Q4 2021 Paper 1

(b) p, $p + 7$, $p + 14$, $p + 21$, … …. is an arithmetic sequence, where $p \in \mathbb{N}$.

(i) Find the nth term, T_n, in terms of n and p, where $n \in \mathbb{N}$.

(ii) Find the smallest value of p for which 2021 is a term in the sequence.

Q4 Paper 1 2022

(a) A sequence u_1, u_2, u_3, ... is defined as follows, for $n \in \mathbb{N}$: $u_1 = 2$, $u_2 = 64$, $u_{n+1} = \sqrt{\frac{u_n}{u_{n-1}}}$ Write u_3 in the form 2^p , where $p \in \mathbb{R}$.

(b) The first three terms in an arithmetic sequence are as follows, where $k \in \mathbb{R}$: $5e^{-k}$, 13, $5e^k$

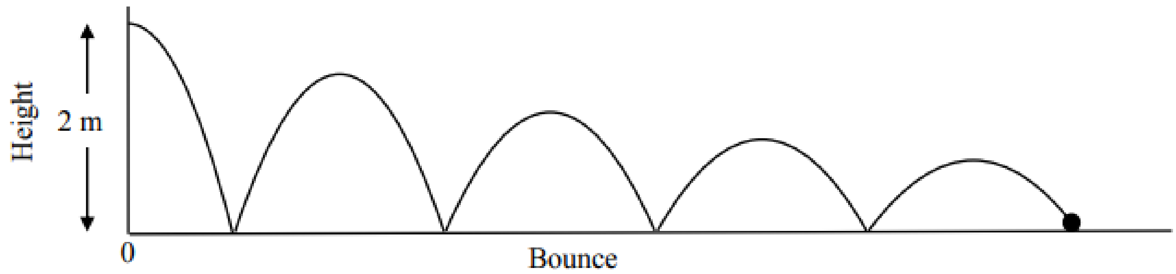
(i) By letting $y = e^k$ in this arithmetic sequence, show that:

$$5y^2 - 26y + 5 = 0$$

(ii) Use the equation in y in part (b)(i) to find the two possible values of k. Give each value in the form ($\ln p$) or ($-\ln p$), where $p \in \mathbb{N}$.

Q1 2015 Paper 1

Mary threw a ball onto level ground from a height of 2 m. Each time the ball hit the ground it bounced back up to $\frac{3}{4}$ of the height of the previous bounce, as shown



(a) Complete the table below to show the maximum height, in fraction form, reached by the ball on each of the first four bounces.

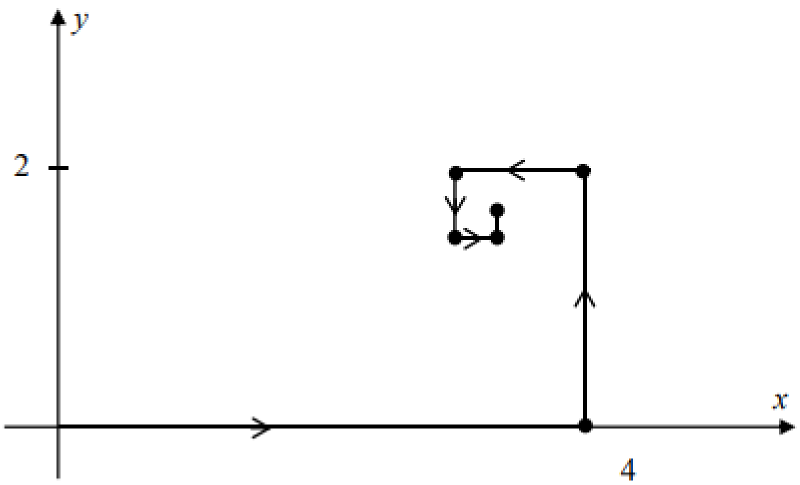
Bounce	**0**	**1**	**2**	**3**	**4**
Height	**2**				

(b) Find, in metres, the total vertical distance (up and down) the ball had travelled when it hit the ground for the 5th time. Give your answer in fraction form

(a) If the ball were to continue to bounce indefinitely, find, in metres, the total vertical distance it would travel.

Q9 2016 Paper 1

(a) At the first stage of a pattern, a point moves 4 units from the origin in the positive direction along the x-axis. For the second stage, it turns left and moves 2 units parallel to the y-axis. For the third stage, it turns left and moves 1 unit parallel to the x-axis. At each stage, after the first one, the point turns left and moves half the distance of the previous stage, as shown



(i) How many stages has the point completed when the total distance it has travelled, along its path, is 7·9375 units?

(ii) Find the maximum distance the point can move, along its path, if it continues in this pattern indefinitely

(iii) Complete the second row of the table below showing the changes to the x co-ordinate, the first nine times the point moves to a new position. Hence, or otherwise, find the x co-ordinate and the y co-ordinate of the final position that the point is approaching, if it continues indefinitely in this pattern.

