Algebra – Review Sheet

Simplifying an expression.

What order should you follow?

Brackets
Exponents
Divide | Multiply
Add | Subtract

To manipulate an equation,

You can:

- Add or subtract the same number to/from both sides
- Multiply or divide both sides by the same number
- Square both sides, cube both sides, etc.
- Take the square root of both sides or cube root, etc

Simple Algebraic Rules

- Multiplying Terms
  (a) x (bc) = abc
  (a) x (b+c) = ab + ac
  (a+b) x (c+d) = a(c+d) + b(c+d)

- Anything to the power of 0 is 1.
  $A^0 = 1$, $473^0 = 1$, $\pi^0 = 1$, $1{,}000{,}000^0 = 1$

- Inverse Powers $X^{-1} = \frac{1}{X}$, $Y^{-6} = \frac{1}{Y^6}$
- Indices
  $Y^3 + Y^2 = (Y \times Y \times Y) + (Y \times Y)$ ..... Can't be simplified

  $(Y^2)^3 = (Y \times Y)^3 = (Y \times Y) \times (Y \times Y) \times (Y \times Y) = Y^6$ ..... multiply the two powers =>2 x 3 = 6

  $Y^2 \times Y^3 = (Y \times Y) \times (Y \times Y \times Y) = Y^5$ ..... add the two powers => 2 + 3 = 5

  $\frac{Y^3}{Y^2} = \frac{Y \times Y \times Y}{Y \times Y} = Y^1$ ..... minus the power below the line from the power above the line => 3 – 2 = 1

Fractions

Addition

$$\frac{1}{A} + \frac{1}{B} = \frac{1}{A} \times \frac{B}{B} + \frac{1}{B} \times \frac{A}{A} = \frac{B + A}{A \times B}$$

Remember, $\frac{A}{A} = 1$ and $\frac{B}{B} = 1$ so we are just changing how we present the terms

Multiplication

$$\frac{1}{A} \times \frac{1}{B} = \frac{1}{A \times B}$$

Division

$$\frac{1}{A} \div \frac{1}{B} = \frac{1}{A} \times \frac{B}{1}$$
$$= \frac{B}{A}$$

We are able to change the division sign to a multiply sign by inverting the second fraction (=> turning it upside down)

So, 'dividing by $\frac{1}{B}$' becomes 'multiplying by $\frac{B}{1}$'

Remember:

$$\frac{A}{A \times B} = \frac{1}{B}$$ ✓

$$\frac{A}{A+B} = \frac{1}{B}$$ ✗

$$\frac{X^2+2X}{X} = \frac{X(X+2)}{X} = \frac{X+2}{1} = X + 2$$ ✓

$$\frac{X+2}{X} = 2$$ ✗

Lowest Common Denominator

When you want to sum (or subtract) two fractions, you need to find a common denominator. The easiest way is usually to multiply the two denominators (this will give you a common denominator, but it won't necessarily be the lowest one). E.g.

$$\frac{1}{7} + \frac{3}{5}$$

Lowest Common denominator = 7 x 5 = 35

Now multiply so that you have the same denominator in each fraction:

$\frac{5}{5} x \frac{1}{7} + \frac{3}{5} x \frac{7}{7}$ (effectively multiplying by 1)

$= \frac{5}{35} + \frac{21}{35}$ (you can combine once they have the same denominator)

$= \frac{26}{35}$

The same applies for algebraic fractions, e.g.

$$\frac{x}{y} + \frac{2x}{z}$$

Lowest Common denominator = yz

$\frac{z}{z} x \frac{x}{y} + \frac{2x}{z} x \frac{y}{y}$

$= \frac{zx}{zy} + \frac{2xy}{zy}$

$= \frac{zx+2xy}{zy}$

Surds

Properties of surds...

$\sqrt{ab} = \sqrt{a}\sqrt{b}$
$\sqrt{\left(\frac{a}{b}\right)} = \frac{\sqrt{a}}{\sqrt{b}}$
$\sqrt{a}\sqrt{a} = a$

To simplify surds, find the largest possible perfect square number greater than 1 that will divide evenly into the number under the square root.

