Warm up questions

Question 1.

$F = P(1+i)^t$

$F = (4000)(1+0.03)^4$

F = €4,502.04

Interest = €4,502.04 - €4,000 = €502.04

Question 2.

$F = P(1+i)^t$

F = 1.12P

$1.12P = P(1 + i)^5$

$1.12 = (1 + i)^5$

$1 + i = \sqrt[5]{1.12}$

1 + i = 1.02292

I = 0.02292 = 2.3% = AER correct to 1 decimal place

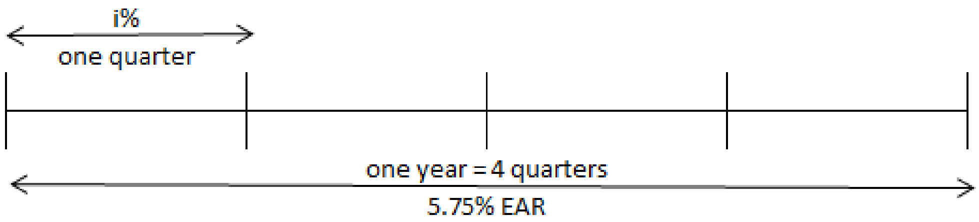
Rule for rounding “correct to 1 dp” is round to nearest number ?

Question 3.

$$\frac{2500}{(1+0.03)^7} = 2032.73$$

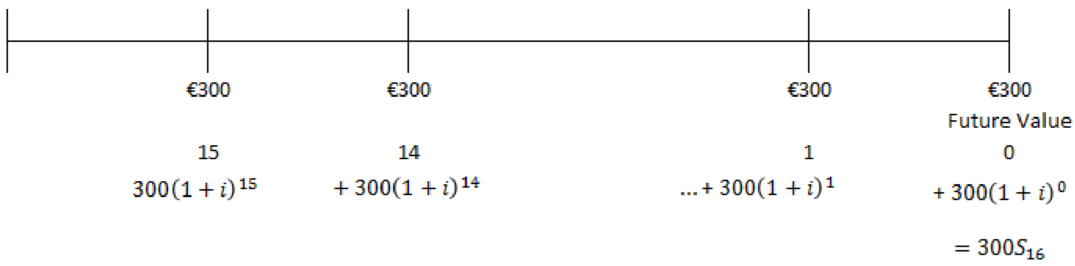
Compound Interest

Question 4.



$$(1+i)^4 = \left(1+\frac{5.75}{100}\right)^1$$

$$1+i = 1.0575^{\frac{1}{4}}$$



$$S_n = \frac{a(1-r^n)}{1-r}$$

$$a = 1$$

$$r = 1+i$$

So, from above, $r = 1.0575^{\frac{1}{4}}$

Hence, $S_{16} = \dfrac{1\left(1-\left(1.0575^{\frac{1}{4}}\right)^{16}\right)}{1-1.0575^{\frac{1}{4}}} = \dfrac{1-1.0575^4}{1-1.0575^{0.25}} = 17.80519556$

Investment is worth $300S_{16} = 300 \times 17.80519556 = 5341.558669$

$$\approx €5341.56$$

(Sense check: $€300\ per\ quarter = €1200\ per\ year = €4800\ in\ 4\ years$, plus interest seems ok.)

Question 5.

$$P(1+i)^t = F$$

$$9950(1+i)^5 = 12700$$

$$(1+i)^5 = \frac{12700}{9950}$$

$$[(1+i)^5]^{\frac{1}{5}} = \left[\frac{12700}{9950}\right]^{\frac{1}{5}}$$

$$1 + i = 1.050016511$$

$$i = 0.050016511 = 5.0016511\%$$

$$i \approx 5.00\%$$

Question 6.

Hint: The unknown (t) is the exponent. So, this involves using logs. Natural logs are easiest on the calculator.

$$P(1+i)^t = F$$

$$10000(1.05)^t = 13000$$

$$1.05^t = \frac{13000}{10000} = 1.3$$

$$\ln(1.05^t) = \ln 1.3$$

$$t \ln 1.05 = \ln 1.3$$

$$t = \frac{\ln 1.3}{\ln 1.05} = 5.377400731 \; years$$

$$t = 5.377400731 \times 12 = 64.52880877 \; months$$

$$t \approx 65 \; months$$

Question 7.

