Probability & Statistics 1 Tutorial Solutions

Exercises:

(i) $_{10}C_4 = 210$

(ii) $_{8}P_3 = 336$

(iii) 1*(1/6) + 2*(1/6) + 3*(1/6) + 4*(1/6) + 5*(1/6) + 6*(1/6) = 3.5

(iv) (a) 26 * 10 * 10 * 10 * 10 = 260,000 (b) 26 * 10 * 9 * 8 * 7 = 131,040

(v) (a) $_{4}C_3 * (3/8)^3 * (5/8)^1 = 0.1318$

(b) Must have picked 3 red marbles in 7 picks followed by a red marble
=> $_{7}C_3 * (3/8)^3 * (5/8)^4 * (3/8) = 0.10561$

(c) 1 – Probability of picking no blue marble in 3 picks.
Probability of picking no blue marbles in 3 picks = $(3/8)^3$ = 0.05273
=> Probability of picking at least 1 blue marble = 1 – 0.05273 = 0.94727

Past Exam Questions:

Question 1:

a)

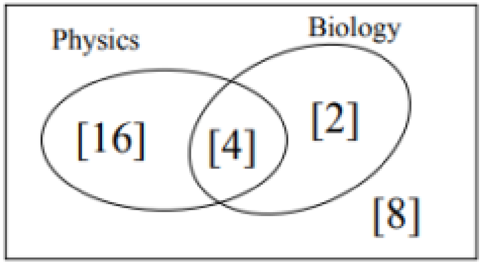
i) Sample space:
the set of all possible outcomes of an experiment

ii) Mutually exclusive events:
Events are mutually exclusive if they have no outcomes in common

iii) Independent events:
Two events are independent if the outcome of one does not depend on the outcome of the other

(b) In a class of 30 students, 20 study Physics, 6 study Biology and 4 study both Physics and Biology.

(i) Represent the information on the Venn Diagram.



A student is selected at random from this class.
The events E and F are:

E: The student studies Physics

F: The student studies Biology.

(ii) By calculating probabilities, investigate if the events E and F are independent.

$P(E \cap F) = \frac{4}{30}$

$P(E) \times P(F) = \frac{20}{30} \times \frac{6}{30} = \frac{4}{30}$

$P(E \cap F) = P(E) \times P(F) \Rightarrow$ E and F are independent events

Question 2:

(i) Find the probability that neither A nor B happens P(A ∩ B') = 0.2 - .15 = 0.05.
The probability that A nor B happens = 1- P(A) – P(B) + P(A ∩ B)

= 1 -0.2 -.75 +.15 = 0.2

(ii) The probability that A happens given that B happens is $\frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.75}$ = 0.2.

(iii) An event is independent of another event if P(A|B)= P(A) i.e. the fact that B has happened doesn't mean that A is more or less likely to happen that would be otherwise and vice versa.

P(A|B) = P(A ∩ B) /P(B) = 0.15/0.75 = .2 = P(A)

P(B|A) = P(A ∩ B) /P(A) = 0.15/0.2 = .75 = P(B)

A and B are independent events as P(A|B) = P(A)=0.2

Question 3:

(a) $\left(\frac{4}{5}\right) * \left(\frac{1}{5}\right) * \left(\frac{4}{5}\right) * \left(\frac{1}{5}\right) * \left(\frac{4}{5}\right) * \left(\frac{1}{5}\right) * \left(\frac{4}{5}\right)$ = 0.0032768

(b) Looking for probability she answers 3 times in first 6 days * Probability of answering on the 7th day

=> ${}_6C_3 * \left(\frac{1}{5}\right)^3 * \left(\frac{4}{5}\right)^3 * \left(\frac{1}{5}\right)$ = 0.016384

(c) Probability of answering at least once in n days = 1 – Probability of never answering in n days.

=> $1 - \left(\frac{4}{5}\right)^n$

(d)

Looking for n such that $1 - \left(\frac{4}{5}\right)^n > 0.99$

=> $1 - 0.99 > \left(\frac{4}{5}\right)^n$

=> $0.01 > \left(\frac{4}{5}\right)^n$

=> $\log_{0.8}(0.01) < n$

=> n > 20.6377

=> n = 21 days

Question 4:

(a) E(loss) = $2000 - 9000 * \left(\frac{1}{20}\right) - 7000 * \left(\frac{1}{10}\right) - 3000 * \left(\frac{1}{4}\right) = 100$

(b) $2000 - (9000 + x) * \left(\frac{1}{20}\right) - (7000 + x) * \left(\frac{1}{10}\right) - (3000 + x) * \left(\frac{1}{4}\right) = 0$

$$2000 - \frac{9000}{20} - \frac{x}{20} - \frac{7000}{10} - \frac{x}{10} - \frac{3000}{4} - \frac{x}{4} = 0$$

$$100 - \frac{x}{20} - \frac{x}{10} - \frac{x}{4} = 0$$

$$100 = \frac{x}{20} + \frac{x}{10} + \frac{x}{4}$$

$$100 = \frac{(x + 2x + 5x)}{20}$$

$$2000 = 8x$$

x = 250

Question 5:

(a) P (€6, €9, €6) = $\left(\frac{5}{12}\right) * \left(\frac{3}{12}\right) * \left(\frac{5}{12}\right) = 0.0434$

(b) P(2 €9 in 7 spins) * P(€9)

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= $_7C_2 * \left(\frac{3}{12}\right)^2 * \left(\frac{9}{12}\right)^5 * \left(\frac{3}{12}\right)$
= 0.0779

(c) Only combination that gives greater than €16 is €9,€9
=> Probability of less than €16 = 1 – P(€9, €9)
= $1 - \left(\frac{3}{12}\right)^2$
= 0.9375

Question 6:

(a) (i)

	Age (years) ≤ 23	Age (years) ≥ 24	Total
Under.	12 785	2922	15 707
Post.	1353	**5654**	**7007**
Total	**14 138**	8576	22 714

(ii)
For independent events P(O) * P(U) = P (O ∩ U)
P(O) = 8576/22714
P(U) = 15707/22714
P (O ∩ U) = $\left(\frac{2922}{22714}\right) = 0.1286$
P(O) * P(U) = $\left(\frac{8576}{22714}\right) * \left(\frac{15707}{22714}\right) = 0.2611$
=> P(O) * P(U) ≠ P (O ∩ U)
=> Events are not independent.

