Society of Actuaries in Ireland

Hints and Tips for Sequences and Series

Sequence is a list of objects in order. Recursion Formula enables finding a term in the sequence from a previous term e.g. $T_{n+1} = T_n + 5$

Arithmetic Sequences

Difference between T_{n+1} and T_n is a <u>constant</u> so first difference between each terms in a sequence is the same. Each term can be written in format:

$T_n = a + (n-1)d$ where $T_1 = a$ and d = first difference ($T_2 - T_1$)

The sum of all the terms cumulatively up to (and including) term n is:

$$S_n = \frac{n}{2}(2a + (n-1)d$$

Geometric Sequences

Difference between T_{n+1} and T_n is a **<u>constant ratio</u>**. So $\frac{\overline{l_n}}{T_{n-1}} = r$ where r is the contract ratio between terms:

$T_n = r.\ T_{n-1} = ar^{n-1}$ where $T_1 = a$ and r is the contact ratio

The sum of all the terms cumulatively up to (and including) term n is:

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{where } r <> 1$$

where -1 < r <1 then $S_\infty = \frac{a}{1-r}$ which can be used to convert recurring decimals into fractions

Limits

$$\lim_{n\to\infty} c = c$$

$$\lim_{n\to\infty}(n+n) = \lim_{n\to\infty} n + \lim_{n\to\infty} n$$

$$\lim_{n\to\infty}\left(\frac{1}{n}\right) = 0$$

$$\lim_{n\to\infty}\left(\frac{1}{x}\right)^n = 0 \ where \ -1 < x < 1$$