Algebra 1 – Solutions

Question 1

(i) Solve for x:

$$2(4-3x)+12=7x-5(2x-7).$$

$$8-6x+12=7x-10x+35$$
$$-15=3x$$
$$x=-5$$

(ii) Verify your answer to **(i)** above.

$x=-5$

$2(4-(-15))+12$	$7(-5)-5(-10-7)$
$38+12$	$-35+85$
50	50

$[50=50]$

Question 2:

$$x=7-y$$
$$(7-y)^2+y^2=25$$
$$y^2-7y+12=0$$
$$(y-4)(y-3)=0$$

$y=4$	$y=3$
$x=7-4$	$x=7-3$
$x=3$	$x=4$
$(3,4)$	$(4,3)$

Question 3:

Factorise the numerator (the top line of the equation) and denominator (bottom line of the equation- difference of two squares)

$$\frac{x(x-y)}{(x+y)(x-y)}$$
$$\frac{x(x-y)}{(x+y)(x-y)}$$
$$=\frac{x}{(x+y)}$$

Question 4:

Express the following as a single fraction in its simplest form:

$$\frac{6y}{x(x+4y)}-\frac{3}{2x}$$

Step 1:

Find the common denominator by multiplying the bottom lines:

$2x * x(x+4y) = 2x^2(x+4y)$. So $2x^2(x+4y)$ is our **common denominator**

Step 2:

Find the numerator by cross multiplying (top lines by bottom lines):

$6y*2x - 3*x(x+4y)$ $= 12xy - 3*(x^2+4xy)$

$=12xy - 3x^2 - 12xy$

$= -3x^2$. So $-3x^2$ is our **numerator**

Step 3:

The answer is the numerator divided by the denominator:

$$\frac{6y}{x(x+4y)} - \frac{3}{2x} = \frac{\textbf{Numerator}}{\textbf{Denominator}}$$
$$= \frac{-3x^2}{2x^2(x+4y)}$$
$$= \frac{-3}{2(x+4y)}$$
$$= \frac{-3}{2x+8y}$$

Question 5:

Solve the simultaneous equations:

$$x^2 + xy + 2y^2 = 4 \quad \textit{Equation (1)}$$
$$2x + 3y = -1. \quad \textit{Equation (2)}$$

Find x in terms of y using the linear equation; Equation (2) *2x + 3y = -1*

$$x = \frac{-3y-1}{2}$$

Substitute $x = \frac{-3y-1}{2}$ *into Equation (1)*

$$\left(\frac{-3y-1}{2}\right)^2 + \left(\frac{-3y-1}{2}\right)y + 2y^2 = 4$$

Multiply across by 4

$$(-3y-1)^2 + (-3y-1)2y + 8y^2 = 16$$

Expand the bracket and take the 16 over to the left-hand side

$$9y^2 + 6y + 1 - 6y^2 - 2y + 8y^2 - 16 = 0$$

Group terms together

$$11y^2 + 4y - 15 = 0$$
$$(11y + 15)(y - 1) = 0$$
$$y = \frac{-15}{11} \text{ or } y = 1$$

Substitute $y = \frac{-15}{11}$ *or* $y = 1$ *into Equation (2) to solve for x*

$2x + 3\left(\frac{-15}{11}\right) = -1$ OR $2x + 3(1) = -1$

$2x + \left(\frac{-45}{11}\right) = -1$ OR $2x + 3 = -1$

$2x = -1 + \frac{45}{11}$ OR 2x = -4

$x = \frac{17}{11}$ OR x = -2

Give the answer matching the appropriate x and y values:

Answer = $\left(\frac{17}{11}, \frac{-15}{11}\right)$ *and* $(-2,1)$

Question 6:

Express the following as a single fraction in its simplest form:

$$\frac{x^2+4}{x^2-4} - \frac{x}{x+2}$$

Hint: $x^2 - 4$ is the difference between two squares i.e. $(x)^2 - (2)^2 = (x + 2)(x - 2)$

Step 1:

Find the common denominator by multiplying the bottom lines:

So $(x^2 - 4)(x+2)$ is our **common denominator**

Step 2:

Find the numerator by cross multiplying (top lines by bottom lines):

$(x^2 + 4)(x+2) - x(x^2 - 4)$ = $(x+2) * [(x^2 + 4) - x(x-2)]$ .... because : $x^2 - 4 = (x + 2)(x - 2)$