Then use the first property above. E.g. $\sqrt{63} = \sqrt{(9x7)}$ = $\sqrt{9}$ x $\sqrt{7}$ = 3 $\sqrt{7}$

Algebra – Review Sheet

Indices and Logarithms (From your Log Tables – Very Important!!!)

Indices

$$a^p a^q = a^{p+q} \qquad a^{\frac{1}{q}} = \sqrt[q]{a}$$

$$\frac{a^p}{a^q} = a^{p-q} \qquad a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$$

$$(a^p)^q = a^{pq} \qquad (ab)^p = a^p b^p$$

$$a^0 = 1 \qquad \left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

$$a^{-p} = \frac{1}{a^p}$$

Logarithms

General rule of logs: $a = b^c \iff \log_b a = c$

$$\log_a(xy) = \log_a x + \log_a y \qquad \log_a\left(\frac{1}{x}\right) = -\log_a x$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y \qquad \log_a(a^x) = x$$

$$\log_a x^q = q \log_a x \qquad a^{\log_a x} = x$$

$$\log_a 1 = 0 \qquad \log_b x = \frac{\log_a x}{\log_a b}$$

Common mistake with Indices

$(A + B)^2 = A^2 + B^2$ ✗

$(A + B)^2 = (A + B) \times (A + B)$
$= A \times (A + B) + B \times (A + B)$
$= A^2 + AB + AB + B^2$
$= A^2 + 2AB + B^2$ ✓

Factorisation

Common forms of algebraic equations you might need to factorise:

Take out common terms $ab + ad = a(b + d)$	**Factorise by grouping** $ab + ad + cb + cd = a(b + d) + c(b + d)$ $= (a + c)(b + d)$
Factorise a trinomial $a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$	**Difference of two squares** $a^2 - b^2 = (a + b)(a - b)$
Difference of two cubes $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	**Sum of two cubes** $a^3 + b^3 = (a + b)(a^2 - 2ab + b^2)$

Solving Equations

Solving Quadratic Equations (i.e. finding the roots of a quadratic equation)

You can either:

A. Factorise & let each factor = 0
   e.g. $x^2 + 3x - 18 = 0$ ...look at the factors of -18 => (-1x18, -2x9, -3x6 etc.)

OR

B. Use the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ...be careful of the sign in front of each of the terms!

OR

C. Complete the square (write in vertex form) and set = 0

All three approaches will work. Some quadratics are easier to factorise than others, but you can always use the formula approach – even for simple quadratics.

Regarding the formula:

- If $b^2 - 4ac > 0$, the equation has two real distinct roots
- If $b^2 - 4ac = 0$, the equation has two equal real roots
- If $b^2 - 4ac < 0$, the equation has no real roots

Factor theorem

"If an algebraic expression is divided by one of its factors, then the remainder is zero. The expression (x – k) is a factor of a polynomial f(x) if the remainder when we divide f(x) by (x – k) is zero."

- If f(k) = 0, then (x – k) is a factor of f(x)
- If (x – k) is a factor of f(x), then f(k) = 0

Solving cubic equations ($ax^3 + bx^2 + cx + d = 0$)

- Find the first root, k, by trial and error
- If x = k is a root, then (x – k) is a factor
- Divide f(x) by (x – k), which always gives a quadratic expression
- Find the factors of the quadratic and then find the roots of the quadratic

Inequalities

Always remember that multiplying or dividing both sides of an inequality by a negative number reverses the direction of the inequality symbol

Quadratic inequalities

Replace >, <, ≥, ≤ with = to make an equation

- Solve the equation to find the roots -> these are called the critical values of the inequality & the solution either will be between these critical values or outside these critical values
- Test a number between the critical values (often 0) in the original inequality

Two possibilities arise:

- If the inequality is true, then the solution lies between the critical values
- If the inequality is false, then the solution does not lie between the critical values

(**Note:** that if the inequality uses < or >, the critical values are not included in the solution set, whereas if the inequality uses ≤ or ≥, the critical values are included in the solution set)

Modulus inequalities

If $|x| \leq a$, then $-a \leq x \leq a$

(**Note:** "mod x" is the same as "|x|")

If $|x| \geq a$, then $x \leq -a$ or $x \geq a$