(i)

$$F = P(1+i)^t = 1(1+0\cdot0035)^{12} = 1\cdot042818$$
$$\Rightarrow i = 4\cdot28\%$$

(ii)

$$F = P(1+i)^t$$
$$1\cdot045 = 1(1+i)^{12}$$
$$1+i = \sqrt[12]{1\cdot045} = 1\cdot0036748$$
$$\Rightarrow i = 0\cdot367\%$$

(iii) Formula is on page 31 of Log Tables

$$A = P\left[\frac{i(1+i)^t}{(1+i)^t - 1}\right]$$
$$= 80000\left[\frac{0\cdot0035(1\cdot0035)^{120}}{(1\cdot0035)^{120} - 1}\right]$$
$$= 80000\left[\frac{0\cdot00532296}{0\cdot520846}\right]$$
$$= 817\cdot59 = €818$$

or

$$80000 = \frac{A}{1\cdot0035} + \frac{A}{1\cdot0035^2} + \ldots + \frac{A}{1\cdot0035^{120}}$$
$$= A\left[\frac{1}{1\cdot0035} + \frac{1}{1\cdot0035^2} + \ldots + \frac{1}{1\cdot0035^{120}}\right]$$
$$= A\left[\frac{\frac{1}{1\cdot0035}\left(1 - \left(\frac{1}{1\cdot0035}\right)^{120}\right)}{1 - \frac{1}{1\cdot0035}}\right]$$
$$= A\left[\frac{0\cdot342471198}{0.0035}\right]$$
$$= A\,[\,97\cdot8489137\,]$$

$$A = 817\cdot58 = €818$$

Present Value

Question 8.

(i)

$$(1+i)^{12} = 1\cdot 04 \Rightarrow 1+i = \sqrt[12]{1\cdot 04} = 1\cdot 003273 \Rightarrow i = 0\cdot 003274$$

Hence, $i = 0\cdot 327\%$

Or

$$(1.00327)^{12} = 1\cdot 039953481$$
$$= 1\cdot 0400$$

$r = 4\%$

Where i is the monthly rate of interest and r is the annual equivalent rate.

(ii) From Sn Formula on page 22 of Log Tables

$$15000 = P\left(1\cdot 00327^{36} + 1\cdot 00327^{35} + \cdots + 1\cdot 00327^{2} + 1\cdot 00327\right)$$
$$\Rightarrow P\left[\frac{1\cdot 00327\left(1\cdot 00327^{36} - 1\right)}{1\cdot 00327 - 1}\right] = 15000$$
$$\Rightarrow P[38\cdot 26326387] = 15000$$
$$\Rightarrow P = 392\cdot 02 \;=\; €392$$

(iii)

$$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$
$$= 15000\left[\frac{0\cdot 00866(1+0\cdot 00866)^{36}}{1\cdot 00866^{36} - 1}\right]$$
$$= 486\cdot 77$$

Monthly payment €487

Question 9. d

(i) $F = P(1 - i)^t$

$F = (20000)(1 - 0.15)^2$

$F = 20000\,(0.85)^2$

$F = €14,450$

(ii) $F = P(1 - i)^t$

$F = 14450(1 - 0.1)^3$

$F = 14450(0.9)^3$

$F = €10,534.05$

Question 10. D

Year	**Project X**	**Project Y**
0	PV = -5000	PV = -3000
1	$PV = \frac{-2000}{1.05^1} = -1904.76$	$PV = \frac{-1000}{1.05^1} = -952.38$
2	$PV = \frac{2000}{1.05^2} = 1814.06$	$PV = \frac{2000}{1.05^2} = 1814.06$
3	$PV = \frac{5000}{1.05^3} = 4319.19$	$PV = \frac{3000}{1.05^3} = 2591.51$
NPV	-771.51	453.19
Advice	Reject	Invest

Amortisation Questions

Question 11. (B)

(i)

$$2{\cdot}5\% \times 5000 = 125$$

(ii) To calculate monthly rate "i"

$$(1+i)^{\frac{1}{12}} = (1{\cdot}2175)^{\frac{1}{12}} = 1{\cdot}016535$$

$$Rate = 1{\cdot}65\%$$

(iii)

Payment number	Fixed monthly payment, €A	€A		New balance of debt (€)
		Interest	Previous balance reduced by (€)	
0				5000
1	125	82·50	42·50	4957·50
2	125	81·80	43·20	4914·30
3	125	81·09	43·91	4870·39

(iv) Source of formulae - Log Tables page 31

$$A = p\left[\frac{i(1+i)^t}{(1+i)^t - 1}\right]$$

$$A[(1+i)^t - 1] = pi(1+i)^t$$

$$A(1+i)^t - A = pi(1+i)^t$$

$$A = (1+i)^t[A - pi]$$

$$\frac{A}{A - pi} = (1+i)^t$$

$$\frac{125}{125 - 5000\left(\frac{1{\cdot}65}{100}\right)} = \left(1 + \frac{1{\cdot}65}{100}\right)^t$$