So $(x+2) * [(x^2 + 4) – x(x-2)]$ is our **numerator**

Step 3:

The answer is the numerator divided by the denominator:

$$\frac{x^2+4}{x^2-4} - \frac{x}{x+2} = \frac{\textbf{Numerator}}{\textbf{Denominator}}$$

$$= \frac{(x+2) * [(x^2+4) - x(x-2)]}{(x^2-4)(x+2)}$$

$$= \frac{(x^2+4) - x(x-2)}{(x^2-4)} \quad \text{...(x+2) cancels}$$

$$= \frac{(x^2+4) - x^2 + 2x}{(x^2-4)}$$

$$= \frac{2x+4}{(x^2-4)}$$

$$= \frac{2(x+2)}{(x+2)(x-2)} \quad \text{...difference of two squares}$$

$$= \frac{\mathbf{2}}{\mathbf{(x-2)}}$$

Question 7:

Find the range of values of x for which $|x - 4| \geq 2$, where $x \in \mathbb{R}$

Method 1:

Expand the bracket $x^2 - 8x + 16 \geq 4$

Take the 4 to the left hand side $x^2 - 8x + 12 \geq 0$

Solve for the roots of the equation $(x-2)(x-6) \geq 0$

$x = 2, \quad x = 6$

Answer $x \leq 2 \; or \; x \geq 6$

Method 2:

Split into 2 separate equations: $+(x-4) \geq 2 \; or \; -(x-4) \geq 2$

Solve each equation separately:

$x - 4 \geq 2$ OR $-(x-4) \geq 2$

$+(x-4) \leq -2$

$x - 4 + 4 \geq 2 + 4$ $x - 4 + 4 \leq -2 + 4$

$x \geq 6$ $x \leq 2$

Question 8:

Find the set of all real values of x for which $2x^2 + x - 15 \geq 0$.

Step 1:

For inequalities, first set the equation = 0 and solve.

$2x^2 + x – 15 = 0$

$(2x-5) (x+3) = 0$

$2x-5=0$ $x+3=0$

$x =2.5$ $x= -3$

Step 2:

Then look at the sign in the inequality in the question.

If the sign is ≤ 0 we are “between the posts” which means the answer will be in the format of

number ≤ x ≤ number e.g. $-3 \leq x \leq 2.5$

If the sign is ≥ 0 we are “outside the posts” which means the answer will be in the format of x ≤ number and x ≥ number e.g. x≤-3 and x ≥ 2.5

Step 3:

Therefore, in this case the answer is x≤-3 and x ≥ 2.5 *(You can sub in values to the original question to check if your answer is correct!)*

Question 9:

$$x=\sqrt{x+6}$$
$$\Rightarrow x^2=x+6$$
$$\Rightarrow x^2-x-6=0$$
$$\Rightarrow (x+2)(x-3)=0$$
$$\Rightarrow x=-2,\quad x=3$$
$$x=-2:\quad -2\neq\sqrt{-2+6}=\sqrt{4}=2 \ \times$$
$$x=3:\quad 3=\sqrt{3+6}=\sqrt{9}=3 \ \checkmark$$

Question 10:

Solve the following for x, y and z.

x + 2y – z = 1 (1)

2x + y + z = 4 (2)

x + 2y + z = 2 (3)

Step 1:

Add two equations together to find equations (4) and (5):

x + 2y – z = 1 .... (1)

x + 2y + z = 2 .... (3)

2x+4y = 3 .... (4)

x + 2y – z = 1 .... (1)

2x + y + z = 4 .... (2)

3x + 3y = 5 .... (5)

Step 2:

Solve equations (4) and (5):

2x+4y = 3 .... (4)

3x + 3y = 5 .... (5)

6x + 12y = 9 .... (4) (x 3)

-6x - 6y = -10 .... (5) (x -2)

6y = -1

y = -1/6

2x+4y = 3 (equation 4)

2x +4(-1/6) = 3

x = 11/6

x + 2y – z = 1 (Equation 1)

(11/6) + 2(-1/6) – z = 1

z = 1/2

Algebra 1 – Solutions

(You can sub in values to the original question to check if your answer is correct!)

Question 11:

Solve the equation |4x – 3| > 5

Step 1:

Set the equation = instead of less than/greater than and solve |4x - 3| = 5. This could mean that

4x - 3= 5	or	4x - 3= -5
4x = 5+3		4x = -5 + 3
4x = 8		4x = -2
x = 2		x = -½

Step 2:

Is the answer "between the roots" or "outside the roots"?

