Society of Actuaries in Ireland

Trigonometry 1

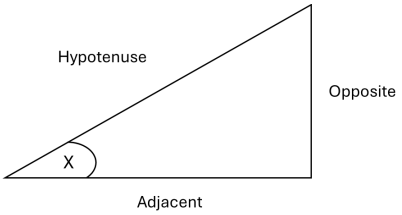
1. Right angled Triangles

1.1 Pythagoras Theorem

Pythagoras Theorem $hyp^2 = opp^2 + adj^2$

1.2 Trigonometric Ratios:

- Trigonometric ratios are the ratios of the sides of a right-angled triangle



$$\text{Sin } x = \frac{Opposite}{Hypotenuse} \quad \text{Cos } x = \frac{Adjacent}{Hypotenuse} \quad Tan\ x = \frac{Opposite}{Adjacent}$$

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Silly **O**ld **H**arry, **C**aught **A** **H**erring, **T**rawling **O**ff **A**merica

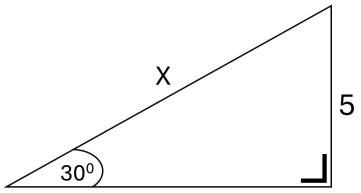
1.3 Uses:

These ratios can be used to find the:

(i) If we know all the angles then we can get the sides of the triangle

Example:

Find the length of the side X in the below triangle



Solution:

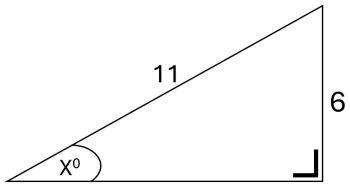
We have the opposite side and are looking for the hypotenuse. The trigonometric ratio that links opposite and hypotenuse is Sin.

- ⇨ Sin(X) = $\frac{O}{H}$
- ⇨ Sin(30) = $\frac{5}{H}$
- ⇨ $\frac{1}{2} = \frac{5}{H}$
- ⇨ H = 10 (Cross multiply)

(ii) If we know all the sides, then we can get the angles of the triangle

Example:

Find the angle X in the below triangle correct to 2 decimal places:



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Solution:

We have the adjacent and hypotenuse. The trigonometric ratio that links adjacent and hypotenuse is Cos.

- ⇨ Cos(X) = $\frac{A}{H}$
- ⇨ Cos(X) = $\frac{6}{11}$
- ⇨ X = $Cos^{-1}(\frac{6}{11})$ (If looking for an angle on it's own, we can use the inverse of the trigonometric function).
- ⇨ X = 56.94^0 (2 decimal places)

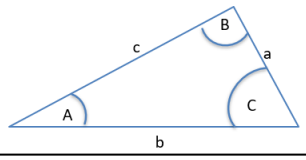
2. All Triangles

The following rules can be used for all triangles. Remember the trigonometric ratios can only be used with right-angled triangles

2.1 Sine Rule

An angle/side pair is the angle and the side directly opposite that angle.

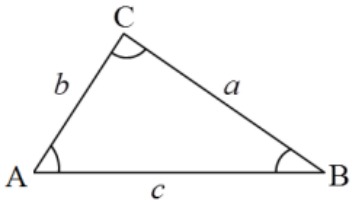
- a, b, c are the lengths of the sides
- A, B, C are the angles
- A opposite a
- B opposite b
- C opposite c



The sine rule is used when you are dealing with 2 pairs of angles and sides and one part is unknown.

If you are trying to find a missing side:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

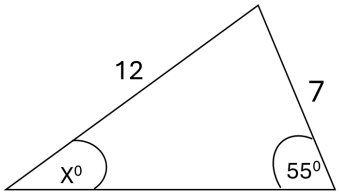


If you are trying to find a missing angle:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Example:

Find the angle X in the below triangle correct to 2 decimal places.