$$\frac{125}{42{\cdot}5} = (1{\cdot}0165)^t$$

$$\log\left(\frac{125}{42{\cdot}5}\right) = t\log(1{\cdot}0165)$$

$$t = \frac{\log\left(\frac{125}{42{\cdot}5}\right)}{\log(1{\cdot}0165)}$$

$$t = 65{\cdot}920$$

$$t = 66 \text{ months}$$

or

$$A = p\left[\frac{i(1+i)^t}{(1+i)^t - 1}\right]$$

$$125 = \frac{5000(0{\cdot}0165)(1{\cdot}0165)^t}{(1{\cdot}0165)^t - 1}$$

$$125 = \frac{82{\cdot}5(1{\cdot}0165)^t}{(1{\cdot}0165)^t - 1}$$

$$\frac{125}{82{\cdot}5} = \frac{1{\cdot}0165^t}{1{\cdot}0165^t - 1}$$

$$\frac{50}{33} = \frac{1{\cdot}0165^t}{1{\cdot}0165^t - 1}$$

$$50(1\cdot0165^t - 1) = 33(1{\cdot}0165^t)$$

$$50(1{\cdot}0165^t) - 50 = 33(1{\cdot}0165^t)$$

$$50(1{\cdot}0165^t) - 33(1{\cdot}0165^t) = 50$$

$$1{\cdot}0165^t(50 - 33) = 50$$

$$1{\cdot}0165^t(17) = 50$$

$$1{\cdot}0165^t = \frac{50}{17}$$

$$t\log 1{\cdot}0165 = \log\frac{50}{17}$$

$$t = \frac{\log\left(\frac{50}{17}\right)}{\log 1{\cdot}0165} = 65{\cdot}92$$

$$t = 66 \text{ months}$$

(v)

$$A = \frac{pi(1+i)^t}{(1+i)^t - 1}$$

$$= \frac{5000\left(1{\cdot}085^{\frac{1}{52}} - 1\right)(1{\cdot}085)^3}{(1{\cdot}085)^3 - 1}$$

$$= €36{\cdot}16$$

OR

Weekly interest rate $(1+i)^{52} = 1{\cdot}085$

$$1 + i = 1{\cdot}085^{\frac{1}{52}}$$

$$1 + i = 1{\cdot}00157$$

$$i = 0{\cdot}00157$$

$$A = \frac{pi(1+i)^t}{(1+i)^t - 1}$$

$$A = \frac{5000(0{\cdot}00157)(1{\cdot}00157)^{156}}{(1{\cdot}00157)^{156} - 1}$$

$$= €36{\cdot}16$$

(vi)

$$125 \times 66 - (36{\cdot}16)(156)$$

=€2609·04

Question 12.

(i) We can use the formula on page 31 as the interest rate and the compounding periods match; i.e. both monthly. (The formula book says "Annual" but we can replace this with "Monthly")

$$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$

$$A = 5000\frac{(0.01)(1.01)^6}{(1.01)^6 - 1}$$

$$A = 862.7418336 \approx €862.74 \text{ p.m.}$$

Monthly payment = €862.74 = (portion to pay interest) + (portion to reduce debt)

(ii)

A	B	C	D = 1% x B	E = C-D	F=B-E
Month	Outstanding Balance at start month	Monthly Payment	Interest	Reduction in Debt	Outstanding Balance at end month
1	5000	862.74	50.00	812.74	4187.26
2	4187.26	862.74	41.87	820.87	3366.39
3	3366.39	862.74	33.66	829.08	2537.31
4	2537.31	862.74	25.37	837.37	1699.94
5	1699.94	862.74	17.00	845.74	854.20
6	854.20	862.74	8.54	854.20	0.00

Question 13

Solution Part (i)

At the end of the first year Tom will save 15% of his salary	60,000 *15% = 9,000
This will accumulate for 19 years at 3% pa	9,000 * [(1.03)^19]
Tom's contribution will increase in the second year (month 24) to	9,000 * 1.04 = 9,360
This will accumulate for 18 years at 3% pa	9,360 * [(1.03)^18]
The final contribution will be paid at the end of year 20	9,000*[1.04^19] = 18,961

So Accumulated Amount will be (from Sn formula on page 22 of Log Tables)

9,000 (1.03^19 + 1.04 *(1.03^18) + …..+ 1.04^19]

= 9,000*[(1.04^19)] *[{(1.03/1.04)}^19 + {(1.03/1.04)}^18+….1]

= 9,000* 2.106849 * (1+i)* [{ 1- **(1/(1+i))**^20 } / i] where 1+i = (1.04/1.03) = 1.009709

= 18,961 * 1.009709 * [1 – ((1/1.009709)^20)/ 0.009709] = 19,145 * 18.09858 = 346,499

Question 13

Solution Part (ii)

The accumulated value in part (i) equals the present value of a payment of 10,000 per annum , increasing at 5% per annum payable for n years. So

346,499 = 10,000 [1 + 1.05/1.03 + (1.05/1.03)^2+….(1.05/1.03)^(n-1)]

34.6499 = [1 + (1+i) + (1+i)^2 + … + (1+i)^(n-1)] where (1+i) = (1.05/1.03) = 1.019417

34.6499 = [{(1+i)^n -1}/i] = [{1.019417^n – 1}/0.019417

0.672814 = {1.019417^n – 1} = > ln 1.672814 = n ln 1.019417 = > n = 26.75

So the accumulated fund will be sufficient to pay the annuity for 26 years with three quarters of a years' payment remaining in the fund.