In this question the sign is > so the answer is outside the roots. So the answer is: **x > 2 and x < -½**

Question 12:

Step 1:

Set the equation = instead of less than/greater than and solve |3x + 2| < 4. This could mean that

3x + 2= 4	or	3x + 2= -4
3x = 4-2		3x = -4 -2
3x = 2		3x = -6
x = ⅔		x = -2

Step 2:

Is the answer "between the roots" or "outside the roots"?

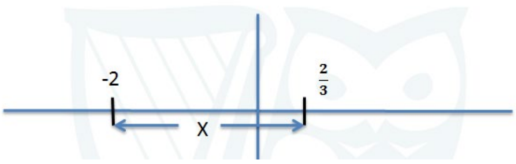
In this question the sign is < so the answer is between the roots. So the answer is: **-2 < x < ⅔**

Step 3:

Is the answer between the roots or outside the roots?

(Note: that x = -2 and x = ⅔ should not be included on the number in the shaded region as inequality does not include equals, it's just less than)

This answer is between the roots so on a graph this looks like:



Question 13:

Step 1:

Set the equation = 0

$$f(x) = 2x^3 - 4x^2 - 22x + 24 = 0$$

Step 2:

To find x, try a few values of x and see if they give you zero.

Try x=0 first:

$$f(0) = 2*0 - 4*0 - 22*0 + 24$$
$$= 24 \neq 0$$

This is not equal to zero so x=0 is not a solution (or "root").

Try x=1:

$f(1) = 2*1 - 4*1 - 22*1 + 24$

$= 0\,!$

This is equal to zero so **x=1** is a solution (or "**root**") which means that **(x-1)** is a **factor**.

Step 3:

Divide the equation by the factor you just found.

$$2x^2 - 2x - 24$$
$$(x-1)\sqrt{2x^3 - 4x^2 - 22x + 24}$$
$$2x^3 - 2x^2$$
$$-2x^2 - 22x + 24$$
$$-2x^2 + 2x$$
$$-24x + 24$$
$$-24x + 24$$

Step 4:

Factorise fully.

$2x^3 - 4x^2 - 22x + 24 \quad = (x-1)(2x^2 - 2x - 24)$

$= (x-1)(2x+6)(x-4)$

Step 5:

Pull out the final answers, also called "roots".

x-1 = 0 => **x=1**

2x+6 = 0

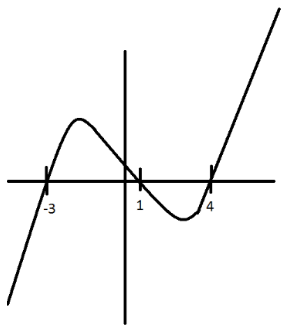
2x = -6 => **x = -3**

x-4 = 0 => **x = 4**

x = -3, 1 ,4 are the solutions

Step 6:

Make sure it crosses the x axis at your solutions x = -3, 1 ,4:



Question 14:

(a)

$x = -3, \quad x = -1, \quad x = 2$

$f(x) = (x+3)(x+1)(x-2) = x^3 + 2x^2 - 5x - 6$

OR

$f(x) = x^3 + 2x^2 - 5x - 6$

$f(-3) = -27 + 18 + 15 - 6 = 0 \Rightarrow (x+3)$ is a factor

$f(-1) = -1 + 2 + 5 - 6 = 0 \Rightarrow (x+1)$ is a factor

$f(2) = 8 + 8 - 10 - 6 = 0 \Rightarrow (x-2)$ is a factor

$f(x) = (x+3)(x+1)(x-2) = x^3 + 2x^2 - 5x - 6$

(b) (i)

$$f(x) = g(x)$$
$$x^3 + 2x^2 - 5x - 6 = -2x - 6$$
$$\Rightarrow x^3 + 2x^2 - 3x = 0$$
$$\Rightarrow x(x^2 + 2x - 3) = 0$$
$$\Rightarrow x(x-1)(x+3) = 0$$
$$\Rightarrow x = 0, \quad x = 1, \quad x = -3$$
$$\Rightarrow y = -6, \quad y = -8, \quad y = 0$$

Points: $(-3, 0), (0, -6), (1, -8)$

(ii)

$$g(x) = -2x - 6$$
$$g(-3) = -2(-3) - 6 = 6 - 6 = 0 \Rightarrow (-3, 0)$$
$$g(0) = -2(0) - 6 = -6 \Rightarrow (0, -6)$$