Stage	**1st**	**2nd**	**3rd**	**4th**	**5th**	**6th**	**7th**	**8th**	**9th**
Change in x	**+4**	**0**	**-1**						
Change in y									

Q2 2023 Paper 1

(b) Find the value of the following limit, where $n \in \mathbb{N}$:

$$\lim_{n\to\infty}\left[\frac{n}{n+1}+\frac{n+1000}{n}+\left(\frac{1}{3}\right)^{n}\right]$$

Q2 2018 Paper 1

(a) The first three terms of a geometric series are x^2, $5x - 8$, and $x + 8$, where $x \ni \mathbb{R}$. Use the common ratio to show $x^3 - 17x^2 + 80x - 64 = 0$.

(b) If $f(x) = x^3 - 17x^2 + 80x - 64$, $x \ni \mathbb{R}$, show that $f(1) = 0$, and find another value of x for which $f(x) = (0)$.

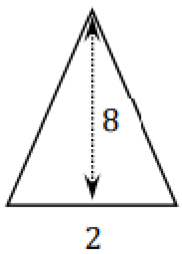
(c) In the case of one of the values of x from part (b), the terms in part (a) will generate a geometric series with a finite sum to infinity. Find this value of x and hence find the sum to infinity.

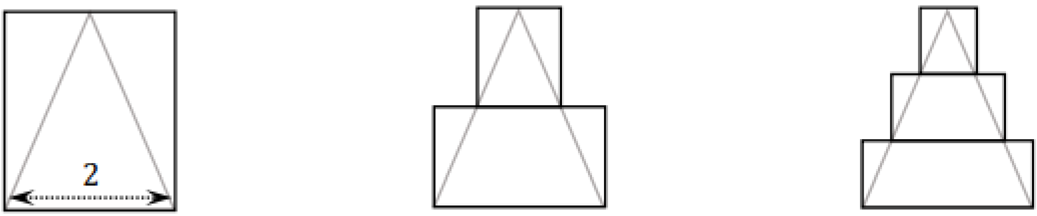
Q2 2020 Paper 1

(b) The complex numbers $3 + 2i$ and $5 - i$ are the first two terms of a geometric sequence. (i) Find r, the common ratio of the sequence. Write your answer in the form $a + bi$ where $a, b \in \mathbb{Z}$

Question 10 Paper 1 2023

A triangle has a base of length 2 units and a perpendicular height of 8 units, as shown in the diagram on the right.
The diagrams below show T_1, T_2, T_3 the first three shapes in a sequence of shapes based on this triangle. For each value of $n \in \mathbb{N}$, the shape T_n is made up of n rectangles of equal height laid on top of each other. T_n is the collection of the smallest such rectangles that completely covers the triangle.





T_1
1 rectangle

T_2
2 rectangles
of equal height

T_3
3 rectangles
of equal height

(a) Show that the total area of the three rectangles in T_3 is $\frac{32}{3}$ square units.

(b) Find the total area of the n rectangles in T_n, for $n \in \mathbb{N}$. Give your answer in square units in terms of n, in its simplest form.

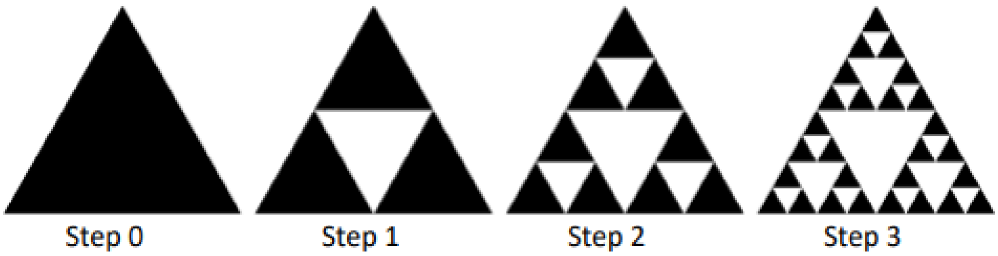
(c) The total area of the rectangles in the nth term of a different sequence of groups of rectangles is as follows, for $n \in \mathbb{N}$:

$$\text{Total area} = A_n = \frac{8(n-1)}{n}$$

Work out the first value of n for which A_n is greater than 95% of the area of the triangle above.

Question 9 Paper 1 2018

The diagram below shows the first 4 steps of an infinite pattern which creates the Sierpinski Triangle. The sequence begins with a black equilateral triangle. Each step is formed by removing an equilateral triangle from the centre of each black triangle in the previous step, as shown. Each equilateral triangle that is removed is formed by joining the midpoints of the sides of a black triangle from the previous step.



(a) The table below shows the number of black triangles at each of the first 4 steps and the fraction of the original triangle remaining at each step. Complete the table.

Step	0	1	2	3
Number of black triangles	1			
Fraction of the original triangle remaining	1		9/16	

(b) (i) Write an expression in terms of n for the number of black triangles in step n of the pattern.

(ii) Step k is the first step of the pattern in which the number of black triangles exceeds one thousand million (i.e. 1×10^9) for the first time. Find the value of k.

(c) (i) Step h is the first step of the pattern in which the fraction of the original triangle remaining is less than $\frac{1}{100}$ of the original triangle. Find the value of h.

