}$ $\lvert RO\rvert = \dfrac{20\sin 87}{\sin 5}$ $\lvert RO\rvert = 229\cdot 16\text{ m}$ $\tan 17 = \dfrac{\lvert HO\rvert}{229\cdot 16}$ $\lvert HO\rvert = 229\cdot 16\tan 17$ $\lvert HO\rvert = 70\text{ [m]}$

Q6 - 2023 Paper 2 Question 9b

Q9	**Model Solution – 50 Marks**
(b) (i)	$4^2 = 6^2 + 8^2 - 2(6)(8)\cos\alpha$ $\cos\alpha = \dfrac{6^2+8^2-4^2}{2(6)(8)}$ $\cos\alpha = \dfrac{84}{96} = \dfrac{7}{8}$ $\alpha = \cos^{-1}\dfrac{7}{8}$
(b) (ii)	$\cos\alpha = \frac{8}{\lvert AC\rvert} = \frac{7}{8}$ so $\lvert AC\rvert = \frac{64}{7}$ $\lvert CD\rvert = \sqrt{\left(\frac{64}{7}\right)^2 - 8^2} = \frac{8\sqrt{15}}{7}$ Area $= 2\left[\frac{1}{2}(8)\left(\frac{8\sqrt{15}}{7}\right)\right]$ $= \frac{64\sqrt{15}}{7} = 35.410... = 35.41\ cm^2$ [2 d.p.] **OR** $\alpha = 28\cdot 955^\circ$ $\lvert AC\rvert = \dfrac{64}{7}$ Area $= 2\left[\frac{1}{2}(8)\left(\frac{64}{7}\right)\sin 28\cdot 96\right] = 35\cdot 410..$ $= 35.41\ cm^2$ [2 d.p.], **OR** $\alpha = 28\cdot 955^\circ$ $\tan 28.955 = \dfrac{\lvert CD\rvert}{8}$ $\lvert CD\rvert = 4.426$ Area $= 2[\frac{1}{2}\times 8\times 4.426] = 35.410\ ...$ $= 35.41\ cm^2$ [2 d.p.]