<u>Question 15</u>

a (i)

$$f(x) = x^3 + kx^2 - 4x - 12$$
$$(x+3) \text{ is factor } \Rightarrow f(-3) = 0$$
$$f(-3) = (-3)^3 + k(-3)^2 - 4(-3) - 12 = 0$$
$$-27 + 9k + 12 - 12 = 0$$
$$9k = 27 \quad \Rightarrow \quad k = 3$$

(ii)

$$\frac{3}{1+x^p} + \frac{3}{1+x^{-p}} = \frac{3(1+x^{-p}) + 3(1+x^p)}{(1+x^p)(1+x^{-p})}$$
$$= \frac{3(1+x^{-p}+1+x^p)}{1+x^p+x^{-p}+x^0}$$
$$= \frac{3(2+x^{-p}+x^p)}{(2+x^{-p}+x^p)}$$
$$= 3$$

Recent Exam Questions

1. **2024 Paper 1 Question 1**

(a)

$$(n-3)^2 = \left(\sqrt{3n+1}\right)^2$$
$$n^2 - 6n + 9 = 3n + 1$$
$$n^2 - 9n + 8 = 0$$
$$(n-8)(n-1) = 0$$
$$n = 8, n = 1$$

Answer: $n = 8$ [as $n = 1$ gives $-2 = \sqrt{4}$]

(b)

$$\frac{4}{2t+1} - \frac{7}{12t} = \frac{4(12t) - 7(2t+1)}{(2t+1)(12t)}$$
$$= \frac{48t - 14t - 7}{(2t+1)(12t)}$$
$$= \frac{34t - 7}{(2t+1)(12t)}$$

(c)

1:	$x + 2y \quad = 143$
2x(-2):	$-2y - 6w = 148$
4:	$x \quad - 6w = 291$
3:	$4x + 5w = 4$
4x(4):	$4x - 24w = 1164$
5:	$29w = -1160$
So	$w = -40$
4:	$x - 6(-40) = 291$
So	$x = 51$
1:	$51 + 2y = 143$
So	$y = 46$

2. **2022 Paper 1 Question 1**

(a)

$b^2 - 4ac = 0$

$m^2 - 4(3)(3) = 0$

$m^2 = 36$

$m = \pm 6$

OR

$(3x - 3)(x - 1) = 0$ so $m = 6$

Or $(3x + 3)(x + 1) = 0$ so $m = -6$

OR

Differentiates: $6x - m = 0$

So $x = \frac{m}{6}$

$3\left(\frac{m^2}{36}\right) - \frac{m^2}{6} + 3 = 0$

$m^2 = 36$

$\boldsymbol{m = \pm 6}$

OR

$x^2 - \frac{m}{3}x + 1 = 0$

Equal roots, α and α:

$2\alpha = \frac{m}{3}$ so $\alpha = \frac{m}{6}$

$\alpha^2 = 1$ so $\frac{m^2}{36} = 1$

So $m = \pm 6$

(b)

$(2x + 3)^2 \geq 0$ for all $x \in \mathbb{R}$

So $(2x + 3)^2 + 7 \geq 7 > 0$

OR

$(2x + 3)^2 = -7$

$2x + 3 = \pm\sqrt{-7}$, which is not real

OR

$4x^2 + 12x + 9 + 7 = 0$

$4x^2 + 12x + 16 = 0$

$b^2 - 4ac = 12^2 - 4(4)(16) < 0$,
so no real roots

OR

The graph of $y = (2x + 3)^2 + 7$ is U-shaped and the minimum value of y is 7, therefore, no real solutions.

(c)

(i) $3(-1)^2 + 2(-1) + 5$

$= 3 - 2 + 5 = 6 \neq 0$

(ii) $(x + 1)(ax + b) + c$

$= ax^2 + (a + b)x + b + c = 3x^2 + 2x + 5$

So $a = 3$,

$3 + b = 2$, so $b = -1$

$-1 + c = 5$, so $c = 6$.

OR

$$\begin{array}{r l} & 3x - 1 \\ x + 1 & \overline{)\,3x^2 + 2x + 5} \\ & \underline{3x^2 + 3x} \\ & \quad -x + 5 \\ & \quad \underline{-x - 1} \\ & \qquad 6 \end{array}$$

Remainder $= 6$

OR

	$3x$	-1
x	$3x^2$	$-x$
1	$3x$	-1

So remainder $= 5 - (-1) = 6$