(ii) What fraction of the original triangle remains after an infinite number of steps of the pattern?

(d) (i) The side length of the triangle in Step 0 is 1 unit. The table below shows the total perimeter of all the black triangles in each of the first 5 steps. Complete the table below.

Step	0	1	2	3	4
Perimeter	3		$\frac{27}{4}$		

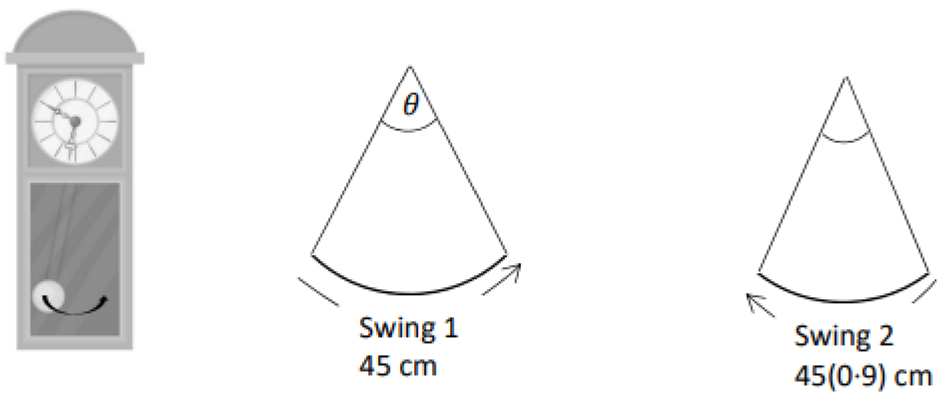
Logic is that a new triangle placed in centre creates 3 equilateral triangles each one is half the perimeter to the original triangle so perimeter increased x 50%

(ii) Find the total perimeter of the black triangles in step 35 of the pattern. Give your answer correct to the nearest unit.

(e) Use your answers to part (c)(ii) and part (d)(ii) to comment on the total area and the total perimeter of the black triangles in step n of the pattern, as n tends to infinity.

Q7 Paper 1 2021

The tip of the pendulum of a grandfather clock swings initially through an arc length of 45 cm. On each successive swing the length of the arc is 90% of the previous length.



(a) (i) Complete the table below by filling in the missing lengths.

Swing	**1**	**2**	**3**	**4**	**5**
Length of Arc (cm)	**45**		$\mathbf{\frac{729}{20}}$		

(ii) $T_n = 45(0{\cdot}9)^{n-1}$ is the arc length of swing n. Find the arc length of swing 25, correct to 1 decimal place.

(iii) Find the total distance travelled by the tip of the pendulum when it has completed swing 40. Give your answer, in cm, correct to the nearest whole number

(iv) Swing p is the first swing which has an arc length of less than 2 cm. Find the value of p.

(b) (i) If the length of the pendulum is 1 m, show that the angle, θ, of swing 1 of the pendulum is 26°, correct to the nearest degree.

(ii) Hence, find the total accumulated angle that the pendulum swings through (i.e. the sum of all the angles it swings through until it stops swinging). Give your answer correct to the nearest degree.

(iii) Hence, or otherwise, find the total distance travelled by the tip of the pendulum when it has moved through half of the total accumulated angle. Give your answer, in cm, correct to the nearest integer.

Q9 Paper 1 2022

Alex gets injections of a medicinal drug. Each injection has 15 mg of the drug. Each day, the amount of the drug left in Alex's body from an injection decreases by 40%. So, the amount of the drug (in mg) left in Alex's body t days after a single injection is given by:

$$15(0\cdot6)^{t} \text{ where } t\in\mathbb{R}.$$

(a) Find the amount of the drug left in Alex's body 2·5 days after a single 15 mg injection. Give your answer in mg, correct to 2 decimal places.

(b) (b) How long after a single 15 mg injection will there be exactly 1 mg of the drug left in Alex's body? Give your answer in days, correct to 1 decimal place.

(c) Alex is given a 15 mg injection of the drug at the same time every day for a long period of time. (c) Explain why the total amount of the drug, in mg, in Alex's body immediately after the 4th injection is given by: $15 + 15(0\cdot6) + 15(0\cdot6)^2 + 15(0\cdot6)^3$

(d) Find the total amount of the drug in Alex's body immediately after the 10th injection. Give your answer in mg, correct to 2 decimal places.

(e) se the formula for the sum to infinity of a geometric series to estimate the amount of the drug (in mg) in Alex's body, after a long period of time during which he gets daily injections.

(f) Jessica also gets daily injections of a medicinal drug at the same time every day. She gets d mg of the drug in each injection, where $d\in\mathbb{R}$. Each day, the amount of the drug left in Jessica's body from an injection decreases by 15%.

(i) Use the sum of a geometric series to show that the total amount of the drug (in mg) in Jessica's body immediately after the nth injection, where $n\in\mathbb{N}$, is: $\frac{20\,d(1-0.85^{n})}{3}$

(ii) Immediately after the 7th injection, there are 50 mg of the drug in Jessica's body. Find the amount of the drug in one of Jessica's daily injections. Give your answer correct to the nearest mg.