Solution:

Using the Sine rule:

- ⇨ $\frac{Sin\ X}{7} = \frac{Sin\ 55}{12}$
- ⇨ Sin X = $\frac{7 * Sin\ 55}{12}$
- ⇨ X = $Sin^{-1}(\frac{7 * Sin\ 55}{12}) = 28.54^0$

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2.2 Cosine Rule

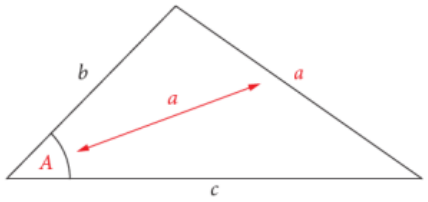
Used when dealing with 3 sides and an angle and one of those is unknown.

To be used when dealing with three sides and an angle.

Let the angle/side pair be the As.

Then,

$$a^2 = b^2 + c^2 - 2bc\cos A$$

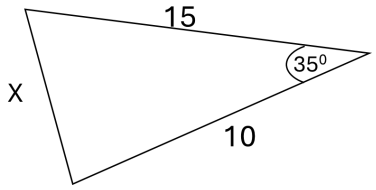


It can help to put a bracket around the last part:

$$a^2 = b^2 + c^2 - [2bc\cos A]$$

Example:

Find the missing length X in the below triangle correct to 2 decimal places.



Solution:

Filling in the values to the Cosine formula:

a (unknown side) = X

b = 10, c = 15, A = 35^0

$a^2 = b^2 + c^2 - (2bc\ CosA)$

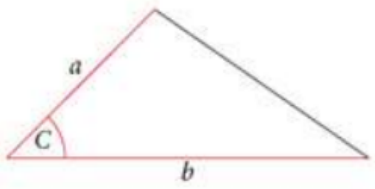
$a^2 = 10^2 + 15^2 - (2 * 10 * 15 * Cos(35))$

$a^2 = 57.7456 \Rightarrow a = 23.89$

2.3 Area of Triangle

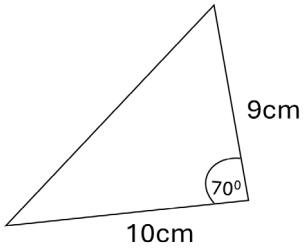
When dealing with a triangle, where we are given two sides, *a* and *b*, and measure of the angle in between these sides, we use the following formula:

$$\text{Area} = \frac{1}{2}ab\sin C$$



Example:

Find the area of the following triangle:



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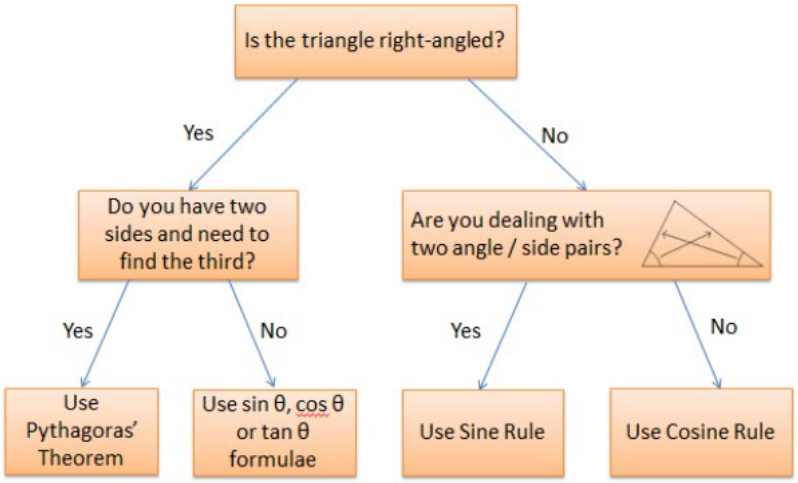
Solution:

Subbing into the equation:

- ⇨ Area = $\frac{1}{2}abSinC$
- ⇨ Area = $\frac{1}{2}10 * 9 * Sin70$
- ⇨ Area = 42.29cm^2

3. Flowchart for Triangle Questions:



4. Degrees and Radians

A radian is the angle when the length of the arc is the same as the length of the radius.

Main piece to know is the following conversion between radians and degrees:

180^o = π radians

360^o = 2π radians

You need to be careful that your calculator is set correctly to degrees or radians depending on the question. Make sure you know how to change between degrees and radians on your calculator.