(b)
1 * $\left(\frac{1}{7}\right) * \left(\frac{1}{7}\right) = 0.0204$

(c)
$\frac{g}{b+g} = \frac{3}{5}$ => 3b + 3g = 5g => 3b – 2g = 0 (1)
$\frac{g+4}{b+g+8} = \frac{4}{7}$ => 7g + 28 = 4b+4g+32 => – 4b + 3g = 4 (2)
Simultaneous Equations:
(1)*3 => 9b – 6g = 0
(2)*2 => -8b + 6g = 8

=> b = 8
(1) 3b – 2g = 0
3(8) = 2g
24 = 2g
g = 12

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Question 7:

(a) Find the probability that Michael is successful (S) with all three of his first three free throws in a game.

$$\text{P}(\text{S},\text{S},\text{S}) = 0\cdot7\times0\cdot8\times0\cdot8 = 0\cdot448$$

(b) Find the probability that Michael is unsuccessful (U) with his first two free throws and successful with the third.

$$\text{P}(\text{U},\text{U},\text{S}) = 0\cdot3\times0\cdot4\times0\cdot6 = 0\cdot072$$

(c) List all the ways that Michael could be successful with his third free throw in a game and hence find the probability that Michael is successful with his third free throw.

S, S, S U, U, S S, U, S U, S, S

$$\text{P}(\text{S},\text{S},\text{S}) = 0\cdot7\times0\cdot8\times0\cdot8 = 0\cdot448$$
$$\text{P}(\text{U},\text{U},\text{S}) = 0\cdot3\times0\cdot4\times0\cdot6 = 0\cdot072$$
$$\text{P}(\text{S},\text{U},\text{S}) = 0\cdot7\times0\cdot2\times0\cdot6 = 0\cdot084$$
$$\text{P}(\text{U},\text{S},\text{S}) = 0\cdot3\times0\cdot6\times0\cdot8 = 0\cdot144$$

$$\text{P} = 0\cdot448 + 0\cdot072 + 0\cdot084 + 0\cdot144 = 0\cdot748$$

(d) **(i)** Let p_n be the probability that Michael is successful with his n^{th} free throw in the game (and hence $(1-p_n)$ is the probability that Michael is unsuccessful with his n^{th} free throw). Show that $p_{n+1} = 0\cdot6 + 0\cdot2p_n$.

$$\begin{aligned} p_{n+1} &= \text{P}(\text{S,S}) + \text{P}(\text{U,S}) \\ &= p_n \times 0\cdot8 + (1-p_n)0\cdot6 \\ &= 0\cdot6 + 0\cdot2p_n \end{aligned}$$

(ii) Assume that p is Michael's success rate in the long run; that is, for large values of n, we have $p_{n+1} \approx p_n \approx p$.
Using the result from part **(d) (i)** above, or otherwise, show that $p = 0\cdot75$.

$$p \approx p_n \approx p_{n+1} = 0\cdot6 + 0\cdot2p_n$$
$$\Rightarrow 0\cdot8p_n = 0\cdot6$$
$$\Rightarrow p_n = \frac{0\cdot6}{0\cdot8} = 0\cdot75 = p$$

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(e) For all positive integers n, let $a_n = p - p_n$, where $p = 0{\cdot}75$ as above.

(i) Use the ratio $\frac{a_{n+1}}{a_n}$ to show that a_n is a geometric sequence with common ratio $\frac{1}{5}$.

$$\frac{a_{n+1}}{a_n} = \frac{p - p_{n+1}}{p - p_n}$$
$$= \frac{0\cdot 75 - (0\cdot 6 + 0\cdot 2p_n)}{0\cdot 75 - p_n}$$
$$= \frac{0\cdot 15 - 0\cdot 2p_n}{5(0\cdot 15 - 0\cdot 2p_n)} = \frac{1}{5}$$

(ii) Find the smallest value of n for which $p - p_n < 0{\cdot}00001$.

$$a_n = p - p_n$$
$$a_1 = p - p_1 = 0\cdot 75 - 0\cdot 7 = 0\cdot 05$$

$$ar^{n-1} = 0\cdot 05(0\cdot 2)^{n-1} < 0\cdot 00001$$
$$(n-1)\ln 0\cdot 2 < \ln 0\cdot 0002$$
$$\Rightarrow n - 1 > \frac{\ln 0\cdot 0002}{\ln 0\cdot 2} = 5\cdot 29$$
$$\Rightarrow n > 6\cdot 29$$

$$n = 7$$

(f) You arrive at a game in which Michael is playing. You know that he has already taken many free throws, but you do not know what pattern of success he has had.

(i) Based on this knowledge, what is your estimate of the probability that Michael will be successful with his next free throw in the game?

Answer: $0{\cdot}75$ **or** p

(ii) Why would it **not** be appropriate to consider Michael's subsequent free throws in the game as a sequence of Bernoulli trials?

Events not independent

